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TRANSVERSE IMPEDANCE AND BEAM STABILITY STUDIES FOR THE MUON COLLIDER RAPID CYCLING SYNCHROTRONS[*†]

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Abstract

The International Muon Collider Collaboration is currently investigating the possibility to build a muon collider with a center-of-mass energy of 3 TeV in a first phase, with an option to build a 10 TeV collider in a second phase. The muon beam decay is the global challenge of such a collider and fast acceleration is required to reach high luminosities. A series of three or four Rapid Cycling Synchrotrons (RCS) are currently proposed as the last acceleration stage before injecting the muon beams into the collider ring. The transverse collective effects in these synchrotrons have been analysed, in particular for the first RCS in the chain. The effect of the higher-order modes of the numerous radio-frequency (RF) cavities, needed for the fast acceleration, have been looked at in detail along with possible mitigation measures. Promising results have been obtained considering for the moment a single muon bunch.

INTRODUCTION

Circular muon colliders offer the perspective to reach high luminosity, multi-TeV center-of-mass (c.o.m) energies, overcoming the limitations from synchrotron radiation encountered with electron-positron colliders [1]. The International Muon Collider Collaboration [2, 3] has been formed to study a 3 TeV c.o.m $\mu^+-\mu^-$ collider, with the option of a following 10 TeV c.o.m stage.

The main challenge to reach high-luminosity in the collider is to counterbalance the natural muon decay to electron, positron and neutrino. The muon lifetime at rest is $\tau_0 = 2.2\,\mu\text{s}$, much smaller than the 33 µs revolution period of a beam at the speed of light in the 10 km long collider ring. However, thanks to relativistic time dilation, the muon lifetime in the laboratory frame increases with γ the Lorentz factor as $\tau = \gamma\tau_0$.

A muon production and acceleration concept, schematized in Fig. 1, was investigated by the US MAP project [4]. After the muon production, a cooling stage reduces the longitudinal and transverse emittances of the muon beam. It is followed by an acceleration stage which provides fast energy increase to the muon beams. The last acceleration stage before the collider ring comprises three Rapid Cycling Synchrotrons (RCS, labelled RCS 1 to RCS 3) which would accelerate the muons from ∼ 60 GeV to 1.5 TeV. For the 10 TeV collider, a fourth RCS would accelerate the muon beams from 1.5 TeV to 5 TeV. These accelerators are designed such that less than 10 % of the muon beam decays during acceleration in each ring.

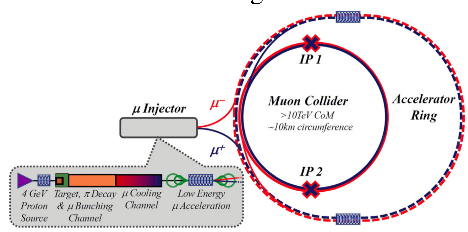

Figure 1: Proposed layout of a muon collider and its muon production and acceleration stages. Picture from Ref. [5].

To reach the beam transmission target, high accelerating gradients are required in each RCS to obtain a quick γ increase. A set of parameters for the longitudinal beam dynamics and the RF system has been proposed in Ref. [6]. The present study uses them as a starting point for transverse impedance and beam stability studies. General transverse stability criteria were derived for the higher-order-modes (HOMs) generated in the RF cavities of RCS 1 using particle tracking simulations. These were applied to a simulation model using TESLA type superconducting RF cavities. The effect of chromaticity was also investigated with the first transverse impedance model.

GENERAL STABILITY CRITERIA FOR RCS 1 HIGH-ORDER-MODES

The RCS 1 will accelerate muon beams from ∼ 60 GeV to 0.3 TeV. To ensure a 90 % survival rate of the muons, an energy gain of 14.8 GeV per turn is required. This translates to the number of RF cavities required in the ring. For the first RCS ∼ 700 TESLA type RF cavities would be required to provide the accelerating gradient [6]. Because of the great number of RF cavities, the parasitic HOMs are a concern for transverse beam stability.

Since the RCS design is still in an early stage, and the RF system parameters might evolve further, general stability criteria for the RF cavity HOMs were derived from simulations. A single HOM can be modeled by a resonator impedance with shunt impedance R_s, resonance angular frequency $\omega_{res} = 2\pi f_{res}$ and quality factor Q. The parameters (R_s, ω_s, Q) are scanned according to Table 1.

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Table 1: Resonator Parameters Scan

Parameter	Value
Shunt impedance R_s	$10^5\,\Omega\,\mathrm{m}^{-1}$ to $10^{17}\,\Omega\,\mathrm{m}^{-1}$
Resonance frequency f_{res}	10^7 Hz to 10^{11} Hz
Quality factor Q	100; 1000; 10 000

For each of these resonators, a time domain simulation tracks the 6D beam motion using the macroparticle code PyHEADTAIL [7]. With time domain simulation the wake function, i.e the Fourier transform of the impedance, must be used. The resonator wake can be written as [8]

$$W(t) = \frac{\omega_{res} R_s}{Q\sqrt{1-\frac{1}{4Q^2}}} \exp\left(-\frac{\omega_{res}}{2Q}t\right) \sin\left(\omega_{res}\sqrt{1-\frac{1}{4Q^2}}t\right).$$

The exponential term in the wake formula provides information on the wakefield decay versus resonator parameters. The beam revolution time is approximately $C_0/c = 20\,\mu\mathrm{s}$ where C_0 is the accelerator circumference and c the speed of light. At $t = 20\,\mu\mathrm{s}$ and when $f_{res}/Q < 10^5$ Hz the wake strength $W(t = 20\,\mu\mathrm{s})$ is 0.2 % of the maximum strength $\omega_{res}R_s/\left(Q\sqrt{1-\frac{1}{4Q^2}}\right)$. The wakefield can be considered fully decayed before the bunch completes a turn. Otherwise, when $f_{res}/Q > 10^5$ Hz multiturn wakefield effects must be accounted for in the simulation.

The machine and simulation parameters are detailed in Table 2. The beam is assumed to have no initial transverse offset generated by the injection system. Because of the extremely large RF voltage, the synchrotron tune $Q_s = 1.52$ at injection energy is much larger than the criterion for stable longitudinal focusing $Q_s \ll 1/\pi$ with one RF system [9]. The RF cavities must be distributed over $n_{RF} \approx 30$ stations along the ring [6]. The PyHEADTAIL longitudinal tracking module was modified to handle these multiple stations, instead of the usual lumping of RF cavities in one location.

Because of the short acceleration time, the muon beam only remains 17 turns in the RCS 1 and any transverse instability generated by HOMs would need to have a growth-rate in the order of a revolution period, i.e. $\sim 20\,\mu\mathrm{s}$.

Table 2: Simulation Input Parameters for RCS 1

Parameter	Value
Circumference C_0	5990 m
Injection energy	63 GeV
Energy gain per turn	14.8 GeV
Number of turns	17
Acceleration time	0.34 ms
Bunch intensity	2.54×10^{12}
Bunch length at injection $1\sigma_z$	13.1 mm
Synchronous phase	45°
Total RF voltage	20.9 GV
Synchrotron tune Q_s	1.52 (at injection)
Transverse normalized emittance	25 µm rad
Number of RF stations n_{RF}	32

Figure 2 summarizes the results of transverse stability simulations with a single resonator wakefield. shaded area corresponds to $(f_{res}, R_s/Q)$ values for which the transverse emittance is at least 20 % larger than the initial one after 17 turns. The black line represent the threshold for which a resonator generates a transverse instability, when single-turn wakefield is used in simulations. The limit on the resonator shunt impedance R_s can be expressed as $R_s < 100[\mathrm{M\Omega\,m^{-1}}] \cdot Q/f_{res}^2\,[\mathrm{GHz}^2]$. The colored lines represent the instability threshold when multi-turn wakefield effects are accounted for. In this case, the threshold is independent of the resonator frequency and can be expressed as $R_s < 10\,\mathrm{T\Omega\,m^{-1}}$.

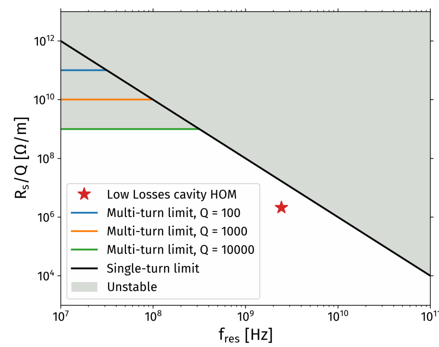

Figure 2: Stability limit for the RCS 1 versus resonator parameters. Black line represents the single-turn limit, colored lines the multi-turn limit and red star the most critical HOM created by 696 TESLA type cavities.

The combination of the single-turn and multi-turn stability limit provides a criterion which can be used to quickly check the impact of a proposed RF system for RCS 1, as described in the following section.

TRANSVERSE IMPEDANCE MODEL WITH RF CAVITIES AND IMPACT ON BEAM STABILITY

A first proposition for the RCS 1 RF system is based on 1.3 GHz TESLA type superconducting RF cavities [6, 10]. These cavities were developed for linear electron-positron colliders, and gradients as high as $50\,\mathrm{MV\,m^{-1}}$ could be reached [10, 11]. Assuming a cavity gradient of $30\,\mathrm{MV\,m^{-1}}$, ~ 700 cavities are required for the RCS 1. To evaluate the impact of HOMs on the transverse beam stability, the Low Loss TESLA cavity design was assumed, a design for which the HOMs were evaluated [12]. The largest transverse HOM is located at a frequency $f_{res} = 2.45$ GHz with $R_s/Q = 3.1\,\mathrm{k\Omega\,m^{-1}}$. With 696 cavities, and assuming the modes of all RF cavities have the exact same frequency, we obtain $(R_s/Q)_{total} = 2.1\,\mathrm{M\Omega\,m^{-1}}$. Using the criterion found previously, the stability threshold for a mode at $f_{res} = 2.45$ GHz is predicted to be at $R_s/Q = 100/2.45^2 = 16.7\,\mathrm{M\Omega\,m^{-1}}$. The most critical HOM of this TESLA type cavity is there-

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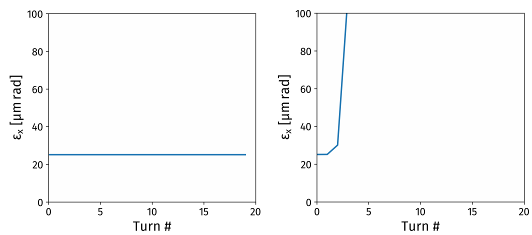
fore predicted to be a factor ~ 8 below the stability threshold, as can also be seen in Fig. 2. This margin was checked in simulation: with a single resonator with f_{res} = 2.45 GHz, Q = 10000 and R_s = 2.1 MΩ m^{-1} · Q = 21 GΩ m^{-1}, no transverse emittance blow-up appeared after 17 turns, as shown in Fig. 3a. Increasing the shunt impedance of this resonator by a factor 7, the beam remains stable. However with a factor 8, the stability limit is exceeded, leading to a fast transverse emittance blow-up, as shown in Fig. 3b.



(a) R_s = 21 GΩ m^{-1} (b) R_s = 168 GΩ m^{-1}

Figure 3: Horizontal emittance evolution versus turn number in the RCS 1, with a single resonator wakefield generated by 696 RF cavities. Left plot assumes the nominal shunt impedance, right plot assumes a factor 8 larger shunt impedance.

The transverse impedance model was extended by including all transverse HOMs listed in Ref. [12]. The resulting horizontal dipolar impedance is plotted in Fig. 4.

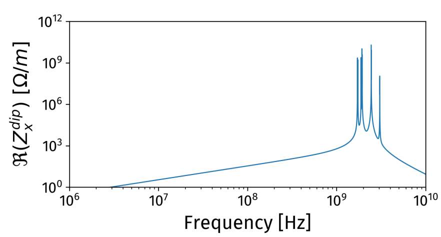
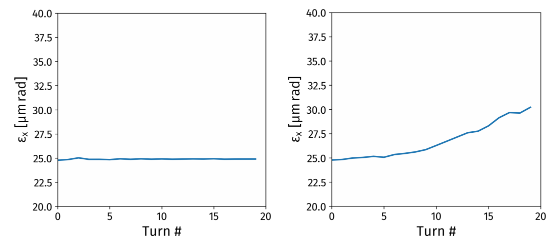


Figure 4: Transverse dipolar impedance (real part) versus frequency. 696 cavities are assumed in the model, and all transverse HOMs from the TESLA type cavity are included.

This model was used to study the impact of chromaticity in the RCS 1. Assuming the RCS lattice will be a FODO type and an average Twiss beta function of $\overline{\beta}_{x,y}$ = 50 m, the transverse tunes can be approximated to $Q_{x,y} \approx C_0/(2\pi\overline{\beta}_{x,y}) = 19$. With a machine operating above transition energy, slightly positive chromaticity mitigates the mode 0 head-tail instability. A negative chromaticity however would drive a mode 0 head-tail instability if no other mitigation method, such as a transverse feedback, is present in the machine. Without sextupoles, the natural machine chromaticity is negative and $Q'_{x,y} \approx -Q_{x,y}$. A machine design without sextupoles for chromaticity correction would save space in the lattice and increase the energy reach of the RCS. However the effects of negative chromaticity must be checked in terms of optics, dynamic aperture, coherent stability *etc.*

The chromaticity $Q'_{x,y}$ was scanned in the −19 to +19 range using the PyHEADTAIL simulation setup described beforehand. With the impedance model pictured in Fig. 4, no transverse emittance blow-up was found after 17 turns of acceleration, both with $Q' = -19$ and $Q' = 0$. Therefore the impedance was multiplied to reach the instability threshold. With a factor 2, the beam is close to instability as can be seen in Fig. 5a where a small emittance growth appears after one turn. With $Q' = -19$ a clear transverse emittance blow-up is present, as figured in Fig. 5b, and a mode 0 instability is observed.



(a) $Q' = 0$ (b) $Q' = -19$

Figure 5: Horizontal emittance evolution versus turn number in the RCS 1, with a factor two on the impedance model pictured in Fig. 4. Left plot assumes a corrected chromaticity, right plot assumes a uncorrected (natural) chromaticity.

A transverse feedback could be a mitigation strategy if the negative chromaticity operation in the RCS is investigated further. This feedback would have to be fast considering the short acceleration time in the first RCS of only 17 turns.

CONCLUSION

The large acceleration gradient required for the muon RCSs, and therefore the number of RF cavities needed in the accelerator, poses challenges in terms of longitudinal and transverse beam stability. General transverse stability criteria were found for the transverse RF cavities HOMs, allowing for quick estimates of the shunt impedance, resonance frequency and quality factor limits. Using the first configuration proposed for the RF system, based on TESLA type superconducting RF cavities, the impact of transverse HOMs on coherent beam stability has been investigated. A factor 8 margin for the most critical HOM was found, and with all transverse HOMs, only a factor 2 margin remains. Including additional impedance contributions coming from the chamber walls would further reduce this margin. Moreover, if an operation with negative chromaticity is chosen for the RCS design, head-tail instabilities would degrade the transverse beam emittance. The effect of an initial transverse offset of the beam is also being studied. Mitigation measures such as using a transverse feedback system will be investigated to preserve the beam quality, depending on design choices.

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