PS/LEA/Note 87-05
March 1987

Formulation of a Flat – Helix Coupler for a Pickup of a Low Velocity Beam

N.Tokuda[1)]

Abstract: This note describes a formulation for the coupling impedance of a flat – helix pickup. For the simplicity of calculation, the structure is assumed infinitely wide, and the helix is replaced with two parallel plates conducting only in the direction of the helix. A design example for a pickup to be used for a 60 MeV/c beam at LEAR is presented. The propriety of the simplified model is subject to experimental check: measurements of phase velocity and coupling impedance.

1) On leave from the Institute for Nuclear Study, the University of Tokyo, Tokyo, Japan.

1. Introduction

A helix pickup worked successfully used in the longitudinal stochastic cooling of a low velocity beam, $\beta = 0.12$, at TARN [1]. The formulation to calculate its phase velocity and coupling impedance was based on a sheath helix model: a round helix is replaced by a cylindrical pipe conducting in the direction of the helix [2]. The coupling impedance reduced from this theory agreed with the experimental data. When the cross sectional shape of a pickup is a circle or a rectangle close to a square, this theory will be reliable. In practice, however, a pickup is often required to have a horizontally wide aperture, or to have a cross sectional shape of a flat rectangle to cover the beam passing region at the location of the pickup. For such a pickup the above theory is not applicable.

For the design of a flat−helix pickup, another formulation is presented here. A simplified mathematical model for a flat−helix was proposed by Johnson [3]; an actual flattened tape helix is represented by two parallel plates conducting only in the direction of the helix. This mathematical model is employed in the present formulation.

2. Mathematical Model

The structure of the mathematical model contemplated here is shown in Fig.1. It consists of infinitely wide plates; the effect at the end parts where the helix is bent over is neglected. The outer plates, located at $y = \pm b$, are the shields of conducting metal plates. The inner ones, located at $y = \pm a$, are helices. Instead of an actual tape helix, we will treat two parallel model plates, conducting only in the direction of the helix with a pitch angle ψ.

On the boundaries, the following conditions are imposed:

(1) on the shields at $y = \pm b$,

i) the electric field components tangent to the shield must be zero,

$$E_x = E_z = 0 \quad ;$$

(2) on the helix at $y = -a$,

ii) the electric field component tangent to the helix must be zero,

$$E_{\parallel} = E_x \cos\psi + E_z \sin\psi = 0;$$

iii) the electric field component perpendicular to the helix must be continuous,

$$E_{\perp} = E_x \sin\psi - E_z \cos\psi \ : \ \text{continuous};$$

iv) the magnetic field component tangent to the helix must be continuous,

$$H_{\parallel} = H_x \cos\psi + H_z \sin\psi \ : \ \text{continuous};$$

(3) on the helix at $y = a$,

v) $E_{\parallel} = -E_x \cos\psi + E_z \sin\psi = 0$;

vi) $E_{\perp} = E_x \sin\psi + E_z \cos\psi$: continuous;

vii) $H_{\parallel} = -H_x \cos\psi + H_z \sin\psi$: continuous.

These condition will be used to determine the coefficients of the electromagnetic filed components, and to derive the dispersion relation.

3. Electromagnetic Field Components and Dispersion Relation

The electromagnetic field propagating the structure is assumed to be independent of x and to be written in the form of

$$\vec{\mathcal{F}} = \vec{F}(y)\, e^{i(\omega t - kz)} , \tag{3.1}$$

where ω is angular frequency, and k wave number. Let β_W be relative phase velocity; then,

$$k = \frac{\omega}{c\beta_w} . \tag{3.2}$$

As a slow wave, $\beta_W < 1$, is treated here, we will use an expression of

$$\beta_W = \sin\phi \tag{3.3}$$

for convenience. Consequently

$$k = k_o \operatorname{cosec}\phi, \tag{3.4}$$

where

$$k_o = \frac{\omega}{c} . \tag{3.5}$$

The electomagnetic field is the superposition of the TM and the TE mode. To obtain the field components, we first solve a Helmholtz equation for the z−component in each mode:

$$\frac{d^2 F_z(y)}{dy^2} - \gamma^2 F_z(y) = 0 \ , \tag{3.6}$$

where F_z is E_z in the TM mode, or H_z in the TE mode, and

$$\begin{aligned} \gamma^2 &= k^2 - k_o^2 \\ &= k_o^2 \cot^2\phi \ . \end{aligned} \tag{3.7}$$

As γ^2 is positive in our case, the solution of Eq.(3.6) is expressed with $e^{\gamma y}$ and $e^{-\gamma y}$ functions. The remaining x− and y−components are automatically derived from the obtained z−components using Maxwell's equations.

The structure is divided into three regions:

Region I, $-b \le y \le -a$,

Region II, $-a \le y \le a$,

Region III, $a \le y \le b$.

The field components in the three regions are expressed as follows by using unknown coefficients A through F, the vacuum impedance $Z_o = 377\ \Omega$, and its inverse Y_o:

(1) Region I $(-b \le y \le -a)$

$$\begin{aligned} E_x &= A\, j\, Z_o \tan\phi \sinh\gamma(y+b) \ , & H_x &= -B\, j\, Y_o \tan\phi \cosh\gamma(y+b) \ , \\ E_y &= B\, j \sec\phi \cosh\gamma(y+b) \ , & H_y &= A\, j \sec\phi \sinh\gamma(y+b) \ , \\ E_z &= B \sinh\gamma(y+b) \ , & H_z &= A \cosh\gamma(y+b) \ ; \end{aligned} \tag{3.8}$$

(2) Region II $(-a \le y \le a)$

$$\begin{aligned} E_x &= C\, j\, Z_o \tan\phi \sinh\gamma y \ , & H_x &= -D\, j\, Y_o \tan\phi \sinh\gamma y \ , \\ E_y &= D\, j \sec\phi \sinh\gamma y \ , & H_y &= C\, j \sec\phi \sinh\gamma y \ , \\ E_z &= D \cosh\gamma y \ , & H_z &= C \cosh\gamma y \ ; \end{aligned} \tag{3.9}$$

(3) Region III $(a \le y \le b)$

$$\begin{aligned} E_x &= E\, j\, Z_o \tan\phi \sinh\gamma(y-b) \ , & H_x &= -F\, j\, Y_o \tan\phi \cosh\gamma(y-b) \ , \\ E_y &= F\, j \sec\phi \cosh\gamma(y-b) \ , & H_y &= E\, j \sec\phi \sinh\gamma(y-b) \ , \\ E_z &= F \sinh\gamma(y-b) \ , & H_z &= E \cosh\gamma(y-b) \ . \end{aligned} \tag{3.10}$$

The above equations satisfies the boundary condition i) on the shield. From the boundary conditions ii) through iv) on the helix at y = −a, we have four homogeneous equations for the four unknown coefficients A through D. To obtain a non−trivial solution, the determinant of the matrix must vanish. A same discussion is repeated also on the helix at y = a. The vanishing determinant leads to the dispersion relation.

$$\frac{\tan\phi}{\tan\psi} = \left(\frac{\tanh\gamma\ell}{\tanh\gamma a}\right)^{1/2} . \tag{3.11}$$

This transcendental equation gives the relation between phase velocity and frequency. Multiplying the both sides by γa, we have

$$k_0 a \cot\psi = \gamma a\left(\frac{\tanh\gamma\ell}{\tanh\gamma a}\right)^{1/2} ; \tag{3.12}$$

the l.h.s. is proportional to frequency, as is obvious from Eq.(3.5). The relation between $\tan\phi/\tan\psi$ (phase velocity) and $k_0 a \cot\psi$ (frequency) is graphically shown in Fig.2 for various values of b/a. At the limit of $f \to \infty$, $\phi = \psi$, and consequently

$$\beta_W = \sin\psi \qquad (f = \infty) . \tag{3.13}$$

At the limit of $f \to 0$,

$$\frac{\tan\phi}{\tan\psi} = \sqrt{\frac{\ell}{a}} \qquad (f = 0) , \tag{3.14}$$

and

$$\beta_w = \left(1 + \frac{a}{\ell}\cot^2\psi\right)^{-1/2} \qquad (f = 0) . \tag{3.15}$$

Therefore, for a large value of b/a, phase velocity varies much with frequency at a low frequency range.

The relations between the unknown coefficients are derived from the boundary conditions. As a result, using $C = H_z(0)$ and $D = E_z(0)$ in Eqs.(3.9), and

$$H_z(0) = -jY_0\sqrt{\coth\gamma a \coth\gamma\ell}\; E_z(0) , \tag{3.16}$$

we have

(1) Region I $(-b \leq y \leq -a)$

$$E_x = -jH_z(0)Z_0 \tan\phi \sinh\gamma a \frac{\sinh\gamma(y+\ell)}{\sinh\gamma(-a+\ell)} ,$$

$$E_y = j E_z(0) \sec\phi \cosh\gamma a \frac{\cosh\gamma(y+\ell)}{\sinh\gamma(-a+\ell)} ,$$

$$E_z = E_z(0) \cosh\gamma a \frac{\sinh\gamma(y+\ell)}{\sinh\gamma(-a+\ell)} , \tag{3.17}$$

$$H_x = -j E_z(0) Y_0 \tan\phi \cosh\gamma a \frac{\cosh\gamma(y+l)}{\sinh\gamma(-a+l)},$$

$$H_y = -j H_z(0) \sec\phi \sinh\gamma a \frac{\sinh\gamma(y+l)}{\sinh\gamma(-a+l)},$$

$$H_z = -H_z(0) \sinh\gamma a \frac{\cosh\gamma(y+l)}{\sinh\gamma(-a+l)};$$

(2) Region II $(-a \leq y \leq a)$

$$\begin{aligned}
E_x &= j H_z(0) Z_0 \tan\phi \sinh\gamma y, \\
E_y &= j E_z(0) \sec\phi \sinh\gamma y, \\
E_z &= E_z(0) \cosh\gamma y, \\
H_x &= -j E_z(0) Y_0 \tan\phi \sinh\gamma y, \\
H_y &= j H_z(0) \sec\phi \sinh\gamma y, \\
H_z &= H_z(0) \cosh\gamma y;
\end{aligned} \tag{3.18}$$

(3) Region III $(a \leq y \leq b)$

$$\begin{aligned}
E_x &= j H_z(0) Z_0 \tan\phi \sinh\gamma a \frac{\sinh\gamma(y-l)}{\sinh\gamma(a-l)}, \\
E_y &= j E_z(0) \sec\phi \cosh\gamma a \frac{\cosh\gamma(y-l)}{\sinh\gamma(a-l)}; \\
E_z &= E_z(0) \cosh\gamma a \frac{\sinh\gamma(y-l)}{\sinh\gamma(a-l)}, \\
H_x &= -j E_z(0) Y_0 \tan\phi \cosh\gamma a \frac{\cosh\gamma(y-l)}{\sinh\gamma(a-l)}, \\
H_y &= j H_z(0) \sec\phi \sinh\gamma a \frac{\sinh\gamma(y-l)}{\sinh\gamma(a-l)}, \\
H_z &= H_z(0) \sinh\gamma a \frac{\cosh\gamma(y-l)}{\sinh\gamma(a-l)}.
\end{aligned} \tag{3.19}$$

4. Coupling Impedance and Characteristic Impedance

The coupling impedance of a travelling wave coupler, with a length of L and a characteristic impedance of R_O, is given by

$$Z_p = \sqrt{\frac{R_0}{8P_T}}\, E_z(0)\, L\, |F(\xi)| \,, \tag{4.1}$$

$$F(\xi) = \frac{\sin\xi}{\xi} \,, \tag{4.2}$$

$$\xi = \frac{1}{2} k_0 L \left(\frac{1}{\beta_b} - \frac{1}{\beta_w} \right) , \tag{4.3}$$

where P_T is the power flow through the coupler, β_b relative beam velocity, and $E_z(0)$ the E_z at the beam position of y = 0 [4]. When the characteristic impedance is different from 50 Ω, a transformer will be inserted between the coupler and a 50 Ω amplifier. In this case, the ratio of the voltage at the 50 Ω system to that at the coupler's output is $\sqrt{(50/R_0)}$. Therefore, a coupling impedance normalized at the 50 Ω system,

$$Z_{pm} = \sqrt{\frac{6.25}{P_T}}\, E_z(0)\, L\, |F(\xi)| \,, \tag{4.4}$$

is more convenient, especially for a comparison between couplers with differenct characteristic impedances.

Our task is to calculate the phase velocity, obtainable from the dispersion relation of Eq.(3.11), and the power flow given by a Poynting's vector. For a coupler of a width of W,

$$P_T = \frac{W}{2} Re \int (E_x H_y^* - E_y H_x^*)\, dy \,. \tag{4.5}$$

In the Regions I and III,

$$P_{T1} = P_{T3} = \frac{|E_z(0)|^2 Y_0}{4} \frac{W}{k_0} \frac{\tan^2\phi}{\cos\phi} \frac{\cosh\gamma a}{\sinh\gamma l} \frac{1}{\sinh\gamma(l-a)} \times \left\{ \sinh\gamma(l+a)\cosh\gamma(l-a) + \gamma(l-a) \right\} \,. \tag{4.6}$$

In the Region II,

$$P_{T2} = \frac{|E_z(0)|^2 Y_0}{2} \frac{W}{k_0} \frac{\tan^2\phi}{\cos\phi} (1 + \coth\gamma a \coth\gamma l)\left(\frac{1}{2}\sinh 2\gamma a - \gamma a\right) . \tag{4.7}$$

Their sum is

$$P_T = |E_z(0)|^2 Y_0 \frac{W}{k_0} \frac{\tan^2\phi}{\cos\phi} \frac{1}{\tanh\gamma l - \tanh\gamma a} \left\{ 1 + \frac{\gamma l}{\sinh 2\gamma l} - \frac{\gamma a}{\sinh 2\gamma a} \right\} . \tag{4.8}$$

This is the power flow to be put into Eq.(4.4). At the limit of $f \to 0$,

$$\lim_{f\to 0} P_{T1,3} = \infty \,, \tag{4.9}$$

$$\lim_{f\to 0} P_{T2} = \frac{|E_z(0)|^2 Y_0}{3} W a \tan\psi \sqrt{\frac{a}{l} + \tan^2\psi} \,. \tag{4.10}$$

At the limit of $f \to \infty$,

$$\lim_{f\to 0} P_{Ti} = \infty \qquad (i = 1, 2, 3) \,. \tag{4.11}$$

Therefore coupling impedance vanishes at f = 0 and ∞.

The characteristic impedance is calculated with

$$Z_c = \frac{|V|^2}{2P_T} \quad , \tag{4.12}$$

where V is the voltage difference between the helix and the shield:

$$\begin{aligned} V &= -\int_a^b E_y \, dy \\ &= j \frac{E_z(0)}{k_o} \frac{\sin\phi}{\cos^2\phi} \cosh \gamma a \quad . \end{aligned} \tag{4.13}$$

The resulting characteristic impedance is

$$\begin{aligned} Z_c = &\frac{Z_o}{2W} \frac{1}{k_o \cos\phi} \cosh^2 \gamma a \,(\tanh \gamma b - \tanh \gamma a) \\ &\times \left\{ 1 + \frac{\gamma b}{\sinh 2\gamma b} - \frac{\gamma a}{\sinh 2\gamma a} \right\}^{-1} \quad . \end{aligned} \tag{4.14}$$

At the limit of f → 0,

$$Z_c = \frac{Z_o}{2} \frac{b-a}{W} \frac{1}{\beta_w(0)} \quad . \tag{4.15}$$

5. Design Example

A design example is presented here for a flat−helix pickup to be installed in LEAR. The aimed beam momentum is 60 MeV/c (β_b = 0.06382). The pickup is to be set in the straight section for beam injection and ejection; therefore, the pickup is required to have a wide aperture, 320 mm or wider. At the optimization of the parameters, the height of the shield (b) is varied, and the following parameters are fixed: the width of the helix (W_h) at 320 mm, the pitch angle (ψ) at $\sin^{-1}0.06$, and the length (L) at 1100 mm. The width of the shield (W_s) is set so that the distance between the shield and the helix is b − a:

$$W_s = W_h + 2(b - a) \quad . \tag{5.1}$$

At the calculation of the Poynting's vector, the width (W) is set at the mean value of W_h and W_s:

$$W = (W_h + W_s) / 2 \quad . \tag{5.2}$$

The result for five values of the shield height (b) is summarized in Table I, where the coupling impedances Z_{pn} of the 1100 mm long pickup for a beam velocity $\beta_b = 0.06$ are listed. The parameter set No. 2 is best, because coupling impedance is high, and characteristics impedance is close to 50 Ω. In Fig. 3(a), coupling impedances are drawn for some values of beam velocity. When beam velocity goes apart 0.06, coupling impedance decreases at high frequencies, because the factor ξ in Eq.(4.3) takes a large value. This effect is prominent at a long coupler designed for a very low velocity beam ($\beta_b \ll 1$).

To gain a capability for a wider range of beam velocity, it is one way to divide the coupler into pieces. When the beam velocity is close to the phase velocity, the divided couplers are connected in series, as shown in Fig.4(a): a signal from a downstream coupler is injected in the upstream coupler. When the beam velocity is quite different from the phase velocity, the couplers are connected in parallel, as shown in Fig.4(b): the signals from the couplers are summed in a power combiner. If we assume that the lengths of the divided couplers are same, and that the signal power is conserved at the summation, the summed coupling impedance from n couplers of a length of L/n is given by

$$Z_{pn}(n) = \frac{2.5}{\sqrt{P_T}} E_z(0) \frac{L}{\sqrt{n}} |F(\xi)| \ . \qquad (5.3)$$

$$\xi_n = \frac{1}{2} k_0 \frac{L}{n} \left(\frac{1}{\beta_\ell} - \frac{1}{\beta_w} \right) . \qquad (5.4)$$

For n = 1, these equations are identical to Eqs.(4.4) and (4.3). Figure 3(b) shows the result for the parameter set No. 2, where the couplers are divided into two pieces, 550 mm long each, and connected in parallel. In this case, the coupling impedance for a beam of $\beta_b = 0.06$ decreases, but those for the other velocities are improved.

Three couplers, however, constitutes the pickup to be installed in LEAR. The overall length is 957 mm = 319 mm × 3. Every coupler has a seven−turn helix; the pitch is 45.6 mm.

$$p = 2(2a + W_h)\tan\psi \ . \qquad (5.5)$$

This division is determined from a mechanical point of view. If the couplers is divided into two pieces, it is difficult to attach feedthroughs to the vacuum chamber, for which the existing chamber is to be used with a slight modification.

The coupling impedances of this three−pieced coupler are shown in Fig.5(a) for the series connection and Fig.5(b) for the parallel one. At the latter connection (Fig.5(b)), the coupling impedance for beam velocity β_b = 0.06 is lower than that of the series−connected cougpler (Fig.5(a)) and that of the parallel−connected coupler of two pieces (Fig.3(b)). But at higher beam velocities ($\beta_b \geq 0.1$), the coupling impedance is improved with the three−pieced coupler. Though the coupling impedances for these beam velocities are lower than that for the synchronous beam, the coupler can be still used, because the ratio U = (amplifier noise power)/(Schottky signal power), which is an important parameter for the cooling time, keeps a value as low as that for the synchronous beam, as the Schottky signal power is proportional to the square of the beam's revolution frequency and consequently to β_b^2.

From the above discussion, a coupler divided into pieces has a capability for a wider range of beam velocity, though the experimental setup becomes more complicated.

References

[1] N. Tokuda et al., "Stochastic Momentum Cooling of a Low Energy Beam at TARN", The 1985 Particle Accelerator Conference, Vancouver, B.C., Canada, 1985: IEEE Trans. on Nucl. Sci., Vol. NS−32, No.5, 1985.

[2] N. Tokuda, "A Helix Coupler for a Pickup of a Low Velocity Beam", CERN report, PS/LEA/Note. 86−5, 1986.

[3] C.C. Johnson, "Impedance and Dispersion Characteristics of the Flattened Helix", IRE Trans. on Electron Devices, Vol. ED−6, p.189, 1959.

[4] D. Boussard and G. Di Massa, "High Frequency Slow Wave Pick−ups", CERN Report, CERN SPS/86−4 (ARF), 1986.

Table I. Summary of the parameter optimization for a flat−helix pickup for a beam velocity β_b = 0.06 (Z_{pn} : coupling impedance normalized at the 50 Ω system, Z_c : characteristic impedance, β_w : phase velocity). Shield height b, and consequently shield width W_s (Eq.(5.1)), are varied; the other parameters are kept constant: shield height a = 30 mm, helix width W_h = 320 mm, length L = 1100 mm, and pitch angle $\sin\psi$ = 0.06.

	Set No.	(1)	(2)	(3)	(4)	(5)
	b (mm)	35.0	37.5	40.0	42.5	45.0
	W_s (mm)	330	335	340	345	350
Z_{pn} (Ω)	(0 MHz)	0	0	0	0	0
	(25 MHz)	200	228	245	254	259
	(50 MHz)	355	391	401	394	376
	(75 MHz)	454	488	486	460	421
	(100 MHz)	503	544	547	527	494
Z_c (Ω)	(0 MHz)	44.7	64.3	82.4	99.2	114.9
	(25 MHz)	44.7	64.3	82.5	99.3	115.0
	(50 MHz)	44.6	64.1	82.1	98.8	114.0
	(75 MHz)	44.0	62.9	80.7	96.1	110.8
	(100 MHz)	42.9	60.5	76.2	90.2	102.6
β_w	(0 MHz)	.0648	.0671	.0692	.0714	.0734
	(25 MHz)	.0646	.0667	.0688	.0708	.0727
	(50 MHz)	.0640	.0658	.0676	.0693	.0709
	(75 MHz)	.0632	.0646	.0659	.0672	.0683
	(100 MHz)	.0624	.0634	.0643	.0651	.0658

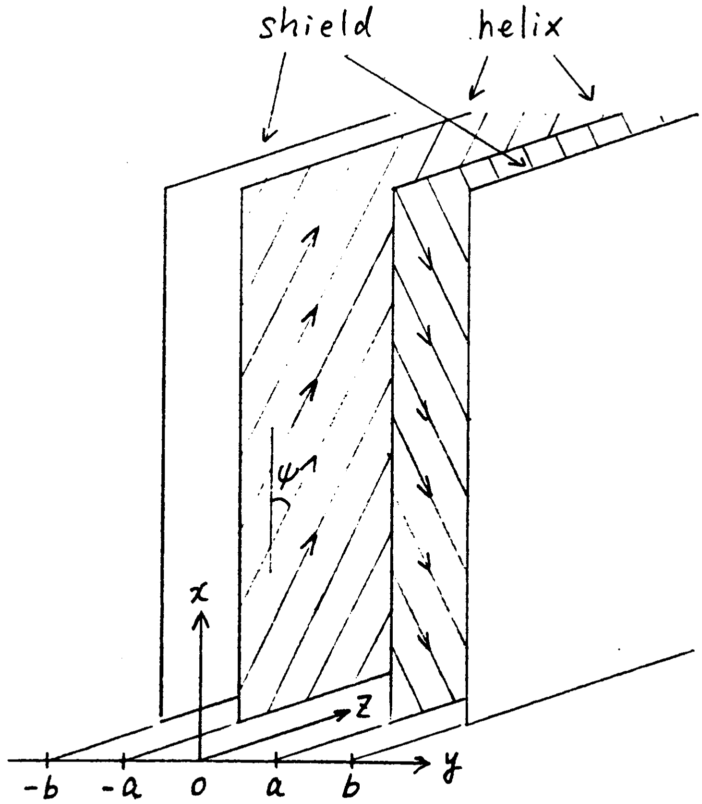

Fig. 1. Model of the flat−helix. The inner plates, located at y = ±a, are conducting only in the direction of the helix, with a pitch angle ψ. The outer plates are the shields, located at y = ±b, of conducting metals. The plates are infinitely wide.

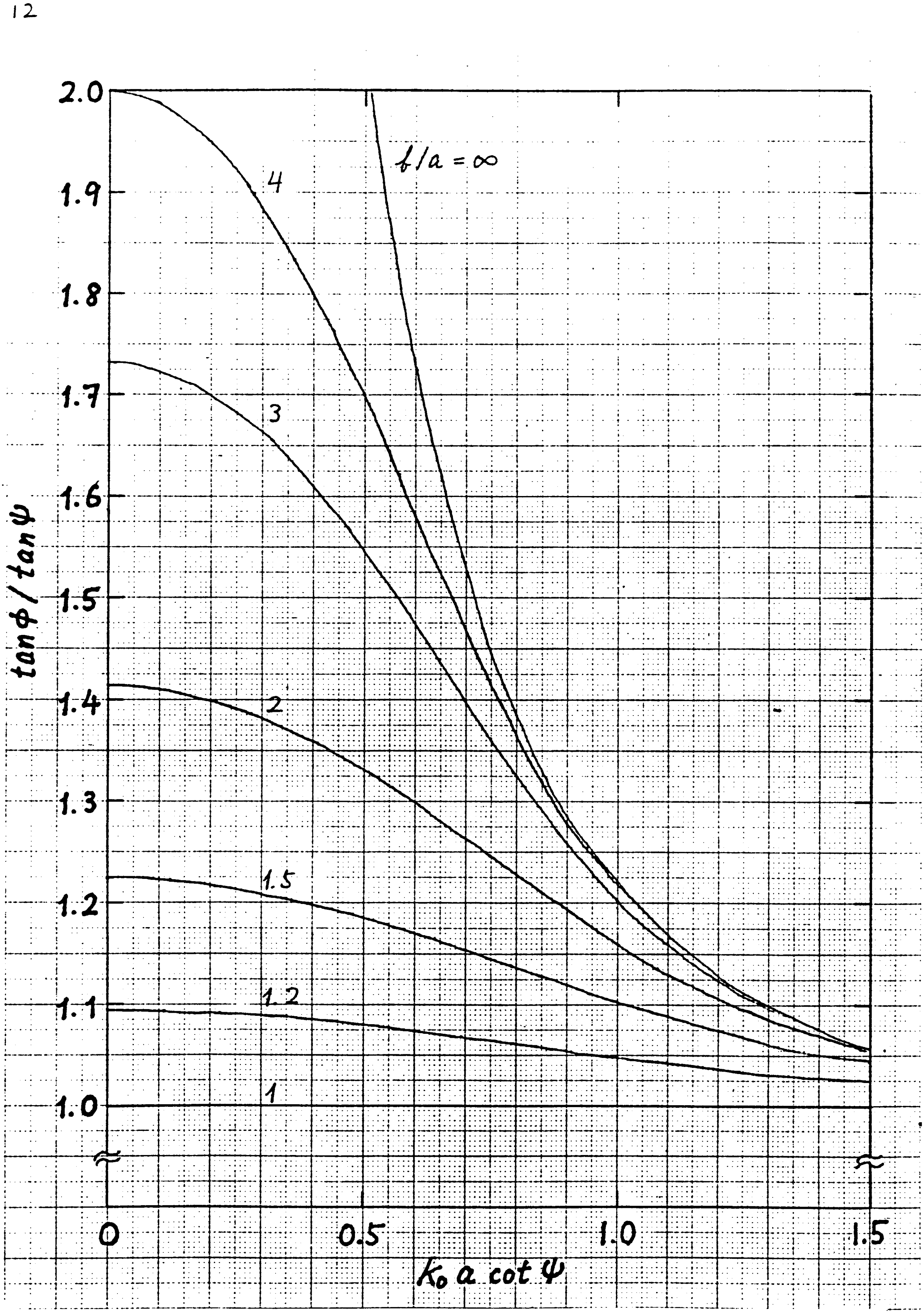

Fig. 2. Dispersion relation for various values of b/a, (shield height)/(helix height), in a flat−helix pickup with a pitch angle ψ. The ordinate means phase velocity ($\beta_W = \sin\phi$), and the abscissa frequency ($k_0 = 2\pi f/c$).

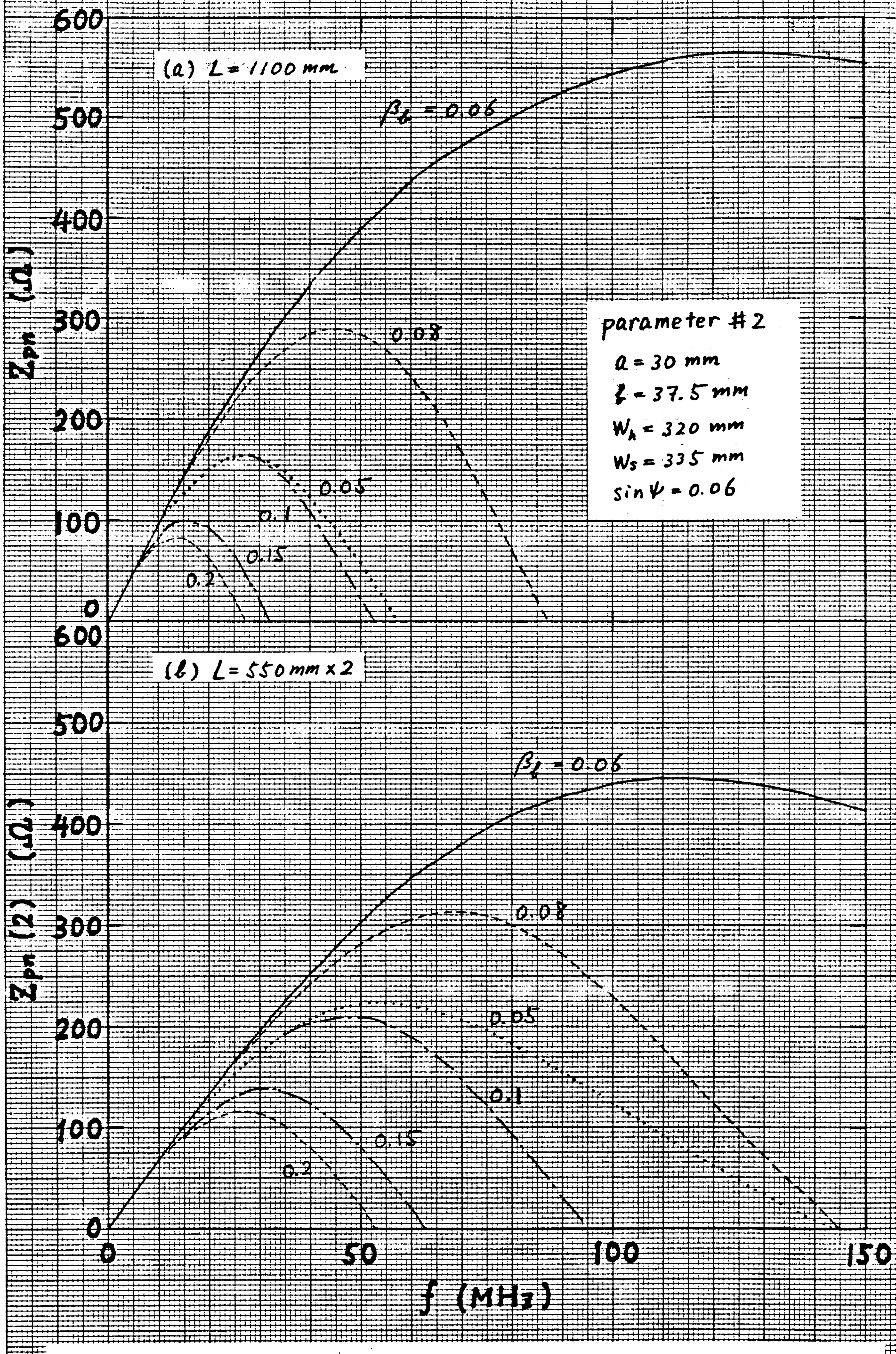

Fig. 3. Coupling impedance for the parameter set No. 2 for various beam velocities. The coupler with an overall length of 1100 mm is divided into two pieces, 550 mm long each: (a) for the series connection (see Fig.4(a)), and (b) for the parallel connection (see Fig.4(b)).

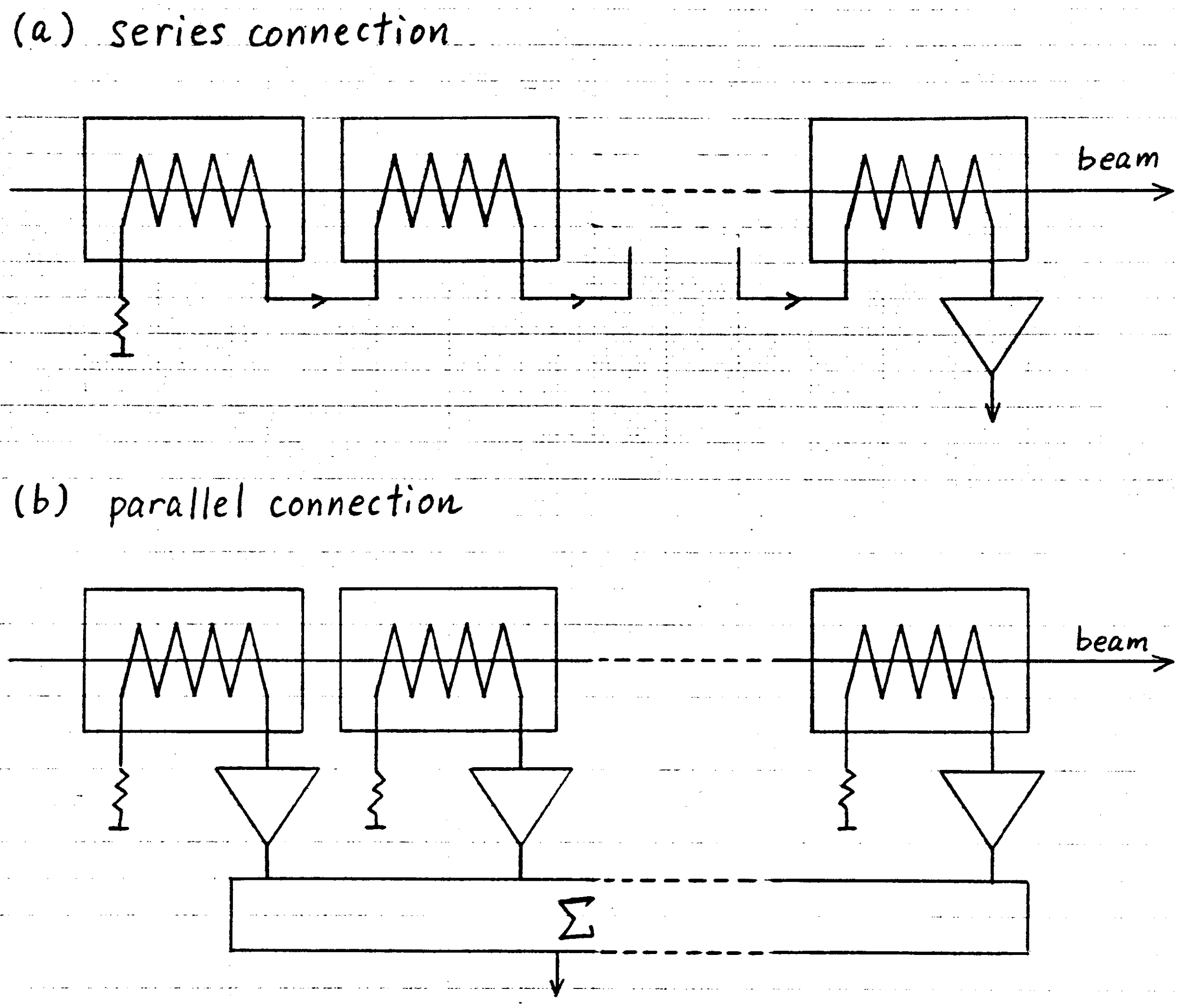

Fig. 4. Connection of the divided couplers: (a) series connection, and (b) parallel connection.

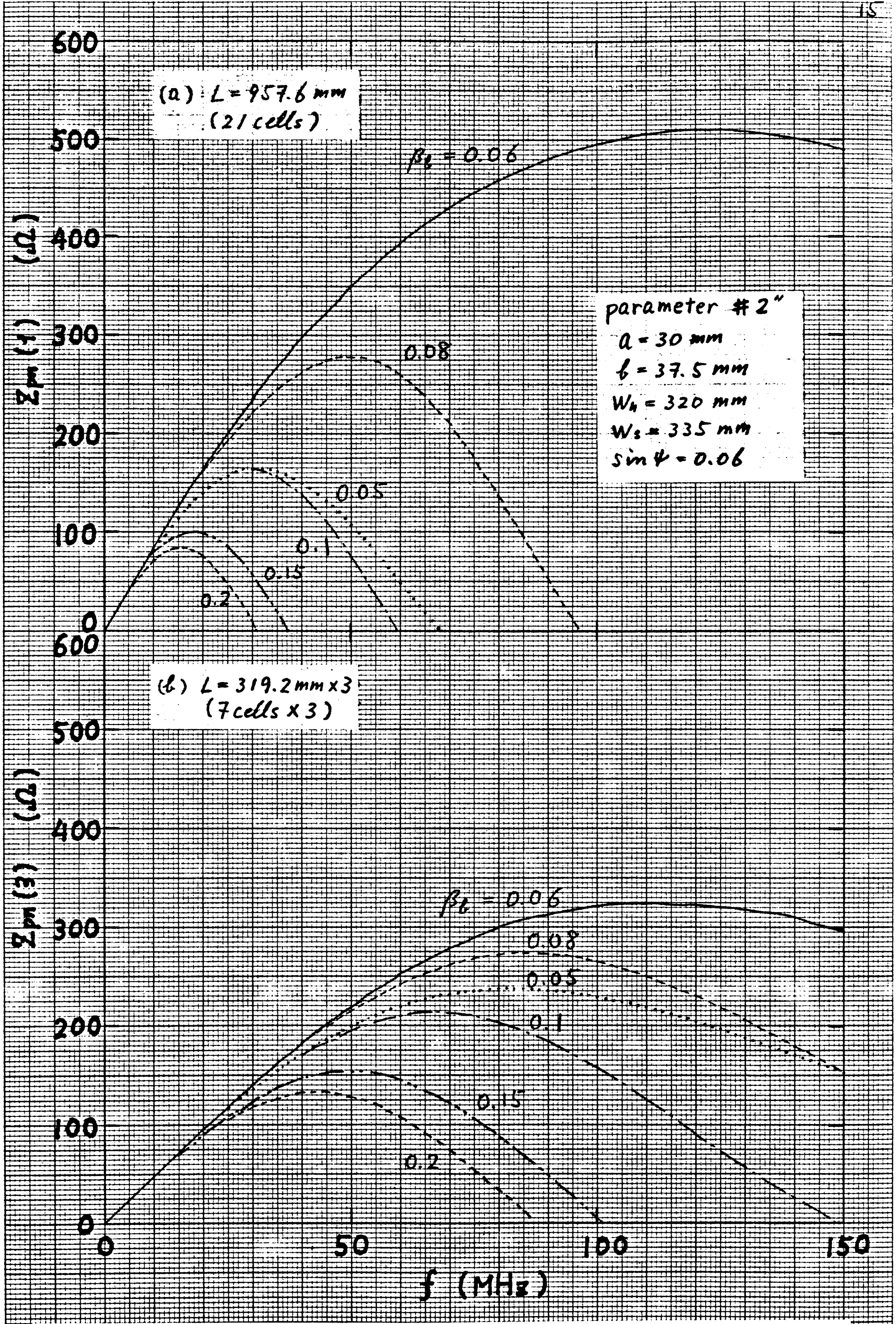

Fig. 5. Coupling impedance of the coupler to be installed in LEAR. The overall length is 957.6 mm, and the other parameters are same as in the parameter set No. 2: (a) for the series connection, and (b) for the parallel connection.

DISTRIBUTION

Groupe LEAR

D. ALLEN
E. ASSEO
S. BAIRD
J. BENGTSSON
M. CHANEL
J. CHEVALLIER
R. GALIANA
R. GIANNINI
P. LEFEVRE
F. LENARDON
R. LEY
D. MANGLUNKI
E. MARTENSSON
J.L. MARY
C. MAZELINE
D. MOEHL
G. MOLINARI
J.C. PERRIER
T. PETTERSSON
P. SMITH
N. TOKUDA
G. TRANQUILLE
H. VESTERGAARD