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THE SECONDARY ELECTRONS IN THE ELECTRON COOLING SYSTEM

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Secondary electrons in the electron cooling system determine the current losses [1]. Their parameters depend on the collector efficiency. In this report the behavior of the secondary electrons in the LEAR electron cooling system with S-shape bending toroid magnets is examined. The parameters of secondary electrons for primary electron beams with a current $I = 0.1\text{-}3$ A and an electron energy 3-30 kV are discussed. The second problem, which takes place with secondary electrons, is related to generation of neutralized electron beams in the electron cooling system [2]. The generation of very dense neutralized electron beams with low energy may be applied for cooling of lead ions dedicated to LHC [3]. Due to the beam-drift coherent instability [2], the upper limit current density is restricted by : [4].

$$j_{\max} = \frac{kvB\varepsilon_0}{L_t}, \tag{1}$$

where

v is the electron velocity,

B is the longitudinal magnetic field,

L_t is the neutralized beam length,

k is a numerical coefficient.

It is a convective instability which appears in stationary conditions due to the feedback. This feedback may be created by secondary electrons [4]. When the wave amplification coefficient K and the feedback coefficient χ satisfy to the condition:

$$\chi K < 1 \tag{2}$$

the neutralization beam is stable [4]. The level of the feedback coefficient is dependent on the density and on the energy distribution of the secondary electrons.

There are three kinds of secondary electrons in the electron cooling system:

1. Electrons reflected from the collector and that make a single pass between it and the gun,
2. Stored electrons performing many oscillations between the collector and the gun,
3. Low-energy electrons created by the ionization of the residual gas.

The trajectory of fast stored and reflected electrons is given in Fig. 1. The density of reflected electrons is determined by:

$$\frac{n_{ref}}{n_b}=\frac{I_{ref}}{I}. \tag{3}$$

Here n_b, I are the beam density and current, n_{ref}, I_{ref} are the density and current of reflected electrons. The density of stored electrons n_{st} is determined by:

$$\frac{n_{st}}{n_b}=\frac{I_{st}}{I}\frac{\tau_{es}}{2\tau_{\parallel}} \tag{4}$$

where I_{st} is the current of stored electrons, τ_{es} is the escape time of stored electrons, $\tau_{\parallel}$ is the half-period of stored electron oscillations between the gun and the collector. For stationary conditions the current of stored electrons is equal to the current losses of fast electrons:

$$I_{st}=I_{los}. \tag{5}$$

In this report we will estimate the current and density of electrons reflected from collector surface, and also the current losses and density of fast stored electrons.

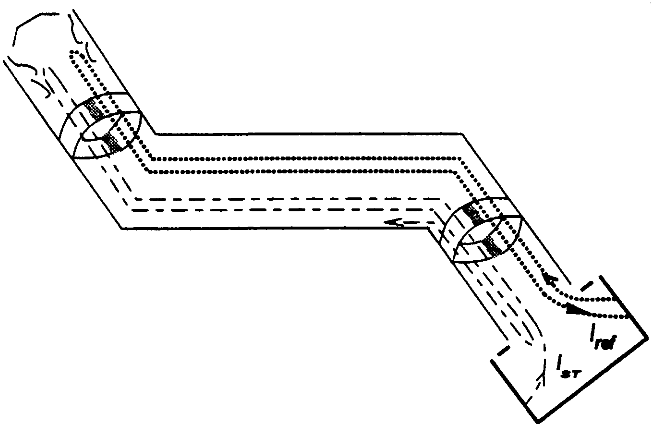

Fig. 1 - The trajectories of fast secondary electrons

1. BEHAVIOR OF THE FAST STORED ELECTRONS IN THE SYSTEM WITH ION TRAPS AND WITH S-SHAPE BENDING MAGNETS

The motion of the fast stored electrons in the electron cooling system is determined by the space charge of partially neutralized electron beam, the transverse electric field of the ion traps [5] and also by the toroid bending magnets. For the purpose of neutralization, two ion traps consisting of two metallic half-cylinders separated by high-resistive glass insulator, have been installed outside the drift space (Fig. 2). The electrodes are polarized by independent positive voltages U_{2t} and U_{1t}, such that a transverse electric field also exists:

$$E_t=\frac{U_{2t}-U_{1t}}{\pi b} \tag{6}$$

where b is the trap radius. When stored electrons pass through these electrodes, their trajectories are deflected by ΔX, due to crossed, transverse electric and longitudinal magnetic fields:

$$\Delta X = l_t \frac{U_{2t} - U_{1t}}{\pi b v_{s1} B} \tag{7}$$

where l_t is the trap length and v_{s1} the stored electron velocity inside traps.

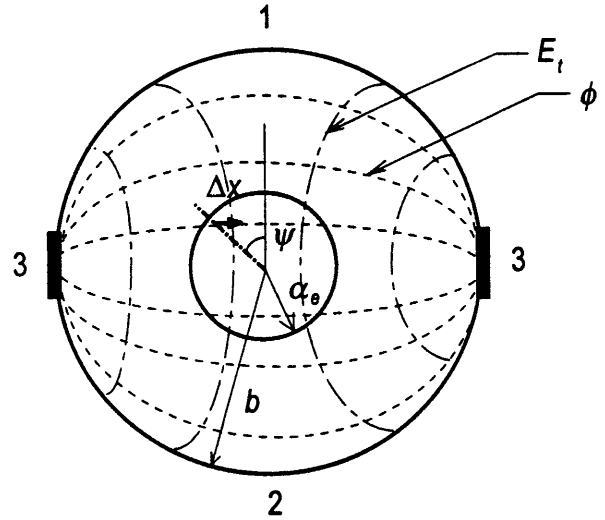

Fig. 2 - The neutralization ion trap. (1,2) half-cylinders, separated by high resistive glass insulators, (3) ·φ- equipotential line, E_t - transverse electric field of trap, ΔX - displacement of electron trajectory inside the trap.

The space charge of the electron beam defines the motion of the stored electrons. Due to the space charge of the neutralization beam, stored electrons rotate with an angular frequency

$$\omega_d = \frac{E_s}{Br}$$

where r is their radial coordinate. E_s is the radial electric field of the neutralized electron beam. The electron beam is neutralized between two traps. We consider the following distribution of electron and ion density, presented in Fig. 3.

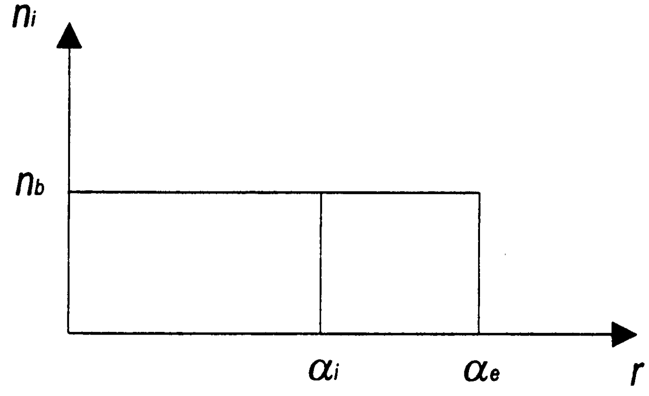

Fig. 3 - The density distribution of ions and primary electrons.
a_i: ion beam radius, a_e: electron beam radius, n_i: ion density.

The radial electric field of neutralized electron beam is equal to

$$E_{sn}=\begin{cases}0 & 0<r<a_i\\ \dfrac{n_b e}{2\varepsilon_0}r\left(1-\eta\dfrac{a_e^2}{r^2}\right) & a_i<r<a_e,\\ \dfrac{n_b e a_e^2}{2\varepsilon_0 r}(1-\eta) & a_e<r<b\end{cases} \tag{8}$$

where η is the neutralization factor, a_i is the ion column radius, a_e is the electron beam radius, b is the drift chamber radius. The electron beam has a space charge (η = 0) between the collector and the neutralization electrode and between the gun and the neutralization gun electrode (see Fig. 1). The radial electric field on the charge electron beam is equal to

$$E_{sc}=\begin{cases}\dfrac{n_b e}{2\varepsilon_0}r & a_i<r<a_e\\ \dfrac{n_b e a_e^2}{2\varepsilon_0 r} & a_e<r<b\end{cases} \tag{9}$$

Due to the beam space charge the stored electrons rotate in the azimuthal direction. The change of azimuthal electron angle between two electron reflections is equal to

$$\Delta\psi(r)=\tau_{\|}\frac{\omega_{dc}L_1+\omega_{dn}(r)L_t}{L}\equiv\tau_{\|}\omega_{def}, \tag{10}$$

where

$\tau_{\|}$ is the half-period of stored electron oscillations,

L_t is the distance between traps,

$\omega_{dn}=E_{sn}/Br$ is the drift frequency of the neutralized electron beam,

L_1 is the length of the charged electron beam,

$\omega_{dc}=E_{sc}/Br$ is the drift frequency of the charged beam.

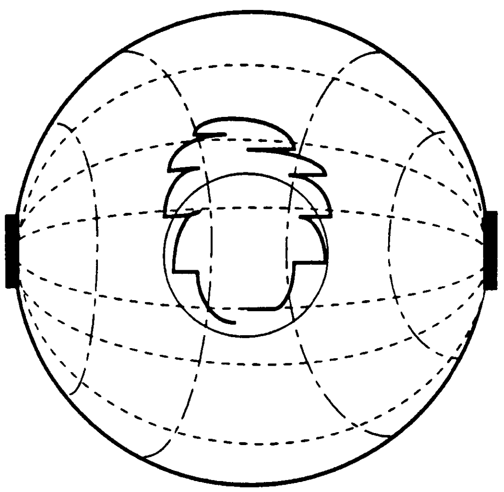

Fig. 4 - The trajectory of the stored electrons in the radial-azimuthal plane

The trajectories of stored electrons in the transverse plane r, ψ are determined by the displacement inside the traps and the azimuthal rotation due to the beam space charge (Fig.4).

The behavior of the stored electrons is also determined by the bending toroid magnets. When stored electrons pass through the bending magnet, they obtain a trajectory displacement (Fig. 5a):

$$\Delta y = \rho\varphi_* \quad (11)$$

where

ρ is the Larmor radius of stored electrons,

$\rho = v_s/\omega_B$, v_s is the stored electron velocity inside the drift tube,

ω_B is the electron cyclotron frequency,

φ_* is the bending angle; for LEAR $\varphi_* = \pi/5$.

When stored electrons pass through the second bending magnet, this displacement is partly compensated by azimuth rotation electrons, which is related to the beam space charge.

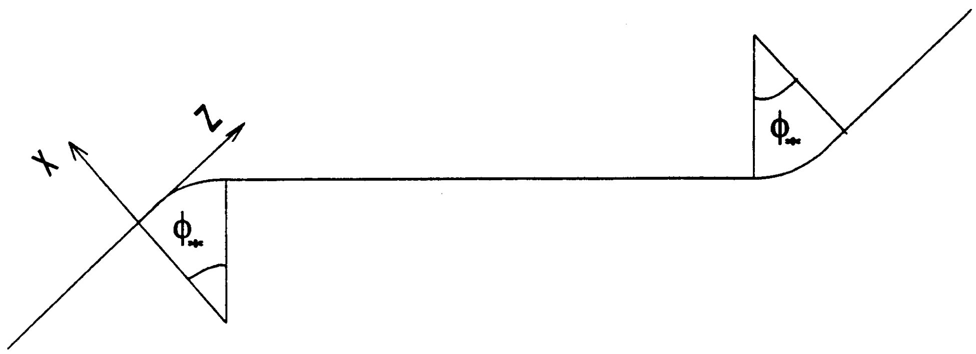

Fig.5a - The system with S-shape bending toroid magnets

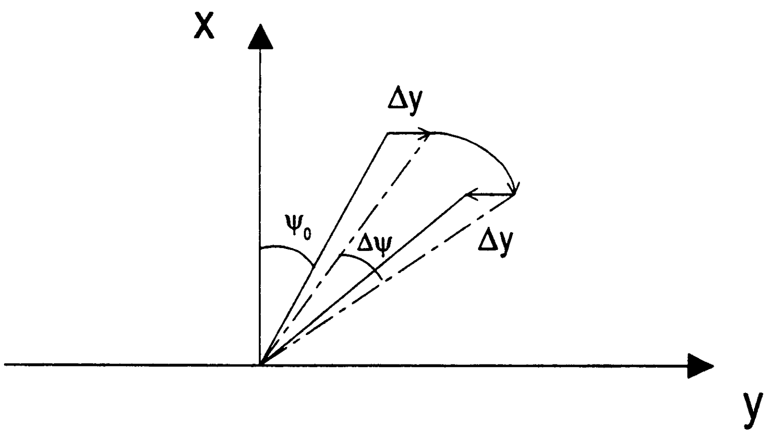

Fig. 5b - Displacement of stored electrons passing trough a system composed of two S-shape bending magnets

The trajectory of stored electrons in the transverse plane for the S-shape electron cooling system without ion traps is presented below.

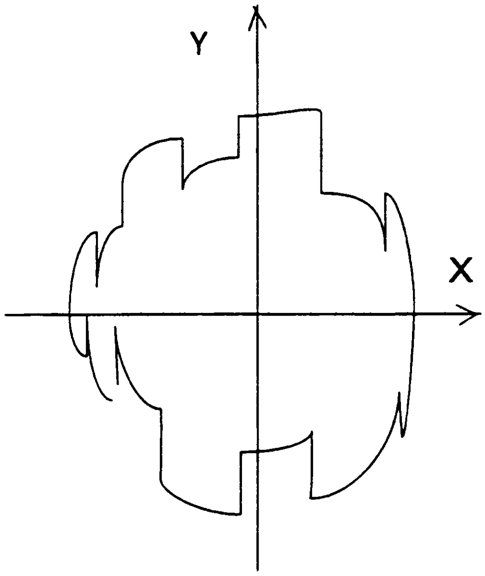

Fig. 6 - The trajectory of stored electrons in the S-shape electron cooling system

2. THE STORED ELECTRON TRAJECTORIES IN THE SYSTEM WITH ION TRAPS

Inside the ion traps (Fig. 2) with different electrode voltages there exists a transverse electric field [5]. The radial and azimuthal components of the electric field are equal to:

$$E_\psi = -\frac{U_{2t} - U_{1t}}{\pi b} \frac{\left(b^2 - r^2\right) b^2 \sin\psi}{\left(b^2 - r^2\right)^2 + (br\cos\psi)^2},$$
$$E_r = \frac{U_{2t} - U_{1t}}{\pi b} \frac{\left(b^2 + r^2\right) b^2 \cos\psi}{\left(b^2 - r^2\right)^2 + (br\cos\psi)^2}, \qquad (13)$$

where

b is the trap radius,

r is the radial coordinate,

ψ is the azimuthal coordinate,

U_{2t}, U_{1t} are the trap electrode voltage.

The equipotential line equation $\varphi = \text{const}$ can be represented as

$$\frac{r\cos\psi}{b^2 - r^2} = \text{const}. \qquad (14)$$

The radial displacement of the electrons inside the two traps during a single pass through the electron cooling system is equal to

$$dr = \frac{2E_\psi}{v_{s1}B} l_t. \qquad (15)$$

The change of the azimuthal angle during this time can be estimated by

$$d\psi = -\frac{2E_r}{v_{s1}B}\frac{l_t}{r} + \omega_{def}\tau_{\parallel}, \tag{16}$$

where E_r, E_ψ are the radial and azimuthal electric fields inside the traps, ω_{def} the drift frequency (Eq. (10)). Let us consider fast stored electrons and suppose

$$\begin{aligned} \Delta\psi \equiv \omega_{def}\tau_{\parallel} &<< 1, \\ \frac{2\Delta X}{b} &<< 1, \end{aligned} \tag{17}$$

where ΔX is the displacement inside the trap (Eq. (7)). For LEAR parameters: L = 4.2 m, L_t = 3.2 m, L_1 =1 m, b = 5 cm, l_t = 10 cm, U_{2t} = U_{4t} = 6 kV, U_{1t} = U_{3t} = 0, a_e = 2.5 cm, a_i = 2 cm, η = 0.64, B =600 Gs,

a) U = 10 kV, I = 1 A, $\Delta\psi(a_i)$ = 0.13, $\Delta\psi(a_e)$ = 0.27, $2\Delta X/b$ = 0.042,

b) U = 3 kV, I = 0.3 A, $\Delta\psi(a_i)$ =0.13, $\Delta\psi(a_e)$ =0 .27, $2\Delta X/b$ =0.076,

where

$\tau_{\parallel}$ = L/v_s, v_s is the stored electron velocity outside traps,

v_s = $(2U/m)^{1/2}$,

U is the cathode voltage,

v_{s1} is the stored electron velocity inside traps:

$$v_{s1} = v_s\left(1 + \frac{U_{2t} - U_{1t}}{2U}\right)^{1/2}, \tag{18}$$

U_{4t}, U_{3t} are the electrode voltages of the second trap and I the beam current. In the case of Eq. (17), Eqs. (15) and (16) can be written in the following form:

$$\begin{aligned} \frac{dr}{d\tau} &= \frac{2E_\psi}{B}\frac{l_t}{v_{s1}} \\ r\frac{d\psi}{d\tau} &= \omega_{def}\tau_{\parallel}r - \frac{2E_r}{B}\frac{l_t}{v_{s1}}, \end{aligned} \tag{19}$$

where $\tau = t/\tau_{\parallel}$, t is the time. The equations describe the electron radial-azimuthal displacement averaged along stored electron trajectory (Fig. 7) for

$$r\frac{d\psi}{d\tau} = r\frac{d\psi}{d\tau}\frac{d\tau}{dr} = -\frac{E_r}{E_\psi} + \omega_{def}\tau_{\parallel}\frac{Bv_{s1}}{E_\psi}\frac{r}{l_t}. \tag{20}$$

The electron trajectory in the transverse plane r, ψ is determined by Eq. (18).

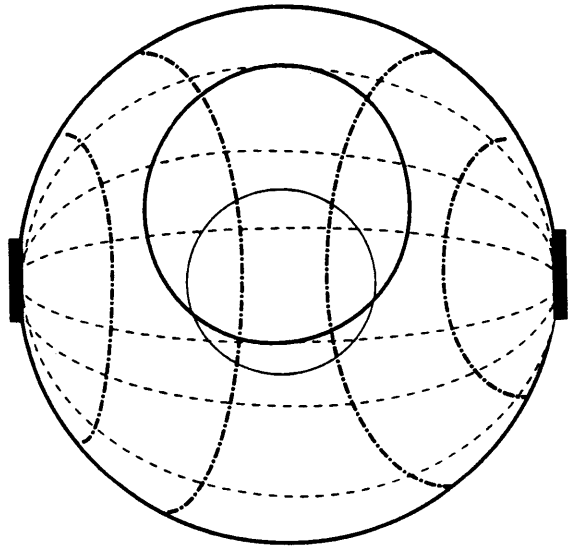

Fig. 7 - Trajectory of stored electrons in the radial-azimuthal plane averaged along trajectory trap displacements

The solution of this equation is represented in the following form for $0 < r < a$:

$$r^2 = r_0^2 + \frac{2b^2}{\alpha}\left(\tilde{U} - \tilde{U}_0\right) \tag{21}$$

where

$$\tilde{U} = \mathrm{arctg}\frac{br\cos\psi}{b^2 - r^2}, \quad \tilde{U}_0 = \mathrm{arctg}\frac{br_0\cos\psi_0}{b^2 - r_0^2}, \quad \alpha = \omega_{def}\tau_{\parallel}\frac{\pi bBv_{s1}}{2\left(U_{2t} - U_{1t}\right)} \tag{22}$$

Here r_0, ψ_0 are the initial electron trajectory radius and azimuthal angle respectively, ω_{def} = $(n_b e/2\varepsilon B)L_1/L$ is the effective drift frequency of the electron beam outside the neutralization system. For a stored electron trajectory of radius $a_e > r > a_i$ and effective drift frequency one can write the solution of Eq. (20) (see Eqs. (8)-(10)):

$$r^2 = r_1^2 + \frac{L_t}{L}\eta a_e^2 \ell n\frac{r^2}{r_1^2} + \frac{2b^2}{\alpha_2}\left(\tilde{U} - \tilde{U}_1\right), \tag{23}$$

where

η is the neutralization factor,

L is the distance between the gun exit and the collector entrance,

L_t is the length of the neutralized beam,

L_1 the length of the charged beam,

$\alpha_2 = \alpha(\omega_{dc})$, $\omega_{dc} = n_b e/2\varepsilon_0 B$,

$$\tilde{U}_1 = \begin{cases}\tilde{U}_0 & a_i < r_0 < a_e \\ \tilde{U}_{01} & 0 < r_0 < a_i\end{cases}, \quad r_1 = \begin{cases}r_0 & a_i < r_0 < a_e \\ a_i & 0 < r_0 < a_i\end{cases}.$$

Here $U_{01} = \mathrm{arctg}(ba_i \cos\psi_{01}/(b^2-a_i^2))$, ψ_{01} is the azimuthal angle for $r = a_i$.

The stored electron trajectory of radius $b > r > a_e$ is described by formula (24)

$$\ell n\frac{r}{r_2} = \frac{1}{\alpha_3}\left(\tilde{U} - U_{02}\right) \tag{24}$$

where

$$\alpha_3 = \alpha\left(\omega_{dn}(b)\right),\quad \omega_{dn}(b) = \frac{n_b e}{2\varepsilon_0 B}\frac{a_e^2}{b^2}\left(1-\eta\frac{L_1}{L}\right),\quad \tilde{U}_{02} = arctg\frac{ba_e\cos\psi_{02}}{b^2-a_e^2}$$

$$U_{02} = \begin{cases}\tilde{U}_0 & a_e < r_0 < b, \\ \tilde{U}_{02} & 0 < r_0 < a_e,\end{cases} \qquad r_2 = \begin{cases} r_0 & a_e < r_2 < b, \\ a_e & 0 < r_0 < a_e,\end{cases}$$

ψ_{02} is the azimuthal angle for $r = a_e$. There are two kinds of stored electron trajectories (Fig. 8). Trajectory (3) corresponds to the crucial parameter α_{3*} ,when single-turn trajectories come into multiturn trajectories. The crucial value of α_{3*}, when $r = b$, $\psi = 0$ in equation (24), is

$$\alpha_{3*} = \frac{\pi}{2}\frac{\left(1-(2/\pi)\tilde{U}_2\right)}{\ell n(b/r_2)}\ . \tag{25}$$

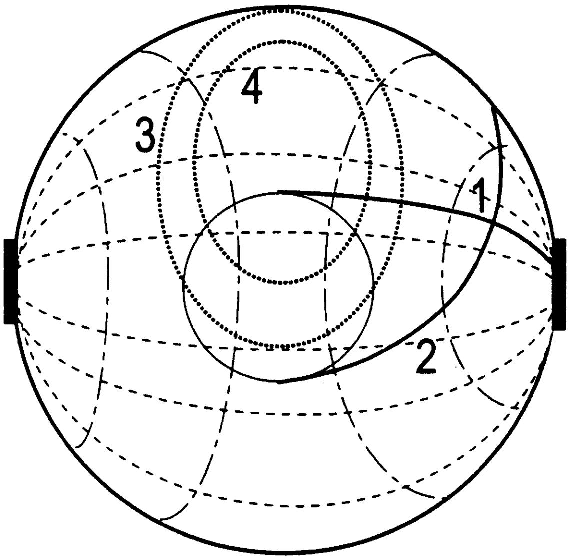

Fig. 8 - Trajectories of stored electrons for the primary electron beam with different space charges.
1-2) Single-turn trajectories, 3-4) multiturn trajectories.

The minimal value of α_{3*},when $r \le a_e$ is equal to

$$\alpha_{3*} = \frac{\pi}{2}\frac{1}{\ell n(b/a_e)}\left(1-\frac{2}{\pi}\mathrm{arctg}\frac{ba_e}{b^2-a_e^2}\right). \tag{26}$$

This value corresponds to the crucial beam density, with multiturn trajectory, for stored electrons produced inside the primary beam,

$$n_{b*} = \alpha_{3*} \frac{l_t}{a_e} \frac{2(U_{2t} - U_{1t})\varepsilon_0}{a_e L e[1 - \eta(L_t/L)]} \frac{1}{[1 + (U_{2t} - U_{1t})/2U]^{1/2}}. \tag{27}$$

For typical LEAR parameters $U_{2t} = 6$ kV, $U_{1t} = 0$, $L = 4.2$ m, $a_e = 2.5$ cm, $l_t = 10$ cm we obtain the crucial value of n_{b*}

$$n_{b*} = \frac{2.3 \times 10^{13}\ \mathrm{m}^{-3}}{(1 - \eta(L_t/L))(1 + (3\ \mathrm{kV}/U\ (\mathrm{kV})))^{1/2}}.$$

This crucial density n_{b*} corresponds to a charged electron-beam current without neutralization ($\eta = 0$)

a) $U = 10$ kV, $I_* = 0.39$ A,

b) $U = 3$ kV, $I_* = 0.15$ A,

or to a neutralized beam current with a neutralization factor $\eta = 0.64$, $a_e = 2.5$ cm, $a_i = 2$ cm, $L_t = 3.2$ m

a) $U = 10$ kV, $I_* = 0.75$ A

b) $U = 3$ kV, $I_* = 0.29$ A.

Single-turn trajectories are realized for low space-charge electron beams, when $n_b < n_{b*}$. The dependence of the trajectory radius for $r < a_i$ on the azimuthal angle and on time is given by following formulae with an accuracy $r^2/b^2 < 1$:

$$r^2 = \left(\frac{b^2}{\alpha^2} \cos^2 \psi + r_0^2 + \frac{2b}{\alpha} r_0 \cos\psi_0 \right)^{1/2} - \frac{b}{\alpha} \cos\psi,$$

$$r^2 = \tilde{r}^2 + \frac{b^2}{\alpha^2} - \frac{2b}{\alpha} \tilde{r} \sin(\omega_{def} t + \tilde{\psi}), \tag{28}$$

where r_0, ψ_0 are the initial trajectory radius and phase, respectively.

$$\tilde{r}^2 = r_0^2 + \frac{2b}{\alpha} r_0 \cos\psi_0 + \frac{b^2}{\alpha^2},$$

$$\tilde{\psi} = \arcsin\left(\frac{r_0 \cos\psi_0 - b/\alpha}{\tilde{r}} \right).$$

For stored electrons, produced at point $r_0 = 0$, we obtain the dependence of the trajectory radius on time

$$r = \frac{2b}{\alpha} \sin \omega_{def} t. \tag{29}$$

The time of escape from the ion beam radius for this stored electron is equal to

$$\tau_{es} = \frac{2}{\omega_{def}} \arcsin \frac{\alpha a_i}{2b} = \frac{2\tau_{\parallel}}{\Delta\psi} \arcsin \frac{\alpha a_i}{2b} \tag{30}$$

For an electron beam with the following parameters $\eta = 0.64$

a) $U = 10$ kV, $I = 0.4$ A, $\tau_{es} = 11\tau_{\parallel}$,

b) $U = 3$ kV, $I = 0.2$ A, $\tau_{es} = 7\tau_{\parallel}$.

The density of stored electrons inside the ion beam radius is estimated to be

$$\frac{n_{st}}{n_b} = \frac{I_{los}}{I_b}\frac{\tau_{es}}{2\tau_{\parallel}} \cong 2\times10^{-3}$$

for $I_{los}/I \approx 5\times10^{-4}$. For the electron beam "without space charge" the time of escape does not depend on the azimuthal turn angle between two stored electron reflection $\Delta\psi$. The time of escape from the ion beam radius, averaged at the initial radius r_0 and the azimuthal phase ψ_0 of the stored electron, is equal to

$$\tau_{es} = \left(\frac{9\pi}{32} + \frac{4}{3\pi}\right)\tau_{\parallel}\frac{a_i}{\Delta X} \tag{31}$$

For an electron beam with $\eta = 0.64$ this time of escape is estimated to be

a) $U = 10$ kV, $I = 0.3$ A, $\tau_{es} = 13\tau_{\parallel}$

b) $U = 3$ kV, $I = 0.1$ A, $\tau_{es} = 9\tau_{\parallel}$.

Considering an electron beam with a space-charge density

$$n_b > n_{b*}, \tag{32}$$

where n_{b*} is the crucial density for single-turn trajectories. This case corresponds to multiturn trajectories of stored electrons. The trajectories of stored electrons in the first approach are closed. This result is obtained when Eqs. (15) and (16) are transformed to differential Eqs. (16) and get the average trajectory of stored electrons. Due to a small kick inside the trap, the stored electron receives an additional displacement from the average trajectory (Fig. 9). We can try to estimate this diffusion displacement. The displacement between trajectory and average trajectory is equal to ΔX (Fig. 9). For an azimuthal angle $\pi > \psi > 0$ we have a drift motion and the full displacement is equal to

$$\Delta Z = \sum \Delta X \approx \frac{\Delta X}{\Delta\psi}.$$

For an azimuthal angle $2\pi > \psi > \pi$ this value is negative

$$\Delta Z \approx -\frac{\Delta X}{\Delta\psi}.$$

The azimuthal phase ψ through one turn is occasional and we obtain diffusion with displacement $\pm\Delta Z$. The square of displacement increases with time as

$$\Delta\bar{r}^2 = \Delta Z^2 \omega_{def} t$$

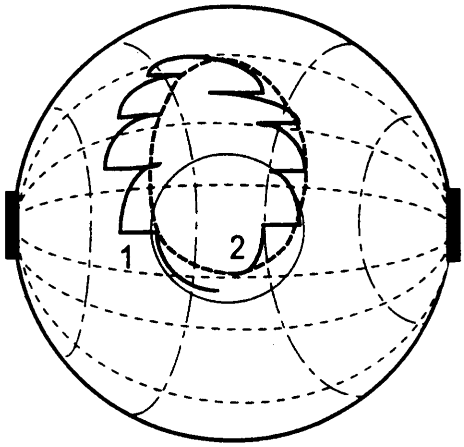

Fig. 9 - The real and average trajectory of stored electron

The dependence of displacement from average trajectory on time is given by:

$$r^2 = \bar{r}_i^2 + \frac{b^2}{\Delta\psi}\left(\frac{\Delta X^2}{b^2}\right)\frac{t}{\tau_{||}}, \tag{33}$$

where $\bar{r}_i$ is the radial coordinate of the average trajectory. The time of escape is estimated to be

$$\tau_{es} = \tau_{||}\left(\frac{b^2}{\Delta X^2}\right)\Delta\psi. \tag{34}$$

When the azimuthal turn angle between two reflections

$$\Delta\psi >> 1,$$

the stored electrons circulate in an azimuthal direction and obtain displacements $\pm\Delta X$ inside traps. The azimuthal angle in this case is occasional during one reflection. The displacement from a circular trajectory increases with time as

$$r^2 = r_0^2 + b^2\left(\frac{\Delta X^2}{b^2}\right)\frac{t}{\tau_{||}}.$$

The time of escape for stored electrons is estimate as

$$\tau_{es} \approx \tau_{||}\left(\frac{b^2}{\Delta X^2}\right). \tag{35}$$

The dependence of time escape for stored electrons on azimuth turn angle $\Delta\psi$ is given in Fig. 10

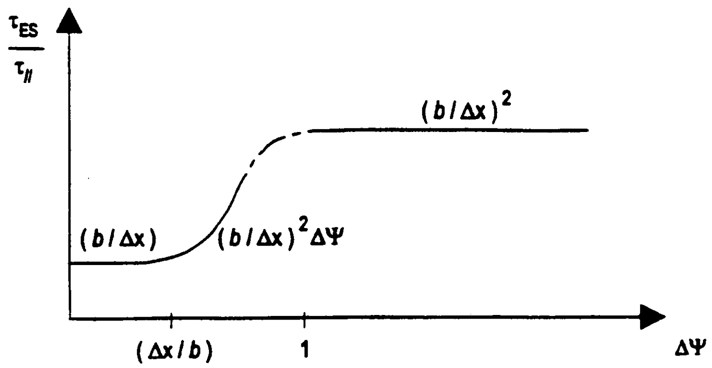

Fig. 10 - Dependence of time escape on the azimuthal angle $\Delta\psi$ between stored electron reflections

The escape time for stored electrons is determined by the relation between two parameters $\Delta\psi$ and $\Delta X/b$. When $\Delta\psi \equiv \Delta X/b$, stored electrons have single-turn trajectories and during time $\tau_{es} \equiv 10\tau_{\parallel}$ escape the system. For typical LEAR parameters the time for multiturn escape is estimated to be:

a) $\eta = 0$, $U = 10$ kV, $I = 0.6$ A, $\tau_{es} = 220\ \tau_{\parallel}$,

b) $\eta = 0$, $U = 3$ kV, $I = 0.25$ A, $\tau_{es} = 97\ \tau_{\parallel}$,

c) $\eta = 0.64$, $U = 10$ kV, $I = 0.8$ A, $\tau_{es} = 117\ \tau_{\parallel}$,

d) $\eta = 0.64$, $U = 3$ kV, $I = 0.3$ A, $\tau_{es} = 50\ \tau_{\parallel}$.

The density of stored electrons in the electron cooling system with ion traps for multiturn trajectories is equal to

cases a, b) $$\frac{n_{st}}{n_b} = \left(\frac{I_{los}}{I}\right)\left(\frac{\tau_{es}}{2\tau_{\parallel}}\right) \equiv 0.1 - 0.02,$$

cases c, d) $$\frac{n_{st}}{n_b} = \left(\frac{I_{los}}{I}\right)\left(\frac{\tau_{es}}{2\tau_{\parallel}}\right) \equiv 0.01 - 0.05,$$

where $I_{los}/I \approx 5 \times 10^{-4}$. Really this density is reduced due to additional escape in the toroid bending magnets.

3. ESCAPE OF STORED ELECTRONS FROM THE SYSTEM WITH S-SHAPE BENDING MAGNETS

The stored electron trajectory in the electron cooling system with S-shape bending magnets is determined by two parameters:

$$\Delta\psi = \omega_d \tau_{\parallel}$$
$$\frac{\Delta r_0}{b} = \frac{2\rho\varphi_*}{b}\sin\frac{\Delta\psi}{2}, \qquad (37)$$

where $\Delta\psi$ is the rotational angle due to space charge effects, Δr_0 is the displacement of the electron when it passes through both bending magnets, ρ is the Larmor radius (see Eq. (11)), and φ_* the bending angle. We consider the case, when

$$\begin{aligned} &\Delta\psi << 1, \\ &\Delta r_0/b << 1 \end{aligned} \qquad (38)$$

For small azimuthal and radial displacement one obtains the following equations

$$\begin{aligned} &\frac{dr}{dt} = \frac{\Delta r_0}{\tau_{\|}} \cos\psi \\ &r\frac{d\psi}{dt} = \omega_d r + \Delta r_0 \sin\psi \end{aligned} \qquad (39)$$

where t is the time, r the radial coordinate, ψ the azimuthal coordinate. The solution of this equation is given by following formula

$$\ell n\frac{r}{r_0} = \Delta r_0\left(\frac{\sin\psi_0}{r_0} - \frac{\sin\psi}{r}\right) \qquad (40)$$

where r_0, ψ_0 are the initial radial coordinate and azimuthal angle, respectively. This is the average stored electron trajectory (Fig. 11).

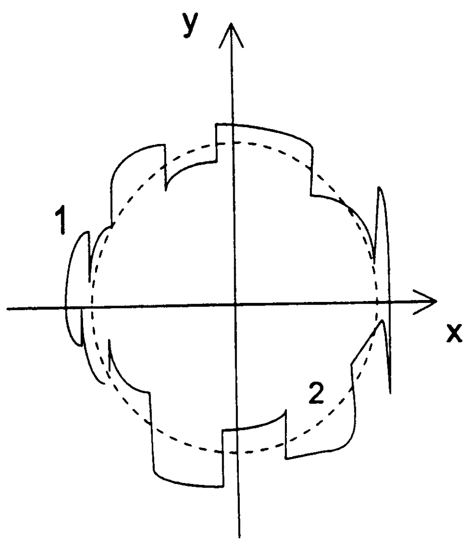

Fig. 11 - 1) Average trajectory of stored electrons; 2)Real trajectory of stored electrons.

The escape of stored electrons from averaged trajectories is connected with diffusion, which appears due to a kick in the bending magnets. The full displacement in both magnets for stored electrons with azimuthal angle $\pi > \psi > 0$ is equal to

$$\Delta Z \equiv \frac{\Delta r_0}{\Delta\psi} \equiv \rho\varphi_*.$$

The full displacement in both magnets for stored electrons with azimuthal angle $2\pi > \psi > \pi$ is estimated as

$$\Delta Z \cong -\frac{\Delta r_0}{\Delta\psi} \cong -\rho\varphi_*.$$

The azimuthal angle through one turn is occasional and one obtains the diffusion escape

$$r^2 = r_0^2 + \Delta Z^2 \omega_d t = r_0^2 + \rho^2 \varphi_*^2 \Delta\psi \frac{t}{\tau_{||}}.$$

The escape time is equal to

$$\tau_{es} = \tau_{||}\left(\frac{b^2}{\rho^2\varphi_*^2}\right)\frac{1}{\Delta\psi}. \qquad (41)$$

When the rotation angle $\Delta\psi >> 1$, the escape time is estimated to be

$$\tau_{es} = \tau_{||}\left(\frac{b^2}{\rho^2\varphi_*^2}\right). \qquad (42)$$

The dependence of the escape time on the azimuthal rotation angle $\Delta\psi$ for *S*-shape and *U*-shape bending toroidal magnets are presented in Fig. 12.

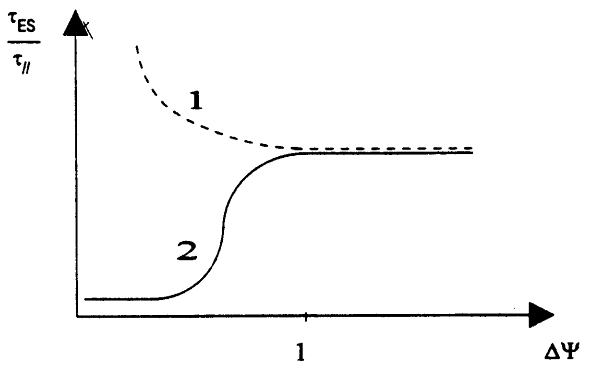

Fig . 12 - The dependence of escape time on the azimuth turn angle $\Delta\psi$.
1) S-shape bending toroid magnets, 2) U-shape magnets.

The values of the escape time for stored electrons in the system with *S*-shape bending magnets and with electrostatic ion traps are given in Table 1. The density of stored electrons for multiturn trajectories in the system with traps and with *S*-shape bending magnets is equal to

$$\frac{n_{st}}{n_b} \cong (1 \div 3)10^{-2}.$$

The density of stored electrons for $I < I_*$ is equal to $n_{st}/n_b \approx 2 \times 10^{-3}$, for beam current $I > I_*$ it increase up to $n_{st}/n_b \approx 2 \times 10^{-2}$.

Table 1

Electron energy KeV	3	3	3	3	10	10	10	10
Beam current A	0.5	0.25	0.5	0.5	0.6	1.5	0.8	1.5
Trap voltage, U_{2tr} kV	6	6	6	6	6	6	6	6
Neutralisation factor	0	0	0.64	0.64	0	0	0.64	0.64
Magnetic field G	600	600	600	600	600	600	600	600
Escape time for traps $\tau_{es}/\tau_{\parallel}$	194	94	77	50	220	550	117	194
Escape time for S-shape bending magnets $\tau_{es}/\tau_{\parallel}$	134	84	69	48	66	35	63	54
Current losses, I_{los}/I_b 10^{-4}	5	5	5	5	5	5	5	5
Density of stored electrons, n_{st}/n_b	3.3×10^{-2}	2.1×10^{-2}	1.7×10^{-2}	1.2×10^{-2}	1.6×10^{-2}	8×10^{-3}	1.5×10^{-2}	1.3×10^{-2}

4. REFLECTED, SINGLE-PASS ELECTRONS

The current of reflected electrons which single pass through the electron cooling system is given in Ref. [6]. The current and energy distribution function for these electrons are calculated here using motion equations. We examine the case where the following conditions are satisfied:

$$U_c >> U_{rep}$$
$$U \geq U_c \quad (43)$$

where U_c is the collector potential with respect to the cathode voltage (U_c = 3kV), U_{rep} is the repeller potential with respect to the cathode (U_{rep} = 0,3-0,5 kV). The dependencies of the magnetic field and longitudinal electric field on the longitudinal coordinate and potential distribution are represented in Fig .13. The current of electrons reflected from the collector surface is determined by the efficiency of the magnetic trap and the electrostatic barrier. The efficiency of the magnetic trap depends on the relationship of the values of the magnetic field on the collector surface and near the repeller.

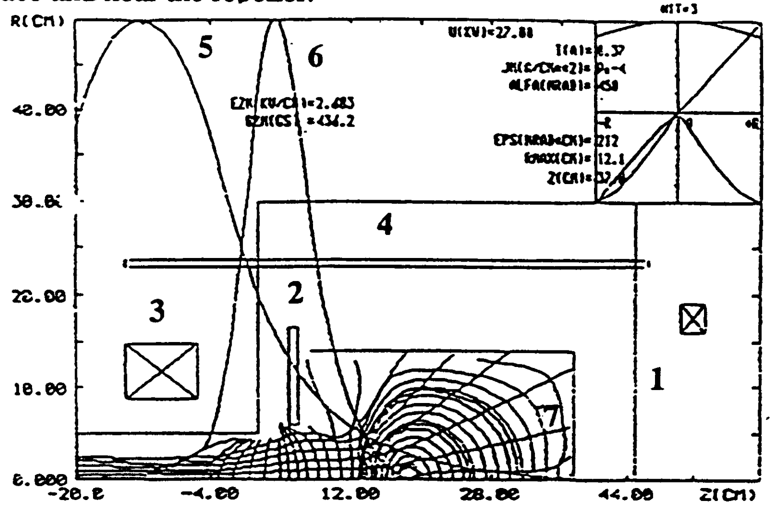

Fig. 13 - 1) Collector, 2) Repeller, 3) Collector coil, 4) Magnetic shielding, 5) Dependence of the magnetic field on the longitudinal coordinate, 6) Dependence of the electric field on the longitudinal coordinate, 7) Equipotential line.

For typical LEAR conditions the magnetic field in the drift tube is equal to B = 600 G, the magnetic field inside the collector coil is about Bc = 800 G, the magnetic field near by the repeller Br =100-150 G, on the collector surface it is equal to Bs =15-20 G. The cosine angle distribution of the secondary electron flux from the collector surface is given by:

$$\frac{dj_{ref}}{d\Omega} = \frac{j_s}{\pi}\sigma\cos\theta, \tag{44}$$

where j_s is the beam density on the collector surface, σ is the secondary emission coefficient, and θ the angle of the secondary electron velocity normal to the collector surface. The energy distribution for these electrons can be approximated by:

$$\frac{d\sigma}{dE} = \frac{a_c}{U_c} + b_0\delta(E - eU_c) \tag{45}$$

where

E is the electron energy,

a_c is the secondary emission coefficient for elastic scattered electrons, $a_c \approx 1$,

b_0 is the secondary emission coefficient for inelastic electrons collisions, $b_0 \approx 0{,}03$.

For magnetized secondary electrons, when

$$\rho_c << L_c \tag{46}$$

the transverse adiabatic invariant along the electron trajectory is conserved, i.e.

$$\frac{v_\perp}{B^{1/2}} = \frac{v_{\perp 0}}{B_s^{1/2}}$$

where

$v_\perp$ is the transverse velocity of electrons with respect to the magnetic line,

$v_{\perp 0}$ is the initial transverse velocity on the collector surface,

B and B_s are the magnetic fields inside the collector and on its surface,

$L_c \approx 30$ cm is the collector length,

$\rho_c = v_c/\omega_{Bs}$ is the Larmor radius of the electrons on the collector surface,

$v_c = \sqrt{2eU_c/m}$ is the secondary electron velocity on the collector surface;

$\omega = e\,B_s/m$.

The energy conservation law, when applied along the secondary electron trajectory, can be represented by:

$$\frac{mv_s^2}{2} + \frac{mv_{\perp 0}^2}{2}\frac{B}{B_s} - e\varphi = \text{const}, \tag{48}$$

where v_s is the longitudinal electron velocity along the magnetic field, φ is the potential, and B the magnetic field in a point inside the collector. The energy of secondary electrons which escape the collector is characterized by the repeller electrostatic barrier

$$eU_c > E \equiv \frac{mv_{\perp 0}^2}{2} + \frac{mv_{sc}^2}{2} > e(U_c - U_{rep}) \tag{49}$$

It is represented by a circle on the phase plane, which is given in Fig. 14. Here $mv_{\perp 0}^2/2$ and $mv_{sc}^2/2$ are the transverse and longitudinal electron energy on the collector surface, $U_1 = U_c - U_{rep}$.

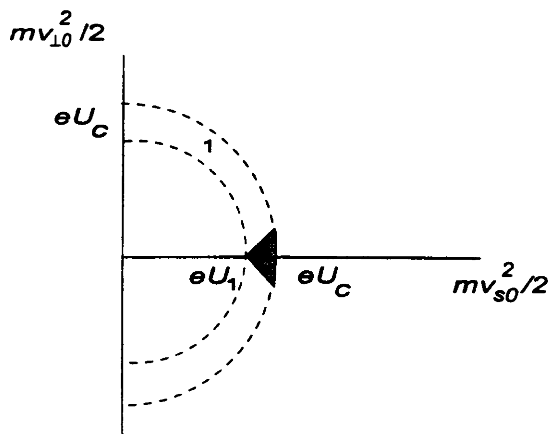

Fig. 14 - The phase plane in energy transverse and longitudinal coordinates on the collector surface

From the law of the energy conservation one obtains the following relation between the longitudinal and transverse energy on the collector surface

$$\frac{mv_{sc}^2}{2} - e(U_c - U_{rep}) - \frac{mv_{\perp 0}^2}{2}\left(\frac{B_r}{B_s} - 1\right) > 0 \tag{50}$$

where B_r and B_s are the magnetic field near the repeller and on the collector surface. The longitudinal velocity near the repeller $v = 0$, hence the transverse velocity here can be estimated as $v_\perp = (Br/B_s)v_{\perp 0}$. Equation (50) on the phase plane respect to sector (2). The maximum transverse velocity of electrons located inside this sector (2) can be calculated from Eqs. (50) and (51)

$$\frac{mv_{sc}^2}{2} + \frac{mv_{\perp 0}^2}{2} = eU_c \tag{51}$$

It is equal to

$$\Delta E = eU_{rep}\frac{B}{B_r}. \tag{52}$$

Only the electrons located inside sector (2) on the phase Plane of Fig. 14 escape the collector. The secondary electron flux from the collector is given by the relationship

$$j_{ref} = \frac{j}{\pi}\int_{s_2}\frac{d\sigma}{dE}2\pi\sin\theta\cos\theta\, d\theta dE \tag{53}$$

where j is the beam current density in the drift tube, j_{ref} the secondary electron current density in the drift tube, S_2 is the sector (2) in Fig. 14. From the expression of σ given by Eq. (45) we obtain, after integration the following result,

$$j_{ref} = 2j\left(a_c \frac{U_{rep}}{U_c} + b_0\right)\frac{U_{rep}}{U_c}\frac{B_s}{B_r}. \tag{54}$$

For a typical LEAR case $U_{rep}/U_c >> b_0$, the secondary electron current density from the collector is equal to

$$j_{ref} = 2ja_c\left(\frac{U_{rep}}{U_c}\right)^2 \frac{B_s}{B_r} \tag{55}$$

The current density of secondary electrons from collector for typical LEAR parameters $U_{rep}/U_c \approx 0{,}1 - 0{,}3$; $B_s/B_r \approx 0{,}1 - 0{,}2$ is equal to:

$$\frac{j_{ref}}{j} \cong 2\times 10^{-3} \div 10^{-2} \tag{56}$$

The longitudinal and transverse energy distribution function is an important characteristic for electrons reflected from the collector. The longitudinal and transverse energy of secondary electrons, which escape the collector, near the repeller, is equal to

$$\begin{aligned} eU_{rep} &> \frac{mv_{sr}^2}{2} > 0, \\ eU_{rep} &> \frac{mv_{\perp r}^2}{2} > 0, \end{aligned} \tag{57}$$

where

$$\frac{mv_{\perp 0}^2}{2} = eU_{rep}\frac{B_s}{B_r}$$

In the drift tube these values can be represented as

$$\begin{aligned} eU_0 > E_s > eU_0 - \Delta E, \\ \Delta E > E_\perp > 0 \\ \Delta E = eU_{rep}\frac{B}{Br} \end{aligned} \tag{58}$$

where B, Br represent the magnetic field in the drift tube and near the repeller, respectively. The longitudinal and transverse energy distribution functions in the drift tube are given in Fig . 15

The Larmor radius of the secondary electrons in the drift tube is equal to

$$\rho_d = \frac{\sqrt{2\Delta E/m}}{\omega_B} \cong 0.3\,\text{cm}$$

for B = 600 G, Br = 150 G, an electron beam radius a_e =2.5 cm and a drift chamber radius b = 5cm, the following condition is satisfied:

$$a_e + \rho_d < b. \tag{60}$$

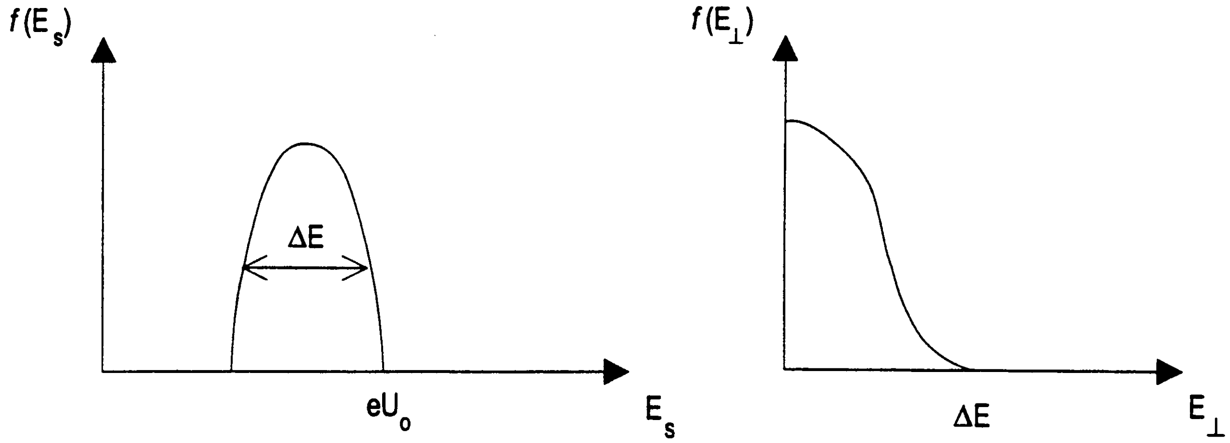

Fig. 15 - The longitudinal and transverse energy distribution in the drift tube

This means that almost all secondary electrons, reflected from the collector, through a single pass in the electron cooling system return to the collector without current losses. In this case the current of reflected electrons from the collector is larger than current losses. The density of reflected electrons is equal to

$$\frac{n_{ref}}{n_b} \cong 2\times10^{-3} + 10^{-2} \tag{61}$$

where n_b is the beam density.

5. THE CURRENT OF STORED ELECTRONS IN THE SYSTEM WITHOUT ION TRAPS

The density of stored electrons is determined by their reflection at the collector entrance and time of escape from the beam. Here the reflection coefficient on the collector entrance,which is characterized by current losses for stationary conditions, is examined.

The phase energy plane represented in Fig. 16 corresponds to secondary electrons, which escape the collector. This phase plane is related to the electrons, which pass through the region near the repeller.

The secondary electrons which escape the collector after reflection from the gun return in the collector. Part of their longitudinal energy is transformed into transverse energy due to non-adiabatic motion on the collector entrance. As a result, the electrons with small longitudinal energy near the repeller

$$E_{sr} < E_* \qquad (62)$$

$$E_{\perp r} < E_*$$

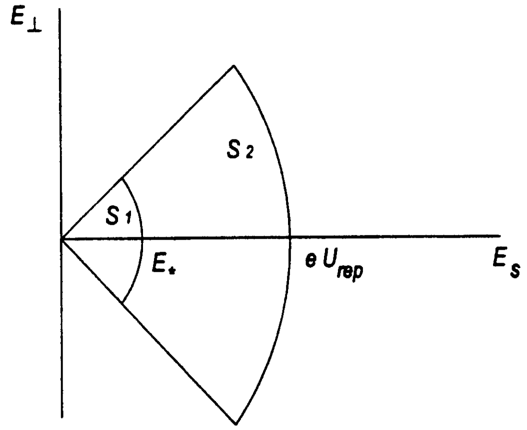

Fig. 16 - The phase plane in the transverse and longitudinal coordinates for electrons which pass through the repeller region

are reflected from the repeller potential barrier and stored in the electron cooling system. The change of the transverse energy after single pass through the system is equal to

$$\Delta E_\perp = \sqrt{E_{\perp r} eU}\,\frac{a_e}{\Delta L}\exp\left[-\frac{\Delta L}{\rho_*}\right] \qquad (64)$$

where

$E_{\perp r}$ is the transverse energy near the repeller for secondary electrons,

U is the cathode voltage,

a_e is the electron beam radius,

ΔL is the length of the scale for decelerated electric field on the collector entrance,

$\rho_* = v/\omega_{Br}$,

$v = \sqrt{2eU/m}$,

$\omega_{Br} = eB_r/m$.

The transverse energy here is comparable to the longitudinal energy.

$$E_{\perp r} \approx \Delta E_\perp \equiv E_* \approx eU\frac{a_e^2}{\Delta L^2}\exp\left[-\frac{2\Delta L}{\rho_*}\right] \qquad (64)$$

For a LEAR case (a_e = 2.5 cm, ΔL = 5 cm, U = 10 kV, ρ_* = 2.5 cm) this value E_* is equal to

$$E_* \approx 50 \text{ eV}.$$

The current density of stored electrons can be calculated from the following formula

$$\frac{j_{St}}{j_{ref}} = \frac{\int\limits_{S_1} f(E) v_\perp dv_\perp dv_\parallel}{\int\limits_{S_2} f(E) v_\perp dv_\perp dv_\parallel} \approx \left(\frac{E_*}{eU_{rep}}\right)^{3/2} \cong 3\times10^{-2} \tag{65}$$

where S_1 is the sector with energy $E < E_*$ and S_2 the sector with energy $E < eU_{rep}$ in Fig. 16. The current of stored electrons is equal to current losses for stationary conditions. The value of the stored electron current is estimated to be

$$I_{st} \equiv I_{los} = I_{ref}\left(\frac{E_*}{eU_{rep}}\right)^{3/2} \approx Ia_c \frac{U^{3/2}U_{rep}^{1/2}}{U_c} \frac{a_e^3}{\Delta L^3} \frac{B_s}{Br} \exp\left[-\frac{3\Delta L}{\rho_*}\right] \tag{66}$$

The value of stored electrons for LEAR condition ($U = 10$ kV, $U_{rep} = 0.5$ kV, $U_c = 3$ kV, $B_s/Br = 0.125$, $a_e = 2.5$ cm, $\Delta L = 5$ cm) is equal to

$$I_{los}/I \approx 10^{-4} \tag{67}$$

6. THE CURRENT OF STORED ELECTRONS IN THE SYSTEM WITH ION TRAPS

The motion of stored electrons in the electron cooling system with traps is presented in Sections 1 and 2 of this report. Here one estimates the coefficient of reflection for the stored electrons from the collector potential barrier (see Fig. 13). The part of electrons with low energy near the barrier that is reflected oscillates between the collector and the cathode. This reflection is related to the electron displacement inside the traps and with radial potential distribution near the potential barrier. The radial potential distribution here is estimated to be

$$\tilde{U}(r) \approx (U - U_{\min}) + \frac{I}{4\pi\varepsilon_0 v_m}\left(1 - \frac{r^2}{a_m^2} + 2\ln\frac{b_c}{a_m}\right), \tag{68}$$

where

$U_{\min}$ is the potential of the collector electrostatic barrier,
v_m $= \sqrt{2eU_{\min}/m}$ is the electron velocity in the electrostatic barrier region,
$U_{\min}$ ≈ 0.2-0.3 kV, is comparable with the repeller potential,
$U_{\min}$ $\le U_{rep}$,
a_m is the beam radius in the potential barrier region,
b_c is the collector radius.

The second number in Eq. (68) is related to the potential distribution due to the electron beam space charge. The potential distribution near the potential barrier is given in Fig.17 a).

The escaping electrons pass through a region with a potential minimum U_1, the returning electrons pass through a region with a potential minimum U_2 due to displacements inside the traps. The electrons with initial azimuthal angle $\pi > \psi > 0$ obtain displacement inside the beam, where the potential minimum U_2 is higher than U_1. These electrons are reflected from the potential barrier, if their longitudinal energy E_* is less than

$$E_* < e(U_1 - U_2) = e\Delta U. \tag{69}$$

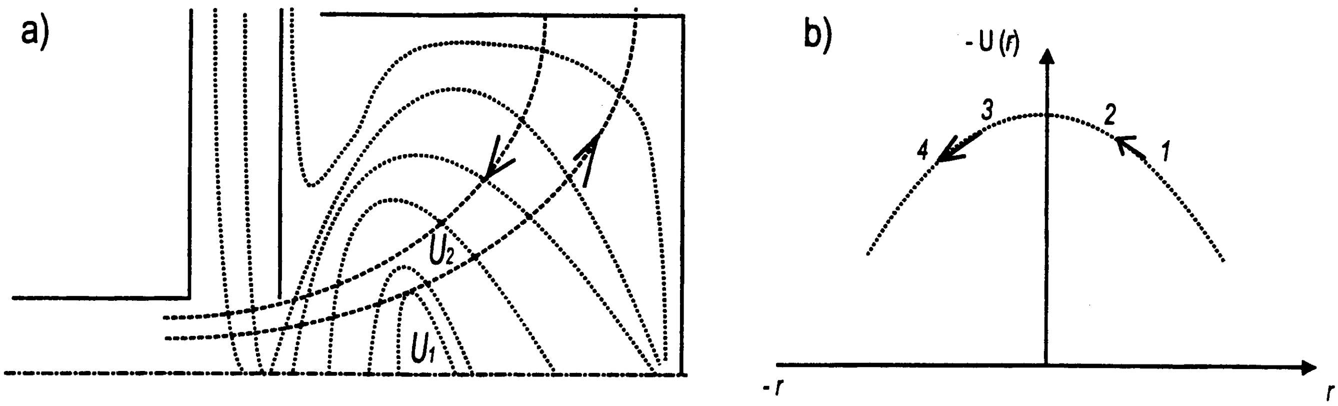

Fig. 17 - a) The potential distribution near the potential barrier.
b) The displacement of stored electrons in the collector potential barrier due to traps. 1, 3: points where electrons escape the collector; 2,4: points where it return in the collector due to displacement inside the traps.

The potential difference ΔU is equal to

$$\Delta U = \frac{I}{2\pi\varepsilon_0 v_m}\frac{r_m \Delta X_m}{a_m^2} \approx \frac{I}{2\pi\varepsilon_0 v_m}\frac{4\Delta X}{a_e} \tag{70}$$

where

ΔX_m is the displacement of the secondary electron in the collector potential barrier,

ΔX is the displacement of electron in the trap,

$4\Delta X$ is the full displacement of the electrons in the traps during one oscillation between the gun and the collector. The current density of stored electrons is equal to

$$\frac{j_{St}}{j_{ref}} \cong \frac{1}{2}\left(\frac{\Delta U}{U_{rep}}\right)^{3/2}, \tag{71}$$

where j_{ref} is the current density of the electrons reflected from the collector surface. The value "1/2" in Eq. (71) is related to electrons which obtain a displacement due to a trap inside the beam. One can estimate the current density of stored electrons

$$\frac{j_{St}}{j_{ref}} \cong \sqrt{2}\left(\frac{I}{4\pi\varepsilon_0 v_m U_{\min}}\right)^{3/2}\left(\frac{4\Delta X}{a_e}\right)^{3/2}. \tag{72}$$

The current loss in the electron cooling system with electrostatic traps is equal to

$$I_{St} \equiv I_{los} = I'_{los} + I_{ref}R, \tag{73}$$

where

$$R = \sqrt{2}\left(\frac{4\Delta X}{a_e}\right)^{3/2}\left(\frac{I}{4\pi\varepsilon_0 v_m U_{\min}}\right)^{3/2},$$

here I'_{los} represents the current losses without ion traps, I_{ref} is the current of electrons, reflected from the collector (see Eq. (55)). The current loss is proportional to the difference of the trap electrode voltage as (U_{2t} - U_{1t}) (see Eqs. (73) and (7)). The value R characterizes the part of the beam reflected from the collector, which oscillate between the gun and the collector. For a typical LEAR case (a_e = 2.5 cm, l_t = 10 cm, U_{2t} = 6 kV, U_{1t} = 0, $U_{\min}$ = U_{rep} = 0.5 kV) this coefficient is equal to

a) U = 10 kV, I = 1 A, R = 0.15; b) U = 3 kV, I = 0.3 A, R = 0.07.

The current losses without traps is estimated to be $I'_{los}/I = (0{,}5\text{-}2) \times 10^{-4}$. With the traps it is 2-4 times more higher $I_{los}/I = (2\text{-}5) \times 10^{-4}$. The density of stored electrons is determined by the current losses and the time of escape (see (4)). When current losses with traps are larger than without traps, the density of stored electrons depends on the trap voltage such that

$$\frac{n_{St}}{n_b} \cong \begin{cases} (U_{2t} - U_{1t})^{1/2} & I < I_*, \\ (U_{2t} - U_{1t})^{-1/2} & I > I_*. \end{cases} \qquad (74)$$

where U_{2t}, U_{1t} represent the trap electrode voltage. The density of stored electrons characterize the feedback coefficient [4]. For minimization of stored electron density is required the reduction of two parameters; the current losses and their time of escape.

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