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MEASUREMENT OF AA APERTURE

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1. Introduction

The available aperture in the AA is measured by injecting a proton beam, blowing it up transversely until a good fraction is lost (presumably on the aperture limits) and then observing the circulating beam intensity vs scraper position. This method works well in the horizontal plane if the injection kicker is used for blowing up the beam. A sharp-edged distribution is then produced and the aperture limit is clearly seen.

In the vertical plane, the only practical way to produce a blow-up is excitation of the betatron oscillations by applying noise to the transverse damper kickers. This results in a distribution with faint tails and the estimation of the exact aperture is not reliable.

It will be shown in the following that the amplitude distribution after blow-up can be calculated and that its dependence on the initial distribution is negligible, provided about 70% of the beam is killed by the blow-up.

2. Calculation of the Distribution

We consider a distribution of particles in phase space, where the position x is normalized to the aperture (i.e. x = 1 at the limit) and x' is normalized so that particles describe circles in the x, x' diagram. The particle density in this plane is called V.

The blow-up by random noise produces small, random angular kicks. Because of the rotation of the particles in phase space the distribution will always be circularly symmetric around the origin. The diffusion in phase space is exactly analogous to heat diffusion in a cylindrical bar towards the outside. The aperture limit corresponds to zero temperature on the outside surface of the cylinder.

For circulating symmetric distributions the diffusion equation is:

$$\frac{\partial V}{\partial t} = D \left(\frac{\partial^2 V}{\partial \hat{x}^2} + \frac{1}{\hat{x}} \frac{\partial V}{\partial \hat{x}} \right) \qquad (1)$$

where $\hat{x}$ is the betatron amplitude.
The solution, for $V = 0$ at $\hat{x} = 1$, is

$$V(x,t) = \sum_{n=1}^{\infty} a_n \ell^{-Dj_n^2 t} J_o (j_n \hat{x}) \qquad (2)$$

where the j_n are the zeros of J_o (in increasing order) and the coefficients a_n depend on the initial distribution $V_o(\hat{x})$ as follows:

$$a_n = \frac{2}{J_1^2(j_n)} \int_0^1 \hat{x} V_o(\hat{x}) J_o(j_n \hat{x}) d\hat{x} \qquad (3)$$

It is clear from (2) that higher-order terms die out rapidly, because of the increasing j_n in the exponential factor.

The total number of particles is proportional to

$$\int_0^1 \hat{x} V d\hat{x} = \sum_{n=1}^{\infty} a_n \ell^{-Dj_n^2 t} J_1(j_n)/j_n \ . \qquad (4)$$

3. Application to the Aperture Measurement

Figure 1 shows the result of numerical calculations, starting from two somewhat extreme initial distributions:

a) all particles have zero amplitude (δ function of V);

b) constant V across the aperture.

The maximum error made by neglecting all terms with $n > 1$ in equation (2), anywhere for $0 < \hat{x} < 1$, divided by the density at $\hat{x} = 0$, is plotted against the remaining fraction of particles after blow-up. Clearly, for any reasonable initial distribution (which must be between the two extremes shown), the final distribution will be known to within better than 2% if the blow-up is continued until 30% of the beam remains.

The distribution vs. amplitude $dN/d\hat{x}$ is related to the phase space density by

$$dN/d\hat{x} \quad = \quad 2\pi\hat{x}V \ .$$

The curve obtained by plotting the remaining intensity vs. scraper position is the integral of this and by using the first term of (2) we find that the result must be proportional to

$$\hat{x}J_1(j_1\hat{x}) \ .$$

A normalized plot of this function is shown in Fig. 2 and tabulated below. By comparing the "scraper curve" to this result, it should be possible to find the aperture limit with better precision.

$\hat{x}$	$\hat{x}J_1(j_1\hat{x})$
.05	.006
.1	.023
.15	.051
.2	.090
.25	.138
.3	.195
.35	.259
.4	.329
.45	.404
.5	.481
.55	.558
.6	.635
.65	.709
.7	.778
.75	.840
.8	.894
.85	.939
.9	.972
.95	.993
1	1.000

References

1. See e.g. Bowman, Introduction to Bessel Functions (Dover, New York, 1958), p. 37.
2. Watson, Theory of Bessel Functions (Camb. Univ. Press, 1962), p. 576.

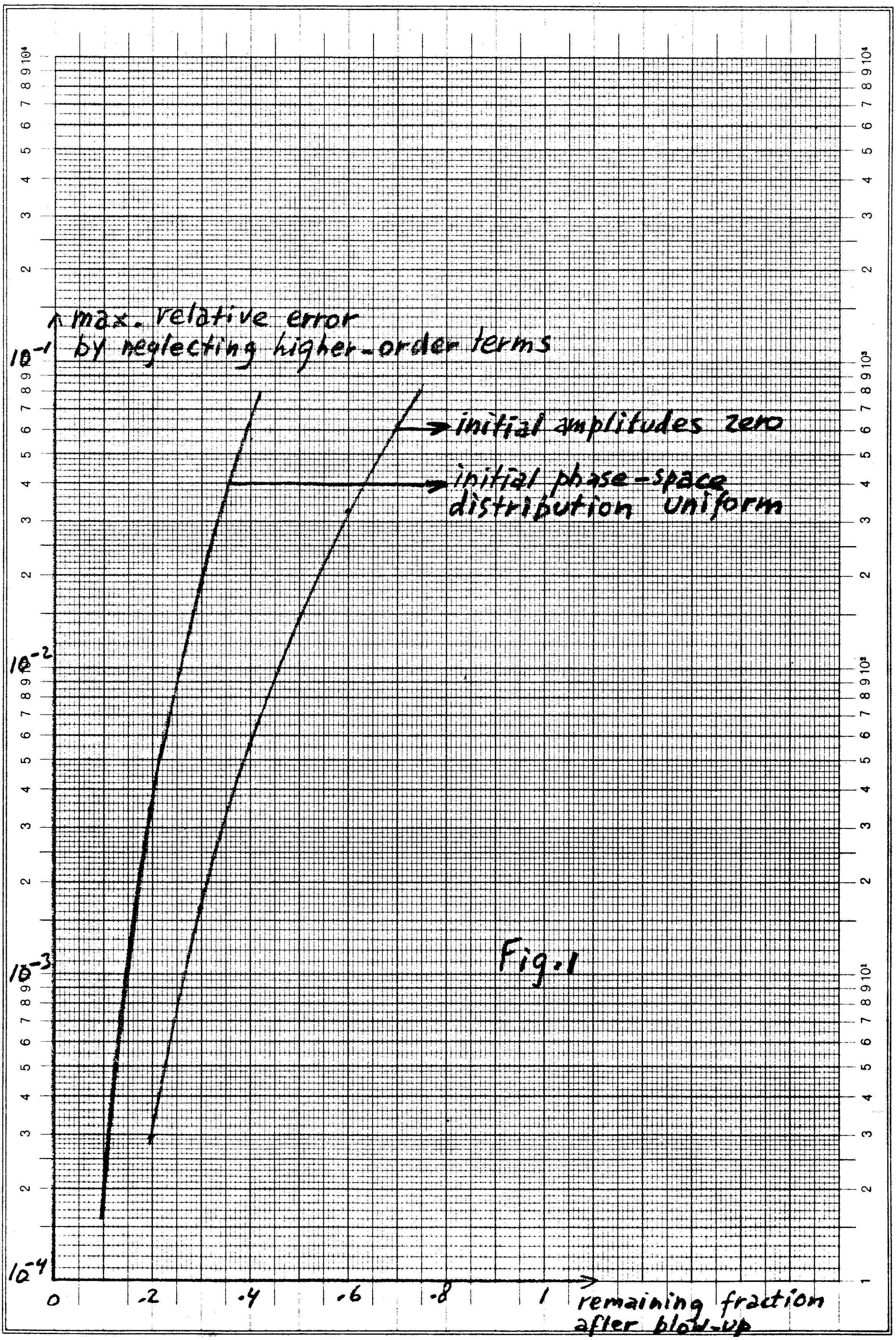
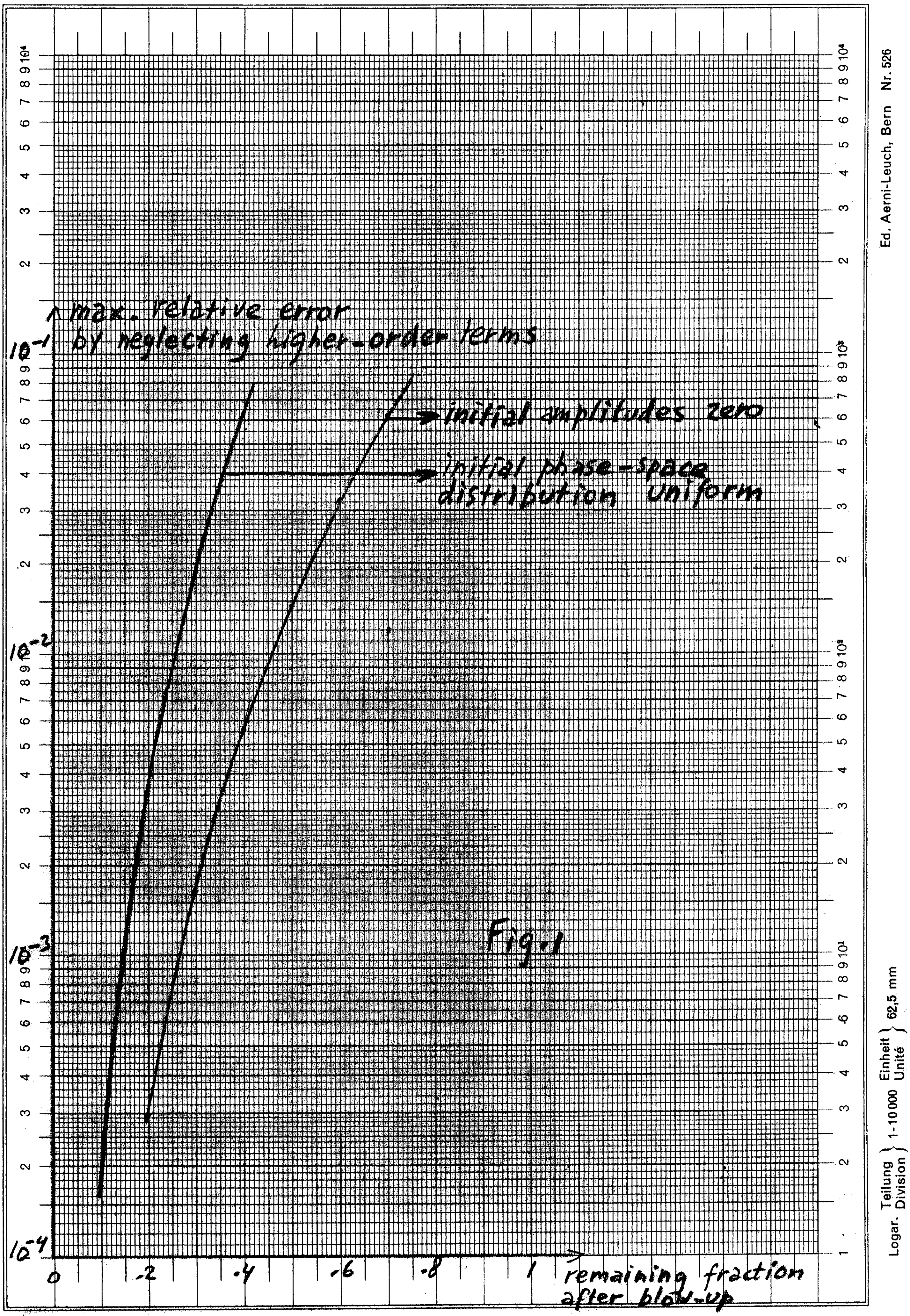

max. relative error
by neglecting higher-order terms
10^{-1}
10^{-2}
10^{-3}
10^{-4}
initial amplitudes zero
initial phase-space distribution uniform
0
.2
.4
.6
.8
1
remaining fraction after blow-up

Fig. 1



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0
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remaining fraction
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Fig. 1

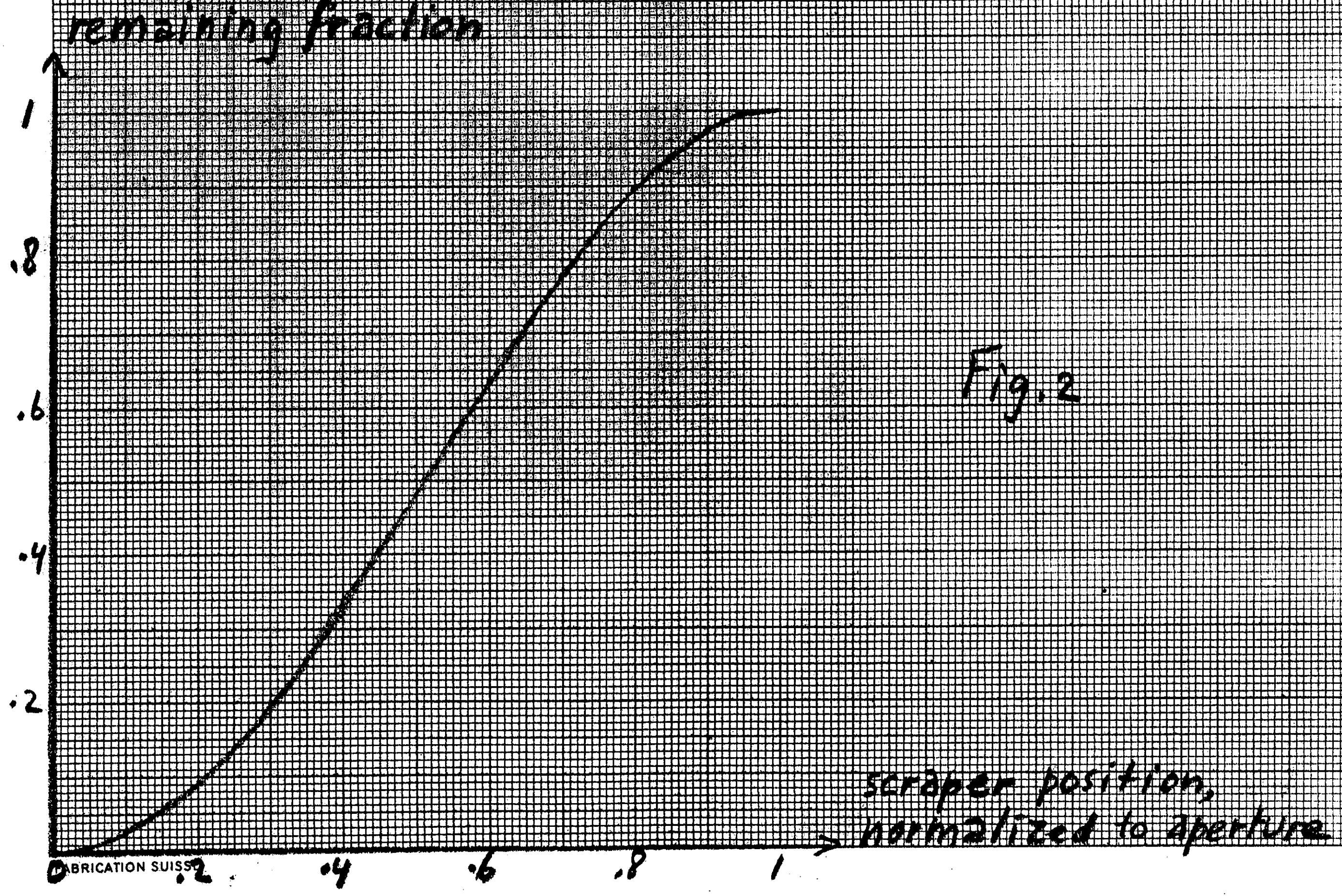
remaining fraction
1
.8
.6
.4
.2
0
.2
.4
.6
.8
1
scraper position, normalized to aperture
ABRICATION SUISS

Fig. 2