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TRANSVERSE BETATRON EMITTANCE

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1 Introduction

A direct measurement of the horizontal betatron amplitude is not possible in the PS, because there is no dispersion-free region. Instead it is needed to measure the horizontal rms width of the beam using a flying wire and subtract the momentum dispersion in quadrature. In this experiment, the validity of this procedure is to be checked.

2 The experiment

The $h = 20$ cycle with 5 bunches injected from one PSB ring was used. The total proton intensity was about $6\,10^{12}$ p. Beam momentum at 3.5 GeV/c ($\beta\gamma = 3.794$) was considered to begin with. The vertical and horizontal profiles of the bunch were measured with the flying wire at 5 RF voltages ranging from 23.7 kV to 202 kV. The longitudinal bunch profile was also measured using the beam observation system, from which the rms bunch length σ_{bunch} and the rms momentum spread σ_δ were computed.

The beam momentum was changed to 26 GeV/c ($\beta\gamma = 27.761$) and the measurements were repeated. The results are displayed in Table 1.

Momentum $[GeV/c]$	V_{RF} $[kV]$	$4\sigma_v$ [mm]	$4\sigma_h$ [mm]	$4\sigma_{bunch}$ [ns]	$2\sigma_\delta$	ϵ_v $[\pi\,\mu m]$	ϵ_h^{linear} [1] $[\pi\,\mu m]$	$\epsilon_h^{quadratic}$ [1] $[\pi\,\mu m]$
3.5	23.7	16.5	21.4	45.4	$1.46\,10^{-3}$	5.44	4.28	8.19
	76.0	16.5	22.2	35.4	$2.10\,10^{-3}$	5.41	3.12	7.93
	121.0	16.4	23.0	31.6	$2.39\,10^{-3}$	5.38	2.86	8.10
	159.2	16.5	22.8	31.4	$2.72\,10^{-3}$	5.45	2.10	7.21
	202.0	16.3	23.3	28.2	$2.77\,10^{-3}$	5.34	2.21	7.55
26.0	20.0	5.3	7.4	44.1	$5.98\,10^{-4}$	0.57	0.43	0.93
	88.0	5.2	8.1	29.1	$8.63\,10^{-4}$	0.54	0.34	0.99
	125.0	5.1	8.4	27.8	$9.85\,10^{-4}$	0.52	0.30	0.99
	184.0	5.1	9.3	26.2	$1.13\,10^{-3}$	0.53	0.33	1.18

Table 1: Measurements at 3.5 GeV/c and 26 GeV/c

[1] The horizontal emittances ϵ_h^{linear} and $\epsilon_h^{quadratic}$ are computed using Eqs. 1–2 and Eqs. 1–3 below respectively.

3 Results

The horizontal and vertical betatron emittances are defined as

$$\epsilon_h = \frac{(2\sigma_{\beta_h})^2}{\beta_h} \qquad \epsilon_v = \frac{(2\sigma_{\beta_v})^2}{\beta_v} \tag{1}$$

where σ_{β_h} and σ_{β_v} are the horizontal and vertical rms betatron amplitude respectively, σ_{β_h} being computed according to either the quadratic formula

$$\sigma_{\beta_h} = \sqrt{\sigma_h^2 - D_h^2\,\sigma_\delta^2} \tag{2}$$

or the linear formula

$$\sigma_{\beta_h} = \sigma_h - D_h\,\sigma_\delta \tag{3}$$

σ_h is the rms beam width obtained from the horizontal beam profile.

In the above, $D_h = 2.3$ m, $\beta_h = 12.6$ m and $\beta_v = 12.5$ m are, respectively, the momentum dispersion and horizontal and vertical betatron functions at the flying wires.

The results are plotted as functions of momentum spreads in Figure 1 and Figure 2 for 3.5 GeV/c and 26 GeV/c respectively (dotted straight lines fit the emittance data). The measurement errors for ϵ_h depends on the measured data (see Appendix A). For both momenta, the quadratic formula gives better fits. The vertical emittances are plotted in Figure 3 and Figure 4.

4 Discussions

(1) During the measurements, the horizontal and vertical emittances of the beam at 26 GeV/c changed suddenly after about half an hour. The reason was unknown. It was suspected that this was due to some unknown changes during injection. In the result analysis, only those measurements taken before the sudden change have been included.

(2) If the normalized horizontal betatron emittance is computed by the multiplication of the factor $\beta\gamma$, it was found that it stayed constant within experimental errors.

(3) Theoretically, if the distribution in betatron amplitude and the distribution in momentum distribution were independent of each other, or, in other words, there were no horizontal and longitudinal coupling, the quadratic formula of Eq. (2) would be exact. Although it is logical to believe in negligible horizontal and longitudinal coupling, the way the measurements were taken could lead to deviation from Eq. (2). While the horizonal rms spread was computed directly from the flying-wire profile, the rms bunch length was taken as the standard deviation of a Gaussian which fit the measured longitudinal profile. The rms momentum spread was then computed from this bunch length and the RF potential. If the fit were not satisfactory, this "measured" rms bunch length would be different from the actual rms bunch length computed directly from the bunch profile. A typical Gaussian fit is shown in Figure 5. The fact that the fitting is in general satisfactory may explain why the agreement with the quadrature formula is good. Also the electronics involved in obtaining the horizontal and longitudinal profiles might lead to distortions of the true profiles. This provides another possible deviation from the quadratic formula.

A Accuracy of the emittance measurements

This appendix is concerned with the precision of the results of emittance computations. The expression for the relative error of the horizontal emittance (Eqs. 1–2) is

$$\frac{\Delta\epsilon_h}{\epsilon_h} = \frac{\Delta\beta_h}{\beta_h} + 2\,\frac{\sigma_h^2}{\sigma_{\beta_h}^2}\,\frac{\Delta\sigma_h}{\sigma_h} + 2\left(\frac{\sigma_h^2 - \sigma_{\beta_h}^2}{\sigma_{\beta_h}^2}\right)\frac{\Delta\sigma_\delta}{\sigma_\delta} + 2\left(\frac{\sigma_h^2 - \sigma_{\beta_h}^2}{\sigma_{\beta_h}^2}\right)\frac{\Delta D_h}{D_h} \qquad (4)$$

where $\Delta\beta_h/\beta_h$, $\Delta D_h/D_h$, $\Delta\sigma_h/\sigma_h$ and $\Delta\sigma_\delta/\sigma_\delta$ are bounds for the relative errors of β_h, D_h, σ_h and σ_δ respectively. A similar expression holds for the relative error of the vertical emittance.

The relative error $\Delta\epsilon_h/\epsilon_h$ is minimum when σ_{β_h} is equal to σ_h. Hence

$$\left(\frac{\Delta\epsilon_h}{\epsilon_h}\right)_{min} = \frac{\Delta\beta_h}{\beta_h} + 2\,\frac{\Delta\sigma_h}{\sigma_h} \qquad (5)$$

If the relative errors of all the optic parameters, the beam width and the momentum spread are identical, Eq. 4 becomes

$$\frac{\Delta\epsilon_h}{\epsilon_h} = \left(6\,\frac{\sigma_h^2}{\sigma_{\beta_h}^2} - 3\right)\frac{\Delta x}{x} \qquad (6)$$

with

$$\frac{\Delta x}{x} = \frac{\Delta\beta_h}{\beta_h} = \frac{\Delta D_h}{D_h} = \frac{\Delta\sigma_h}{\sigma_h} = \frac{\Delta\sigma_\delta}{\sigma_\delta} \qquad (7)$$

For instance, assuming $\Delta x/x = 4\%$ and considering the data in Table 1, Eq. 6 yields relative errors of the horizontal emittance within the range 94%–128%.

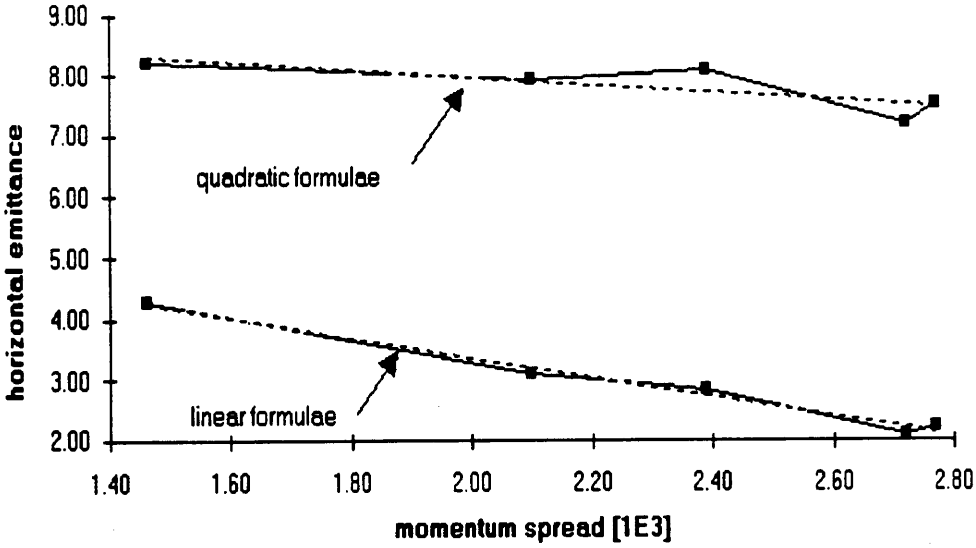

Figure 1: Horizontal emittance measurements at 3.5 GeV/c

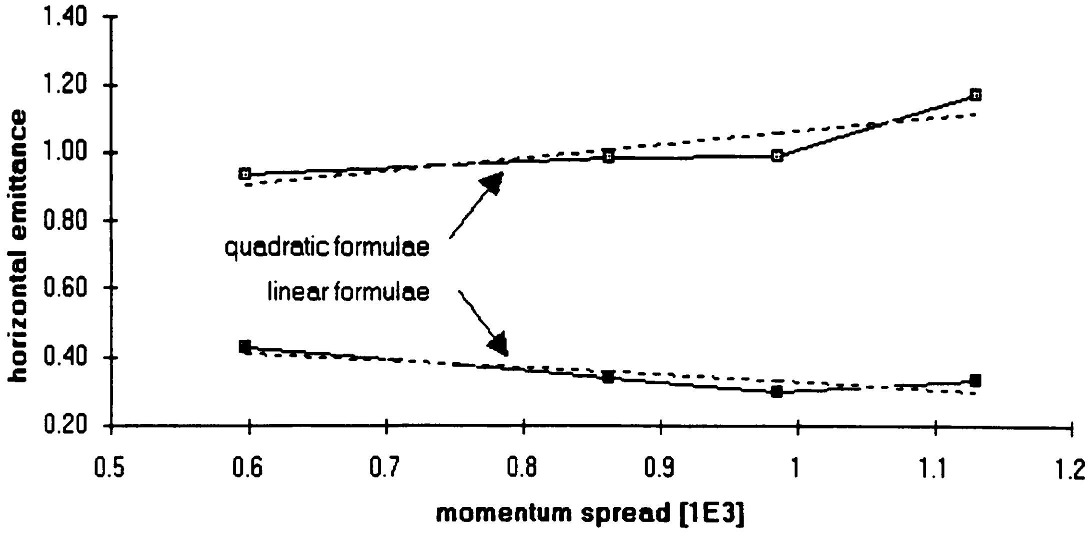

Figure 2: Horizontal emittance measurements at 26 GeV/c

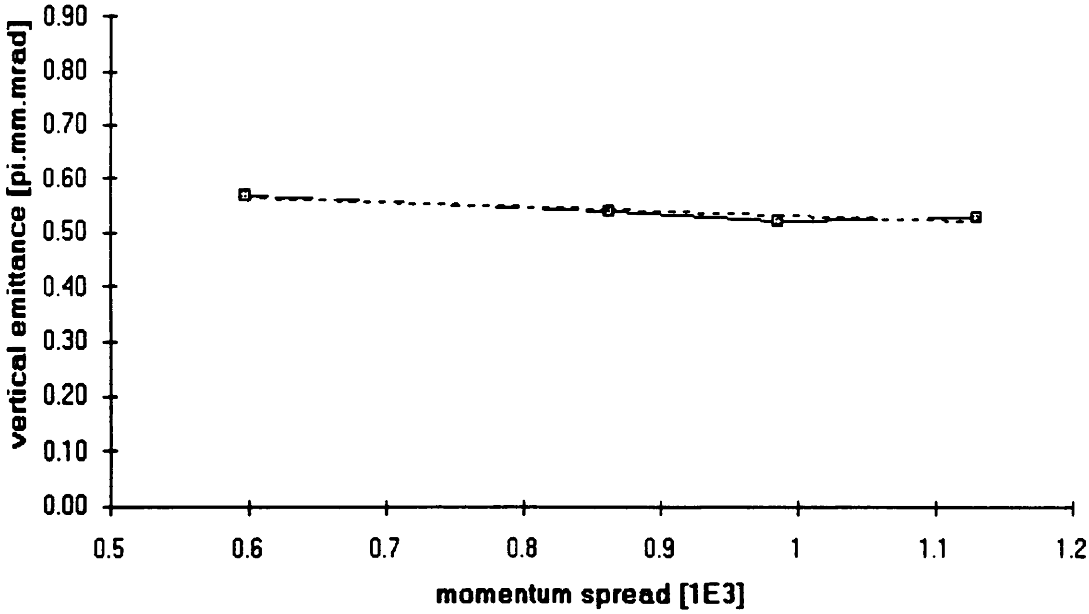

Figure 3: Vertical emittance measurements at 3.5 GeV/c

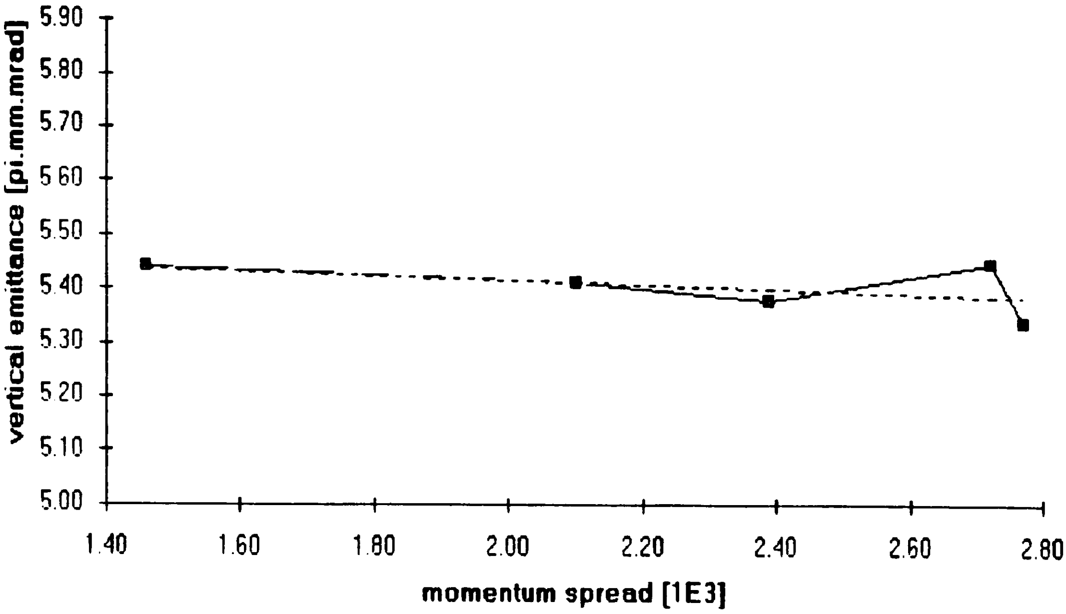

Figure 4: Vertical emittance measurements at 26 GeV/c

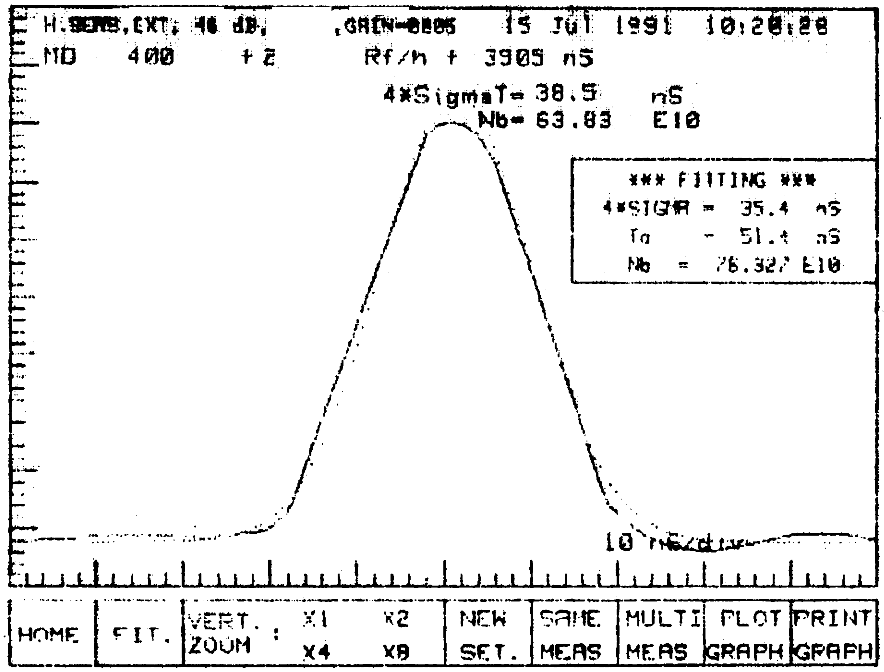

Figure 5: Longitudinal bunch profile at 3.5 GeV/c

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