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Systematic Errors of a ”frozen Spin” EDM measurement with electric and magnetic Fields due to imperfect Alignment of Fields

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Summary

Many of the rings proposed to measure the Electric Dipole Moment (EDM) of charged particles rely on a combination of magnetic and electric fields bending the beam to fulfil the ”frozen spin” condition. Imperfect alignment of the two fields bending the beam is known to rotate the spin from the longitudinal into the vertical direction. This effect is very similar to the signature of a finite EDM and, thus, a potential limitation for the sensitivity of the measurement. The concept of a ”spin transparent” quadrupole is to superimpose magnetic and electric contributions for focusing elements as well to increase the spin coherence time. A vertical offset between the two quadrupole contributions leads as well to rotations of the spin from the longitudinal to the vertical direction. In this report, simple expressions to compute the angular frequencies of these rotations around a radial axis are given.

Contents

1 Introduction

Several schemes to measure the Electric Dipole Moment (EDM) of charged particles are discussed at present. Most of these proposals foresee to run a synchrotron satisfying the "frozen spin" condition. This condition requires that, in absence of an EDM and with the well known Magnetic Dipole Moment (MDM) in a perfect machine, bunches with initial longitudinal polarization (parallel or antiparallel to the direction of movement) remain longitudinally polarized. This implies that the spin of a reference particle (reference energy and reference orbit in perfect machine) rotates together with the direction of the trajectory. This is achieved by an appropriate choice of the electric and magnetic fields of bending elements [1, 2, 3, 4, 5, 6]. The effect of a finite EDM is a rotation of the spin around a radial direction from the longitudinal direction into the vertical direction. In an EDM ring the resulting vertical spin build up, which is very small for the smallest EDM to be detected in typical proposals, is measured with a polarimeter. A special case not further treated here and possible only for particles with positive anomalous magnetic moment $G > 0$ as for example protons, is operation with beams at the "magic energy", where the "frozen spin" condition is met with only electric fields [7].

Imperfect alignment of field components in "frozen spin" EDM rings combining magnetic and electric fields to bend the beam is known to generate rotations of the spin from the longitudinal to the vertical direction, an effect potentially limiting the feasible sensitivity of such experiments. The resulting vertical spin components can be misinterpreted as the result of a finite EDM and lead to systematic errors of the measurements. The resulting measurement error can in principle be mitigated or compensated by operating the ring with Clock-Wise (CW) and Counter-Clock-Wise (CCW) beams. However, this compensation will be limited by imperfect inversion of the magnetic field and the polarimeter. In this report simple expressions to estimate the angular frequencies describing these rotations of the spin around a radial axis are given, which is typically very fast for realistic assumptions on the alignment of the bending fields. Different cases as a tilt (rotation around the longitudinal axis) of the bending magnetic field only, a tilt of the bending electric field only and a tilt of both the magnetic and electric field such that the will be treated.

The use of a "spin transparent quadrupole" superimposing electric and magnetic fields to fulfil the "frozen spin" condition as well in focusing elements has been proposed with the aim to lengthen the spin coherence time [8]. A vertical offset between the magnetic and electric contributions to the focusing strengths generates as well a rotation of the spin from the longitudinal to the vertical direction and is studied.

2 Angular Frequencies describing the Rotation of the Spin and "frozen Spin" condition

Spin rotations of charged particles with both a MDM and an EDM aligned with the particle spin are described by the Thomas-BMT equation with additional terms due to the EDM. The vector $\vec{\zeta}$ describes the direction of the spin in a coordinate system moving with the particle. The time derivative of this spin direction $\vec{\zeta}$ of a particle with charge q and mass m in a magnetic field $\vec{B}$ and an electric field $\vec{E}$ is described with the help of an angular frequency $\vec{\Omega}_S$ given for example in [9, 10] and becomes in SI units:

$$\frac{d\vec{\zeta}}{dt} = \vec{\Omega}_S \times \vec{\zeta} = \left(\vec{\Omega}_M + \vec{\Omega}_E\right) \times \vec{\zeta} \tag{1}$$

$$\begin{aligned}\vec{\Omega}_M &= -\frac{q}{m}\left[\left(G+\frac{1}{\gamma}\right)\vec{B} - G\,\frac{\gamma-1}{\gamma}\frac{\vec{B}\cdot\vec{\beta}}{\beta^2}\vec{\beta} - \left(G+\frac{1}{\gamma+1}\right)\vec{\beta}\times\frac{\vec{E}}{c}\right] = \\ &= -\frac{q}{m}\left[\left(G+\frac{1}{\gamma}\right)\vec{B}_\perp + (G+1)\,\frac{\vec{B}_\parallel}{\gamma} - \left(G+\frac{1}{\gamma+1}\right)\vec{\beta}\times\frac{\vec{E}}{c}\right]\end{aligned} \tag{2}$$

$$\vec{\Omega}_E = -\frac{q\,\eta}{2\,m}\left(\frac{\vec{E}}{c} - \frac{\gamma-1}{c\,\gamma}\frac{\vec{E}\cdot\vec{\beta}}{\beta^2}\vec{\beta} + \vec{\beta}\times\vec{B}\right) = -\frac{q\,\eta}{2\,m}\left(\frac{\vec{E}_\perp}{c} + \frac{1}{\gamma}\frac{\vec{E}_\parallel}{c} + \vec{\beta}\times\vec{B}\right) \tag{3}$$

where $\vec{\Omega}_S$ is expressed as the sum of $\vec{\Omega}_M$ describing the effect with the well known MDM[1] plus a term $\vec{\Omega}_E$ due to a possible EDM. q and m are the charge and mass of the particle, $\vec{B}$ and $\vec{E}$ the magnetic and electric field, β and γ the relativistic factors and $\vec{\beta} = \vec{v}/c$ a vector with length β and a direction parallel to the velocity ($\vec{v}$ and c are the velocity of the particle and the velocity of light). The quantities G and η describe the well known magnetic moment and the EDM to be measured. For the case of protons and Deuterons $G = G_p = 1.79285$ and $G = G_D = -0.142987$, respectively. Note that for a proton or Deuteron EDM of $d_s = 10^{-29}\,e$ cm, which is often quoted as expected sensitivity of experiments proposed to identify a possible proton EDM, η is as low as $\eta_s = 1.9 \cdot 10^{-15}$ (almost identical for both particle types!). The indices $\perp$ and $\parallel$ denote the component of a vector perpendicular and parallel to the direction of movement; for example for the electric field $\vec{E}_\perp = \vec{E} - (\vec{t}\cdot\vec{E})\,\vec{t}$ and $\vec{E}_\parallel = (\vec{t}\cdot\vec{E})\,\vec{t}$ with $\vec{t} = \vec{\beta}/\beta$ a unit vector pointing in the direction of the movement.

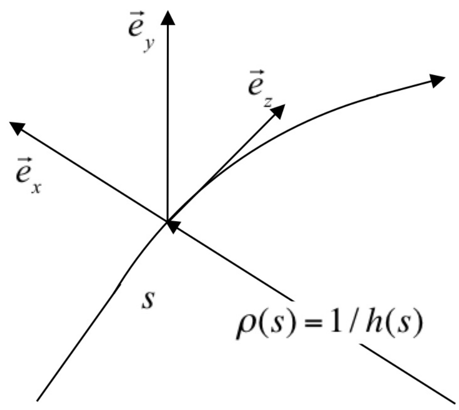

Figure 1: Coordinate system used with positive curvature $h > 0$ for the CW beam.

The frozen spin condition and all other relations will be derived using the coordinate system sketched in Fig. 1 for the CW beam with the radial direction $\vec{e}_x$ pointing outwards and a positive curvature $h = 1/\varrho$. For CCW beams, the curvature $h < 0$ is negative and the radial x-axis points towards the inside of the ring. A particle circulating without betatron

[1]Comprising a contribution of the magnetic field in the particle rest frame coupling to the MDM and a contribution describing the Thomas-Wigner rotation due to the acceleration of the particle.

oscillations along the reference orbit ($x = y = 0$) in a perfect machine is exposed only to a vertical magnetic field B_y and a radial electric field E_x. The curvature $h = 1/\varrho$ is written as a sum of a contribution $\kappa_E h$ from the electric field and a contribution $\kappa_M h$ from the magnetic field:

$$h = \frac{1}{\varrho} = \kappa_M h + \kappa_E h \quad \text{with} \quad \kappa_M h = \frac{q\,B_y}{m\gamma\beta c} \quad \text{and} \quad \kappa_E h = -\frac{q\,E_x}{m\gamma\beta^2 c^2} \quad .$$

Both the angular frequency $\vec{\Omega}_p = -h\,\beta c\,\vec{e}_y$ describing the rotation of the direction of motion and the angular frequency $\vec{\Omega}_M$ describing the rotation of the spin with an MDM only have only a vertical component. The "frozen spin" condition is satisfied if, without an EDM ($\vec{\Omega}_E = 0$), both the spin and the particle direction rotate around the vertical axis with the same angular frequency leading to the condition:

$$\Omega_{p,y} = -h\,\beta c = \Omega_{M,y} = -\frac{q}{m}\left[\left(G + \frac{1}{\gamma}\right) B_y - \left(G + \frac{1}{\gamma+1}\right)\frac{\beta\,E_x}{c}\right] \quad .$$

The solution to these equations is given by:

$$\kappa_E = \frac{\gamma^2 G}{G+1} \quad , \quad \kappa_M = \frac{1-\beta^2\gamma^2 G}{G+1} \quad , \tag{4}$$

$$E_x = -\frac{\gamma^2 G}{G+1}\,\frac{\gamma\beta^2 mc^2}{q}\,h \quad \text{and} \quad B_y = \frac{1-\beta^2\gamma^2 G}{G+1}\,\frac{\gamma\beta mc}{q}\,h \quad . \tag{5}$$

From above expressions, the following relation between magnetic and electric field, given in some report on EDM measurement proposals, can be easily derived:

$$B_y = -\frac{1-\beta^2\gamma^2 G}{\beta^2\gamma^2 G}\,\frac{\beta}{c}\,E_x \quad . \tag{6}$$

Note that for particles with negative anomalous magnetic moment $-1 < G < 0$ as for example Deuterons, the sign of B_y and E_x is the same and positive for CW beams. This means that the deflections due to magnetic and electric fields have opposite sign (the absolute value of the deflection due to the magnetic field B_y is stronger than the one from the electric field E_x generating a force towards the outside of the ring). For particles with positive $G > 0$, the radial electric field E_x dominates over the magnetic bending field and points towards the inside of the ring ($E_x < 0$ for CW beam). The vertical magnetic field B_y is positive below the so-called "magic energy" and negative above (counter-acting the deflection due to the electric field).

A possible EDM generates a rotation of the spin around a radial axis. The angular frequency of this rotation, averaged over the circumference C by replacing h by $2\pi/C$, is obtained by inserting the fields into Eq. 3 :

$$\bar{\Omega}_{E,x} = -\frac{q\eta}{2mc}\left(E_x - \beta c\,B_y\right) = -\frac{\eta\gamma\beta^2 c}{2}\left(-\frac{\gamma^2 G}{G+1} - \frac{1-\beta^2\gamma^2 G}{G+1}\right)\frac{2\pi}{C} = \frac{\pi\eta\gamma\beta^2 c}{C} \tag{7}$$

which is small for typical EDM ring proposals.

3 Imperfect Alignment of electric and/or magnetic fields bending the beam

3.1 Tilt of the electric and magnetic field without a vertical deflection of the beam

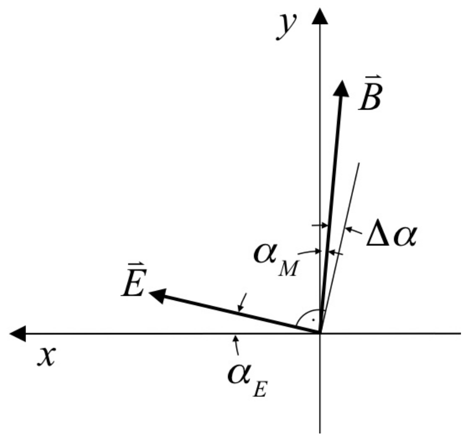

Figure 2: Small rotations of both the magnetic and electric fields of combined bendings. Frozen spin configuration (Eq. (6)). Situation for negative $-1 < G < 0$ with the electric field counter-acting the (stronger) deflection due to the electric field.

As a first case, a tilt (rotation around the longitudinal axis) of both the electric and magnetic field components such that the beam is not deflected in vertical direction is considered. The magnetic and electric fields are rotated as indicated in Fig. 2 by small angles α_M and α_E, respectively. The requirement that the vertical force due to the resulting field components $B_x = -\alpha_M B_y$ and $E_y = \alpha_E E_x$ vanishes gives:

$$F_y = q\left(\alpha_E E_x - \beta c \alpha_M B_y\right) = -\alpha_E \frac{\gamma^2 G}{G+1}\gamma\beta^2 mc^2 h - \alpha_M \frac{1-\beta^2\gamma^2 G}{G+1}\gamma\beta^2 mc^2 h = 0 \quad .$$

This last last expression leads to the relation $\alpha_E\kappa_E + \alpha_M\kappa_M = 0$. Using in addition the relative rotation of the electric field w.r.t. the magnetic field given by $\Delta\alpha = \alpha_E - \alpha_M$ and making use of the relation $\kappa_E + \kappa_M = 1$, one finds:

$$\alpha_M = -\kappa_E \Delta\alpha \quad \text{and} \quad \alpha_E = \kappa_M \Delta\alpha \tag{8}$$

and finally the additional field components

$$B_x = -\alpha_M B_y = \frac{\gamma^2 G\left(1-\beta^2\gamma^2 G\right)}{(G+1)^2}\frac{m\beta\gamma c}{q} h\,\Delta\alpha \tag{9}$$

$$E_y = \alpha_E E_x = -\frac{\gamma^2 G\left(1-\beta^2\gamma^2 G\right)}{(G+1)^2}\frac{m\beta^2\gamma c^2}{q} h\,\Delta\alpha \tag{10}$$

which act as a Wien filter rotating the spin around an axis in radial direction. The angular frequency averaged over the circumference (again replacing h by the average value $2\pi/C$) is obtained by inserting the field components into Eq. 2 and given by:

$$\bar{\Omega}_{M,x,\Delta\alpha} = -\frac{G\,(1-\beta^2\gamma^2 G)}{(G+1)}\,\gamma\,\beta c\,\frac{2\pi}{C}\overline{\Delta\alpha} \tag{11}$$

where $\overline{\Delta\alpha}$ denotes the average of $\Delta\alpha$ over all bendings of the lattice.

3.2 Tilt of the electric field for a machine with magnetic focusing

In a machine with magnetic focusing the vertical deflection of the beam due to a tilt α_E of the electric field in bendings has to be compensated by radial magnetic fields (in a machine with electric quadrupoles, these deflections would be compensated by electric fields such that, in first order, the resulting rotations of the spin into the vertical plane vanish). The average vertical electric field due to tilts of the electrical bendings is given by (factor $2\pi/(h\,C)$ to compute average around circumference from average inside bendings):

$$\bar{E}_y = \frac{2\,\pi}{h\,C}\bar{\alpha}_E\,E_x = -\frac{\gamma^2 G}{G+1}\,\frac{2\pi}{C}\,\frac{\gamma\beta^2 mc^2}{q}\,\bar{\alpha}_E$$

with $\bar{\alpha}_E$ the average of α_E over all bendings. The resulting deflection has to be compensated by deflection due to the magnetic field of the quadrupoles leading to an average magnetic field experienced by a particle of

$$\bar{B}_x = -\frac{1}{\beta c}\bar{E}_y = \frac{\gamma^2 G}{G+1}\,\frac{2\pi}{C}\,\frac{\gamma\beta mc}{q}\,\bar{\alpha}_E \quad .$$

The resulting average angular frequency around the radial axis is obtained by inserting the field components into Eq. 2 and given by:

$$\bar{\Omega}_{M,x,\alpha_E} = -G\,\gamma\beta c\,\frac{2\pi}{C}\,\bar{\alpha}_E \quad . \tag{12}$$

3.3 Tilt of the magnetic field for a machine with electric focusing

In a machine with electric focusing, the vertical deflection of the beam due a tilt α_M of the magnetic field in bendings has to be compensated by vertical electric fields (in a machine with magnetic quadrupoles, these deflections would be compensated by magnetic fields such that, in first order, the resulting rotations of the spin into the vertical plane vanish). The average radial magnetic field due to tilts of the magnetic bendings is given by:

$$\bar{B}_x = -\frac{2\,\pi}{h\,C}\bar{\alpha}_M\,B_y = -\frac{1-\beta^2\gamma^2 G}{G+1}\,\frac{2\pi}{C}\,\frac{\gamma\beta mc}{q}\,\bar{\alpha}_M$$

with $\bar{\alpha}_M$ the average of α_M over all bendings. The resulting deflection has to be compensated by deflection due to the magnetic field of the quadrupoles leading to an average magnetic field experienced by a particle of

$$\bar{E}_y = -\beta c\bar{B}_x = \frac{1-\beta^2\gamma^2 G}{G+1}\,\frac{2\pi}{C}\,\frac{\gamma\beta^2 mc^2}{q}\,\bar{\alpha}_M \quad .$$

The resulting average angular frequency around the radial axis is obtained by inserting the field components into Eq. 2 and given by:

$$\bar{\Omega}_{M,x,\alpha_M} = \left(1 - \beta^2\gamma^2 G\right) \frac{\beta\, c}{\gamma} \frac{2\pi}{C} \bar{\alpha}_M \quad . \tag{13}$$

4 Imperfect alignment of electric and magnetic parts of a "spin transparent quadrupole"

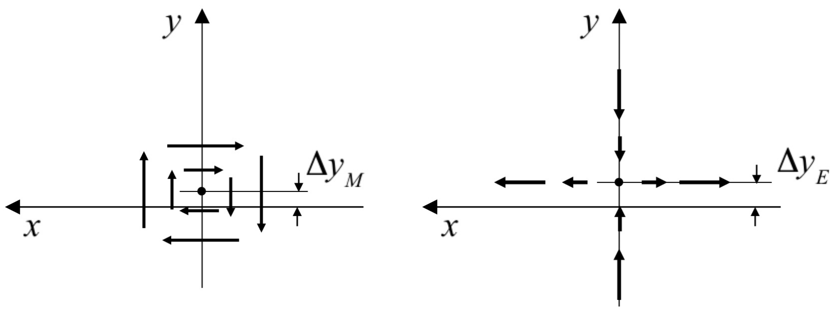

Figure 3: Sketch of the magnetic (left image) and electric (right image) field of a "spin transparent" quadrupole with vertical offsets of the two components. Situation sketched is correct for an horizontally focused magnetic quadrupole and for negative $-1 < G < 0$ ($\kappa_M > 1$ and $\kappa_E < 0$). The effect from the magnetic F quadrupole is reduced by the weaker electric D quadrupole.

A proposal named "spin transparent quadrupole" to fulfil the "frozen spin" condition as well inside quadrupoles has been studied [8]. The conclusion of this study assuming perfect alignment of electric and magnetic contributions of the quadrupoles was that, indeed, the spin coherence time can be increased. Here, the effect of a vertical offset between the magnetic and electric part, such that a particle at the theoretical center is not deflected, is studied. Extrapolating from the derivations for the ratio between the electric and magnetic contributions to the bending, the quadrupole gradients are a superposition of electric and magnetic contributions $k_E = \kappa_E\, k$ and $k_M = \kappa_M\, k$. Vertical offsets of the magnetic and electric components of a quadrupole, as sketched in Fig. 3, generate additional field components given by

$$\Delta B_x = -\Delta y_M \frac{d\,B_x}{dy} = -k_M \frac{m\gamma\beta c}{q} \Delta y_M \quad \text{and} \quad \Delta E_y = -\Delta y_E \frac{d\,E_y}{dy} = -k_E \frac{m\gamma\beta^2 c^2}{q} \Delta y_E$$

The assumption that the offsets do not generate a vertical deflection leads to the condition:

$$\Delta F_y = q\left(\Delta E_y + \beta c\, \Delta B_x\right) = -m\gamma\beta^2 c^2 \left(k_M \Delta y_M + k_E \Delta y_E\right) = 0 \quad .$$

The last expression leads to the condition $k_M \Delta y_M + k_E \Delta y_E = 0$. Introducing in addition the relative offset between the electric and magnetic quadrupoles defined by $\Delta y = y_E - y_M$ and making use of the relation $\kappa_M + \kappa_E = 1$, one obtains:

$$\Delta y_M = -\kappa_E\, \Delta y \quad \text{and} \quad \Delta y_E = \kappa_M\, \Delta y$$

and finally the additional field components

$$\Delta B_x = \frac{\gamma^2 G\,(1-\beta^2\gamma^2 G)}{(G+1)^2}\,\frac{m\gamma\beta c}{q}\,\Delta y \quad \text{and} \quad \Delta E_y = -\frac{\gamma^2 G\,(1-\beta^2\gamma^2 G)}{(G+1)^2}\,\frac{m\gamma\beta^2 c^2}{q}\,\Delta y$$

which act as Wien filter rotating the spin around an axis in radial direction, very similar to the effect of an EDM in a perfect ring. The angular frequency describing this rotation, averaged over the circumference (additional factor l_{eq}/C with l_{eq} the equivalent length of the quadrupole), is obtained by inserting the field components into Eq. 2 and given by:

$$\bar{\Omega}_{M,x,\Delta y} = -\frac{G\,(1-\beta^2\gamma^2 G)}{(G+1)^2}\,\gamma\beta c\,\frac{l_{eq}k}{C}\,\Delta y \quad . \tag{14}$$

5 Application to proposals of EDM rings with bendings combining electric and magnetic fields

5.1 Deuteron EDM ring

For numerical evaluations parameters of the "frozen spin" proposal described in [4, 5, 6] are used. The rest energy of Deuterons is $E_r = 1875.61$ MeV and their anomalous magnetic moment $G = -0.142987$. The kinetic energy of circulating Deuterons is $E_{kin} = 270$ MeV giving relativistic factors $\gamma = 1.1439$ and $\beta = 0.485635$. At this energy, the sharing between magnetic and electric deflection to fulfil the frozen spin condition is given by $\kappa_M \approx 1.2183$ and $\kappa_E \approx -0.2183$ (force towards the outside of ring counteracting stronger force from magnetic field).

Detailed lattice descriptions is given in [4, 5, 6]. The plots show a rather regular FODO lattice with 44 half-cells, 32 combined bendings sections and a circumference of about $C = 150$ m. The integrated strength of the quadrupoles is estimated to $k\,l_{eq} = 0.25$ m^{-1}.

- An EDM of $10^{-29}\,e\cdot$cm, which is quoted as the sensitivity aimed at by the proposal, corresponds to $\eta = 1.9\cdot 10^{-15}$ and an angular frequency rotating the spin from the longitudinal into the vertical direction of $\bar{\Omega}_{E,x} = 3.2$ nrad/s.

- Inserting an average tilt of the electric bend w.r.t the magnetic bend of $\overline{\Delta\alpha} = 0.1$mrad into Eq. 11 results in a rotation of the spin from the longitudinal into the vertical direction of $\bar{\Omega}_{M,x,\Delta\alpha} = 122$ rad/s.

- Using Eq. 12, one finds for an average tilt of the electric bends of $\bar{\alpha}_E = 0.1$ mrad in a lattice with magnetic focusing a rotation of the spin from the longitudinal into the vertical direction of $\bar{\Omega}_{M,x,\alpha_E} = 100$ rad/s.

- Using Eq. 13, one finds for an average tilt of the magnetic bends of $\bar{\alpha}_M = 0.1$ mrad in a lattice with electric focusing a rotation of the spin from the longitudinal into the vertical direction of $\bar{\Omega}_{M,x,\alpha_M} = 557$ rad/s. This result is in agreement (apart from the sign, which may be caused by different coordinate systems or a different convention to describe a tilt) with tracking studies presented in Fig. 2 of [6].

- Using Eq. 14, one finds for a relative offset $\Delta y = 0.1$ mm between the electric and magnetic contributions of a "spin transparent" quadrupole a rotation of the spin from the longitudinal direction of $\bar{\Omega}_{M,c,\delta y} = 7.3$ rad/s.

Note that the effects described do not mimic EDM for operation with CW and CCW beams (in distinct machine cycles) in case the magnetic fields can be perfectly inverted (and the electric fields remain identical). This means that operation of the ring with CW and CCW beams allows, in principle, to differentiate the effect from a real EDM. Nevertheless, operation with such large rotations of the spin around a radial axis (many revolutions over the expected duration of one machine cycle) is not consistent with the initial "frozen spin" concept and has practical implications.

5.2 EDM "frozen spin" prototype ring with fourfold periodicity

For numerical evaluations, parameters of the "frozen spin" proposal with fourfold periodocity as described in [11] are used. The circumference of the ring is 100 m and the kinetic energy of circulating protons is $E_{kin} = 45$ MeV. This gives relativistic factors $\gamma = 1.048$ and $\beta = 0.299$. At this energy, the sharing between magnetic and electric deflection to fulfil the frozen spin condition is given by $\kappa_E \approx 0.7050$ and $\kappa_M \approx 0.2950$.

- An EDM of $10^{-26}\, e \cdot$ cm, which has been mentioned as the sensitivity aimed at by the proposal, corresponds to $\eta = 1.9 \cdot 10^{-12}$ and an angular frequency rotating the spin from the longitudinal into the vertical direction of $\bar{\Omega}_{E,x} = 1.68$ μrad/s.
- An average tilt of the electric bend w.r.t the magnetic bend of $\overline{\Delta\alpha} = 0.1$mrad results in a rotation of the spin from the longitudinal into the vertical direction of $\bar{\Omega}_{M,x,\Delta\alpha} = -312$ rad/s.
- An average tilt of the electric bends of $\bar{\alpha}_E = 0.1$ mrad in a lattice with magnetic focusing results in a rotation of the spin from the longitudinal into the vertical direction of $\bar{\Omega}_{M,x,\alpha_E} = -1058$ rad/s.
- An average tilt of the magnetic bends of $\bar{\alpha}_M = 0.1$ mrad in a lattice with electric focusing results in a rotation of the spin from the longitudinal into the vertical direction of $\bar{\Omega}_{s,x,\alpha_M} = 443$ rad/s.

5.3 EDM "frozen spin" prototype ring with fivefold periodicity

The lattice taken into account for the simulation results is the periodicity five proton PTR proposal [12] with additional magnetic bending fields for "frozen spin" operation for a first direct Proton EDM measurement. The circumference of the ring for this proposal is C=91.79 m and the kinetic energy of circulating protons is, for the frozen spin operation, E_{Kin}= 37.034 MeV. The relativistic factors are γ= 1.03947 and β=0.2730 respectively. In particular, two cases have been studied: a tilt (rotation around the longitudinal axis) of one out of $N_B = 20$ bending elements by 0.1 mrad with magnetic focusing and a tilt of one bending by 0.1 mrad with electric focusing. The resulting average tilts are $\bar{\alpha}_M = 0.05$ mrad

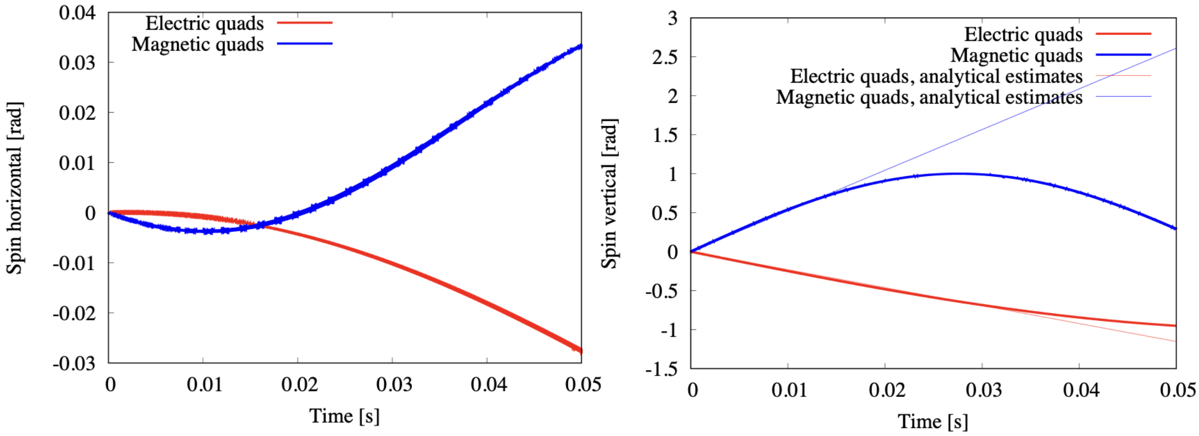

Figure 4: Spin tracking results for a long-term simulation. The tangential at the start of the simulation expected from analytical estimates is plotted for comparison.

and $\bar{\alpha}_E = 0.05$ mrad, respectively. Before adding the tilt to one of the bending elements, an energy correction has been applied to suppress the spin rotations around the vertical axis. Furthermore, an orbit correction, launching the particle such that it follows the closed orbit (with the latter computed from the phase space coordinates at start and the end of a turn and the transfer matrix) was done to avoid betatron oscillations. The closed orbit has been computed from the change of trajectory between the start and the end of the turn plus the transfer matrix (over one turn). In the following, the computation of the vertical spin component and comparison with analytical estimates will be presented.

5.3.1 Simulation Results

For the simulations, an initially longitudinally polarized beam is considered. The first simulation results are showing the case of a tilt of one out of 20 bending elements. The spin tracking results for long-term simulations are shown in Fig. 4, while in Fig. 5 a zoom in the first turns is shown, in order to see the different spin evolution for a proton executing betatron oscillations and a proton following the closed orbit. We can clearly see that we have very fast spin rotations around the radial axis (from longitudinal to vertical), which is a concern. The horizontal spin components are orders of magnitude smaller (and could anyhow be taken care of by a spin feedback system). These rotations have very important implications for the operation of the ring.

The second simulation results are obtained for a tilt of the electric and magnetic field of one bending such that there is no vertical deflection of the beam. Also for this other case a simulation for an initially longitudinally polarized beam has been performed. In Fig. 6 the spin tracking results for the CW beam for the longterm simulations are shown, while in Fig. 7 we can see the spin tracking results for the CCW beam. Again very fast spin rotations around the radial axis (from longitudinal to vertical) are observed.

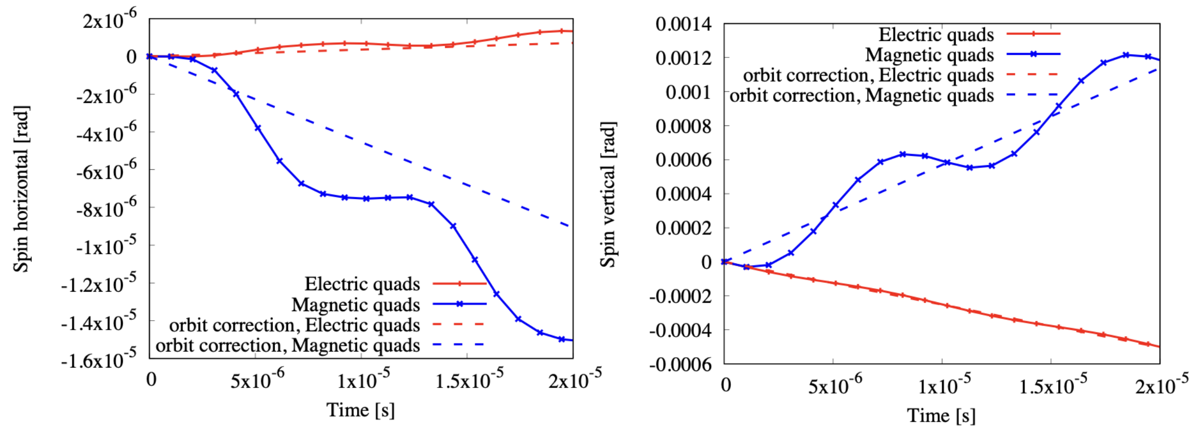

Figure 5: Spin tracking results for just the first turns and comparison with the results obtained after applying the orbit correction (with a revolution period of 1.024 μs).

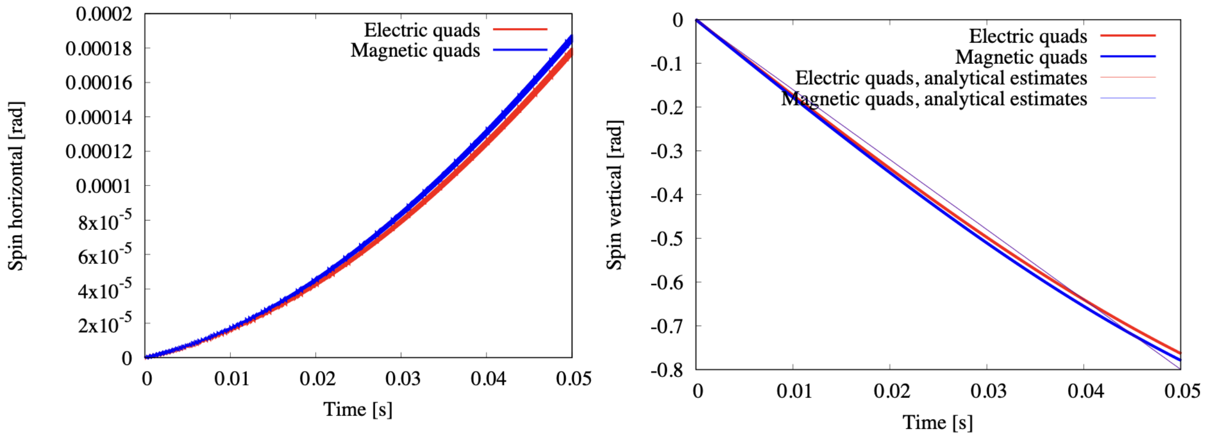

Figure 6: Spin tracking results for a long-term simulation for the CW beam. The tangential at the start of the simulation expected from analytical estimates is plotted for comparison.

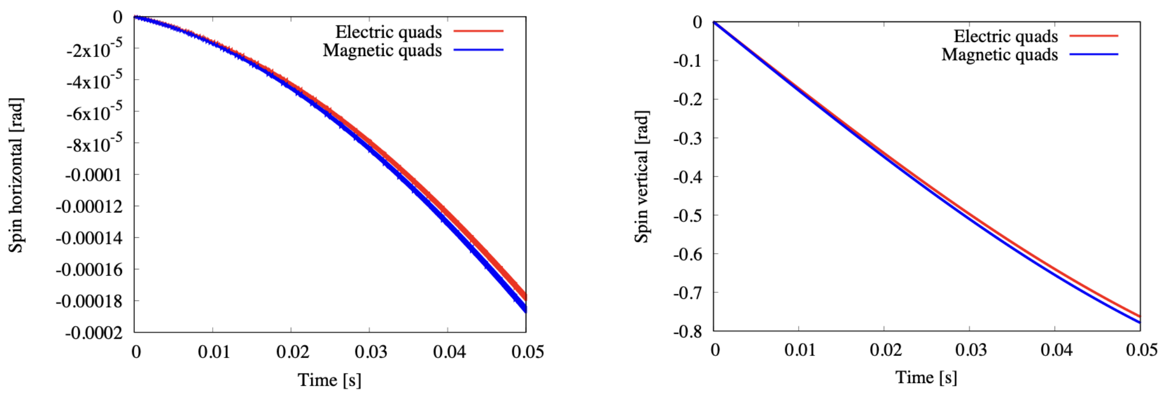

Figure 7: Spin tracking results for a long-term simulation for the CCW beam.

5.3.2 Comparison between Analytical Estimates and Simulation Results

For the comparison with analytical estimates, the average tilt of the bend is the tilt of the single misaligned bending divided by the number of bendings, in this case 20.

- An EDM of $10^{-24}\, e \cdot \mathrm{cm}$ corresponds to an angular frequency rotating the spin from the longitudinal into the vertical direction of $\bar{\Omega}_{E,x} = 1.51\ \mu\mathrm{rad/s}$.

- Using Eq. 12, one finds for an average tilt of the electric field for a machine with magnetic focusing of $\bar{\alpha}_E = 0.05$ mrad a rotation angular frequency of $\bar{\Omega}_{M,x,\alpha_E} = -51.5$ rad/s. From simulations, we have a vertical spin component after one turn of 5.828×10^{-5} rad, which corresponds to a rotation angular frequency of $\bar{\Omega}_{M,x,\alpha_E} = -51.3$ rad/s.

- Using Eq. 13, one finds for the tilt of the magnetic field for a machine with electric focusing of $\bar{\alpha}_M = 0.05$ mrad a rotation angular frequency of $\bar{\Omega}_{M,x,\alpha_M} = 22.9$ rad/s. From simulations, we have a vertical spin component after one turn of -2.5758×10^{-5} rad, which corresponds to a rotation angular frequency of $\bar{\Omega}_{M,x,\alpha_M} = 22.7$ rad/s.

- Inserting an average tilt of the electric bend w.r.t the magnetic bend of $\overline{\Delta\alpha} = 0.1$mrad into Eq. 11 results in a rotation of the spin from the longitudinal into the vertical direction of $\bar{\Omega}_{M,x,\Delta\alpha} = 15.6$ rad/s. From simulations, we have that the vertical spin component after one turn is -1.7748×10^{-5} rad, which corresponds to a rotation angular frequency of $\bar{\Omega}_{M,x,\Delta\alpha} = 17.3$ rad/s.

In all the cases, we have very good agreement between the analytical and the simulation results.

6 Summary and Conclusion

Imperfect alignment of electric and magnetic bends in "frozen spin" EDM rings can generate very fast rotations of the spin around a radial axis. Vertical offsets between the magnetic and electric parts of "spin transparent quadrupoles" proposed for some variants of "frozen spin" EDM rings generate similar effects. These effects do not mimic EDM in the sense that they can be distinguished from an EDM by operating the ring with CW and CCW beams, provided that the magnetic field can be perfectly reversed. At a first glance, focusing using magnetic quadrupoles seems to be preferable (slower spin rotations for given tilts of bending field components) for the Deuteron ring proposal used for numerical evaluations. For the proposed proton EDM measurement in a prototype ring, proposed to test ideas for a fully electric "magic energy" ring, electric focusing seems preferable.

Operating an EDM ring with fast rotations of the spin around a radial axis (many thousands of revolutions during a typical duration of a measurement and not just a few turns as for the Koop wheel concept) is very different from the initial "frozen spin" concept. These fast unintended rotations of the spin around a radial axis have important implications for the operation of the ring. Thus, the effects described should be taken into account for the design of EDM rings using magnetic and electric fields and, in particular, discussing the beam preparations and measurements.

In principle, feedback systems acting e.g. on Wien filters rotating the spin around a radial axis could be implemented to slow down the effects described. Another possible mitigation measure is to estimate relative rotations between the magnetic and electric bending components with the beam. Time modulation of the electric and magnetic fields bending the beam (such that the total deflection remains constant) with imperfect alignment would induce vertical orbit modulations [2].

Even though, in case of operation with CW and CCW beams with perfect inversion of the magnetic fields, the contributions to the final result cancel, this may be impractical: the spin rotation angular frequencies have to be measured with very high relative precision, which may not be feasible with realistic polarimeter. Moreover, imperfect inversion of magnetic fields (with e.g. presence of stray fields, which do not invert) are a likely limitation for the achievable accuracy.

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[2]Note that variations of the electric and magnetic contributions to the bendings would as well change the focusing (additional focusing terms for non-relativistic beams in electro-static bends). Even though for bendings without gradients, the effect mainly acts in the horizontal plane, this may perturb such investigations.

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