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LIMITATIONS OF RADIAL MAGNETIC FIELD ESTIMATES FROM COUNTER-ROTATING BEAMS IN AN ELECTRO-STATIC EDM RING

C. Carli, V. Cilento, CERN, Geneva, Switzerland

Abstract

Proposals to measure a possible Electric Dipole Moment (EDM) of protons in an electro-static machine are studied by a world-wide community. The machine is operated at the so-called magic energy to satisfy the "frozen spin" condition such that, without imperfections and the well-known magnetic moment of the particle, the spin is always oriented parallel or antiparallel to the direction of movement. The effect of a finite EDM is a build-up of a vertical spin component. A small average radial magnetic field leads as well to a build-up of a vertical spin component, which cannot be disentangled from the effect due to a finite EDM, and thus generates a systematic error of the measurement. Essential ingredients of the concept are to install the machine inside a state-of-the-art magnetic shielding and to measure the vertical orbit separation of two counter-rotating beams, enhanced by choosing a very low vertical tune, with high precision pick-ups. In this paper, we analyse limitations of this method and, in particular, the impact of wanted ("strong focusing" lattice) and unwanted variations of the betatron functions.

INTRODUCTION

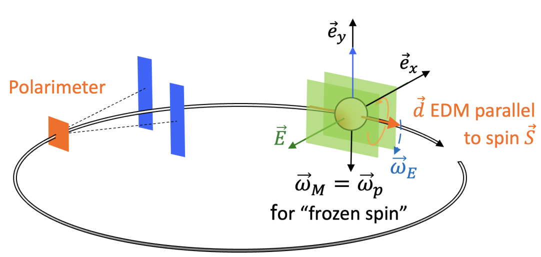

Figure 1: Sketch of a frozen spin EDM measurement ring.

For a "frozen spin" storage ring to measure a possible Electric Dipole Moment (EDM) as sketched in Fig. 1, parameters are adjusted such that, with only the well known Magnetic Dipole Moment (MDM) and in the absence of imperfections, an initial longitudinal polarization of a bunch is maintained. Thus, the angular frequencies describing the the rotation of the spin with an MDM only and the direction of motion must be identical. A finite EDM generates a slow rotation around the radial direction from the horizontal plane into the vertical direction. Typical proposals for proton EDM measurements foresee the use of only electric fields for bending [1–5] with the frozen spin conditions satisfied by operating the machine at the "magic energy".

Any other effect related to machine imperfections, such as unwanted residual magnetic fields and component misalignment generating vertical spin components with an MDM only are systematic errors limiting the sensitivity of the measurement. Probably the most significant systematic error source for magic energy rings using electrical gradients for focusing is residual radial magnetic fields. A scheme, based on the observation of the orbit difference between two counter-rotating beams, proposed to estimate the effect is analysed leading to the conclusion that the target sensitivity of $10^{-29}\, e \cdot \mathrm{cm}$ of typical proposals cannot be reached.

BASIC EQUATIONS AND MAGIC ENERGY

The time evolution of the particle spin is described by the Thomas-BMT equation complemented by terms due to a possible EDM [6] as $d\vec{S}/dt = (\vec{\omega}_M + \vec{\omega}_E) \times \vec{S}$ and

$$\vec{\omega}_M = -\frac{q}{m}\left[\left(G+\frac{1}{\gamma}\right)\vec{B}_\perp + (G+1)\frac{\vec{B}_\parallel}{\gamma} - \left(G+\frac{1}{\gamma+1}\right)\vec{\beta}\times\frac{\vec{E}}{c}\right]$$

$$\vec{\omega}_E = -\frac{q}{m}\frac{\eta}{2}\left(\frac{\vec{E}_\perp}{c} + \frac{1}{\gamma}\frac{\vec{E}_\parallel}{c} + \vec{\beta}\times\vec{B}\right)$$

with $\vec{S}$ a (unit) vector pointing in the direction of the spin, $\vec{\omega}_M$ an angular frequency describing the particle with only an MDM and $\vec{\omega}_E$ an angular frequency describing spin rotation due an EDM. q and m are the particle charge and mass, γ and β the relativistic parameters and $\vec{B}$ and $\vec{E}$ the magnetic and electric fields. Indices $\perp$ and $\parallel$ denote the component perpendicular and parallel to the particles direction of motion. G is the anomalous magnetic moment factor and η describes a possible EDM. For protons with an EDM of $10^{-29}\, e \cdot \mathrm{cm}$ (target sensitivity), these values are $G = 1.7928...$ and $\eta = 1.9 \cdot 10^{-15}$.

The angular frequency describing the rotation of the direction of motion along the reference orbit has only a vertical component given for a Clock-Wise (CW) beam by $\omega_{p,y} = -\beta c/\rho = (q/m)E_x/(\gamma\beta c)$ with ρ the bending radius. The angular frequency describing the rotation of the spin with an MDM only is $\omega_{M,y} = (q/m)(G + 1/(\gamma+1))\beta E_x/c$. The frozen spin condition is satisfied for $\omega_{p,y} = \omega_{M,y}$ leading to the condition $\beta^2\gamma^2 = G^{-1}$. The magic energy evaluated for protons is given by $E_m = (\sqrt{1 + 1/G} - 1)mc^2 = 232.8\,\mathrm{MeV}$.

For a $C = 500\,\mathrm{m}$ circumference machine operated with magic energy protons, the average electric field is $\bar{E}_x = -5.27\,\mathrm{MV/m}$. The resulting angular frequency generating the vertical spin component for an EDM of $10^{-29}\, e \cdot \mathrm{cm}$ is given by $\omega_{E,x} = -(q/m)(\eta/2)\bar{E}_x/c = 1.6\,\mathrm{nrad/s}$.

RADIAL MAGNETIC FIELD

An average radial magnetic field $\bar{B}_x$ is probably the main sensitivity limitation for magic energy proton EDM rings with electric focusing. To estimate the resulting spin rotations, one has to take into account that, on average, the

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particle experiences no vertical acceleration (disregarding gravity leading to well understood small effects) leading to the condition $q\,\beta c\bar{B}_x + q\bar{E}_y = 0$. The angular frequency caused by both the magnetic and electric fields and for an MDM only is given by:

$$\omega_{M,x,B} = -\frac{q}{m}\left[\left(G+\frac{1}{\gamma}\right)-\left(G+\frac{1}{\gamma+1}\right)\beta^2\right]\bar{B}_x = \frac{1}{m}\frac{G+1}{\gamma^2}\bar{B}_x$$

The average magnetic field resulting in the same spin rotations as for a given EDM is found by equating this expression with the angular frequency due an EDM given in the previous section resulting in

$$\bar{B}_x = \frac{\gamma^2}{G+1}\frac{\eta}{2}\frac{\bar{E}_x}{c} \quad .$$

For a $C = 500\,\mathrm{m}$ circumference ring this average radial magnetic field giving the same spin rotations as an EDM of $10^{-29}\,e\cdot\mathrm{cm}$ is $\bar{B}_{x,l} = 9.3\cdot10^{-18}\,\mathrm{T}$ and about seven orders of magnitude lower than residual fields of around 1 nT inside state-of-the-art magnetic shields. Moreover, in a ring with electric focusing, the effect generated by radial magnetic fields cannot be disentangled from a finite EDM by combining spin rotations measured for two counter-rotating beams.

ORBIT SEPARATION OF COUNTER-ROTATING BEAMS

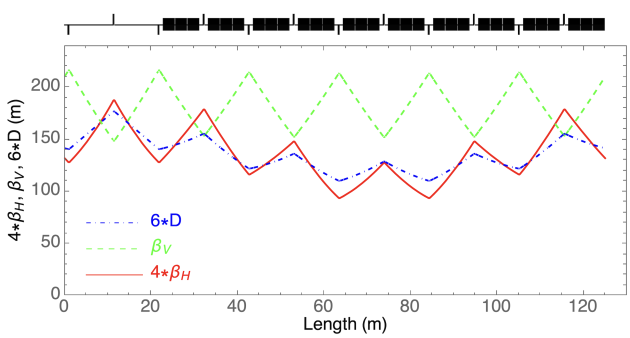

Figure 2: Lattice functions for the strong focusing purely electrostatic EDM ring proposal.

An essential ingredient for fully electro-static magic energy proton EDM rings is an estimate and correction of the average radial magnetic field based on the measurement of the vertical orbit separation between counter-rotating beams. This orbit separation at longitudinal position s due to an unwanted integrated radial magnetic field $B_x(\hat{s})d\hat{s}$ at position $\hat{s}$ is given by the well known expression for the closed orbit perturbation with an additional factor two (as the magnetic fields deflects the two beams in opposite direction):

$$\Delta y(s) = \frac{B_x(\hat{s})d\hat{s}}{p/q}\frac{\sqrt{\beta(s)\beta(\hat{s})}}{\sin(\pi Q_V)}\cos\left(|\mu(s)-\mu(\hat{s})|-\pi Q_V\right)$$

with $\beta(s)$ and $\mu(s)$ the vertical Twiss betatron function and vertical betatron phase at longitudinal position s. The Twiss function for one quarter of the strong focusing lattice [7] used in [2, 3] are given in Fig. 2. Limitations related to the number of orbit separation pick-ups and their precise positioning at the correct betatron phases have been analysed in reference [7]. The variations of the vertical Twiss betatron functions lead to further limitations. A situation with positive (negative) values of $B_x(s)$ at positions with large (small) $\beta(s)$ such that the average radial $\bar{B}_x$ field vanishes obviously results in a vertical orbit separation, which would be misinterpreted.

For a quantitative analysis, the closed orbit separation is expressed by a Fourier series expansion and extrapolated from a localised magnetic field to perturbations around the ring circumference:

$$\Delta y(s) = \sqrt{\beta(s)}\left[\sum_{m=0}^{\infty} c_m\cos\left(\frac{m}{Q_V}\mu(s)\right)+\sum_{m=1}^{\infty} s_m\sin\left(\frac{m}{Q_V}\mu(s)\right)\right]$$

with

$$c_m = \frac{(2-\delta_{m0})}{(p/q)\pi Q}\frac{1}{1-\left(\frac{m}{Q}\right)^2}\int_0^C d\hat{s}B_x(\hat{s})\sqrt{\beta(\hat{s})}\cos\left(\frac{m}{Q}\mu(\hat{s})\right)$$

$$s_m = \frac{1}{(p/q)\pi Q}\frac{1}{1-\left(\frac{m}{Q}\right)^2}\int_0^C d\hat{s}B_x(\hat{s})\sqrt{\beta(\hat{s})}\sin\left(\frac{m}{Q}\mu(\hat{s})\right)$$

Assuming that N orbit difference pick-ups are located at vertical betatron phases $\mu_i = 2\pi Q\,(i-1)/N$ with the index i denoting the i-th pick-up, one defines the quantity:

$$\Delta\hat{y} = \left\langle\sqrt{\beta}\right\rangle\frac{1}{N}\sum_{i=0}^{N-1}\frac{\Delta y_i}{\sqrt{\beta_i}} = \left\langle\sqrt{\beta}\right\rangle\sum_{\nu=0}^{\infty}\frac{c_{\nu N}}{1-\left(\frac{\nu N}{Q}\right)^2} \tag{1}$$

were Δy_i denotes the vertical orbit difference measured at the i-th pick up with a betatron function β_i and $\left\langle\sqrt{\beta}\right\rangle$ the average of the square root of the betatron function around the circumference. The factor in front of the sum has been chosen such that, for a smooth focusing lattice, $\Delta\hat{y}$ becomes the average of the observed orbit separations.

Smooth Focusing Lattices

For smooth focusing lattices with constant focusing gradient and, thus, the vertical betatron function independent of of the position and given by $\beta = C/(2\pi Q)$, the above relation for $\Delta\hat{y}$ becomes

$$\Delta\hat{y} = \frac{1}{N}\sum_{i=0}^{N-1}\Delta y_i = \tag{2}$$

$$= \frac{C^2}{\frac{p}{q}2\pi^2Q^2}\left[\bar{B}_x + \sum_{\nu=1}^{\infty}\frac{1}{1-\left(\frac{\nu N}{Q}\right)^2}\oint d\hat{s}B_x(\hat{s})\cos\left(\frac{\nu N}{Q}\hat{s}\right)\right]$$

with $\bar{B}_x$ the average radial field around the circumference. The dominating term is proportional to $\bar{B}_x$ and $1/Q^2$. For this reason, very low vertical tunes of $Q = 0.1$ and $Q = 0.44$ have been proposed for the weak focusing [1] and strong focusing [3, 7] lattice proposals respectively. Even with these low tunes and for a magnetic field of $\bar{B}_{x,l} = 9.3\,\mathrm{a}T$, the expected

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orbit separation would become as low as $\Delta\hat{y} = 5$ pm and $\Delta\hat{=}0.26$ pm for these two lattice options. There are claims that with SQUID based orbit separation pick-ups and over very long measurement times (duration of one run or longer) a sufficient measurement accuracy can be reached.

The terms in the sum in Eq. (2) give additional contributions generating an error of the average field estimate underlined in [7]. The ν-th term is about a factor $2/(1-(\nu N/Q)^2) \approx 2/(\nu N/Q)^2$ smaller than the dominating term $\bar{B}_x$. Retaining (for an order of magnitude estimate) only the first term of the sum, one finds a relative error of the magnetic field estimate of $\Delta\bar{B}_x/\bar{B}_x \approx 2/(N/Q)^2$.

Assuming optimistically $n = 48$ equally spaced position difference pick-ups (due to straight sections, the lattice does not have exact 48 fold periodicity), the relative error for the radial field estimate becomes $\Delta\bar{B}_x/\bar{B}_x \approx 1/6000$.

Lattices with Variations of the Betatron Functions

The lattices of real storage rings have wanted and unwanted variations of the Twiss parameters around the circumference. The consequence for known and intentional variations of the betatron functions are evaluated. The following estimate for the average radial magnetic field is motivated by Eqs. (1) and (2):

$$\begin{aligned}\bar{B}_{x,e} &= \frac{p}{q}\frac{\pi Q}{C}\frac{1}{\left\langle\sqrt{\beta}\right\rangle^2}\Delta\hat{y} \\ &\approx \frac{p}{q}\frac{\pi Q}{C}\frac{1}{\left\langle\sqrt{\beta}\right\rangle}c_0 = \frac{1}{\left\langle\sqrt{\beta}\right\rangle}\frac{1}{C}\int d\hat{s}B_x(\hat{s})\sqrt{\beta(\hat{s})}\end{aligned}$$

where for the transformation from the first to the second line only the dominating term of Eq. (1) with $m = 0$ in the sum has been retained. The uncertainty of the radial magnetic field estimate caused by neglecting higher terms with $m > 0$ has already been treated and is independent of the effect described here. To estimate the uncertainty $\Delta\bar{B}_{x,\beta}$ of $\bar{B}_x$ due to variations of the betatron function, the last expression is rewritten as

$$\bar{B}_{x,e} \approx \bar{B}_x + \underbrace{\frac{1}{\left\langle\sqrt{\beta}\right\rangle C}\int_0^C d\hat{s}B_x(\hat{s})\left(\sqrt{\beta(\hat{s})} - \left\langle\sqrt{\beta}\right\rangle\right)}_{\Delta\bar{B}_{x,\beta}} .$$

For a quantitative order of magnitude estimate and based on measurements [8], we assume that the residual magnetic field inside the state-of-the-art shield has an rms value of $\sigma_{B_x} \approx 1\, nT$. There will be strong correlation between residual fields within a distance in the longitudinal direction of less than the transverse extension of the shield, assumed to be $d = 1$ m. Thus, for an order of magnitude estimate, the circumference is divided into C/d pieces with length d. The magnetic field estimate error becomes:

$$\Delta\bar{B}_{x,\beta} = \frac{1}{\left\langle\sqrt{\beta}\right\rangle}\sum_j \frac{d}{C}B_{x,j}\left(\sqrt{\beta_j} - \left\langle\sqrt{\beta}\right\rangle\right)$$

with j an index running over the C/d pieces of the ring. The rms of this quantity, i.e., the uncertainty of the magnetic field estimate becomes:

$$\sigma_{B_{x,\beta}} = \frac{1}{\left\langle\sqrt{\beta}\right\rangle}\sqrt{\frac{d}{C}}\sigma_{B_x}\sigma_{\sqrt{\beta}} \quad .$$

From the lattice functions of the strong focusing version shown in Fig. 2, one can estimate $\left\langle\sqrt{\beta}\right\rangle \approx 13.9\,\mathrm{m}^{1/2}$ and $\sigma_{\sqrt{\beta}} \approx 0.73\,\mathrm{m}^{1/2}$ giving an uncertainty of the radial magnetic field estimate of $\sigma_{B_{x,\beta}} = 2.3$ pT, i.e. orders of magnitude above what is required for the sensitivity goal. Note that the smooth focusing lattice still has as variation of the betatron functions due to straight sections yielding a significant uncertainty of the magnetic field estimate.

SUMMARY AND CONCLUSIONS

Limitations of a scheme to assess and correct an average radial magnetic field, which is expected to be the main sensitivity limitation for purely electrostatic magic energy EDM rings, have been analysed. The conclusion is that variations of the betatron functions together with the residual magnetic fields to be expected inside a state-of-the-art magnetic shielding generate an uncertainty of this average magnetic field estimate limiting the sensitivity of purely electrostatic EDM rings to a value well above the target of $10^{-29}\, e\cdot$cm. The sensitivity limit is higher for the strong focusing lattice proposal than for the weak focusing lattice (featuring variations of the betatron functions due to the required straight sections and anyhow ruled out due to excessive IBS growth rates [7]). There are additional effects limiting the accuracy of the assessment of the radial magnetic field from vertical orbit separation measurements such as a horizontal beam separation at locations with unwanted skew quadrupolar components (rotated electric quadrupoles or bendings with plates not exactly parallel).

The considerations presented here have been the trigger for the proposal of the hybrid ring concept [4]. If electric quadrupoles are replaced by magnetic ones, additional radial fields in an otherwise perfect machine do not generate spin rotations around a radial axis proportional to this perturbation. Thus, the hybrid ring proposal is an effective mitigation measure. However, other severe systematic effects have to be expected due to (i) unwanted vertical electric fields generating very fast spin rotations, which can in principle be disentangled from a finite EDM by combining observations from counter-rotating beams, (ii) separation of counter-rotating beams and electric (skew) quadrupolar components and (iii) a combination of horizontal bending misalignments and residual longitudinal magnetic fields [9].

Thorough studies combining analytical estimates and simulations are required to come to realistic estimates of the sensitivity of magic energy EDM rings both with electric and with magnetic focusing. This will allow the preferred option to be chosen along with a limit on the smallest EDM that can be identified.

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