PS/AA/Note 85-10

8.8.85

Determination of Electrostatic Beam Potentials by a Finite-Element Numerical Method

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Abstract

A method is presented for determining the electrostic beam potential due to the passage of a Gaussian beam through simple 2- and 3- dimensional structures. The method uses the powerful finite-element program DOT[1], which is described briefly, followed by a summary of how to implement various programs that have been developed to run DOT on CERN's CDC computers.

Introduction

The electrostatic potential, U, due to the presence of a beam of charged particles in a stainless-steel vacuum chamber, may be deduced by solution of the familiar equation

$$\nabla^2 U(r) = \frac{-\rho(r)}{\varepsilon_o} \qquad (1)$$

where $\rho(r)$ is the charge density as a function of position and ε_o the permittivity of the vacuum.

This potential gives rise to electric fields which can play an influential rôle in determining the dynamical behaviour of the beam itself. In addition, the fine structure of these fields for a negative space charge potential (such as is the case with $\bar{p}$ and e beams) governs various exotic phenomena linked to the trapping of residual gas ions and charged dust particles in the beam potential well[2]. These phenomena, believed to include the excitation of non-linear resonances and multiple Coulomb scattering[3], give rise to sometimes violent emittance growths, particularly at high stack intensities, in the AA. With the introduction of ACOL, the AA will be expected to handle large stacks; thus it is desirable to determine the electrostatic potential accurately in order to understand (and hopefully prevent) the unwanted effects of trapped positive charges.

There are, of course, simple situations for which the electric fields can be calculated exactly. For simple chamber geometries, and a uniform charge density throughout the beam, equation (1) reduces to Poisson's equation, which may in this case be solved rigorously. However, attempts to solve (1) for a more realistic beam profile and/or a complex chamber geometry result in series solution and Bessel-function approximations for U[4], which are not sufficiently accurate to provide the required information on the fine-structure of the resulting electric field. The obvious alternative is to use a numerical method to solve

equation (1), where (r) may take on any distribution, and boundary and initial conditions can be imposed as required. The program chosen, DOT, is a powerful finite-element program developed for use in the "Determination Of Temperatures". However, because of the complexity of finite-element programs, pre- and post-processing software had to be developed to convert DOT into the useful, easily handled tool for accelerator physicists it will hopefully prove to be.

The finite-element program DOT

DOT is designed to solve the non-linear heat transfer equation

$$\nabla^2 T + \frac{Q}{K} = \frac{1}{d}\frac{\partial T}{\partial t} \qquad (2)$$

where T is the temperature as a function of position and time, Q is the heat generation rate, K is the thermal conductivity and d the diffusivity, by means of the so-called "finite-element" formulation, wherein a solid continuum is idealised by an ensemble of discrete elements. These elements may be of variable shape and size, and are connected via node points which form the mesh. Any or all of the elements or nodes may carry boundary or initial condition parameters, which may be altered on a given node independently of all the others. DOT can handle planar 2-dimensional and axisymmetric geometries, both of which have been used here. (Each geometry can be generated in rectangular or curvilinear coordinates, though for this study only the former have been used. Hence we treat only rectangular cross-sectional and cylindrical chambers).

Clearly the use of DOT to solve equation (1) requires the RHS of (2) to be set to zero. We then substitute potential for temperature, charge density for heat generation rate, and electrical permittivity for thermal conductivity to recover the form of (1). It should be noted that there is thus considerable scope within DOT to improve the modelling of chambers and their beams (such as monitoring the effect of transverse movement of the beam with time) than has been exploited so far. It is hoped, however, that the facilities presented here will provide a framework around which future improvements can be made.

Formulation of the problem for the AA beam

Because of its complexity and flexibility, DOT requires a vast amount of input for each system studied. Even the moderately simple mesh shown in figure 1 generates some 2200 data entries if we set the outer rim to zero potential and all elements in the dense central (beam) portion are ascribed a number of charges. Since this is precisely the kind of thing we wish to do, we clearly require a pre-processing program which will generate all these data from a few input variables. The wish to run this pre-processor (and some of the other programs developed) interactively has resulted in all the computing facilities described being written and run on the CDC machines at CERN.

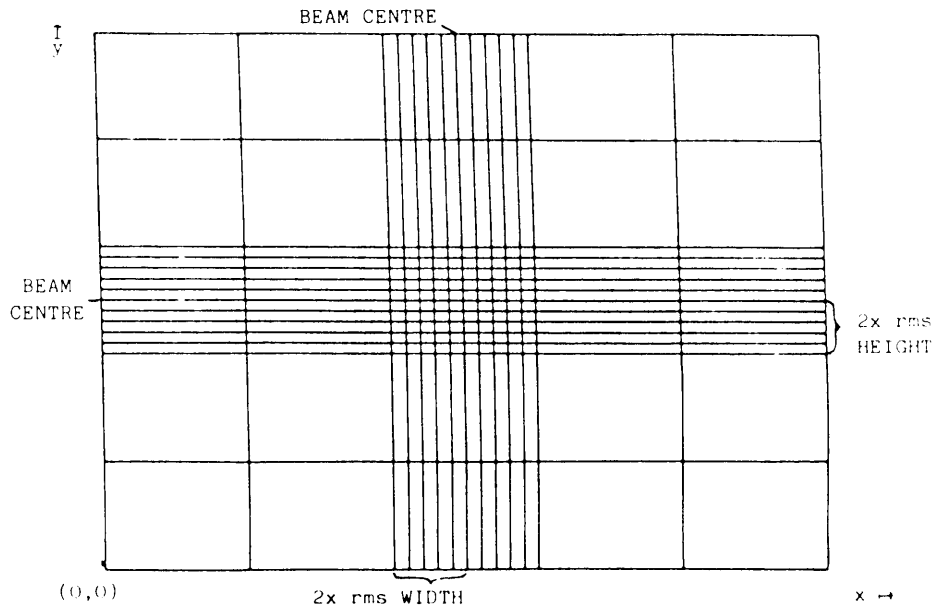


Figure1. Typical rectangular mesh used for generating DOT input.

The pre-processing program in its present form is a development of Alain Poncet's "MESHDOT" program called "NEAT". It is capable of dealing with the two geometries mentioned earlier. The first is a rectangular section transverse to the beam, such as in figure 1, with a dense portion of 10 x 10 elements* each ascribed a characteristic charge according to the uncorrelated bi-Gaussian beam distribution

$$\rho(x,y)dxdy = \frac{Ne}{2\pi C\sigma_H\sigma_V} e^{-\frac{x^2}{2\sigma_H^2} - \frac{y^2}{2\sigma_V^2}} dxdy \qquad (3)$$

This dense portion of the mesh extends over 2σ each side of the beam centre coordinates. (It is observed that the balance between natural or forced cooling effects, and coherent diffusive heating effects seems to produce beams in the AA and electron machines that are approximately Gaussian.) NEAT thus requires for input the chamber dimensions, the position and r.m.s. size of the beam, and the number of "non-beam" elements required. The user is also given the option of placing a clearing voltage on all node points along y=0, thereby allowing an analysis of the clearing necessary to prevent ion pockets around the machine.

The second geometry is that of axisymmetry. Again the generated mesh (such as in figure 2) is two-dimensional, but with the x=0 line taken as an axis of symmetry. Hence we may model a longitudinally uniform, radially Gaussian beam

$$\rho(r)drdz = \frac{Ner}{2\sqrt{2\pi}C\sigma_r^3} e^{-\frac{r^2}{2\sigma_r^2}} drdz \quad \text{for} \quad r \leq 2\sigma_r \qquad (4)$$

passing along the centre of a length l of cylindrical chamber. Again

* This may be changed to 13x13, 16x16, 19x19 etc,by changing the value of NBEL in NEAT.

the walls (that is r=a) are set to zero potential, and NBEL determines the number of beam elements. Input variables this time are chamber radius and length, the number of longitudinal and "non-beam" radial elements required and the r.m.s. beam dimension σ_r.

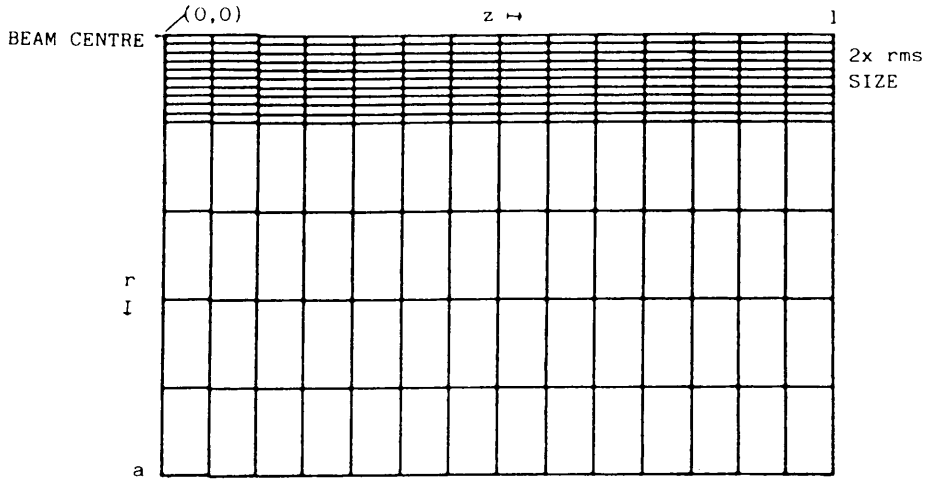


Figure 2. Typical mesh forming the DOT model of an axisymmetric structure

Clearly all this input information is easily attainable, but for the r.m.s. beam sizes. These can be deduced from beam emittances which in turn are found from scraper measurements of the horizontal and vertical beam sizes X and Y at QDN13 in the AA. The conversion of X and Y into the σ is outlined in annex 2 of reference 2 and gives

$$\sigma_x = \frac{X}{2.4477} \quad ; \quad \sigma_y = \frac{Y}{2.4477} \qquad (5)$$

It may be noted that the input variables to NEAT produce meshes which cover a fairly limited range of situations. However, once the DOT input has been generated, it may be edited "by hand", thus introducing a far greater flexibility. For example, the effect of a charged dust particle in the beam can be emulated by changing the beam charge at just one node point, giving the chance to study situations which are far from easy to treat analytically[3,5].

The use of DOT and its peripheral programs on the CDC

The running of DOT (and the various programs that have been written to aid its running) requires a good deal of laborious file handling. Therefore, with considerable help from Tony Shave of the TIS division, two interactive macros have been developed to make much of this manipulation transparent to the user.

Once logged on to the CDC machine, one should "ATTACH,MENU1,ID=PSØ26PØNC", and then type "MENU1" and return. This will run the interactive DOT pre-processor NEAT which produces the DOT

input file, TAPE1. MENU1 automatically calls the main menu, MENU2, which offers 9 options, outlined briefly below:

1. RE-RUN PROGRAM TO GENERATE DOT INPUT. This simply re-calls MENU1.

2. PLOT THE GENERATED MESH. This facility uses a graphics program AMBIT written by Juli Hargreaves in 1982, which produces plots of the generated mesh similar to those in figures 1 and 2. AMBIT still contains a few bugs, but these are confined to the more refined options offered by the program.

3. EDIT DOT INPUT FILE (TAPE1). Instructions are sent to the terminal on how this should be done. The format required in TAPE1 can be found in appendix A1 of the DOT manual (reference 1).

4. RUN DOT. This will batch the most recent TAPE1 and bring the output file to the terminal. The DOT output file is extremely copious, but does include the sometimes useful job history on the last page. For this reason, TAPE11 is generated in NEAT and DOT, and forms a concise output file containing the NEAT input information, the mesh node coordinates and the resulting potential in volts at each of those points. The DOT output file and TAPE11 can be studied at the terminal by opting for numbers 5 and 6 respectively.

7. OBTAIN A LIST OF THE LOCAL FILES. All files which are brought into a user's space are given a "local file name", which must be known in order to perform option 8;

8. SEND A LOCAL FILE TO A PRINTER using the CDC LP facility.

9. EXIT. To re-enter the main menu, simply type "MENU2" and return.

Options 1, 2, 4, 7 and 8 automatically return the user to the main menu after (or sometimes, thanks to the idiosyncrasies of the CDC, before or during) execution. To finish the session, one should EXIT the menu (option 9) and LOGOUT. Since there can be a considerable amount of information on the screen at once, and AMBIT requires a terminal capable of handling graphics, it is recommended that a Tektronix T2 terminal be used when running this system. (As a further note, the macro requires that TAPEs 1 and 11 are stored as permanent files each time DOT is batched; since only 5 cycles of a permanent file are allowed, one should be careful not to crash the menu by having too many cycles at any one time.)

Preliminary Results

Unfortunately, time has not permitted a thorough comparison of the DOT output with previous work, though the preliminary results appear to be encouraging. Figure 3 shows a handplot of beam potential vertically through the centre of a beam in a square chamber. By changing the

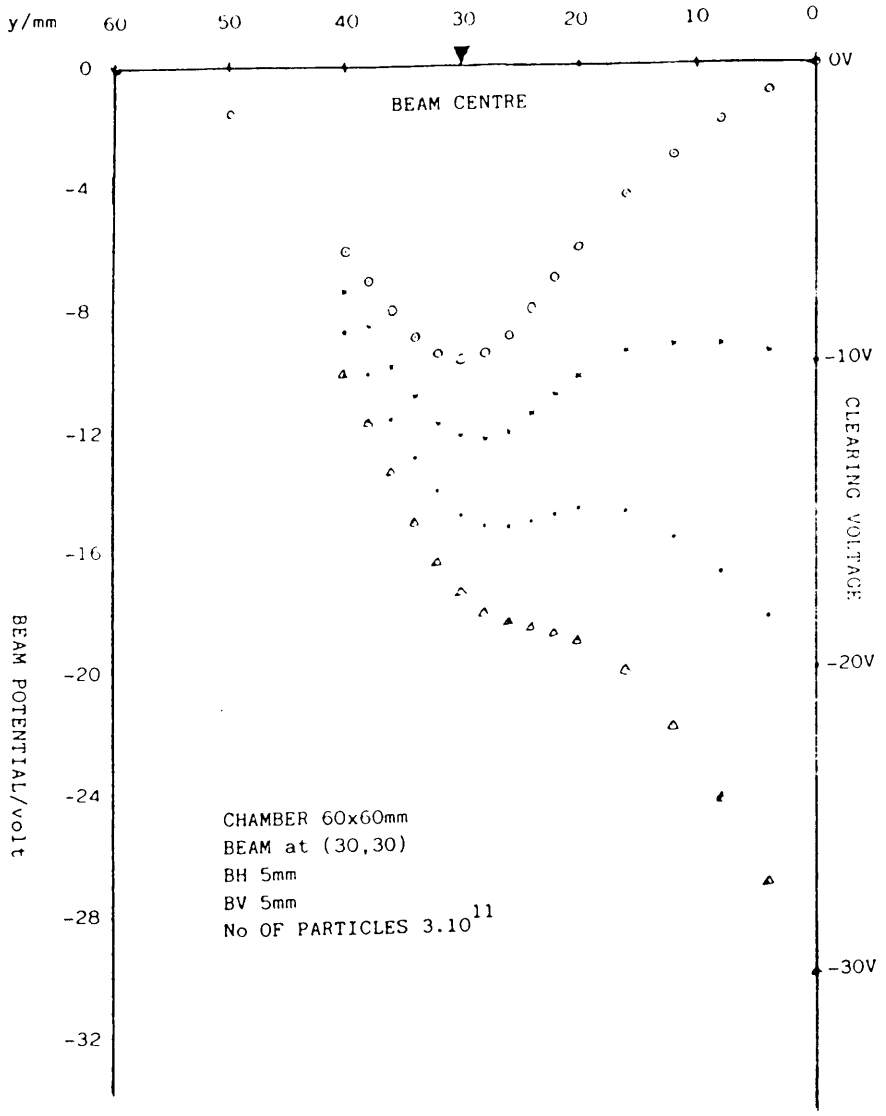


Figure 3. Handplots from DOT output showing the effect of a clearing voltage on the beam potential well

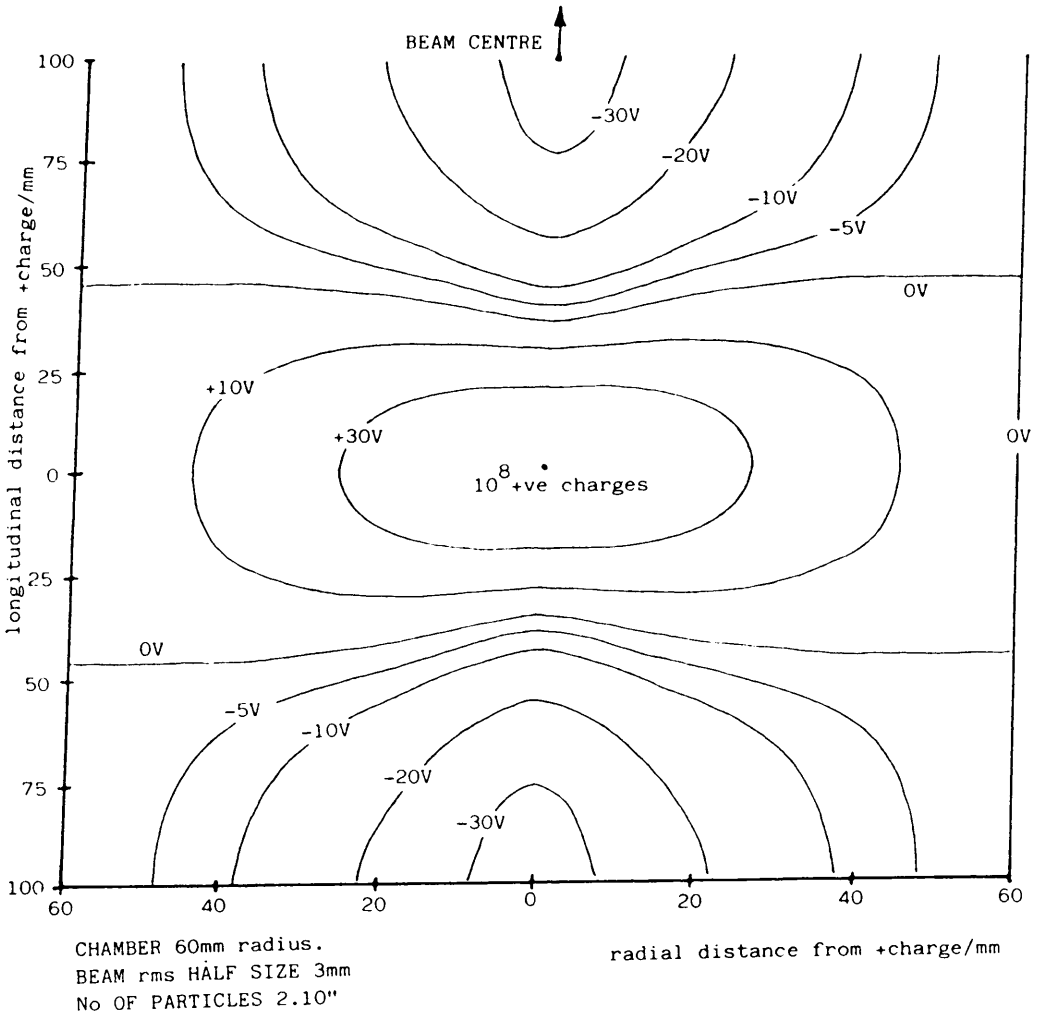
clearing voltage at y=0, one may deduce that for this configuration -30V will remove the possibility of an ion pocket.

Figure 4 shows an example of the use of axisymmetric geometry. The effect of a large number of oppositely charged particles in a beam can be treated analytically as long as the region of neutralisation does not extend to the chamber walls[5]; this imposes a restriction which is not necessary in numerical analysis, which can thus treat any number of charges placed anywhere in the beam chamber.

Suggested improvements and developments to the existing programs

The most glaring omission in the facilities provided is that of a post-processing graphics program to plot the beam potentials to the terminal. Unfortunately, SIGMA, one of the more usable graphics

(a)
right



(b)
below

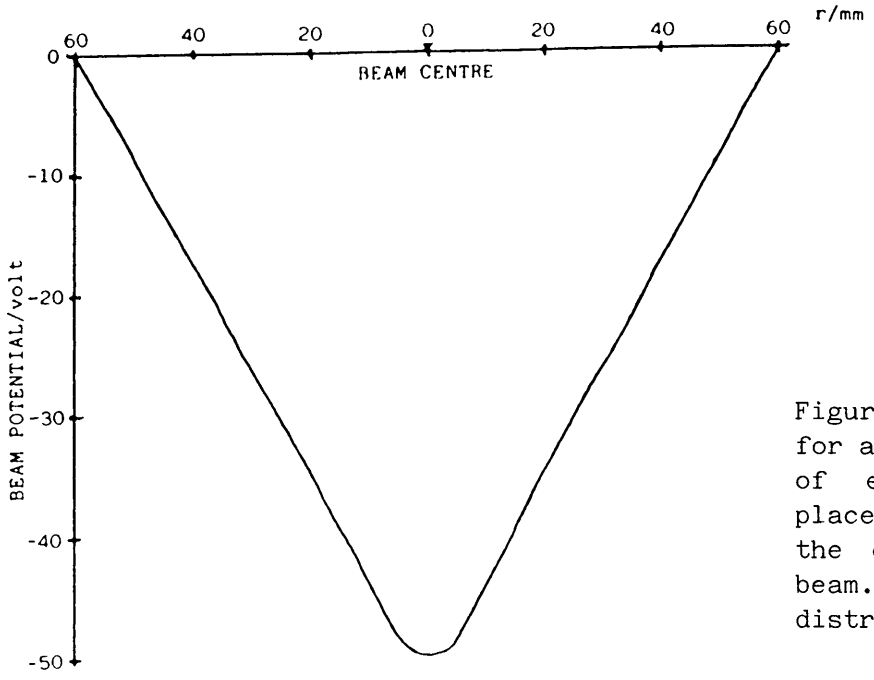


Figure 4. Handplots from DOT output for an axisymmetric case. (a) Lines of equi-potential caused by the placement of 10^8 positive charges in the centre of a negative Gaussian beam. (b) The radial potential distribution of the undisturbed beam.

packages available, which we hoped to use for this purpose is to be phased out in the near future. Probably the safest option is to try to develop a program using the PLOT10 library, as used in AMBIT. AMBIT takes TAPE12 from NEAT as its input file; clearly a version of TAPE11 (which already contains the node numbers, coordinates and potentials) in the correct format could be used as a similar input file to an interactive potential plotting program. It is then a relatively simple task to add this facility to the options in MENU2.

The other most obvious improvement to the system that could be made is the expansion of NEAT to offer a range of beam charge distributions and also to make use of DOT's ability to handle curvilinear coordinates. By this means some of the more awkward sections of beam chamber (and for example that of LEP) could be modelled. This could be particularly useful for analysing the electric fields in suspected ion-pocket regions.

Conclusions

DOT should provide a useful tool in the analysis and understanding of electrostatic beam potentials. In their present form the programs developed around DOT can model situations which should be of use in specifying clearing voltages and in coming to terms with the effects on the beam of single-site scatterers such as dust particles and residual gas ions.

Acknowledgements

I would like to thank Alain Poncet for all his help, advice and explanation concerning particle beams and finite element programs. Thanks are also due to Tony Shave for helping with the computing and to Tracy Marshall for typing the manuscript.

References

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(5) Private communication, F.Pederson.