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SOME REMARKS ON THE TRANSPORT OF AN EJECTED P.S. PROTON BEAM OVER A LONG DISTANCE

I. INTRODUCTION

Up till now, beams ejected from the P.S. machine are focused on targets placed inside the ring area and require beam transport over distances of about 30 m only.

Proposals have been made to construct new experimental areas located several hundreds of metre away from the P.S. Ejected beams will have to be transported into these areas over long distances. To do this job, two methods have been considered :

first : periodic repetition of the phase space properties of the beam;

second: periodic repetition of focal spots.

The first method has been studied by taking a system of alternating focusing and defocusing quadrupole magnets separated by equal drift lengths so a periodic system of single elements.

The second method has been looked into by a system of triplets separated by equal drift lengths, with the focal spot in the middle between two triplets.

2. DESIGN CRITERIA FOR A TRANSPORT SYSTEM

1. Matching of the ejected beam into the transport system should be convenient.
2. Small fluctuations of the properties of the ejected beam should not result in a fast blow up of the beam in the transport system (radiation aspects).
3. The chromatic aberration of the system should be small.
4. Good beam confinement. It should be possible to use either standard quadrupoles or rather small aperture ones (saving power)
5. A small number of quadrupoles per 100 m of transport. Low power consumption.

3. PROPERTIES OF EJECTED BEAMS

The size of the beam ejected towards the neutrino experiment has been measured (ref. 1).

The reported measured emittances are (at 25 GeV/c) :

horizontal	$7\pi \cdot 10^{-7}$ rad. m
vertical	$3\pi \cdot 10^{-7}$ rad. m

The beam is divergent in both planes. The divergence in the horizontal plane, after leaving the septum magnet, is about 0.15 mrad.

The spread in momentum of the ejected protons is very small and $\frac{\Delta p}{p}$ is thought to be in the order of 0.03 o/o. This implies that the requirement for the transport system of having a low chromatic aberration is not very stringent.

4. THE PERIODIC SYSTEM OF SINGLE ELEMENTS

The properties of these periodic systems are described by A. Citrc (ref. 2) and by J.W. Gardner (ref. 3).

To discuss the use of a periodic system, some properties of it will be recalled.

Let the transfer matrix transforming the vector x , x' through one magnetic period be represented by
(x = displacement, x' = divergence)

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$$

When the magnetic period has a length ℓ , one has :

$$\begin{matrix} x\,(z+\ell) \\ x'(z+\ell) \end{matrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} x\ (z) \\ x'\ (z) \end{pmatrix}$$

At symmetry within a period one has : $M_{11} = M_{22}$. A beam which is defined in the x , x' phase space at the symmetry by an upricht ellipse and having an ellipse axis ratio $R = \sqrt{(M_{12})/(-M_{21})}$, will repeat its properties after each period (see ref. 2, p. 14 and ref. 3, p.14-15). In other words : the same upright ellipse will be found after each period. However, this does not mean that individual points on the ellipse are found on the same position after each period.

The ellipse axis ratio $R = a/b$ where a is the displacement half-axis and b the divergence half-axis.

In a system of alternating focusing and defocusing quadrupoles separated by drift lengths, points of symmetry are found at the mid-plane of the quadrupoles. Producing an upright ellipse in the mid-plane of the first element of the system will result in a periodic repetition of this ellipse when the R of the ellipse is equal to $\sqrt{(M_{12})/(-M_{21})}$. The upright ellipse with maximum displacement are found in the focusing plane.

In order to discuss the ratio R which is required to **preserve upright** ellipses in the periodic system of single quadrupoles we use the same terminology as used in ref. 3.

L = drift length between quadrupoles

d = effective length quadrupoles

Θ = ωd , where ω is the lens strength

α = L/d

Given α , one can find for a value of ω the ratio R of the upright ellipse at the mid F-plane which will be repeated periodically. Similar for mid D-plane. Curves of R versus Θ have a minimum. This minimum can be selected as operating value of the system as there the transport of the ellipse is less strongly affected by variation of the lens strength or particle momentum.

Taking the foregoing as criteria and using the curves on Fig. 18 of Ref. 3, one finds that there exists in first approximation a linear relation between R of the upright ellipse and L ($R = 5/14 \cdot L$, mid F-plane, R in cm/mrad, L in m). This means that if one wants R to be low, one should keep L low. Or, better confinement with closely packed systems.

Having R fixed, one can still select the effective length of the quadrupoles, d . Small d's will require high lens strengths. Design criteria (5) asked for a small number of quadrupoles per 100 m. If one takes, arbitrarily, for $L = 8$ m as minimum, this implies that $R = 3.0$ cm/mrad. For a phase space area of $0.7\ \pi$ cm/mrad (ejected beam) this means

$$a = 0.46 \text{ cm}$$
$$b = 0.15 \text{ mrad.}$$

$R = 3.0$ cm/mrad can be regarded as the minimum value which can be obtained practically. This means that in order to use a periodic system of quadrupoles as transport system, one has to shape the ejected beam to nearby parallel beam.

Some other properties of the periodic system will be discussed by working out a numerical example (calculations done with TRAMP). Let

L = 21.6 m

d = 2.16 m

Length period = 47.52 m

Ellipses with $R = 7.8$ cm/mrad in the mid F-plane are preserved and in the D-plane for $R = 2.2$ cm/mrad (R being minimum obtainable value again)

For a phase space area of $0.7\ \pi$ cm/mrad this means :

$a = 0.74$ cm
$b = 0.095$ mrad in the horizontal plane and

$a = 0.26$ cm
$b = 0.12$ mrad in the vertical plane.

Required field gradient = 193 gauss/cm

Tracking ellipses with the values found through 6 periods (~ 285 m) showed that the ellipse properties were well preserved (in the horizontal plane the ellipse had turned over 0.07 mrad).

The influence of changes in particle mementum :

	Change in beam envelope for p/p = 0.4 o/o	
	Horizontal plane	Vertical plane
After 2 periods	0.03 o/o	0.3 o/o
" 4 "	0.07 o/o	0.7 o/o
" 6 " (285 m)	0.01 o/o	0.6 o/o

The influence reported is independent of the values of a and b , R is determining.

The quadrupole apertures can be rather small (say 5 cm) as even under non-perfect matching conditions the beam remains rather confined. Small aperture lenses are acceptable. The table below illustrates this aspect.

Phase space 0.7 π cm/mrad	R	a cm	b mrad	Maximum half width of beam envelope cm
6 periods (285 m)	7.8	0.74	0.095	0.75
	7.0	0.70	0.10	0.79
	6.2	0.66	0.11	0.83
	2.0	0.37	0.19	1.4
	0.5	0.18	0.4	2.9

5. TRIPLET SYSTEMS

The triplets are placed at equal distances and the focal point in both planes are found in the middle of each drift length (2 L) between two triplets. The magnification is equal to one in both planes (calculated with programme of reference 4).

The confinement properties of such a triplet system can be judged by considering two initial conditions = a point source and a parallel beam. The confinement of a beam from a point source is independent of the number of triplets in the system and the acceptance angle is determined by the useful aperture of the quadrupoles (2 v) and L (angle = v /L). The situation is different for a parallel beam.

The boundary ray of a parallel beam with initial width 2 w , makes an angle of

$\alpha = \frac{2\,w}{L}$ with the axis after passing the first triplet and

$\alpha = n \cdot \frac{2\,w}{L}$ " " n triplets.

The maximum displacement of the boundary ray is found inside the triplets and amounts in the n-th triplet to : $w\,(1 + 2\,(n-1)\,)$. The limiting triplet is found by : $w\,(1 + 2\,(n-1)\,) \geqslant v$. So, the confinement of an initially parallel beam deteriorates by passing through the triplets (this in contrast with a periodic system). To limit this effect one favours a system with wide aperture quadrupoles and by placing the triplets wide apart, requiring a nearby parallel beam at the entrance of the system.

One arrives at a requirement similar to the one found above when considering the effects of chromatic aberration. The chromatic aberration, starting from a point source is proportional with the initial emission angle and approximately with the total length travelled through the system but is independent of the number of triplets. For parallel beams, the chromatic aberration is determined by the number of triplets passed through.

Was the periodic system of singlets rather chromatic aberration free this cannot be obtained with the triplet system. As numerical example two triplet systems have been calculated.

System 1 : distance between focal spots 141 m; 1 m, 2 m, 1 m quadrupoles
Field gradients = 785, 825, 788 gauss/cm
Four triplets were comprised.

System 2 : distance between focal spots 282 m; 1 m, 2 m, 1 m quadrupoles
Field gradients = 551, 582, 545 gauss/cm
Two triplets were comprised.

Tracking with trajectory considered in periodic system also : 0.7 cm 0.1 mrad, gives for p/p = 0.4 o/o a displacement at the focal spot of :

System 1	after	141 m	2.7 o/o	System 2	after	282 m	4.2 o/o
	"	282 m	8 o/o		"	564 m	11 o/o
	"	423 m	15 o/o				
	"	564 m	25 o/o				

Maximum displacement of the 0.7 cm, 0.1 mrad trajectory in system 1, 2 respectively 5.6 cm, 3.5 cm. This means that the system needs correction when extended over greater distances.

CONCLUSIONS

Proton beams ejected from the P.S. can be shaped into nearby parallel beams. Due to the small phase space area of the ejected beam, transporting this nearby parallel beam onto a target about 100 m away, does not pose a particular problem. With a single coublet or triplet one can solve this problem.

When the ejected beam has to travel several hundreds of metres, the confinement becomes more difficult and small variations of initial conditions will become stressed at the end of the beam. Two solutions have been looked at the periodic system of single quadrupoles and the periodic triplet system. The former system has certain advantages with respect to the latter. It offers good confinement, some fluctuations of initial conditions can be accepted and a modest number of small aperture quadrupoles is only needed (about 5 quadrupoles per 100 m is reasonable). The periodic triplet system is less suited for very long distance beam transport (above 1000 m). However, it requires less civil engineering investments as the distance between triplets with a large aperture can be greater.

Finally, it has been found very useful to design periodic systems with the graphs given in ref. 3.

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