CERN/MPS-MU/EP 71-1
EJ/41 RG/ld - 12. 1.1971

FAST EXTRACTION FROM THE 300 GeV SYNCHROTRON

BY "BEAM SHAVING"

II. Technical Notes

R. Gouiran

I. INTRODUCTION

The main idea of this type of ejection has been already described, in a qualitative way, in a previous note (ref. 1). We present now some detailed computations in order to justify the conclusion, for those who could be interested.

I. LATTICE MR 47 (Ref. 2)

The region where the ejection will take place is as in Fig. 1. (The main magnets have been omitted, as well as the bumpers).

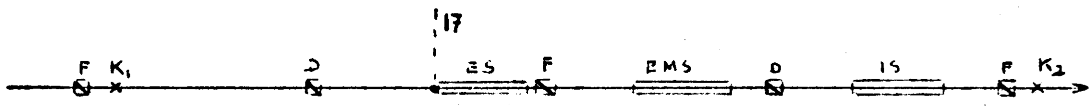


Fig. 1

F and D are the machine quadrupoles, K_1 , K_2 are the ferrite dipoles, E.S. is the electrostatic septum, 7 m long, EMS is the electromagnetic septum and IS is the iron extraction septum. The entrance of E.S. is at the so-called position 17 , 7.20 m in front of an F quadrupole. The centre of K_1 is at 3.80 m after the end of an F quadrupole.

A period FODO has a phase angle of 92.5^0 in both planes. There are 18 such periods in one superperiod, whose phase angle is consequently $5\pi/4 + 4x2\pi$. There are 6 superperiods in one turn , whose phase angle is consequently $3\pi/2 + 27x2\pi$. So the normal machine has $Q_H = Q_V = 27.75$. At position 17, $\beta_{17} = 71.64$, $\alpha_{17} = -1.955$ in H plane.

For normalization, we use the matrix

$$N = \beta^{-\frac{1}{2}} \begin{pmatrix} 1 & 0 \\ \alpha & \beta \end{pmatrix} .$$

III. THE FAST KICKS

K_1 and K_2 are located half a wave length apart, or in any other configuration in order to eliminate all orbit deformation in the rest of the machine. The usual orbit deformation by means of classical bumpers brings the orbit close to the septa (ref. 2) and K_1 gives the last kick for the extraction. K_1 and K_2 have the same current but are out of phase by a time corresponding to a particle transit from K_1 to K_2.

Characteristics of K_1 and K_2 (Ref. 3) :

Material	:	Ferrite Ferrite length : 2.70 m Total length : 3 m
Magnet aperture	:	100 x 100 mm
Maximum bending power	:	0. 35 T.m
Peak current	:	3000 A
Peak voltage	:	16 kV
Rise time	:	11.5 μs
Maximum flat top	:	23 μs

For a complete extraction the current will be as designed in Fig. 2 :

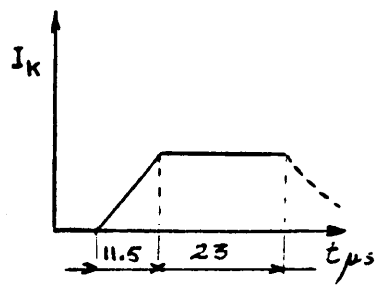


Fig. 2

For a partial extraction the flat-top time, as well as the fall time are controlled.

The extracted beam current, for a complete extraction, will be as described in Fig. 3

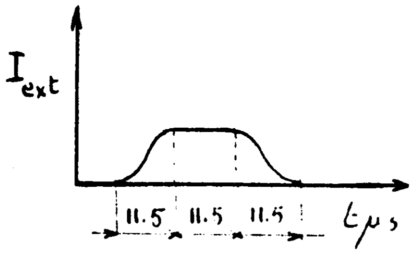


Fig. 3

with a fine RF structure at 183 MHz.

The matrix elements between K1 and 17 are : $m_{12} = 77.33$, $m_{22} = 2.16$. An angular kick k at K1 gives a phase space motion at 17 which is $\binom{m_{12}}{m_{22}} k$. Let us call "kick vector" the direction in the phase space motion, produced by k at position 17 ; its slope is m_{22}/m_{12} . In the normalized phase space, this direction is inclined by an angle ζ_k such that $tg\zeta_k = \alpha_{17} + \beta_{17} \frac{m_{22}}{m_{12}} = +0.1$. So $\zeta_k = 3.5°$ which is nearly zero.

(we note that, by pushing K1 3 m downstream, ζ_k becomes 11° at position 17)*.

IV. NORMAL BEAM CHARACTERISTICS

a) at 400 GeV

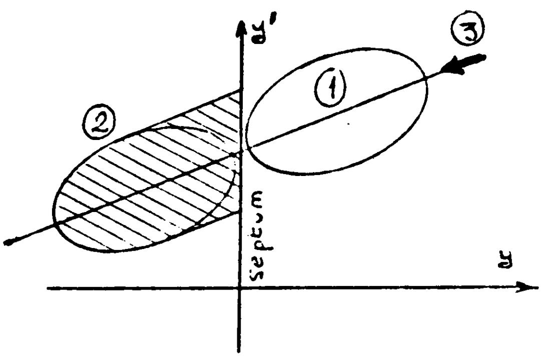
In ref. 2 the beam emittance is $E_H = 0.3 \cdot 10^{-6}$ m.rad with $\Delta p/p = \pm 3.10^{-4}$. The beam width is $2\sqrt{\beta_{17}E} = 9.3$ mm at position 17.

For the normal beam, that is to say in a machine without perturbation, the emittance ellipse is always a circle in the normalized phase space. As ζ_k is nearly zero, one can say that the kick verctor is parallel to the ellipse axis; this means that, during the extraction, the emittance ellipse slides along its own axis. This is exactly what we need for minimizing the extracted emittance as shown in Fig. 4 a) and b).

1) is the emittance before the extraction,
2) is the emittance after the extraction,
3) is the kick vector direction.

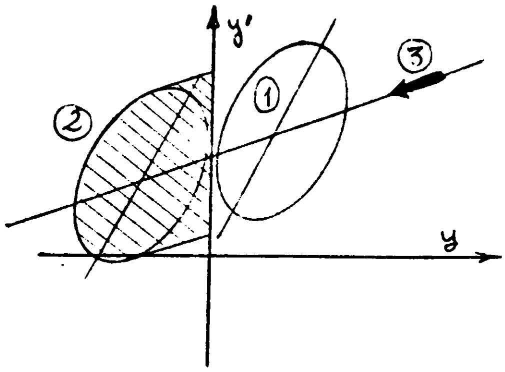
---

*) The tilt angles are counted anticlockwise, though the betatronic phases are counted clockwise, for historical reasons.



Good extraction

Fig. 4 a)



Bad extraction

Fig. 4 b)

This is a typical feature of any beam shaving extraction : the emittance ellipse should slide alongs its axis; if not the extracted emittance takes an unacceptable triangular shape, as shown in Fig. 4 b).

At position 17 (E.S. entrance), the normal α_p function is a vector $\binom{e}{e'} = \pm \binom{0.32}{0.9\times10^{-2}}$ for $\Delta p/p = \pm\, 3\cdot10^{-4}$ in mm.mrad). So the total beam width is 9.3 + 0.64 = 10. mm and the total emittance "thickness" (its angular width) is 0.13 + 0.018 = 0.148 mrad.

b) <u>at 200 GeV</u>

$E_H = 0.6\cdot10^{-6}$. $\frac{\Delta p}{p} = \pm\, 5 \cdot 10^{-4}$. So, including the momentum dispersion, the total beam width is 13.7 mm, and the total emittance "thickness" is 0.213 mrad.

V. LOSSES WITH A NORMAL BEAM

When the ellipse emittance sweeps over the septum edge, the relative number of particles hitting this edge is $H = \frac{t}{T}\frac{d}{w}$, where t is the time of the sweep, T the time of a turn, d the apparent septum thickness, w the beam width. For a complete extraction $\frac{t}{T} = \frac{11.5}{23} = \frac{1}{2}$ and $\frac{d}{w} = \frac{0.15}{10}$ at 400 GeV).

So at 400 GeV, H = 0.75 o/o and, at 200 GeV , H = 0.55 o/o. Though H does not correspond to a loss (some particles are scattered by the septum and recovered, particularly if a wire array is used), we shall consider it as a pessimistic limit of a loss. One has also to consider the loss produced by particles grazing the inner part of the septum, as shown below.

Let us consider a straight septum giving a deflection α per unit length. If we considered it as parallel to the beam axis, a particle entering the septum with phase space coordinates y_0 , y_0' (as in Fig. 5) will graze the septum after a distance $L_1 = y_0'/\alpha$. We shall see that, in our case, $L_1 \approx 3.6$ m. One can also see on Fig. 5 that, after a distance L_2 , the septum can be incurved and, in our case, $L_2 \approx 5.20$ m (for a 7 m long septum, giving a total deflection of about 0.18 mrad). The minimization of this extra loss can be done according to the following procedure.

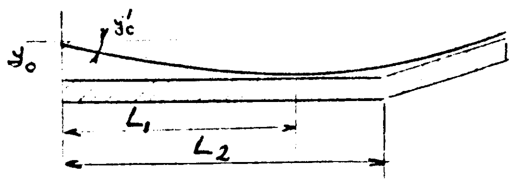


Fig. 5

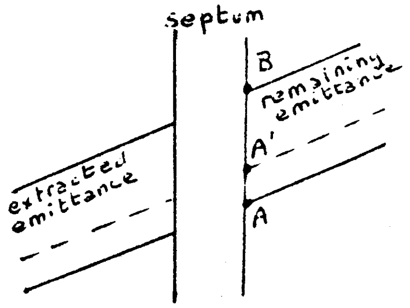


Fig. 6 a)

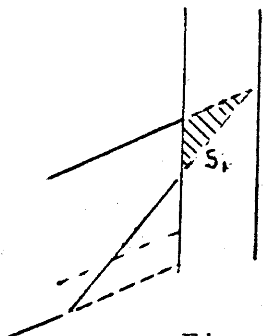


Fig. 6 b)

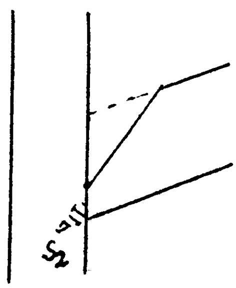


Fig. 6 c)

If the septum is aligned in such a way that the particle in A grazes the septum (fig. 6 a), not any particle of the emittance remaining in the machine will be lost further on in the septum. If now we aligne it on the particle A′ , and if we look the phase space situation after the distance L_1 (Fig. 6 b), the dashed area of the extracted emittance (S1) is lost. If we now look the phase space situation after the distance L_2 (Fig. 6 c), the dashed area of the internal remaining emittance (S2) is lost. As the particle density is much lower on the emittance edges than in the centre, and as L_2 can be made smaller than the total septum length, one can look for the axis A′ where the total loss (S1 + S2) is minimized.[x)] Using a gaussian distribution for the particle density in phase space, one finds that this extra loss (S1 + S2) is a fraction 8 o/o of the main loss H . These arguments show also that the septum thickness is critical only on its first half and its alignment is also critical only on its first two third. So the total number of particles hitting the septum are L = H + S1 + S2; at 400 GeV L = 0.81 o/o, and at 200 GeV L = 0.59 o/o.

VI. THE JUMPS

a) Angular Kick from K_1

The beam displacement at E.S. should be equal to the beam total width plus the septum thickness = 10 + 0.15 = 10.15 mm. This should be equal to m_{12} k (k being the kick produced by k_1). As m_{12} = 77.33, one finds that k = 0.132 mrad, corresponding to a bending power of 0.176 T · m for k_1 , at 400 GeV.

b) Deflection from the electrostatic septum

This deflection should be equal to the total emittance "thickness" plus the electromagnetic copper septum thickness reported back at the electrostatic septum exit. This last value happens to be 0.09 mrad for a 2 mm copper septum (see Fig. 9). So the necessary jump is, at 400 GeV, 0.148 + 0.09 ≈ 0.24 mrad. As the beam widths, at the E.S. septum, are 10 mm at the entrance

---

x) This can be achieved by varying the angle of the septum relative to the beam.

and 11.9 mm at the exit, one can operate the septum with an aperture of 13 mm at the entrance and 15 mm at the exit. For a 7 m long septum at 400 GeV, these values will require a mean field of 110 kV/cm.

VII. IMPROVED BEAM

The losses and the field requirement can be minimized by increasing locally the β function and by reducing the momentum dispersion as well as the local α_p function.

a) The β function

We suppose that the unperturbed machine has a $Q_H = 27.75$ and we introduce a defocusing quadrupole of strength δ (= 1/f) exactly one superperiod before the E.S. septum entrance. The figure 7a shows the new emittance ellipse in the normalized phase space at the quadrupole. (We shall use for normalization the values of the unperturbed machine). One understands immediately that the emittance is now an ellipse inclined by -45^0 before the quadrupole (B) and by $+45^0$ after the quadrupole (A).

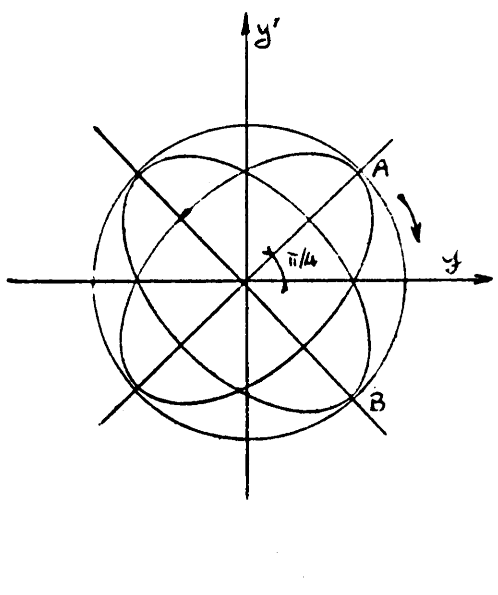


Fig. 7 a)

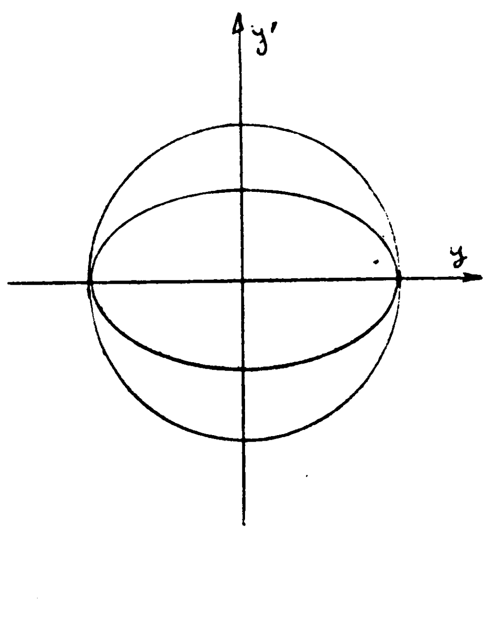


Fig. 7 b)

Then, at the septum entrance, one superperiod after, this ellipse has rotated by $\frac{5\pi}{4}$ as shown in Fig. 7b. This is exactly what we wanted : an elongated ellipse to minimize the loss and the jump, and an ellipse flat in the normalized phase space in order that its main axis remains parallel to the "kick vector" whose tilt angle in this phase space is nearly zero, as we have seen in paragraph III.

Now the new β function shows a beating with three maxima around the machine and it looks as on Fig. 8. This figure is just a schematic illustration and the number of oscillations are not exactly represented..

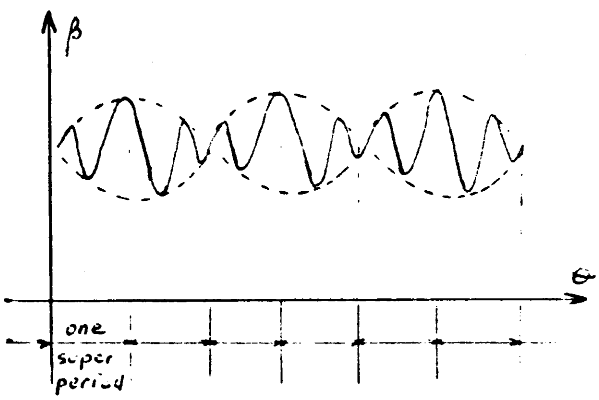


Fig. 8

The matrix through one machine turn, starting just after the quadrupole is $\begin{pmatrix}1 & 0\\ \delta & 1\end{pmatrix}\begin{pmatrix}-\alpha & -\beta\\ \gamma & \alpha\end{pmatrix}=\begin{pmatrix}-\alpha & -\beta\\ \gamma-\delta\alpha & \alpha-\beta\delta\end{pmatrix}$

where α, β, γ are the original parameters at position 17.

The phase per turn becomes $2k\pi+\mu'$ with $\cos\mu' = -\beta\delta/2$; the new β function becomes $\beta' = -\beta/\sin\mu'$ with $\alpha' = -\text{cotg}\,\mu' - \alpha/\sin\mu'$, $Q'_H = 27 + \mu'/2\pi$.

Let us put a star to these values counted in the phase space normalized by $N_{17} = \beta_{17}^{-\frac{1}{2}}\begin{pmatrix}1 & 0\\ \alpha_{17} & \beta_{17}\end{pmatrix}$. So $\beta^* = -1/\sin\mu'$, $\alpha^* = -\text{cotg}\,\mu'$, $\gamma^* = \beta^*$. After a rotation of $5\pi/4$ the ellipse emittance is flat on the y axis and its maximum corresponds to its main axis, that is to say

$$\beta^*_{max} = -\frac{1}{\sin \mu'} + \frac{1}{tg\ \mu'} \quad .$$

If we want to increase β by a factor 2.4 at the septum entrance, at 400 GeV, we set $\beta^*_{max} = 2.4$, and we find $\mu' = 5\pi/4$, with $\sin \mu' = \cos \mu' = -1/\sqrt{2}$. As $\cos \mu' = -\beta_{17}\ \delta/2$ one gets $\delta = \sqrt{2}/\beta_{17} = 0.0195$. This corresponds to 42 o/o of a main machine quadrupole. One finds $Q'_H = 27.624$ and $Q'_V = 27.798$. If, for 200 GeV, one wants to increase β by a factor 2.8 at the same point, we set $\beta^*_{max} = 2.8$, and one finds $\mu' = \pi + 39°$ and $\delta = 0.0217$ (46 o/o of a main machine quadrupole); then $Q'_H = 27.611$, $Q'_V = 27.802$.

b) <u>The perturbed closed orbit</u>

We call $\binom{e}{e'}$ the phase space vector of the perturbed closed orbit (so called "α_p"); we have

$$\begin{pmatrix} e \\ e' \end{pmatrix} = \frac{1}{2(1-\cos\mu')} \begin{pmatrix} 1-m_{22} & m_{12} \\ m_{21} & 1-m_{11} \end{pmatrix} \begin{pmatrix} m_{13} \\ m_{23} \end{pmatrix} \qquad (1)$$

where m_{ij} are the matrix elements for one machine turn, m_{13} and m_{23} being the dispersive terms.

In the non-perturbed machine, where $\cos \mu = 0$, one deduces that

$$\left| \begin{array}{l} m_{13} = (1+\alpha)\ e + \beta e' \\ m_{23} = (1-\alpha)\ e' - \gamma e \end{array} \right. \qquad (2)$$

Introducing the perturbative quadrupole, the equation (1) becomes, for one turn starting just <u>after</u> the quadrupole, :

$$\begin{pmatrix} \bar{e} \\ \bar{e}' \end{pmatrix} = \frac{1}{2(1-\cos\mu')} \begin{pmatrix} 1-\alpha+\beta\delta & -\beta \\ \gamma-\delta\alpha & 1+\alpha \end{pmatrix} \begin{pmatrix} m_{13} \\ \delta m_{13}+m_{23} \end{pmatrix} \qquad (3)$$

where α, β, γ are the non-perturbed machine parameters at 17 (the E.S. septum entrance), δ is the quadrupole strength and $\cos\mu' = -\beta\delta/2$. One obtains the new values :

$$\left|\begin{array}{l} \bar{e} = e/(1-\cos\mu') \\ \bar{e}' = e' - \dfrac{\cos\mu'}{1-\cos\mu'}\,\dfrac{(1+\alpha)}{\beta}\,e \end{array}\right. \qquad (4)$$

At the septum entrance, the function α_p becomes :

$$\begin{pmatrix} e_s \\ e'_s \end{pmatrix} = \begin{bmatrix} \dfrac{1}{\sqrt{2}}\begin{pmatrix} 1+\alpha & \beta \\ -\gamma & 1-\alpha \end{pmatrix} & \begin{matrix}\bar{m}_{13} \\ \bar{m}_{23}\end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 1 \end{bmatrix} \begin{bmatrix} \bar{e} \\ \bar{e}' \\ 1 \end{bmatrix} \qquad (5)$$

where $\bar{m}_{13}$ and $\bar{m}_{23}$ are the dispersive term for one superperiod. These terms are obtained from the equation (1) for one superperiod, that is to say :

$$\begin{pmatrix} e \\ e' \end{pmatrix} = \frac{1}{2(\sqrt{2}-1)} \begin{bmatrix} \sqrt{2}-1+\alpha & \beta \\ -\gamma & \sqrt{2}-1-\alpha \end{bmatrix} \begin{pmatrix} \bar{m}_{13} \\ \bar{m}_{23} \end{pmatrix} \qquad (6)$$

One finally finds the new perturbed orbit at the septum entrance :

$$\left|\begin{array}{l} e_{es} = e \\ e'_{es} = e' + e\,\dfrac{\sqrt{2}}{\beta}\,\dfrac{\cos\mu'}{1-\cos\mu'} \end{array}\right. \qquad (7)$$

So e is unchanged and e' could be decreased when $\cos\mu'$ is < 0 (in the case of a defocusing quadrupole); this is an advantage because it reduces the angular jump required at the E.S. septum.

As $e = 1.06 \cdot 10^{+3}\ \Delta p/p$, $e' = 30\ \Delta p/p$ (mm/mrad) in the non perturbed machine, one finds that, at 400 GeV, with $\cos\mu' = -0.71$, $e_{es} = 1.06 \cdot 10^{+3}\ \Delta p/p$, $e'_{es} = 22\ \Delta p/p$ (at 200 GeV, with $\cos\mu' = -0.76$, these values are practically unchanged).

VIII. THE LOSSES WITH THE IMPROVED BEAM

a) At 400 GeV

By increasing β by a factor 2.4, as we have seen, we obtain a beam width of 14.3 mm. Using an adiabatic RF debunching on a small flat top of 10 ms (zero phase angle, reduced voltage), one can expect a decrease of $\Delta p/p$ by a factor three; so the total beam width, with momentum dispersion, will be 14.6 mm. Consequently, the relative number of particles hitting or grazing the septum will be

$$\left(\frac{1}{2}\ \frac{0.15}{14.3}\right) \cdot 1.08 = \underline{0.57\ o/o.}$$

b) At 200 GeV

With the same conditions as above, but with a β increase of 2.8, one obtains a total beam width of 22.5 mm. The relative number of particles hitting or grazing the septum will be 0.36 o/o.

IX. THE JUMPS WITH THE IMPROVED BEAM

At 400 GeV the horizontal beam displacement at the septum entrance will be now 14.5 mm. The angular kick produced by k_1 will be 14.5/77.5 = 0.187 mrad, resulting in a bending power of 0.25 T·m.

At the electrostatic septum, the angular jump should be : copper septum thickness (0.09 mrad) + beam emittance (0.0835 mrad) + reduced momentum dispersion (0.005 mrad) = 0.18 mrad at 400 GeV. Using an aperture of 17 mm at the entrance and 19 mm at the exit, the corresponding field, for a 7 m long septum, happens to be 103 kV/cm, at 400 GeV

X. THE APERTURES

It is difficult to say something definite about the acceptance and the particle trajectories as long as no field map is available in the fringe field region of the machine quadrupole. We suppose that the quadrupole field remains ideal and we draw the acceptance polygone at the exit of the electrostatic septum, that is to say 0.20 m before the following F quadrupole of Fig. 1.

As a typical example the polygone is drawn for the improved beam with a β increase of 2.8 on Fig. 9. One sees three couples of lines : (1) electrostatic septum, (2) electromagnetic copper septum, (3) iron extraction septum. The line (4) shows an aperture limit coming from the following F quadrupole and sets at 65 mm from a quadrupole axis.

This figure shows that this extraction is very tight indeed. To make a good use of it, it could be preferable to open the electromagnetic septum at 22 mm (instead of 20 mm) and to ask for a fairly good quadrupole field in the following F quadrupole up to 70 mm off centre (instead of 62 mm). If the electric field of the septum could be pushed up to 120 kV, or if it could be lengthened further more, one could open it a little more. In order to guarantee a good parallelism between the kick vector and the beam emittance axis, the kicker K_1 should be pushed a little more upstream, if possible.

A small difficulty will arise at 400 GeV when we want to use this extraction in the middle of a resonant slow extraction. The E.S. septum aperture will be set already at 18 mm; so, with a field of 110 kV/cm, the angular jump could not exceed 0.19 mrad, instead of the 0.24 required in paragraph VI b. If the jump is not sufficient, then a small corner of the beam emittance will hit the copper septum. To figure it, we just slide down the line (2) on Fig. 9 by 0.05 mrad and we see that about 0.4 o/o of the particles can be lost that way (taking account of the smaller particle density on emittance edges). (Of course this extra loss can be minimize by the procedure already described in paragraph V, but applied now to the copper septum).

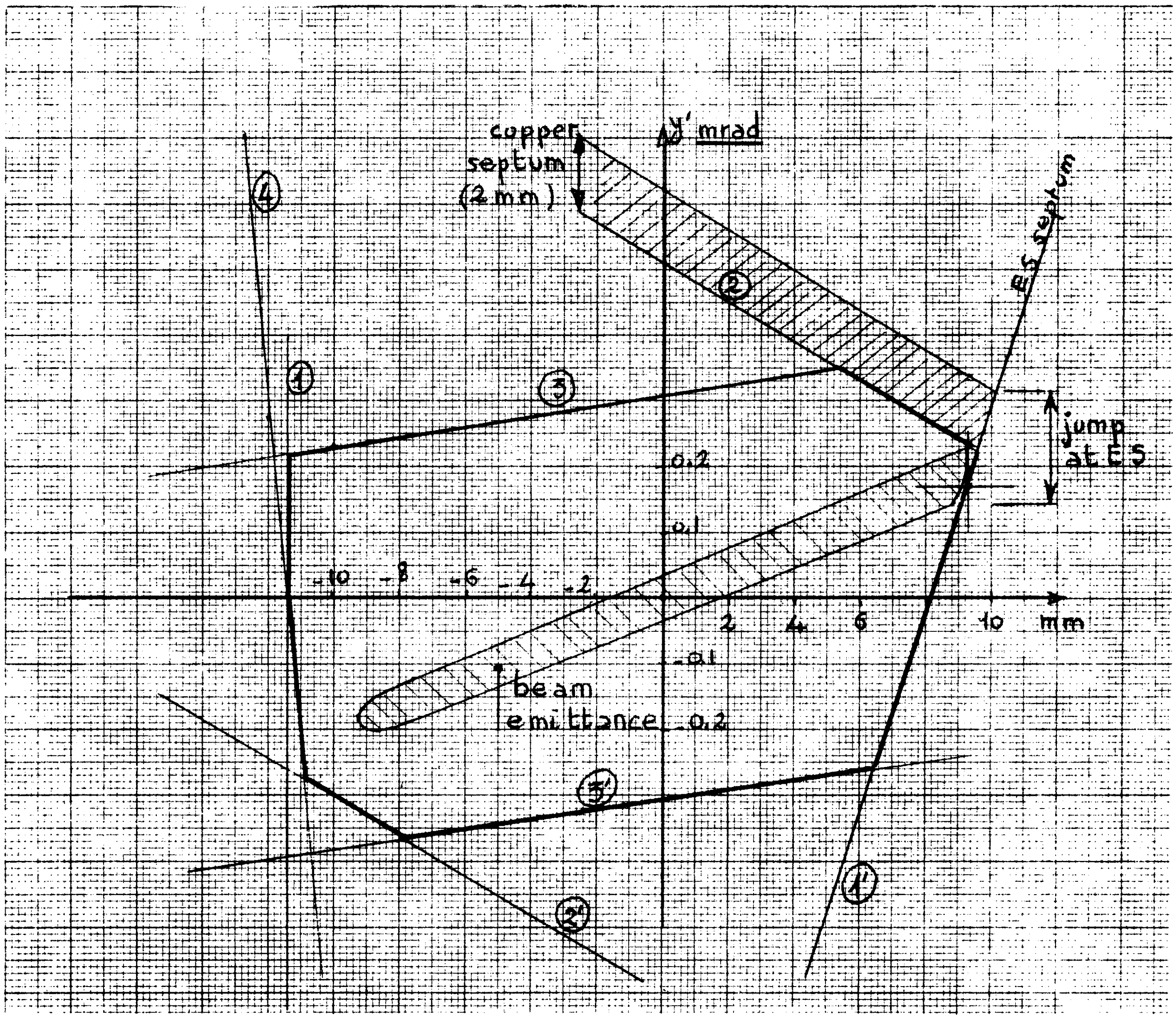

copper septum (2 mm)
y' mrad
ES septum
jump at ES
beam emittance
mm
0.2
0.1
-0.1
-0.2
-10
-8
-6
-4
-2
2
4
6
10

Fig. 9

In this particular case the relative number of particles hitting or grazing a septum will be in total = 0.81 + 0.4 $\approx$ 1.2 o/o. at 400 GeV.

ACKNOWLEDGEMENT

I should like to thank particularly O. Barbalat, C. Bovet, A. Brückner, J. Parain and E.J. Wilson, for interesting discussion on this subject.

Distribution

300 GeV Machine Committee
300 GeV Ejection Study Group

D. Bloess
A. Brückner
K.H. Kissler
J.H.B. Madsen
G.L. Munday
G. Plass
P.H. Standley
C. Bovet

and on request

REFERENCES

1) Fast Extraction from the 300 GeV Synchrotron by "Beam Shaving"
R. Gouiran - CERN/MPS-MU/EP 70-3

2) MR 47 Lattice
(Note MC/42 by E.J.N. Wilson)

3) Beam extraction from the 300 GeV Synchrotron
Report by the Ejection Study Group
MPS/DL Note 70-26

4) Fast extraction from the 300 GeV Synchrotron by beam shaving
Proposal for a kicker magnet system by
A. Brückner - EJ/38 - SI/Note MAE 70-10

SUMMARY OF REPORT

FAST EXTRACTION FROM THE 300 GeV SYNCHROTRON

BY "BEAM SHAVING"

II. Technical Notes

R. Gouiran

(Ref.: CERN/MPS-MU/EP 71- 1 - 12. 1.1971)

SUMMARY : A qualitative description of this type of resonance has already been given in a previous report (EJ 39 - CERN/MPS-MU/EP 70-3). Here we just give the technical details and the computation principles for those who could be interested, particularly concerning the modification of the β function and of the perturbed orbit, and the acceptance polygones.

This report is available to those interested; please fill in slip below.

Distribution : Scientific Staff MPS - SI

- -

To : Miss L. Deberti - MPS Division

From : Division

Would you please send me ... copy(ies) of the report CERN/MPS-MU/EP 71-1, dated 12.1.1970, by R. Gouiran.