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INVESTIGATION OF PHASE DENSITY REDISTRIBUTION IN REAL CHARGED PARTICLE BEAMS WITH THE HELP OF NUMERICAL CALCULATIONS

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INTRODUCTION

p. 419 The distribution function of the phase space density of particles in a beam satisfies the system of equations for the self-consistent field. Two approaches to determine the particle phase density distribution function may be presented /1/.

In the first case the law of particle distribution in phase space reamins unknown for the motion of the beam in the self-consistent field (stationary problem). I.M. Kapchinskij /1/ showed that stationarity is ensured for a microcanonical particle distribution in 4-dimensional phase space in linear external fields. Then the charge density in an arbitrary beam cross-section does not depend on the transversal coordinates. The non-stationary problem has been considered in refs. /2,3,4/.

When neglecting the dispersion of longitudinal particle velocities, then the stationary 6-dimensional problem may be reduced to a non-stationary problem in 4 dimensions, corresponding to the transversal particle coordinates velocities. This problem is more interesting since in this case one may find the particle density distribution function in phase space for arbitrary cross-sections of real beams if the phase density distribution is known for a certain arbitrary initial cross-section.

THE SYSTEM OF EQUATIONS AND THE METHOD OF CALCULATION

In the case of an axially symmetrical beam it is convenient to use as coordinates in geometrical space radius R, angle θ and longitudinal coordinate z of a cylindrical system. It is well known that the particle phase density stays constant along particle trajectories only in the space of canonically conjugate variables /5/. We chose a phase space with variables R, θ, z, v, p, w; where v, w are the radial and axial particle velocities; and where $p = R^2\theta$. For non-relativistic velocities v, w, p differ from the generalized momenta canonically conjugate to the generalized coordinates R, z, θ only by a constant factor equal to the particle mass. The phase density for particle motion in the space R, θ, z, v, p, w is conserved. Mathematically this may be written as :

$$\frac{\partial F}{\partial t} + w\frac{\partial F}{\partial z} + v\frac{\partial F}{\partial R} + \dot{\theta}\frac{\partial F}{\partial \theta} + \dot{w}\frac{\partial F}{\partial w} + \dot{v}\frac{\partial F}{\partial v} + \dot{p}\frac{\partial F}{\partial p} = 0, \qquad (1)$$

where F(R,θ,z,v,p,w) is the particle phase density.

For the stationary axially symmetrical case and in the absence of azimuthal forces equation (1) may be written as :

$$w\frac{\partial F}{\partial z} + v\frac{\partial F}{\partial R} + \dot{w}\frac{\partial F}{\partial w} + \dot{v}\frac{\partial F}{\partial v} = 0. \qquad (2)$$

If the dispersion of longitudinal particle velocites and the dependence of the longitudinal velocity on the radial coordinate are neglected, then the longitudinal particle velocity is a single-valued function of the coordinates.

After deviding equation (2) by w, we rewrite it in the following way :

$$\frac{\partial F}{\partial z} + \frac{dR}{dz}\frac{\partial F}{\partial R} + \frac{dw}{dz}\frac{\partial F}{\partial w} + \frac{dv}{dz}\frac{\partial F}{\partial v} = \frac{dF}{dz} = 0. \qquad (3)$$

p. 420 The solution of equation (3) may be looked for as the solution of a Cauchy problem. As initial condition we may take the value of phase density at z = 0, which is written as :

$$F_0 = F\rbrack_{z=0} = f_0(R_0, v_0, p_0)\delta(w_0 - w_0{}^*), \qquad (4)$$

where $w_o^* = \text{const}$.

Along the particle trajectories, which are the characteristic curves of equation (3), the following relation holds :

$$F(R, v, p, z, w) = f_0(R_0(R, v, p, z), v_0(R, v, p, z), p_0)\,\delta\,(w_0(z, w) - w_0{}^*). \qquad (5)$$

We introduce the function :

$$L(w) = w_0(z, w) - w_0{}^*. \qquad (6)$$

From (6), regarding z as a parameter, we have :

$$dL = \frac{w}{w_0}\, dw. \tag{7}$$

We assume that there is no longitudinal overtaking of particle. Then we have from (5) with the help of (7) :

$$\int_{-\infty}^{+\infty} F dw = \frac{f_0 w_0}{w}. \tag{8}$$

Using the result (8), inserting (5) into equation (3) and integrating the last one, we get :

$$\frac{\partial g}{\partial z} + \frac{dR}{dz}\frac{\partial g}{\partial R} + \frac{dv}{dz}\frac{\partial g}{\partial v} = \frac{dg}{dz} = 0, \tag{9}$$

where $g = f.w$; $\iint g\, dv\, dp = 2\pi R j$, j is beam current density.

From (9) it follows that in the general case in the presence of an external electrical field the particle density f in phase space is not conserved, but the quantity g. This is a consequence of the fact that we neglected the longitudinal velocity dispersion.

Particle trajectories have been found with the help of the equation :

$$\frac{d^2R}{dz^2} = \frac{e}{2\pi\varepsilon_0 m}\,\frac{I(R, z)}{Rw^3} + \frac{p^2}{R^3 w^2} + \varphi(R', R, z), \tag{10}$$

where :

$$I(R, z) = \iiint g\, dp\, dv\, dr \tag{11}$$

is the beam current, e, m are particle charge & masse, ε_o is the permeativity of the vacuum, $\phi(R',R,z)$ is the term related to the presence of the external fields.

Equation (10) has been solved numerically. The function I(R,z) has been computed in each step by numerical integration with the help of the expression (11).

RESULTS OF THE CALCULATION AND DISCUSSION

With the help of the described method of calculation the redistribution of phase density in the proton beam of the injector I-100 has been investigated. The values of the distribution function in the initial cross-section, denoted in Fig. 1 as the cross-section with coordinate z = 0, have been obtained by measuring the ion density by the method of 4 slits using films for recording the beam. The results of the calculation is given in Figs. 1 - 3. Fig. 1 where the beam envelopes and current density distributions in some cross-sections are given illustrates some properties and peculiarities of real beams.

First, the current density distribution in a beam cross-section is essentially different from a uniform one and has qualitatively different form for different cross-sections, so that one may treat it as a redistribution of the current density over the beam's cross-section. The presence of the redistribution of the current density supports the appropriateness of the non-stationary problem.

Second, the beam envelopes are not symmetrical with respect to the cross-section corresponding to the minimal size of the current of charged particles. This is related to the non-uniform velocity structure of the proton current , as if it consisted of several beams with different average velocities.

In Fig. 2 projections of the phase volume on the plane of Cartesian coordinates x,x' are given for several beam cross-sections. The contour of the projection of the phase volume continuously changes its form and for cross-sections sufficiently distant from the cross-over current it has "protrusions" observed in several experimental investigations /6,7,8/. But in the given case these "protrusions" may be related to the extended character of the example of the initial contour of the phase volume.

Let us say a few words on the comparison of computed results with experiments. In Fig. 3 the computed current density distribution in the cross-section corresponding to the entrance of the accelerating tube of the linac injector /9/ is shown.

In Fig. 4 an enlarged representation of the beam is given, cut out from the primary current by a diaphragm with a width of 0.06 mm. As is seen from a comparison between the photography and Fig. 3, the calculated current density distribution qualitatively agrees with the measured one. The beam sizes, approximately determined with the help of a grid as scale, equally agrees with the computed one (mesh size 3,2 x 3,2 mm^2).

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Distribution (open)

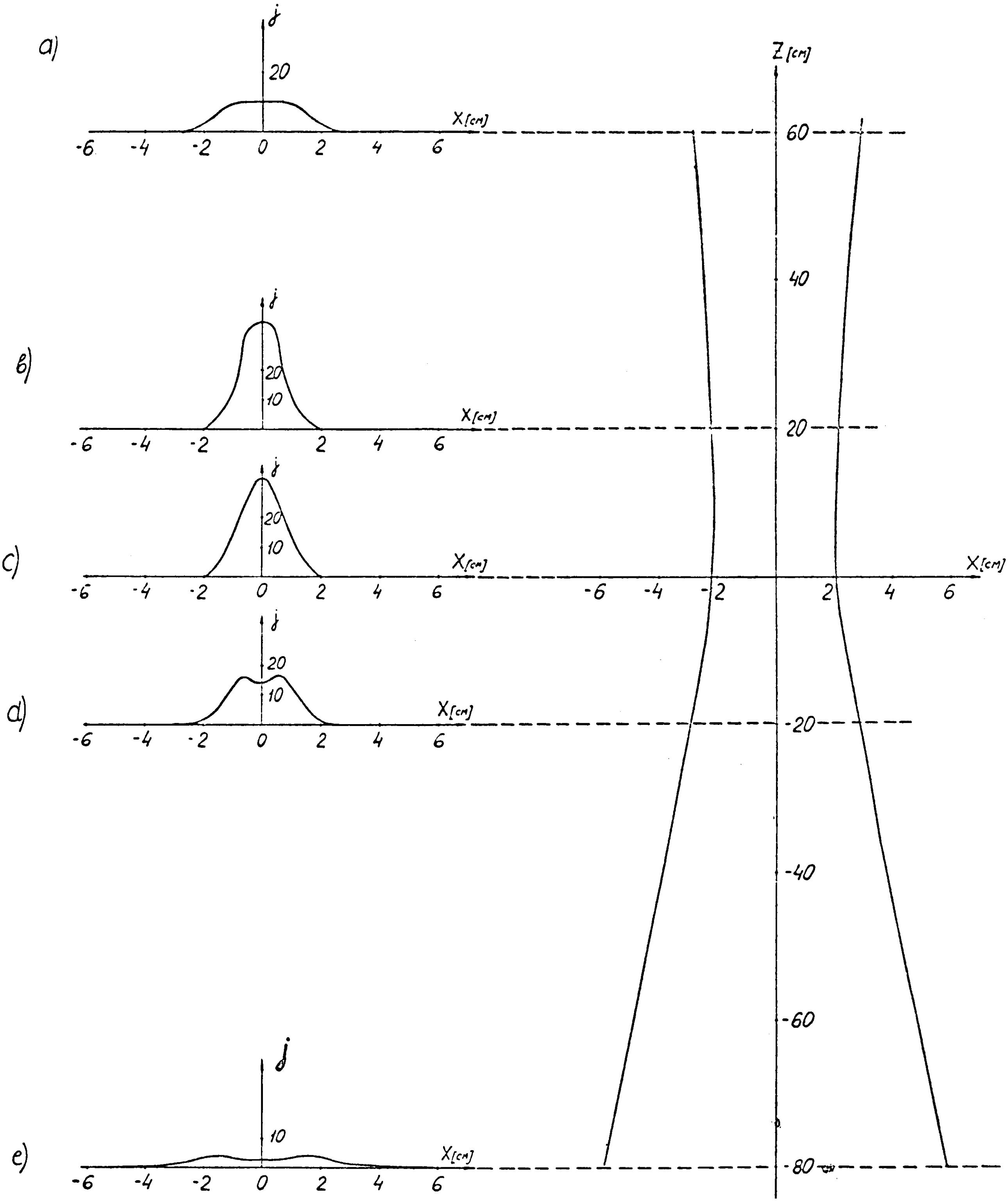

Fig. 1 : Beam envelope and current density distribution
a) z = 60 cm b) z = 20 cm c) z = 0
d) z = -20 cm e) z = -80 cm

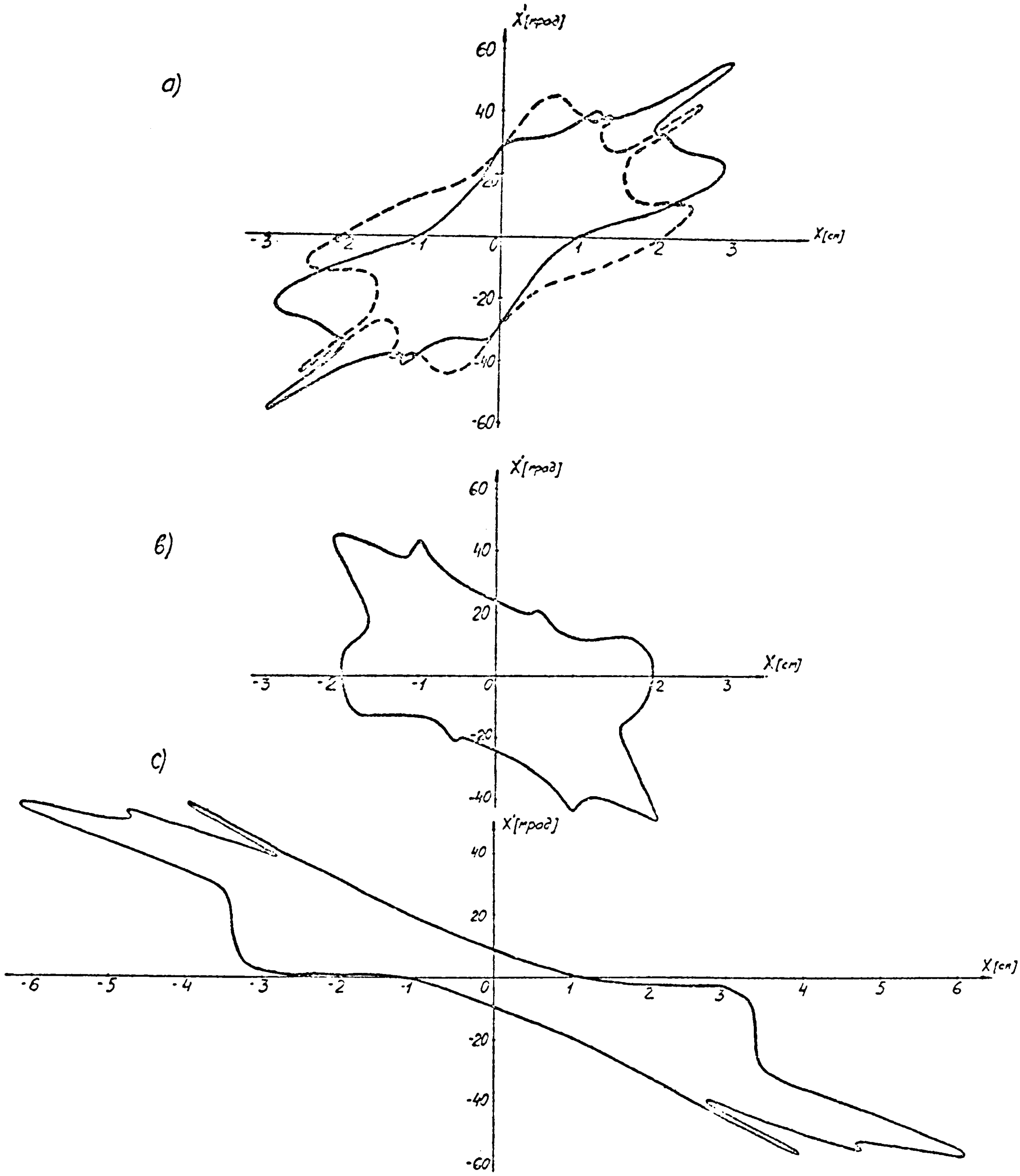

Fig. 2. : Projections of the beam's phase volume on the xx' plane.

a) z = 60 cm b) z = 0 c) z = −80 cm

(dotted curve gives the projection computed without consideration of Coulomb forces).

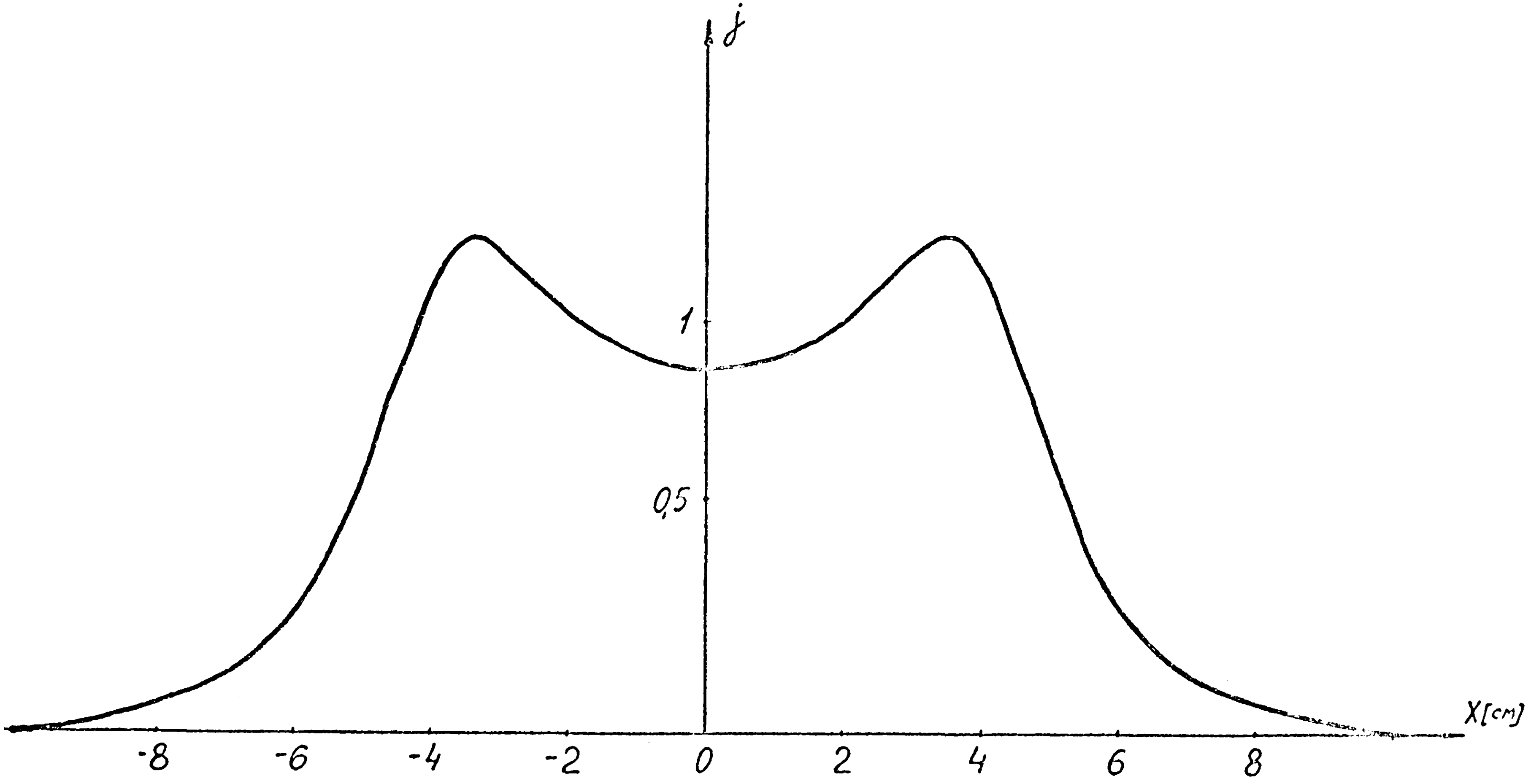

Fig. 3. : Current density distribution at the tube entrance

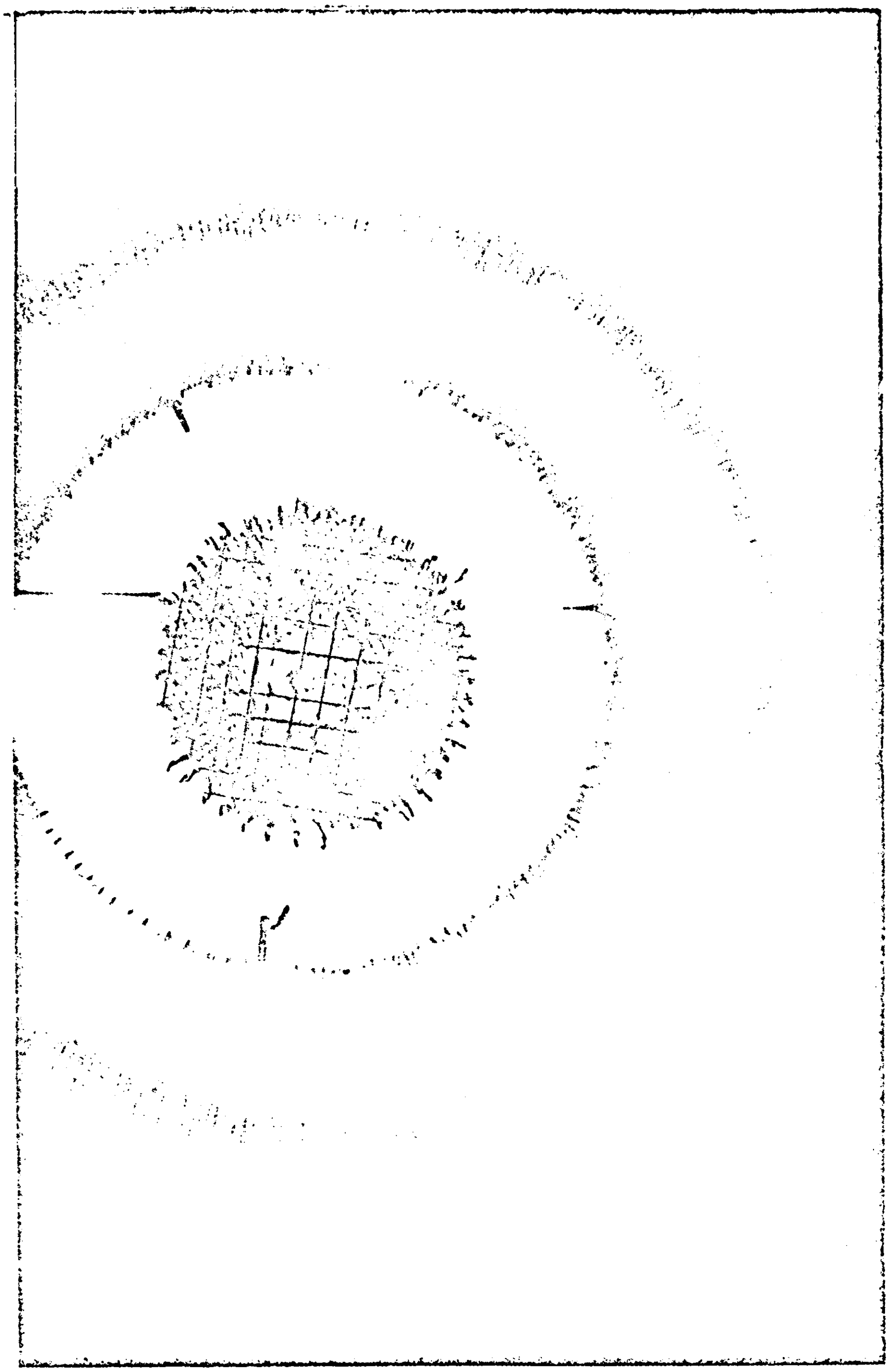

Fig. 4. : Image of the beam cut out from the primary current by a fine diaphragm