

Beyond the Standard Model prospects for kaon physics at future experiments

G. D'Ambrosio ^a, F. Mahmoudi^{b,c,d} and S. Neshatpour ^b

^a*INFN-Sezione di Napoli, Complesso Universitario di Monte S. Angelo,
Via Cintia Edificio 6, 80126 Napoli, Italy*

^b*Université de Lyon, Université Claude Bernard Lyon 1, CNRS/IN2P3,
Institut de Physique des 2 Infinis de Lyon, UMR 5822,
F-69622, Villeurbanne, France*

^c*Theoretical Physics Department, CERN,
CH-1211 Geneva 23, Switzerland*

^d*Institut Universitaire de France (IUF),
75005 Paris, France*

E-mail: gdambros@na.infn.it, nazila@cern.ch, s.neshatpour@ip2i.in2p3.fr

ABSTRACT: Rare kaon decays offer a powerful tool for investigating new physics in $s \rightarrow d$ transitions. Currently, many of the interesting decay modes are either measured with rather large uncertainties compared to their theoretical predictions or have not yet been observed. The future HIKE programme at CERN will provide unprecedented sensitivity to rare kaon decays, allowing for strong constraints on new physics scenarios with lepton flavour universality violation. We present the overall picture that emerges from a study of the different decay modes with a global analysis considering projections based on the HIKE programme, both with and without KOTO-II future measurements. We also highlight the most relevant decays and identify that in addition to the “golden channel”, $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, the rare $K_L \rightarrow \pi^0 \ell \bar{\ell}$ decay, especially in the electron sector offers strong constraints on short-distance physics.

KEYWORDS: Kaons, Lepton Flavour Violation (charged)

ARXIV EPRINT: [2311.04878](https://arxiv.org/abs/2311.04878)

Contents

1	Introduction	1
2	Theoretical framework	2
2.1	Observables considered in the global analysis	3
2.2	Comments on $K_L \rightarrow e^+e^-$	4
3	Global analyses	5
3.1	Projections for NA62 and HIKE	5
3.2	Impact of KOTO-II	8
4	Conclusions	9

1 Introduction

In the ever-evolving landscape of particle physics, the study of kaons occupies certainly an important role, offering new insights and an improved sensitivity to physics beyond the Standard Model (SM). At present, the field of kaon physics finds itself at a crossroads, characterised by ongoing experiments, such as NA62 and KOTO, at the highest level of activity studying rare kaon decays. The future trajectory of kaon physics unfolds with the contemplation of upcoming experiments such as HIKE [1, 2], KOTO-II [3], and LHCb upgrade II [4, 5], which can not only advance the current experiments but also anticipate new avenues for exploration.

In this context, it is well-founded to address the achievements of ongoing experiments in a global analysis to assess the current sensitivity to new physics parameters, together with discussing in detail the role of each of the kaon decays that have been measured, and to provide a detailed analysis of future sensitivities of the upcoming experiments [6]. This is precisely the aim of this paper.

A specific category within the realm of rare processes, known as “golden modes”, holds a distinct role in the indirect exploration of new physics. Golden modes involve rare decays where we can measure theoretically clean observables. This allows us to uncover contributions from new physics that compete favourably with SM processes. Within the framework of the future experiment programme, the focus is on precisely measuring two key golden modes: $K^+ \rightarrow \pi^+\nu\bar{\nu}$ and $K_L \rightarrow \pi^0\nu\bar{\nu}$. The accurate measurement of the branching ratios for these decay processes serves as a model-independent benchmark, providing reliable constraints for beyond the Standard Model (BSM) scenarios.

The decay $K^+ \rightarrow \pi^+\nu\nu$ is very precisely measured but it does not distinguish between flavour families. However, its interplay with other rare kaon decays provides strong constraints for lepton flavour universality violating (LFUV) effects. Other rare kaon decays include $K_L \rightarrow \pi^0\ell\bar{\ell}$ which is planned to be measured within the HIKE programme for both electron and muon channels. This decay mode although not theoretically as clean as the golden modes, can offer a

strong probe of new physics. In addition, the $K_L \rightarrow \mu\bar{\mu}$ decay has been measured with better than 2% uncertainty, and although the long-distance dominated SM prediction has rather large uncertainty and depends on the sign of the two-photon contribution, it still offers valuable information on short-distance physics. Unlike the $K_L \rightarrow \mu\bar{\mu}$ decay, which exhibits sign ambiguity, the $K_S \rightarrow \mu\bar{\mu}$ decay does not. However, its experimental upper bound is approximately two orders of magnitude larger than the SM prediction, a situation that the upcoming LHCb upgrade aims to improve [4]. Another relevant decay mode is $K^+ \rightarrow \pi^+ \ell\bar{\ell}$ which although currently lacks a reliable theoretical prediction can test lepton flavour universality violation.

Currently, the primary obstacle in investigating BSM scenarios, and more broadly, in the quest for new physics using kaons, lies in the limited statistical precision of kaon decay measurements. To a lesser extent, this limitation is also influenced by theoretical uncertainties in the SM predictions. Encouragingly, efforts are underway to reduce these uncertainties in the coming years, and we address in this study their impacts in global fits to new physics parameters.

We examine the potential for rare kaon measurements to reveal departures from the accidental symmetries of the SM (LFUV), in upcoming experiments. We evaluate different benchmark points, incorporating the final precision expected from NA62, followed by the target precision for HIKE Phase 2, and finally, we include the ultimate precision anticipated for KOTO-II within the same timeframe as the HIKE programme.

The results presented in this work are obtained using the SuperIso public programme [7–10].

The paper is organised as follows: in section 2 we present briefly the theoretical framework and the relevant observables that we consider in this study. Section 3 presents the global analyses and our main results for future experiment prospects. Finally, our conclusions are given in section 4.

2 Theoretical framework

The $s \rightarrow d$ transitions are parameterised using the following effective Hamiltonian:

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} \lambda_t^{sd} \frac{\alpha_e}{4\pi} \sum_k C_k^\ell O_k^\ell. \quad (2.1)$$

Here $\lambda_t^{sd} \equiv V_{ts}^* V_{td}$, and the relevant operators are

$$\begin{aligned} O_9^\ell &= (\bar{s}\gamma_\mu P_L d) (\bar{\ell}\gamma^\mu \ell), \\ O_{10}^\ell &= (\bar{s}\gamma_\mu P_L d) (\bar{\ell}\gamma^\mu \gamma_5 \ell), \\ O_L^\ell &= (\bar{s}\gamma_\mu P_L d) (\bar{\nu}_\ell \gamma^\mu (1 - \gamma_5) \nu_\ell), \end{aligned} \quad (2.2)$$

with $P_L = (1 - \gamma_5)/2$. The Wilson coefficients C_k^ℓ are parameterised as:

$$C_k^\ell = C_{k,\text{SM}}^\ell + \delta C_k^\ell. \quad (2.3)$$

In general, besides the operators given in eq. (2.2), new physics effects also contribute via (pseudo)scalar operators and those with right-handed quark currents which are not relevant for the SM. For this work, we only consider the above-mentioned subset, and consider new

physics contributions in the chiral basis, assuming the neutral leptons to be connected to their charged counterparts via $SU(2)_L$ gauge symmetry, such that $\delta C_L^\ell \equiv \delta C_9^\ell = -\delta C_{10}^\ell$. Furthermore, we investigate lepton flavour universality violation, considering new physics effects for electrons to be different compared to muons and taus, $\delta C_L^e \neq \delta C_L^\mu (= \delta C_L^\tau)$.

2.1 Observables considered in the global analysis

In this work, we consider rare kaon decays which are sensitive to new physics effects and for which a reliable theoretical prediction exists in the SM. The seven observables we consider are (see refs. [11, 12] for more detailed descriptions):

- The branching ratio of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ which are theoretically very clean [13–20] and predicted with less than 8 and 12% uncertainties, respectively.¹ The dominant short-distance (SD) contributions in these decays make them very sensitive probes of new physics effects. However, experimentally the branching ratios are measured as the sum over the three neutrino flavours and it is not possible to differentiate between new physics contributions to electrons, muons and taus. On the experimental side, $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ has already been measured by NA62 [24] with $\sim 40\%$ precision which will be improved to 20% by the end of its run in 2025. The HIKE programme plans to measure this decay with an impressive 5% precision at phase 1. On the other hand, $K_L \rightarrow \pi^0 \nu \bar{\nu}$ has not yet been observed and the current upper bound by KOTO [25, 26] is two orders of magnitude larger than its SM prediction.
- Lepton flavour universality violating effects in the $K^+ \rightarrow \pi^+ \ell \bar{\ell}$ decays. For this channel, although a precise theoretical determination is not yet available [27–33], the comparison of the form factors of the electron and the muon modes offers a test on LFUV effects [34]. For this decay mode any disagreement among the experimental determination of the form factor parameters $a_+^{\mu\mu}$ and a_+^{ee} indicates short-distance LFUV effects among electrons and muons via $a_+^{\mu\mu} - a_+^{ee} = -\sqrt{2} \text{Re} [V_{td} V_{ts}^* (C_9^\mu - C_9^e)]$. Experimentally a_+^{ee} has been measured by E865 [35] and NA48/2 [36] (we use their combination as given in [37]) while $a_+^{\mu\mu}$ was recently measured by NA62 [38].² The HIKE programme aims to reduce the uncertainty in $a_+^{ee} - a_+^{\mu\mu}$ to less than half of what is currently measured.
- The branching ratios of $K_L \rightarrow \mu \bar{\mu}$ and $K_S \rightarrow \mu \bar{\mu}$. On the theory side, in these decays, the large long-distance contributions are dominant resulting in large theoretical uncertainties [41–61]. However, short-distance contributions can lead to sizeable effects, making it possible to extract constraints on new physics parameters. This is especially relevant for $K_L \rightarrow \mu \bar{\mu}$ which has been measured with less than 2% uncertainty [62–65]. The predicted $\text{BR}(K_L \rightarrow \mu \bar{\mu})$ depends on whether the short-distance and long-distance contributions interfere constructively or destructively which depends on the unknown sign of $\mathcal{A}(K_L \rightarrow \gamma\gamma)$.³ For $K_S \rightarrow \mu \bar{\mu}$, the situation is different, while the theory

¹In refs. [21–23] the SM predictions are done with $|V_{cb}|$ extracted from different observables as a function of β and γ of the unitarity triangle, resulting in more precise predictions.

²The combination of ref. [37] for a_+^{ee} was done with the external parameters from ref. [39] while the NA62 determination of $a_+^{\mu\mu}$ is done with the external parameters from the latest fit to $K \rightarrow 3\pi$ data [40].

³In this work, for the $K_L \rightarrow \mu \bar{\mu}$ decay, unless otherwise stated, for the long-distance contribution via $\mathcal{A}(K_L \rightarrow \gamma\gamma)$, we consider the sign to be both positive and negative, denoted by LD+ and LD–, respectively.

Observable	SM prediction	Experimental results	Reference	NA62 final	HIKE Phase 2	HP2 + KOTO-II
$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$	$(7.86 \pm 0.61) \times 10^{-11}$	$(10.6^{+4.0}_{-3.5} \pm 0.9) \times 10^{-11}$	[24]	20%	5%	5%
$\text{BR}(K_L^0 \rightarrow \pi^0 \nu \nu)$	$(2.68 \pm 0.30) \times 10^{-11}$	$< 3.0 \times 10^{-9}$ @90% CL	[25]	current	current	20%
$\text{LFUV}(a_+^{\mu\mu} - a_+^{e\mu})$	0	-0.014 ± 0.016	[37, 38]	current	± 0.007	± 0.007
$\text{BR}(K_L \rightarrow \mu\mu) (+)$	$(6.82^{+0.77}_{-0.29}) \times 10^{-9}$	$(6.84 \pm 0.11) \times 10^{-9}$	[62]	current	1%	1%
$\text{BR}(K_L \rightarrow \mu\mu) (-)$	$(8.04^{+1.47}_{-0.98}) \times 10^{-9}$					
$\text{BR}(K_S \rightarrow \mu\mu)$	$(5.15 \pm 1.50) \times 10^{-12}$	$< 2.1(2.4) \times 10^{-10}$ @90(95)% CL	[66]	current	current	current
$\text{BR}(K_L \rightarrow \pi^0 e e)(+)$	$(3.46^{+0.92}_{-0.80}) \times 10^{-11}$	$< 28 \times 10^{-11}$ @90% CL	[93]	current	20%	20%
$\text{BR}(K_L \rightarrow \pi^0 e e)(-)$	$(1.55^{+0.60}_{-0.48}) \times 10^{-11}$					
$\text{BR}(K_L \rightarrow \pi^0 \mu\mu)(+)$	$(1.38^{+0.27}_{-0.25}) \times 10^{-11}$	$< 38 \times 10^{-11}$ @90% CL	[94]	current	20%	20%
$\text{BR}(K_L \rightarrow \pi^0 \mu\mu)(-)$	$(0.94^{+0.21}_{-0.20}) \times 10^{-11}$					

Table 1. The SM predictions [7–12], current experimental values, and projected precisions. In the last three columns, “current” indicates that the measurement precision or the upper bound is kept to the current experimental value.

prediction is not dependent on the sign ambiguity, on the experimental side, currently the upper bound by LHCb [66] is about two orders of magnitude larger than the SM prediction making it difficult to obtain constraints on short-distance physics.

- The branching ratio of $K_L \rightarrow \pi^0 \ell \bar{\ell}$ both in the electron and the muon channels. This decay mode has the following main contributions [29, 30, 47, 49, 50, 59, 67–89]: 1) The CP-conserving two-photon contribution via $K_L \rightarrow \pi^0 \gamma^* \gamma^* \rightarrow \pi^0 \ell \bar{\ell}$. 2) The direct CP-violating term sensitive to short-distance physics and proportional to λ_t . 3) The indirect CP-violating contributions via $K_S \rightarrow \pi^0 \ell \bar{\ell}$ [90–92] which are proportional to the CP violating parameter ϵ . 4) The interference between the latter two contributions which can be both destructive and constructive.⁴ The electron mode is more sensitive to new physics effects compared to its muon counterpart which is due to the larger available phase-space as well as the vanishing two-photon contribution for the electron mode. On the experimental side, current bounds [93, 94] are one order of magnitude larger than the SM predictions.

The SM predictions and experimental measurements or upper bounds are collected in the second and third columns of table 1, respectively.

2.2 Comments on $K_L \rightarrow e^+ e^-$

Following the discussion of the relative contributions of long- versus short-distance contributions in $K_L \rightarrow \mu \bar{\mu}$, here we would like to address the precision at which $\text{BR}(K_L \rightarrow e^+ e^-)$ has to be measured in order to have some interesting physics impact. We will repeat the $K_L \rightarrow \ell \bar{\ell}$ calculation done in previous literature [43, 45, 53, 54, 95, 96] (in particular ref. [45]) but focusing on the difference for $K_L \rightarrow e^+ e^-$, and what is interesting to measure. Generically for K_L decaying into final lepton pairs we separate the long-distance two-photon intermediate state contribution, $A_{\gamma\gamma}$, from a pure short distance one, A_{short} :

$$A(K_L \rightarrow \ell \bar{\ell}) = (A_{\gamma\gamma} + \text{Re}A_{\text{short}}) \bar{\ell} \gamma_5 \ell . \tag{2.4}$$

⁴All through this paper, for the $K_L \rightarrow \pi^0 \ell \bar{\ell}$ decay, we assume constructive interference which is theoretically more favoured than destructive interference.

The long-distance two-photon intermediate state contribution, has a large imaginary piece, $\text{Im}A_{\gamma\gamma}$. It is then convenient to write $\Gamma(K_L \rightarrow \ell\bar{\ell})$ in terms of the experimental value of $\Gamma(K_L \rightarrow \gamma\gamma)$ and write the sum of the absorptive (R_{abs}) and dispersive (R_{disp}) contributions [45]

$$\Gamma(K_L \rightarrow \ell\bar{\ell}) = \frac{2\alpha_{\text{em}}^2 r_\ell \beta_\ell}{\pi^2} [R_{\text{abs}} + R_{\text{disp}}] \Gamma(K_L \rightarrow \gamma\gamma), \quad (2.5)$$

$$R_{\text{abs}} = (\text{Im}C_{\gamma\gamma})^2 = \left[\frac{\pi}{2\beta_\ell} \ln \frac{1 - \beta_\ell}{1 + \beta_\ell} \right]^2, \quad (2.6)$$

$$R_{\text{disp}} = \left[\chi(\mu) - \frac{5}{2} + \frac{3}{2} \ln \left(\frac{m_\ell^2}{\mu^2} \right) + \text{Re}C_{\gamma\gamma} \right]^2, \quad (2.7)$$

where $r_\ell = m_\ell^2/m_K^2$, $\beta_\ell = \sqrt{1 - 4r_\ell}$ and

$$C_{\gamma\gamma} = \frac{1}{\beta_\ell} \left[\text{Li}_2 \left(\frac{\beta_\ell - 1}{\beta_\ell + 1} \right) + \frac{\pi^2}{3} + \frac{1}{4} \ln^2 \left(\frac{\beta_\ell - 1}{\beta_\ell + 1} \right) \right]. \quad (2.8)$$

In eq. (2.5), compared to the muon mode, r_e suppresses $\text{BR}(K_L \rightarrow e^+e^-)$ while enhances R_{abs} and R_{disp} , as the small electron mass (with β_e close to 1) makes large logarithms. Also, we need to have experimental information on the real coefficient $\chi(\mu) = \chi_{\text{short}} + \chi_{\gamma\gamma}(\mu)$ since then we can uncover short and long-distance information; $\chi_{\gamma\gamma}(\mu)$ depends on the $K_L \rightarrow \gamma\gamma$ form factor for which an analytic expression can be found in ref. [54]. Based on the above arguments we have evaluated the experimental precision required for $\text{BR}(K_L \rightarrow e^+e^-)$ in order to get a handle on $\chi(\mu)$: our finding indicates that percent level measurement (assuming the SM evaluation for χ_{short}) will achieve this goal.

3 Global analyses

We present global analyses of the relevant rare kaon decay observables given in section 2.1, considering both the current experimental measurements and the projected situation as given in table 1. For the projections, we do not assume an improvement in the precision of the theoretical predictions.

3.1 Projections for NA62 and HIKE

The global fits to the rare kaon decays are given in the $\{\delta C_L^e, \delta C_L^\mu (= \delta C_L^\tau)\}$ plane. In figure 1 the 95% confidence level region of the fit to the current data is shown with the solid purple contour. The SM prediction of $\text{BR}(K_L \rightarrow \mu\bar{\mu})$ depends on the long-distance (LD) contribution of $K_L \rightarrow \gamma\gamma$, and due to its sign ambiguity we consider fits both when the LD sign is negative (as given in the left plot of figure 1) as well as positive (the right plot of figure 1). In the fit to current data, the main constraining observable is the branching ratio of the decay $K^+ \rightarrow \pi^+ \nu\bar{\nu}$, but it does not distinguish between the electron and muon modes. It is the branching ratio of the decay $K_L \rightarrow \mu\bar{\mu}$ that breaks this degeneracy. Overall, these two decays are currently the main constraining observables in the global fit (a detailed investigation of the impact of individual observables on this fit can be found in ref. [11]).

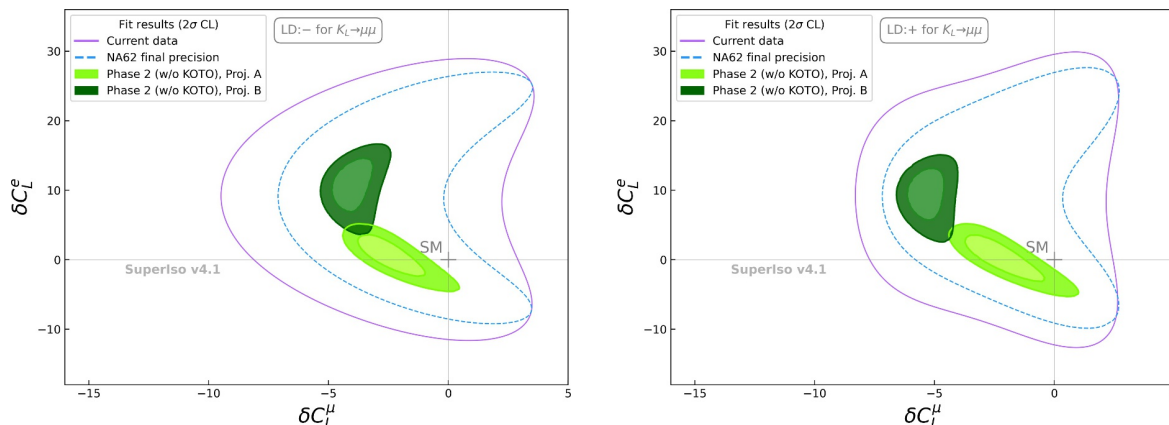


Figure 1. Global fit to kaon observables with current data (purple solid contour) and final NA62 projection (dashed blue contour) at 95% CL. The global fit for the projected HIKE sensitivity (corresponding to the penultimate column of table 1) is shown at 68 and 95% CL with two shades of light (dark) green for scenario A (B). For further details, see the text.

For the projections, an obvious benchmark point is the final precision of NA62, at which point $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ will be measured with 20% uncertainty [97]. For this case, we assume that the future central value remains the same as the current measurement. The 95% confidence level contour of the fit is shown in figure 1 with the dashed blue outline. The shrinking of the 2σ region compared to the current fit is only due to the better experimental precision of $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$, where the rest of the data are kept unchanged.

We also present the global fit results for projections based on HIKE Phase 2 target precision (penultimate column in table 1). In this case, the precision of $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ will be improved to 5%, and the precision of $\text{BR}(K^+ \rightarrow \pi^+ \ell \bar{\ell})$ is expected to be improved by more than a factor of two. Most significantly, the challenging ultra-rare decays $K_L \rightarrow \pi^0 \ell \bar{\ell}$ will be measured with 20% precision. Here, we assume there will be no improvement in the measurement of $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$, although within the same time frame, the KOTO-II collaboration is expected to have observed and measured this decay (see section 3.2 below for fit projections including KOTO-II precision).

The target precision of the projected data can be read off from table 1, however, since currently some observables have not been measured, for the central values we assume projections for two different scenarios:

- **Projection A:** for the observables that have been measured we consider their current central values and for the others we consider their SM predictions.
- **Projection B:** we project all central values based on the best-fit values of the Wilson coefficients from the fit to the current data.

The results of the fits at 68 and 95% confidence levels for future HIKE sensitivity are given in figure 1 for Projection A with the two shades of light green and for Projection B with the two shades of dark green. For Projection A, we find consistency with the SM at the 3σ level. This is expected since the decays that have not yet been experimentally observed are assumed to be measured with central values corresponding to their SM predictions. For

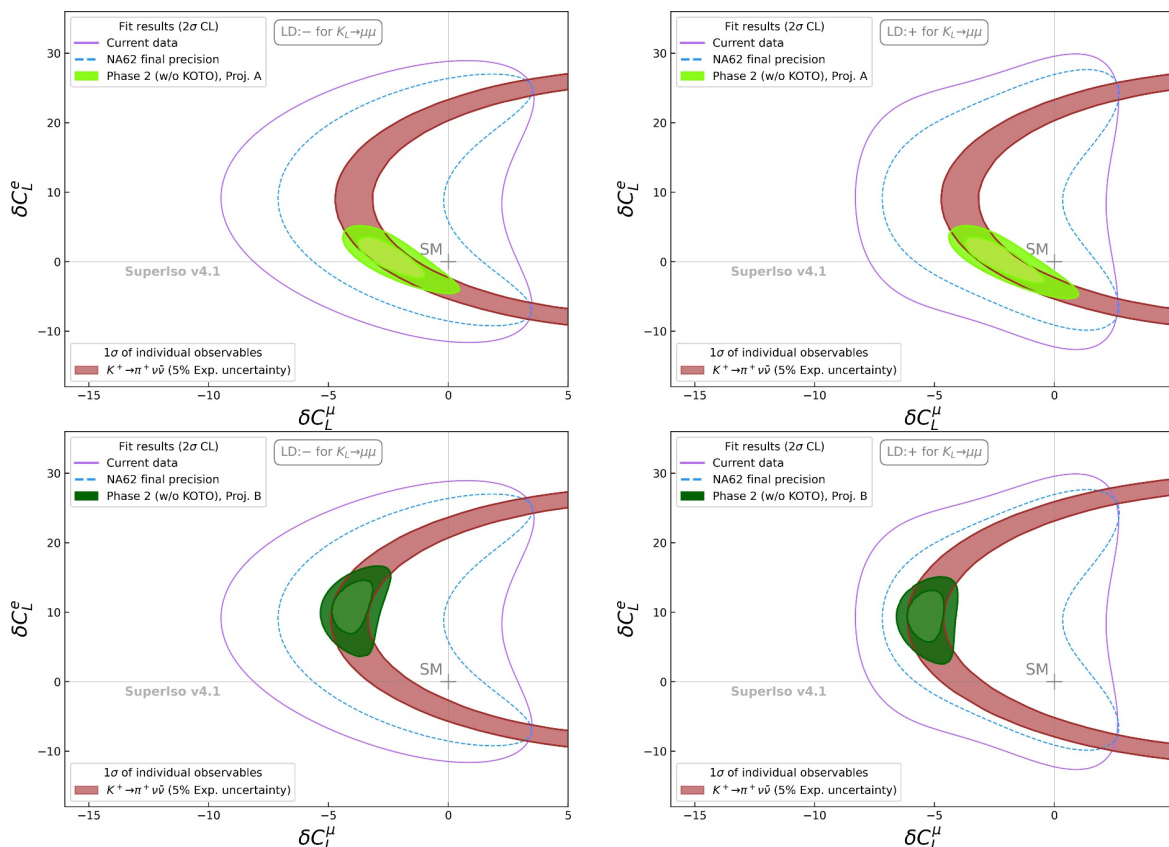


Figure 2. Impact of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ on the projected fits for HIKE sensitivity. See the text and the caption of figure 1 for further detail.

Projection B, there is a clear departure from the SM, which reflects the fact that the current best-fit point (with which the yet unobserved decays have been projected) does not coincide with the SM. There is only a small overlap at 2σ level between the two scenarios for the case of LD– contributions to $K_L \rightarrow \mu \bar{\mu}$, this overlap is even smaller for the case of LD+.

In a similar way to the fit to current data, also for the projections one of the main constraining observables is $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ which is projected with 5% uncertainty. This is visible in figure 2, where we show the two different projections in separate plots (for either sign of LD contributions to $K_L \rightarrow \mu \bar{\mu}$). The brown half-oval shape which indicates the 1σ bound due to $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ coincides nicely with the 1σ CL of the fit. However, clearly there are other observables that have a strong impact on the fits. While in the fit to current data, the main observable disentangling C_L^e and C_L^μ is in the muon mode via $\text{BR}(K_L \rightarrow \mu \bar{\mu})$, for the projections this is taking place in the electron mode via $\text{BR}(K_L \rightarrow \pi^0 e \bar{e})$. In figure 3, in addition to the bound due to $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ the impact of $\text{BR}(K_L \rightarrow \pi^0 \ell \bar{\ell})$ is also shown at 1σ level (for the case of LD+ contributions to $K_L \rightarrow \mu \bar{\mu}$). While for both projections the strong impact of $K_L \rightarrow \pi^0 e \bar{e}$ is visible, for Projection B (right plot) it is more prominent; this is by construction, as all the projected data have been produced with the same (current) best-fit point. For both projections, the main two constraining decays are $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 e \bar{e}$.

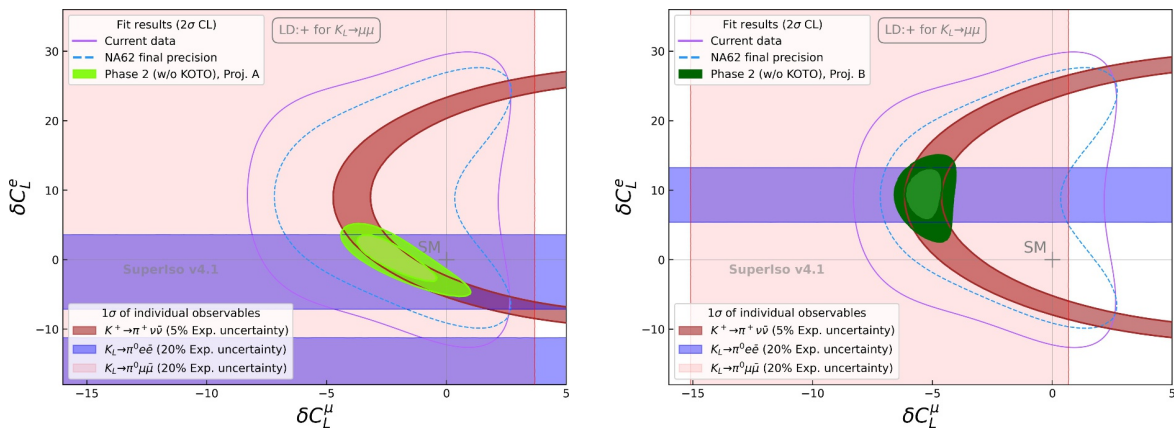


Figure 3. The impact of $K_L \rightarrow \pi^0 e^+ e^-$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ on the projected fits for HIKE sensitivity. The left (right) plot corresponds to the upper (lower) right plot of figure 2.

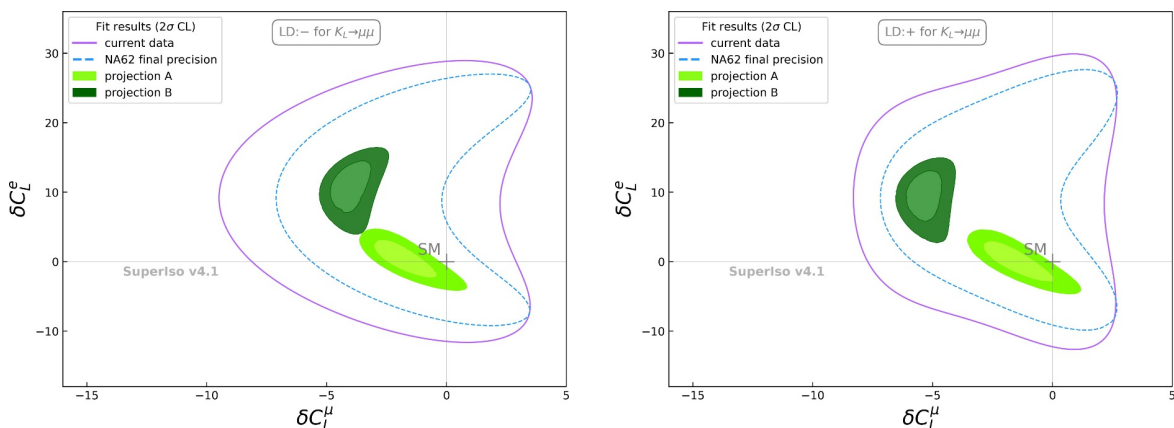


Figure 4. Projected fits for HIKE sensitivity together with KOTO-II (as given in the last column of table 1).

3.2 Impact of KOTO-II

In this subsection, in addition to the future HIKE sensitivity, we also consider 20% precision measurement of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ by KOTO-II. We use the same setup as described in the previous subsection for only HIKE sensitivity. The projected fits are displayed in figure 4 with the light and dark green shades for Projections A and B, respectively. The two scenarios no longer have any overlap at the 2σ level, and in the case of LD+ contributions to $K_L \rightarrow \mu \bar{\mu}$, the distinction of the two scenarios is more pronounced. Comparing figure 4 with figure 1, the fits now indicate a more constrained region which is due to the additional bound from KOTO-II. Furthermore, Projection A is slightly more consistent with the Standard Model because in this scenario it is assumed that the central value of $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ will match the SM prediction. The impact of the 20% KOTO-II measurement is demonstrated in figure 5 with the yellow half-oval shape where the 1σ bound of $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ is shown.

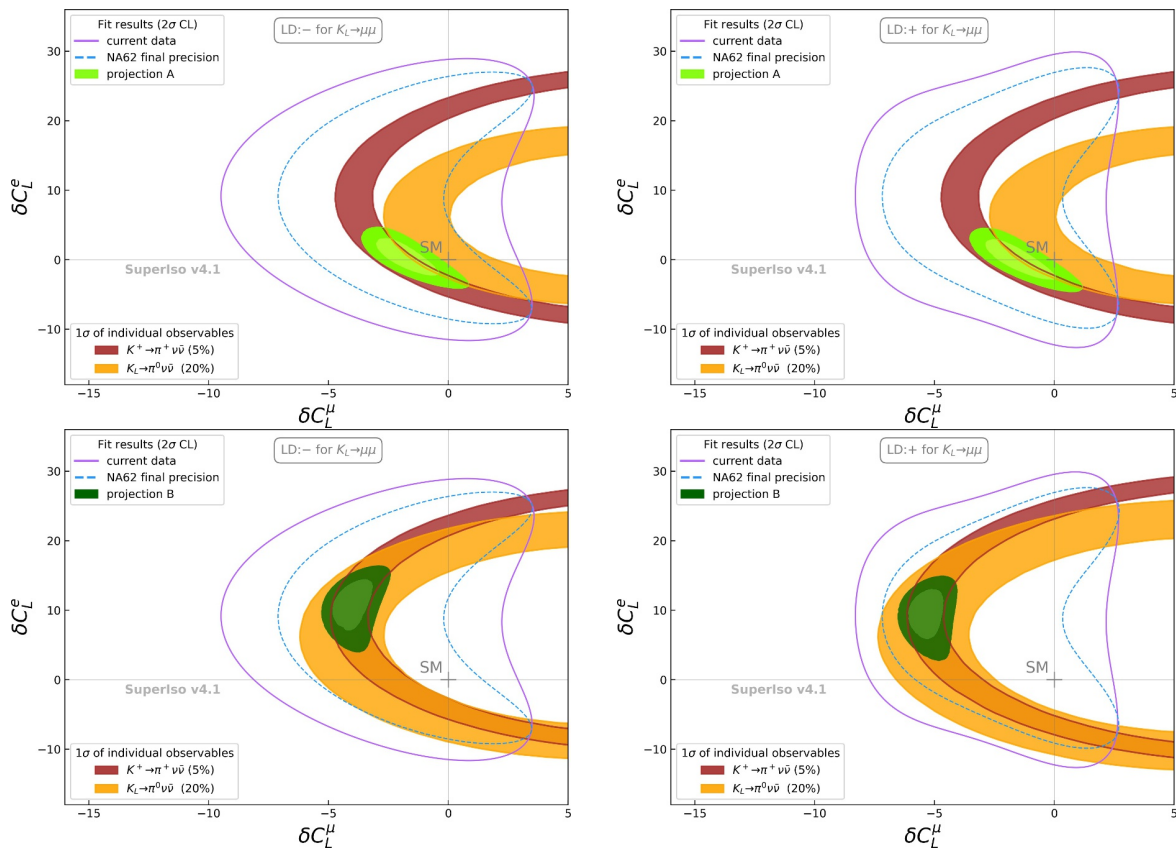


Figure 5. Impact of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ on the projected fit for HIKE and KOTO-II sensitivities.

4 Conclusions

In this work, we investigated the potential physics reach of rare kaon decay measurements with the future HIKE programme at CERN. We studied the constraining power of rare kaon decays on short-distance physics in a model-independent approach for beyond the Standard Model scenarios with lepton flavour universality violation.

We displayed the fit to current data and also made projections for the final precision of NA62 experiment considering 20% uncertainty for the golden channel $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. Furthermore, we performed projected fits for HIKE Phase 2 target precision. In addition to the improvements in the golden channel and $K^+ \rightarrow \pi^+ \ell \bar{\ell}$ that have been measured, HIKE will provide the first measurement of processes that have not yet been observed.

Therefore, we considered two different scenarios: A) considering future measurements for yet unobserved decays to match the Standard Model predictions, B) all observables are projected to match the best fit with current data. We showed that future measurements of rare kaon decays by HIKE offer very powerful constraints on lepton flavor universality violating new physics effects, leading to clearly different results.

Furthermore, we highlighted the decays that are most impactful in the global fit and demonstrated that besides the golden channel, the $K_L \rightarrow \pi^0 \ell \bar{\ell}$ decays, especially in the electron channel put strong constraints on new physics scenarios. This is in contrast to the

fits with current data where the strongest constrain besides $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is from $K_L \rightarrow \mu \bar{\mu}$ (which disentangles the electron and muon contributions).

Moreover, we performed global fits including the future $K_L \rightarrow \pi^0 \nu \bar{\nu}$ measurement with 20% precision by KOTO-II in addition to future HIKE measurements resulting in more constrained fits. However, from the projected fits it is clear that HIKE Phase 2 offers strong sensitivity to new physics, even without improvements in the experimental measurement of $K_L \rightarrow \pi^0 \nu \bar{\nu}$.

To clearly highlight the impact of future experimental measurements, we have not taken into account here any improvement in the theoretical precision of the considered observables. Our results are therefore rather conservative as it is reasonable to expect better theoretical precisions in the calculation of the relevant observables, resulting in even stronger constraints and bounds for new physics.

Acknowledgments

We would like to thank Cristina Lazzeroni, Gino Isidori, Marc Knecht, Urs Wiedemann and Karim Massri for fruitful discussions and suggestions. The work of GD was supported in part by the INFN research initiative Exploring New Physics (ENP). This research was funded in part by the National Research Agency (ANR) under project ANR-21-CE31-0002-01.

Open Access. This article is distributed under the terms of the Creative Commons Attribution License ([CC-BY4.0](https://creativecommons.org/licenses/by/4.0/)), which permits any use, distribution and reproduction in any medium, provided the original author(s) and source are credited.

References

- [1] HIKE collaboration, *HIKE, High Intensity Kaon Experiments at the CERN SPS: letter of intent*, [arXiv:2211.16586](https://arxiv.org/abs/2211.16586) [[INSPIRE](#)].
- [2] HIKE collaboration, *High Intensity Kaon Experiments (HIKE) at the CERN SPS proposal for phases 1 and 2*, [arXiv:2311.08231](https://arxiv.org/abs/2311.08231) [[INSPIRE](#)].
- [3] K. Aoki et al., *Extension of the J-PARC hadron experimental facility: third white paper*, [arXiv:2110.04462](https://arxiv.org/abs/2110.04462) [[INSPIRE](#)].
- [4] LHCb collaboration, *Physics case for an LHCb upgrade II — opportunities in flavour physics, and beyond, in the HL-LHC era*, [arXiv:1808.08865](https://arxiv.org/abs/1808.08865) [[INSPIRE](#)].
- [5] A.A. Alves Junior et al., *Prospects for measurements with strange hadrons at LHCb*, *JHEP* **05** (2019) 048 [[arXiv:1808.03477](https://arxiv.org/abs/1808.03477)] [[INSPIRE](#)].
- [6] C. Ahdida et al., *Post-LS3 experimental options in ECN3*, [arXiv:2310.17726](https://arxiv.org/abs/2310.17726) [[INSPIRE](#)].
- [7] F. Mahmoudi, *SuperIso: a program for calculating the isospin asymmetry of $B \rightarrow K^* \gamma$ in the MSSM*, *Comput. Phys. Commun.* **178** (2008) 745 [[arXiv:0710.2067](https://arxiv.org/abs/0710.2067)] [[INSPIRE](#)].
- [8] F. Mahmoudi, *SuperIso v2.3: a program for calculating flavor physics observables in supersymmetry*, *Comput. Phys. Commun.* **180** (2009) 1579 [[arXiv:0808.3144](https://arxiv.org/abs/0808.3144)] [[INSPIRE](#)].
- [9] F. Mahmoudi, *SuperIso v3.0, flavor physics observables calculations: extension to NMSSM*, *Comput. Phys. Commun.* **180** (2009) 1718 [[INSPIRE](#)].

- [10] S. Neshatpour and F. Mahmoudi, *Flavour physics with SuperIso*, *PoS TOOLS2020* (2021) 036 [[arXiv:2105.03428](#)] [[INSPIRE](#)].
- [11] G. D'Ambrosio, A.M. Iyer, F. Mahmoudi and S. Neshatpour, *Anatomy of kaon decays and prospects for lepton flavour universality violation*, *JHEP* **09** (2022) 148 [[arXiv:2206.14748](#)] [[INSPIRE](#)].
- [12] S. Neshatpour and F. Mahmoudi, *Flavour physics phenomenology with SuperIso*, *PoS CompTools2021* (2022) 010 [[arXiv:2207.04956](#)] [[INSPIRE](#)].
- [13] C. Bobeth and A.J. Buras, *Leptoquarks meet ϵ'/ϵ and rare kaon processes*, *JHEP* **02** (2018) 101 [[arXiv:1712.01295](#)] [[INSPIRE](#)].
- [14] G. Buchalla, A.J. Buras and M.E. Lautenbacher, *Weak decays beyond leading logarithms*, *Rev. Mod. Phys.* **68** (1996) 1125 [[hep-ph/9512380](#)] [[INSPIRE](#)].
- [15] G. Buchalla and A.J. Buras, *$K \rightarrow \pi$ neutrino anti-neutrino and high precision determinations of the CKM matrix*, *Phys. Rev. D* **54** (1996) 6782 [[hep-ph/9607447](#)] [[INSPIRE](#)].
- [16] G. Buchalla and A.J. Buras, *The rare decays $K \rightarrow \pi\nu\bar{\nu}$, $B \rightarrow X\nu\bar{\nu}$ and $B \rightarrow l^+l^-$: an update*, *Nucl. Phys. B* **548** (1999) 309 [[hep-ph/9901288](#)] [[INSPIRE](#)].
- [17] G. Isidori, F. Mescia and C. Smith, *Light-quark loops in $K \rightarrow \pi\nu\bar{\nu}$* , *Nucl. Phys. B* **718** (2005) 319 [[hep-ph/0503107](#)] [[INSPIRE](#)].
- [18] F. Mescia and C. Smith, *Improved estimates of rare K decay matrix-elements from K_{l3} decays*, *Phys. Rev. D* **76** (2007) 034017 [[arXiv:0705.2025](#)] [[INSPIRE](#)].
- [19] M. Misiak and J. Urban, *QCD corrections to FCNC decays mediated by Z penguins and W boxes*, *Phys. Lett. B* **451** (1999) 161 [[hep-ph/9901278](#)] [[INSPIRE](#)].
- [20] J. Brod, M. Gorbahn and E. Stamou, *Two-loop electroweak corrections for the $K \rightarrow \pi\nu\bar{\nu}$ decays*, *Phys. Rev. D* **83** (2011) 034030 [[arXiv:1009.0947](#)] [[INSPIRE](#)].
- [21] A.J. Buras and E. Venturini, *Searching for new physics in rare K and B decays without $|V_{cb}|$ and $|V_{ub}|$ uncertainties*, *Acta Phys. Polon. B* **53** (2021) 6 [[arXiv:2109.11032](#)] [[INSPIRE](#)].
- [22] A.J. Buras and E. Venturini, *The exclusive vision of rare K and B decays and of the quark mixing in the standard model*, *Eur. Phys. J. C* **82** (2022) 615 [[arXiv:2203.11960](#)] [[INSPIRE](#)].
- [23] A.J. Buras, *Standard model predictions for rare K and B decays without new physics infection*, *Eur. Phys. J. C* **83** (2023) 66 [[arXiv:2209.03968](#)] [[INSPIRE](#)].
- [24] NA62 collaboration, *Measurement of the very rare $K^+ \rightarrow \pi^+\nu\bar{\nu}$ decay*, *JHEP* **06** (2021) 093 [[arXiv:2103.15389](#)] [[INSPIRE](#)].
- [25] KOTO collaboration, *Search for the $K_L \rightarrow \pi^0\nu\bar{\nu}$ and $K_L \rightarrow \pi^0 X^0$ decays at the J-PARC KOTO experiment*, *Phys. Rev. Lett.* **122** (2019) 021802 [[arXiv:1810.09655](#)] [[INSPIRE](#)].
- [26] KOTO collaboration, *Study of the $K_L \rightarrow \pi^0\nu\bar{\nu}$ decay at the J-PARC KOTO experiment*, *Phys. Rev. Lett.* **126** (2021) 121801 [[arXiv:2012.07571](#)] [[INSPIRE](#)].
- [27] G. D'Ambrosio, G. Ecker, G. Isidori and J. Portoles, *The decays $K \rightarrow \pi\ell^+\ell^-$ beyond leading order in the chiral expansion*, *JHEP* **08** (1998) 004 [[hep-ph/9808289](#)] [[INSPIRE](#)].
- [28] F.J. Gilman and M.B. Wise, *$K \rightarrow \pi e^+e^-$ in the six quark model*, *Phys. Rev. D* **21** (1980) 3150 [[INSPIRE](#)].
- [29] G. Ecker, A. Pich and E. de Rafael, *Radiative kaon decays and CP violation in chiral perturbation theory*, *Nucl. Phys. B* **303** (1988) 665 [[INSPIRE](#)].

- [30] G. Ecker, A. Pich and E. de Rafael, $K \rightarrow \pi \ell^+ \ell^-$ decays in the effective chiral Lagrangian of the standard model, *Nucl. Phys. B* **291** (1987) 692 [INSPIRE].
- [31] G. D'Ambrosio, G. Ecker, G. Isidori and H. Neufeld, *Radiative nonleptonic kaon decays*, [hep-ph/9411439](#) [INSPIRE].
- [32] B. Ananthanarayan and I. Sentitemsu Imsong, *The 27-plet contributions to the CP-conserving $K \rightarrow \pi \ell^+ \ell^-$ decays*, *J. Phys. G* **39** (2012) 095002 [[arXiv:1207.0567](#)] [INSPIRE].
- [33] RBC and UKQCD collaborations, *Simulating rare kaon decays $K^+ \rightarrow \pi^+ \ell^+ \ell^-$ using domain wall lattice QCD with physical light quark masses*, *Phys. Rev. D* **107** (2023) L011503 [[arXiv:2202.08795](#)] [INSPIRE].
- [34] A. Crivellin, G. D'Ambrosio, M. Hoferichter and L.C. Tunstall, *Violation of lepton flavor and lepton flavor universality in rare kaon decays*, *Phys. Rev. D* **93** (2016) 074038 [[arXiv:1601.00970](#)] [INSPIRE].
- [35] E865 collaboration, *A new measurement of the properties of the rare decay $K^+ \rightarrow \pi^+ e^+ e^-$* , *Phys. Rev. Lett.* **83** (1999) 4482 [[hep-ex/9907045](#)] [INSPIRE].
- [36] NA48/2 collaboration, *Precise measurement of the $K^\pm \rightarrow \pi^\pm e^+ e^-$ decay*, *Phys. Lett. B* **677** (2009) 246 [[arXiv:0903.3130](#)] [INSPIRE].
- [37] G. D'Ambrosio, D. Greynat and M. Knecht, *On the amplitudes for the CP-conserving $K^\pm(K_S) \rightarrow \pi^\pm(\pi^0)\ell^+\ell^-$ rare decay modes*, *JHEP* **02** (2019) 049 [[arXiv:1812.00735](#)] [INSPIRE].
- [38] NA62 collaboration, *A measurement of the $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ decay*, *JHEP* **11** (2022) 011 [Addendum *ibid.* **06** (2023) 040] [[arXiv:2209.05076](#)] [INSPIRE].
- [39] J. Bijnens, P. Dhonte and F. Borg, *$K \rightarrow 3\pi$ decays in chiral perturbation theory*, *Nucl. Phys. B* **648** (2003) 317 [[hep-ph/0205341](#)] [INSPIRE].
- [40] G. D'Ambrosio, M. Knecht and S. Neshatpour, *Determination of the structure of the $K \rightarrow \pi\pi\pi$ amplitudes from recent data*, *Phys. Lett. B* **835** (2022) 137594 [[arXiv:2209.02143](#)] [INSPIRE].
- [41] G. D'Ambrosio and D. Espriu, *Rare decay modes of the K mesons in the chiral lagrangian*, *Phys. Lett. B* **175** (1986) 237 [INSPIRE].
- [42] G. Ecker and A. Pich, *The longitudinal muon polarization in $K_L \rightarrow \mu^+ \mu^-$* , *Nucl. Phys. B* **366** (1991) 189 [INSPIRE].
- [43] D. Gomez Dumm and A. Pich, *Long distance contributions to the $K_L \rightarrow \mu^+ \mu^-$ decay width*, *Phys. Rev. Lett.* **80** (1998) 4633 [[hep-ph/9801298](#)] [INSPIRE].
- [44] M. Knecht, S. Peris, M. Perrottet and E. de Rafael, *Decay of pseudoscalars into lepton pairs and large N_c QCD*, *Phys. Rev. Lett.* **83** (1999) 5230 [[hep-ph/9908283](#)] [INSPIRE].
- [45] G. Isidori and R. Unterdorfer, *On the short distance constraints from $K_{L,S} \rightarrow \mu^+ \mu^-$* , *JHEP* **01** (2004) 009 [[hep-ph/0311084](#)] [INSPIRE].
- [46] G. D'Ambrosio and T. Kitahara, *Direct CP violation in $K \rightarrow \mu^+ \mu^-$* , *Phys. Rev. Lett.* **119** (2017) 201802 [[arXiv:1707.06999](#)] [INSPIRE].
- [47] F. Mescia, C. Smith and S. Trine, *$K_L \rightarrow \pi^0 e^+ e^-$ and $K_L \rightarrow \pi^0 \mu^+ \mu^-$: a binary star on the stage of flavor physics*, *JHEP* **08** (2006) 088 [[hep-ph/0606081](#)] [INSPIRE].
- [48] C. Quigg and J.D. Jackson, *Decays of neutral pseudoscalar mesons into lepton pairs*, UCRL-18487 (1968) [INSPIRE].
- [49] B.R. Martin, E. De Rafael and J. Smith, *Neutral kaon decays into lepton pairs*, *Phys. Rev. D* **2** (1970) 179 [INSPIRE].

- [50] M.J. Savage, M.E. Luke and M.B. Wise, *The rare decays $\pi^0 \rightarrow e^+e^-$, $\eta \rightarrow e^+e^-$ and $\eta \rightarrow \mu^+\mu^-$ in chiral perturbation theory*, *Phys. Lett. B* **291** (1992) 481 [[hep-ph/9207233](#)] [[INSPIRE](#)].
- [51] G. D'Ambrosio, M. Miragliuolo and P. Santorelli, *Radiative nonleptonic kaon decays*, LNF-92-066-P (1992) [[INSPIRE](#)].
- [52] G. D'Ambrosio and G. Isidori, *CP violation in kaon decays*, *Int. J. Mod. Phys. A* **13** (1998) 1 [[hep-ph/9611284](#)] [[INSPIRE](#)].
- [53] G. Valencia, *Long distance contribution to $K_L \rightarrow \ell^+\ell^-$* , *Nucl. Phys. B* **517** (1998) 339 [[hep-ph/9711377](#)] [[INSPIRE](#)].
- [54] G. D'Ambrosio, G. Isidori and J. Portoles, *Can we extract short distance information from $B(K_L \rightarrow \mu^+\mu^-)$?*, *Phys. Lett. B* **423** (1998) 385 [[hep-ph/9708326](#)] [[INSPIRE](#)].
- [55] V. Cirigliano et al., *Kaon decays in the standard model*, *Rev. Mod. Phys.* **84** (2012) 399 [[arXiv:1107.6001](#)] [[INSPIRE](#)].
- [56] G. Colangelo, R. Stucki and L.C. Tunstall, *Dispersive treatment of $K_S \rightarrow \gamma\gamma$ and $K_S \rightarrow \gamma\ell^+\ell^-$* , *Eur. Phys. J. C* **76** (2016) 604 [[arXiv:1609.03574](#)] [[INSPIRE](#)].
- [57] A. Dery, M. Ghosh, Y. Grossman and S. Schacht, *$K \rightarrow \mu^+\mu^-$ as a clean probe of short-distance physics*, *JHEP* **07** (2021) 103 [[arXiv:2104.06427](#)] [[INSPIRE](#)].
- [58] J. Brod and E. Stamou, *Impact of indirect CP violation on $Br(K_S \rightarrow \mu^+\mu^-)_{\ell=0}$* , *JHEP* **05** (2023) 155 [[arXiv:2209.07445](#)] [[INSPIRE](#)].
- [59] Y. Zhao and N.H. Christ, *Calculating $K \rightarrow \gamma\gamma$ using lattice QCD*, *PoS LATTICE2021* (2022) 451 [[INSPIRE](#)].
- [60] N.H. Christ et al., *Lattice QCD calculation of the two-photon contributions to $K_L \rightarrow \mu^+\mu^-$ and $\pi^0 \rightarrow e^+e^-$ decays*, *PoS LATTICE2019* (2020) 128 [[INSPIRE](#)].
- [61] V. Chobanova et al., *Probing SUSY effects in $K_S^0 \rightarrow \mu^+\mu^-$* , *JHEP* **05** (2018) 024 [[arXiv:1711.11030](#)] [[INSPIRE](#)].
- [62] PARTICLE DATA GROUP collaboration, *Review of particle physics*, *PTEP* **2022** (2022) 083C01 [[INSPIRE](#)].
- [63] E791 collaboration, *Measurement of the branching ratio for the rare decay $K_L^0 \rightarrow \mu^+\mu^-$* , *Phys. Rev. D* **51** (1995) 985 [[INSPIRE](#)].
- [64] T. Akagi et al., *Experimental study of the rare decays $K_L^0 \rightarrow \mu e$, $K_L^0 \rightarrow ee$, and $K_L^0 \rightarrow \mu\mu$ and $K_L^0 \rightarrow eeee$* , *Phys. Rev. D* **51** (1995) 2061 [[INSPIRE](#)].
- [65] E871 collaboration, *Improved branching ratio measurement for the decay $K_L^0 \rightarrow \mu^+\mu^-$* , *Phys. Rev. Lett.* **84** (2000) 1389 [[INSPIRE](#)].
- [66] LHCb collaboration, *Constraints on the $K_S^0 \rightarrow \mu^+\mu^-$ branching fraction*, *Phys. Rev. Lett.* **125** (2020) 231801 [[arXiv:2001.10354](#)] [[INSPIRE](#)].
- [67] G. Buchalla, G. D'Ambrosio and G. Isidori, *Extracting short distance physics from $K_{L,S} \rightarrow \pi^0 e^+e^-$ decays*, *Nucl. Phys. B* **672** (2003) 387 [[hep-ph/0308008](#)] [[INSPIRE](#)].
- [68] G. Isidori, C. Smith and R. Unterdorfer, *The rare decay $K_L \rightarrow \pi^0 \mu^+\mu^-$ within the SM*, *Eur. Phys. J. C* **36** (2004) 57 [[hep-ph/0404127](#)] [[INSPIRE](#)].
- [69] J.F. Donoghue, B.R. Holstein and G. Valencia, *$K_L \rightarrow \pi^0 e^+e^-$ as a probe of CP violation*, *Phys. Rev. D* **35** (1987) 2769 [[INSPIRE](#)].
- [70] G. Ecker, A. Pich and E. de Rafael, *$K^0 \rightarrow \pi^0 \gamma\gamma$ decays in chiral perturbation theory*, *Phys. Lett. B* **189** (1987) 363 [[INSPIRE](#)].

- [71] L.M. Sehgal, *CP violation in $K_L \rightarrow \pi^0 e^+ e^-$: interference of one photon and two photon exchange*, *Phys. Rev. D* **38** (1988) 808 [INSPIRE].
- [72] J. Flynn and L. Randall, *The CP conserving long distance contribution to the decay $K_L \rightarrow \pi^0 e^+ e^-$* , *Phys. Lett. B* **216** (1989) 221 [INSPIRE].
- [73] L. Cappiello and G. D'Ambrosio, *$K_L \rightarrow \pi^0 \gamma \gamma$ decay in the chiral effective lagrangian*, *Nuovo Cim. A* **99** (1988) 155 [INSPIRE].
- [74] T. Morozumi and H. Iwasaki, *The CP conserving contribution in the decays $K_L \rightarrow \pi^0 \gamma \gamma$ and $K_L \rightarrow \pi^0 e^+ e^-$* , *Prog. Theor. Phys.* **82** (1989) 371 [INSPIRE].
- [75] G. Ecker, A. Pich and E. de Rafael, *Vector meson exchange in radiative kaon decays and chiral perturbation theory*, *Phys. Lett. B* **237** (1990) 481 [INSPIRE].
- [76] L. Cappiello, G. D'Ambrosio and M. Miragliuolo, *Corrections to $K \rightarrow \pi \gamma \gamma$ from $K \rightarrow 3\pi$* , *Phys. Lett. B* **298** (1993) 423 [INSPIRE].
- [77] P. Heiliger and L.M. Sehgal, *Analysis of the decay $K_L \rightarrow \pi^0 \gamma \gamma$ and expectations for the decays $K_L \rightarrow \pi^0 e^+ e^-$ and $K_L \rightarrow \pi^0 \mu^+ \mu^-$* , *Phys. Rev. D* **47** (1993) 4920 [INSPIRE].
- [78] A.G. Cohen, G. Ecker and A. Pich, *Unitarity and $K_L \rightarrow \pi^0 \gamma \gamma$* , *Phys. Lett. B* **304** (1993) 347 [INSPIRE].
- [79] A.J. Buras, M.E. Lautenbacher, M. Misiak and M. Munz, *Direct CP violation in $K_L \rightarrow \pi^0 e^+ e^-$ beyond leading logarithms*, *Nucl. Phys. B* **423** (1994) 349 [hep-ph/9402347] [INSPIRE].
- [80] G. D'Ambrosio and J. Portoles, *Vector meson exchange contributions to $K \rightarrow \pi \gamma \gamma$ and $K_L \rightarrow \gamma \ell^+ \ell^-$* , *Nucl. Phys. B* **492** (1997) 417 [hep-ph/9610244] [INSPIRE].
- [81] J.F. Donoghue and F. Gabbiani, *$K_L \rightarrow \pi^0 \gamma e^+ e^-$ and its relation to CP and chiral tests*, *Phys. Rev. D* **56** (1997) 1605 [hep-ph/9702278] [INSPIRE].
- [82] KTeV collaboration, *Measurement of the decay $K_L \rightarrow \pi^0 \gamma \gamma$* , *Phys. Rev. Lett.* **83** (1999) 917 [hep-ex/9902029] [INSPIRE].
- [83] K. Murakami et al., *Experimental search for the decay mode $K_L \rightarrow \pi^0 \gamma e^+ e^-$* , *Phys. Lett. B* **463** (1999) 333 [hep-ex/9907007] [INSPIRE].
- [84] KTeV collaboration, *First observation of the decay $K_L \rightarrow \pi^0 e^+ e^- \gamma$* , *Phys. Rev. Lett.* **87** (2001) 021801 [hep-ex/0011093] [INSPIRE].
- [85] M.V. Diwan, H. Ma and T.L. Trueman, *Muon decay asymmetries from $K_L^0 \rightarrow \pi^0 \mu^+ \mu^-$ decays*, *Phys. Rev. D* **65** (2002) 054020 [hep-ph/0112350] [INSPIRE].
- [86] F. Gabbiani and G. Valencia, *$K_L \rightarrow \pi^0 \gamma \gamma$ and the bound on the CP conserving $K_L \rightarrow \pi^0 e^+ e^-$* , *Phys. Rev. D* **64** (2001) 094008 [hep-ph/0105006] [INSPIRE].
- [87] F. Gabbiani and G. Valencia, *Vector meson contributions do not explain the rate and spectrum in $K_L \rightarrow \pi^0 \gamma \gamma$* , *Phys. Rev. D* **66** (2002) 074006 [hep-ph/0207189] [INSPIRE].
- [88] NA48 collaboration, *Precise measurement of the decay $K_L \rightarrow \pi^0 \gamma \gamma$* , *Phys. Lett. B* **536** (2002) 229 [hep-ex/0205010] [INSPIRE].
- [89] G. Isidori, G. Martinelli and P. Turchetti, *Rare kaon decays on the lattice*, *Phys. Lett. B* **633** (2006) 75 [hep-lat/0506026] [INSPIRE].
- [90] NA48 collaboration, *Search for the decay $K_S \rightarrow \pi^0 e^+ e^-$* , *Phys. Lett. B* **514** (2001) 253 [INSPIRE].
- [91] NA48/1 collaboration, *Observation of the rare decay $K_S \rightarrow \pi^0 \mu^+ \mu^-$* , *Phys. Lett. B* **599** (2004) 197 [hep-ex/0409011] [INSPIRE].

- [92] NA48/1 collaboration, *Observation of the rare decay $K_S \rightarrow \pi^0 e^+ e^-$* , *Phys. Lett. B* **576** (2003) 43 [[hep-ex/0309075](#)] [[INSPIRE](#)].
- [93] KTEV collaboration, *Search for the rare decay $K_L \rightarrow \pi^0 e^+ e^-$* , *Phys. Rev. Lett.* **93** (2004) 021805 [[hep-ex/0309072](#)] [[INSPIRE](#)].
- [94] KTEV collaboration, *Search for the decay $K_L \rightarrow \pi^0 \mu^+ \mu^-$* , *Phys. Rev. Lett.* **84** (2000) 5279 [[hep-ex/0001006](#)] [[INSPIRE](#)].
- [95] G. Isidori, *Standard model versus new physics in rare kaon decays*, in the proceedings of the 1999 Chicago conference on kaon physics (K 99), (1999), p. 355 [[hep-ph/9908399](#)] [[INSPIRE](#)].
- [96] M. Hoferichter, B.-L. Hoid and J.R. de Elvira, *Improved standard-model prediction for $K_L \rightarrow \ell^+ \ell^-$* , [arXiv:2310.17689](#) [[INSPIRE](#)].
- [97] S. Pirandola et al., *Theory of channel simulation and bounds for private communication*, *Quantum Sci. Technol.* **3** (2018) 035009 [[INSPIRE](#)].