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# SINGLE BUNCH COLLECTIVE EFFECTS IN BFI

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This note makes a first order estimate of the most important single bunch effects that have been considered in the feasibility study of a B-Factory machine in the CERN ISR tunnel. At present, specific details of the machine are not known. Chances are that the figures given will not be the final ones; however, they probably will not be very different. Space charge and resistive wall impedance effects have been assumed small and will be neglected. Also, free space impedance has been neglected, but with a question mark as the subject requires more detailed studies.

## Longitudinal broad band impedance

As is common, the integrated effect of all the impedances, beam pipe discontinuities, bellows, valves, pick-up's, etc. and the higher order modes of the RF cavities, are assumed to be equivalent to a single resonator with impedance given by [1]

$$Z_{/\!/} = \frac{R_s}{1 - j\,Q\,(\omega / \omega_r - \omega_r / \omega)} \qquad 1)$$

where
$R_s$ is the shunt resistance
Q is the quality factor (set equal to 1 in our computations)
$\omega_r$ is the resonant angular frequency (chosen to be close to the cut-off frequency of the beam) pipe $\omega_c \approx c/b$, and b is the equivalent beam pipe radius. We have set b equal to the half height of the vacuum chamber.

When studying longitudinal instabilities, it is convenient to consider the value $|Z_{/\!/}/n|$, where $n = \omega / \omega_0$ is the ratio of the frequency divided by the revolution frequency. In the frequency interval from zero to $\omega_r$ : $| Z_{/\!/} / n | \approx | Z_{/\!/} / n |_0 \approx R_s\,\omega_0 / \omega_r$, while above $\omega_r$, $|Z_{/\!/}/n|$ decays to zero as $\approx \omega^{-1.68}$ (see below).

The beam impedance interaction is proportional to the product of the beam spectrum and the impedance frequency response. Let $\sigma_t$ and $\sigma_s$ be the rms bunch duration and length respectively. For short bunches, i.e. when $1/\sigma_t > \omega_r$ (equivalently $\sigma_s < b$), a large amount of the beam energy, viewed in the frequency domain, is located well above $\omega_r$ ($\approx\omega_i$). At these high frequencies, the impedance is low. This effect is taken into account by applying the SPEAR scaling law [2,4]. This leads to defining an effective impedance

$$| Z_{/\!/} / n |_{eff} = | Z_{/\!/} / n |_0 \,(\sigma_s / b)^{1.68} \qquad 2)$$

An important contribution to the value of $| Z_{/\!/} / n |$ is given by the higher order modes in the many RF cavities. This effect is counted separately as

$$| Z_{/\!/} / n | = | Z_{/\!/} / n |_{pipe} + N_{cell} | Z_{/\!/} / n |_{RF}$$

$$= | Z_{/\!/} / n |_{pipe} + N_{cell} \Sigma_j \{ R_s / Q \omega_r \}_j \omega_0$$

where $N_{cell}$ is the number of RF cells, $| Z_{/\!/} / n |_{RF}$ is the integrated effect of the h.o. modes[3] and the sum is over all parasitic cavity modes. At present, a final version of the RF system has not been chosen. For our computations we have assummed a single resonator. Similar comments apply for the transverse impedance.

## Transverse Impedance

The transverse impedence is approximated as [5]

$$| Z_{\perp} | = \frac{2R}{b^2} | Z_{/\!/} / n | \qquad 3)$$

where

R is the machine radius.

No scaling law, similar to the SPEAR scaling law for short bunches, has been applied in our calculations for the transverse impedance of BFI.

## Longitudinal microwave instability [6]

Anticipating the results of the following computations, this is the effect which imposes the most stringent constraint on the impedance value for all the machines considered. The instability manifests itself as a bunch lengthening (and bunch widening in energy spread) once a certain bunch particle density threshold is exceeded. Typically, no direct beam losses are observed; however, the peak luminosity may decrease due to the longer bunches. To be safe, the impedance must be less than

$$| Z_{/\!/} / n |_{eff} < \frac{(2 \pi)^{3/2} \alpha \; E/e \, (\sigma_E / E)^2 \sigma_s}{e \, c \, N_b} \qquad 4)$$

where :

$\alpha$ is the momentum compaction factor
E is the particle energy
$\sigma_E$ / E is the rms bunch energy spread
$\sigma_s$ is the rms bunch length in m
e is the electron charge
c is the speed of light

$N_b$ is the number of particles in the bunch $= \dfrac{2 \pi R I_0}{K_b e c}$

$I_0$ is the total DC beam current
$K_b$ is the number of bunches

The maximum acceptable values of the impedance are listed in table 1) for the three machine options foreseen (the values shown are those corresponding to the low frequency scaled impedance $| Z_{/\!/} / n |_0$ ).

## Transverse mode coupling [7,10]

For short bunches ($\sigma_s < b/2$ as is our case) this effect has the lowest instability threshold and is particularly severe since one can expect large beam losses above the current threshold. For such an instability, the stability condition is

$$| Z_\perp | < \frac{8 \pi^{3/2} E/e \, Q_s \, \sigma_s}{e \, c \, N_b \, \langle\beta\rangle} \qquad 5)$$

where :

$Q_s$ is the synchrotron tune $= \alpha R ( \sigma_E / E ) / \sigma_s$
$\langle\beta\rangle$ is the average beta function.

The maximum acceptable values of the transverse impedance, converted into the corresponding longitudinal one with formula 3), are shown in table 1).

## Potential well bunch lengthening (or shortening)

This effect is due to the interaction between the beam and the imaginary part of the effective longitudinal impedance. It can be computed by solving the following equation [8,9]

$$s^3 - s - k\ I_0\ \{\mathrm{Im}[\ Z_{/\!/} / \ n]\}_{eff} = 0 \tag{6}$$

where :

$s = \sigma_s / \sigma_{s0}$ is the ratio between the actual bunch length $\sigma_s$ and the bunch length $\sigma_{s0}$ for vanishing current

$I_0$ is the total DC beam current

$$k = \frac{\alpha R^3}{(2\pi)^{1/2} K_b Q_{s0}^2 E/e\ \sigma_{s0}^3}$$

and $\{\mathrm{Im}[\ Z_{/\!/} / \ n]\}_{eff}$ is the normalized product, in the frequency domain, of the m=1 oscillation mode power spectrum of the bunch and the spectrum of the imaginary part of the broad band longitudinal impedance, i.e.

$$\{\mathrm{Im}[\ Z_{/\!/} / \ n]\}_{eff} = \frac{\sum_{p=1}^{\infty} \mathrm{Im}\left[Z_{/\!/}(\omega_p)/n\right] h_1(\omega_p)}{\sum_{p=1}^{\infty} h_1(\omega_p)} \tag{7}$$

where :

$$\omega_p = p\ K_b\ \omega_0$$

$$\mathrm{Im}[Z_{/\!/}(\omega_p) / n] = | Z_{/\!/} / n |_0 \frac{Q(1 - x^2)}{x^2 + Q^2 (1 - x^2)^2} \tag{8}$$

and

$$x = \omega_p / \omega_r$$

$h_1(\omega)$ is the power spectrum of the m=1 bunch oscillation given by

$$h_1(\omega_p) = 1.128\,(\omega_p / \omega_{cr})^2 \exp[-(\omega_p / \omega_{cr})^2] \quad 9)$$

$\omega_{cr} = 1 / \sigma_t = c / \sigma_s$ is $2\pi$ times the rms frequency spectrum of the bunch.

If the bunch is long ($\sigma_s > b$), most of its power lies in the low frequency region where $\mathrm{Im}[Z_{/\!/} / n] \sim |Z_{/\!/} / n|_0 > 0$ (i.e. inductive). The value of s is then > 1 and we will obtain bunch lengthening. On the other hand, if the bunch is short ($\sigma_s < b$) most of the power will be located above $\omega_r$ where the impedance is < 0 (capacitive). One then expects a bunch shortening (s < 1).

Eq. 6) must be solved numerically with a computer since $\{\mathrm{Im}[Z_{/\!/}/n]\}_{eff}$ is dependent on $\sigma_s$. Some typical results are shown on Fig. 1.

## Conclusions

The potential well effects are relatively small (~ 10%).

The longitudinal microwave instability begins at lower currents than the transverse mode coupling instability; however, we have not taken into account (since they are not yet known) the localized impedances and beta functions at the RF cavities. With this knowledge, the situaion could change[10] N.B.: reduction of the beta (and dispersion) values at the RF cavity locations is essential to avoid transv. mode coupling (and synchro-betatron) instabilities.

The most difficult to achieve impedance values belong, of course, to the 3.5 GeV rings. These values are $|Z_{/\!/} / n|_0 = 0.33\ \Omega$ and $1.1\ \Omega$ which seem quite ambitious to obtain.

Since the bunches are short (spectra > 5 GHz), the impedance behaviour above the cutoff frequency (~ 1.4 GHz) becomes important . Here we have considered the SPEAR scaling law that foresees an impedance "decay" as $\omega^{-.68}$, but other models advocate decays of $\omega^{-1/2}$ or $\omega^{-3/2}$[11,12]. Consequently, we suggest that computations and measurements of each machine component be carefully performed in this high frequency range in order to better estimate the beam behaviour.

## Acknowledgments

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| L [ $cm^{-2}$ $s^{-1}$] | $10^{33}$ | | $6 * 10^{33}$ | $10^{34}$ | |
|---|---|---|---|---|---|
| E [GeV] | 3.5 | 8 | 2 x 5.3 | 3.5 | 8 |
| long. $\mid Z_{/\!/} / n \mid_0$ [Ω] | 0.33 | 4.5 | 2.3 | 1.1 | 14 |
| transv. $\mid Z_{/\!/} / n \mid_0$ [Ω] | 2.2 | 11 | 4.6 | 3.5 | 22 |

<u>Table 1)</u> The different machine options ( parameter list from the feasibility study) are differentiated in the 1st and 2nd row with the Luminosity and Energy values.

The 3rd row shows the maximum acceptable values of $\mid Z_{/\!/} / n \mid_0$ given by the longitudinal microwave instability (for ex.: 0.33 Ω in the 3.5 GeV machine).

The 4th row shows the maximum acceptable values of $\mid Z_{/\!/} / n \mid_0$ given by the transverse mode coupling instability (what is actually listed is the transverse impedance converted into the corresponding longitudinal one with formula 3).

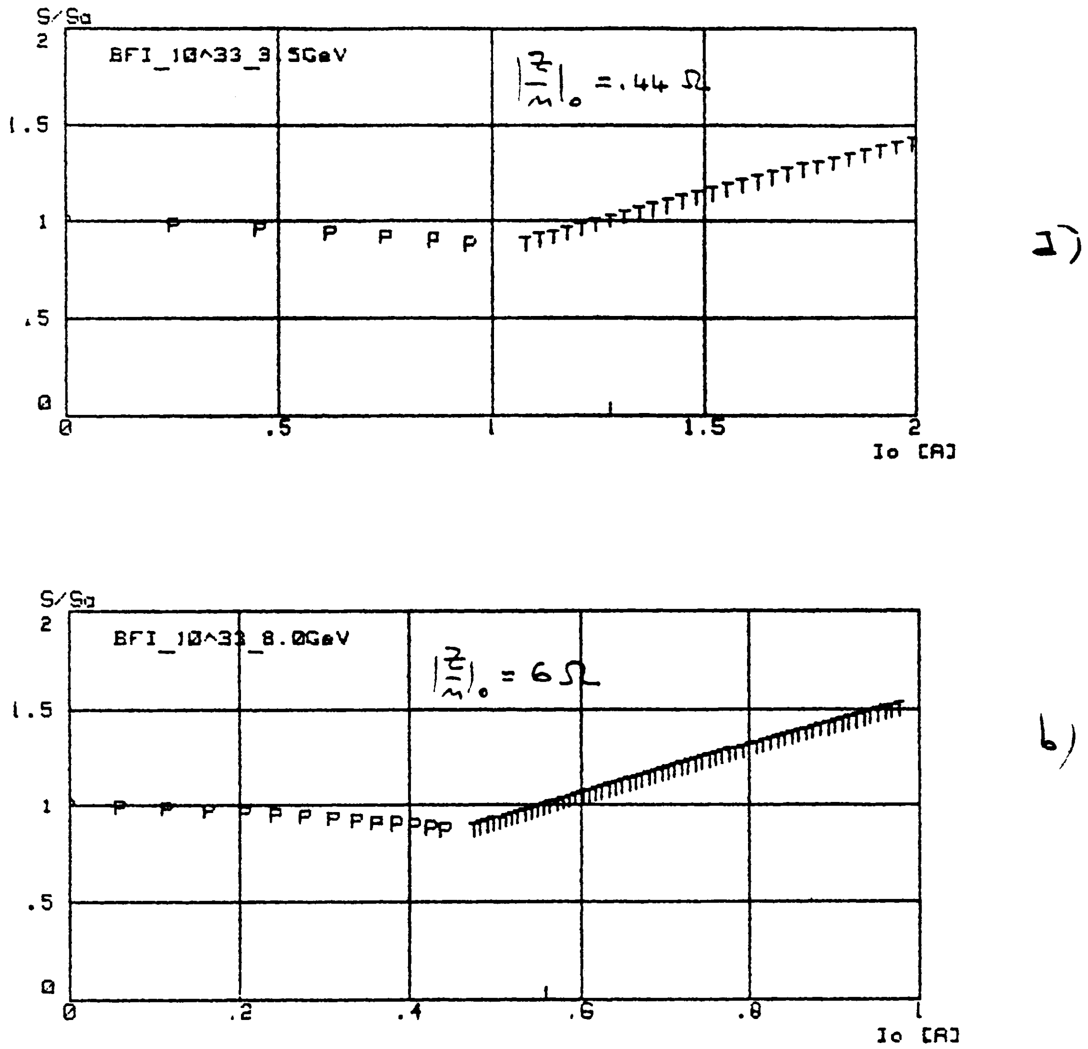


Fig. 1 : Plots of $\sigma/\sigma_{s0}$ versus DC beam current in the $10^{33}$ machine
a) 3.5 GeV ring b) 8 GeV ring
"P" and "T" mean potential well or turbulence regime.