MPS/DL-Note 69-20
11 July 1969

EFFECT OF BEAM-INDUCED RF ON DEBUNCHING IN THE 300 GeV MACHINE

The travelling-wave structure proposed by Schnell[1] for the 300 GeV machine would have an efficient bandwidth of half a percent and operate on a harmonic number of 4500, so there will be more than 20 other possible harmonic numbers that fall within its band. Under these conditions the risk of the coasting beam rebunching itself, at the frequency which makes the structure impedance worst, gives probably the strictest[2] debunching criterion.

The in-phase (decelerating) component of voltage induced in such a structure by sinusoidal beam current modulation is given[3] by

$$\frac{1}{4}\;\frac{1-\cos\tau}{\tau^2}\;Z_s I\, L^2$$

where Z_s is the so-called series impedance, I is the a.c. beam current, L is the length of the structure, and τ is the phase slip between beam and wave in the whole length L, for the frequency and wavelength in question. The quadrature component is

$$\frac{1}{4}\;\frac{\tau-\sin\tau}{\tau^2}\;Z_s I\, L^2$$

The total voltage amplitude has its largest value * at $\tau = 0$, where it is

$$\frac{1}{8}\, Z_s I\, L^2$$

So we must require stability in the presence of an equivalent gap impedance

---

* The quadradure component is biggest at $\tau = \pm\pi$, where $(\tau-\sin\tau)/\tau^2$ reaches $\pm\, 1/\pi$.

which may be as big as

$$|Z_g|_{max} = \frac{1}{8} Z_s L^2$$

for each of the n RF structures in the machine.

From the definitions one has

$$Z_s L^2 = \left[\frac{U^2}{P}\right]_{\text{at midband}}$$

and a structure that has been designed to have optimum group velocity (for given length and bandwidth) has[1)]

$$\left[\frac{U^2}{P}\right]_{\text{at midband}} = 4.66 \left(\frac{R}{Q}\right)\left(\frac{f}{\Delta f}\right) L$$

where Δf is the full width of the band for which it is optimised and

$$\frac{R}{Q} = \frac{1}{\omega} \frac{(\text{accelerating field})^2}{\text{stored energy per unit length}}$$

We take the values from the 1964 design study[1)*],
$R/Q = 400\ \Omega/m$, $\Delta f/f = 5.5\ 10^{-3}$, and the total length of accelerating structure in the machine $nL = 160$ m. This gives

$$n|Z_g|_{max} = 0.58 \left(\frac{R}{Q}\right)\left(\frac{f}{\Delta f}\right) nL$$

$$= 6.8 \times 10^6\ \Omega$$

---

* The present trend is in the direction of higher R/Q and less nL, with rather little change in their product [3)].

As criterion for stability against rebunching of the coasting beam we take [4]

$$\left(\frac{\Delta E}{E}\right)^2 > \frac{1}{0.7}\ \frac{1}{4\pi^2}\ \frac{\beta^3}{\eta\,\gamma\,h}\ \frac{Ne^2}{mc^2}\ \frac{c}{R}\ n|Z_g|_{max}$$

where ΔE is the halfwidth of the spectrum of the coasting beam and we put in the values $\gamma = 320$, $\eta = -d\log\omega/d\log p = \gamma_{tr}^{-2} - \gamma^{-2} = 1.43 \times 10^{-3}$, $R = 1200$ m, $N = 3 \times 10^{13}$, $e^2/mc^2 = 1.7 \times 10^{-28}$ Farad, $h = 4500$, to find

$$\frac{\Delta E}{E} > 0.58 \times 10^{-3}\ (\pm)$$

This will require a considerable dilution compared with the ideal energy spread of the 300 GeV beam ($\pm\ 0.1 \times 10^{-3}$ in bunches $\pm\ 5.6^0$ long[1]), but is not big enough to create any obvious difficulties. It is about ± 2 mm at α_{pmax}.

H.G.Hereward

/jdw

References

1) The design study of a 300 GeV..... AR/Int.SG/64-15. See pages 121 and 445.

2) H.G. Hereward. "Effects of cavities on debunching....." CERN/MPS/DL 69-7. Equ.(29).

3) W. Schnell, private communication.

4) We are being more stringent by a factor of 1/0.7 than the criterion used in 2). According to Keil (private communication) this factor is a good summary of the computed results of 5), for several reasonable shapes of spectrum, with the width being measured at half height.

5) Ruggiero and Vaccaro. "Solution of the dispersion.....". ISR-TH/68-33.

Distribution: (open)

Ejection study group
Machine Studies Team

E.Keil
W.Schnell