

# Model Predictive Control of Induced Draft Cooling Towers in a Large Scale Cooling Plant

Faiq Ghawash\* Morten Hovd\* Brad Schofield\*\*

\* National Norwegian University of Science and Technology, Trondheim, Norway ([faiq.ghawash](mailto:faiq.ghawash@ntnu.no), [morten.hovd](mailto:morten.hovd@ntnu.no))

\*\* Beams Department, Industrial Controls and Safety Systems Group, CERN, Meyrin, Switzerland ([brad.schofield@cern.ch](mailto:brad.schofield@cern.ch))

**Abstract:** We study the problem of designing a model predictive control strategy for multiple induced draft cooling towers. The common objective of the cooling towers is to regulate the outlet water temperature of the common collection basin to the desired set point. Each cooling tower can be operated in different modes (bypass, showering, ventilation) to meet the heat rejection requirement. To deal with the interacting dynamics and logic, we propose a model predictive control strategy based on mixed integer programming (MIP) which accounts for the changing weather conditions and active heat load to simultaneously determine the best operational mode and optimal fan speed for each cooling tower. Dwell time constraints are added to avoid excessive switching between different operational modes. Although the proposed formulation can account for different operational efficiencies of the cooling towers, an adaptive penalty is designed to balance the run time among identical cooling towers. The inherent over-actuated structure of the cooling process is exploited to provide robustness against the unavailability of an operational mode of a cooling tower. The efficacy of the proposed approach is demonstrated on experimentally validated models of induced draft cooling towers. Simulation results show significant performance improvements and energy savings over conventional heuristic solutions.

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*Keywords:* Model Predictive Control, Mixed Integer Programming, Switched System, Process Control.

## 1. INTRODUCTION

In a large scale cooling plant multiple induced draft cooling towers are used to meet the cooling requirements of different clients (e.g chillers, cryogenics etc.). Depending on the active heat load, weather conditions and different operational efficiencies, the cooling towers can be operated in different modes to meet the heat rejection requirement. Advanced control design techniques are required to determine the optimal configuration which can meet the control specifications while minimizing the energy consumption of the cooling towers (see Ghawash et al. (2021); Viljoen et al. (2020)).

For a constant mass flow rate of water, fan speed modulation is used to vary the mass flow rate of air entering the cooling tower. For a large scale industrial cooling tower, the cooling and lubrication requirement of the motor and gear box assembly imposes minimum fan speed requirement and in some cases the minimum fan speed requirement is as high as 60% of the rated fan speed (as in Peljo (2018); Schofield et al. (2019)). Hence, the change in the operational mode of a single cooling tower can cause a significant change in the overall cooling capacity and energy consumption of the cooling towers. In such cases, an optimal control strategy must simultaneously determine the best operational mode and optimal fan speed for each

cooling tower to enable energy optimal operation of the cooling towers.

The control design for multiple cooling towers has been studied as part of the chilled water cooling plant (e.g Braun et al. (1989)). A large scale chilled water cooling plant consists of multiple chillers, variable speed condenser water pumps and cooling towers equipped with variable speed fans. Due to the dynamic coupling among the elements of the cooling plant, the power consumption of different elements is strongly correlated. For instance, decreasing the condenser water supply temperature (equivalently decreasing the outlet water temperature of the cooling towers) and increasing the condenser water mass flow rate will decrease the power consumption of the chillers at the expense of increasing the power consumption of the fans and pumps. In such a setting, optimal and near optimal control strategies have been presented to optimize the outlet water temperature of the cooling towers which minimizes the overall power consumption of the cooling plant (see Braun and Diderrich (1990); Li et al. (2012)). However, the results on determining optimal (or near optimal) outlet water temperature are scarce in the cases where the outlet water of the cooling towers is supplied to multiple clients having different energy consumption patterns. The investigation in this regard is reserved for future work. In this study, we assume that the optimal set point for the outlet water temperature is available and focus

on designing an optimal control strategy for achieving the desired outlet water temperature of the common collection basin of the cooling towers.

In this paper, we present a mixed integer model predictive control strategy to enable optimal operation of multiple induced draft cooling towers. The proposed strategy can simultaneously determine the best operational mode and optimal fan speed for each cooling tower to provide the required cooling capacity. In addition, the developed framework provides robustness against mode unavailability, ensures run time balance among identical cooling towers and prevents excessive mode switching. Such characteristics make the developed control strategy suitable for practical implementation. We also present a comparison that shows a considerable improvement in performance and energy usage over the conventional heuristics solutions.

The rest of the paper is organized as follows: Section 2 provides a brief overview of the working principle and mathematical models of different operational modes of the cooling towers. Section 3 defines the control objectives, highlights the physical and operational constraints and details the development of model predictive control strategy. Section 4 provides numerical simulations and compares the performance and energy usage of model predictive control strategy with conventionally used solutions. Finally, section 5 concludes the paper.

## 2. INDUCED DRAFT COOLING TOWERS

In a large scale cooling plant, multiple induced draft cooling towers are used to cool the incoming hot water stream by rejecting the excess heat into the atmosphere. Fig.1 shows the setup under consideration where multiple induced draft cooling towers are used to reject the heat from the incoming hot water stream. The mass flow rate of the incoming water is equally distributed among the cooling towers. Depending on the weather conditions, each cooling tower can be operated in ventilation, showering or bypass mode to provide the required cooling capacity (Peljo (2018)). Due to the piping layout, the bypass mode is required to be activated together for all the cooling towers. The cooled water from the individual cooling towers goes to a shared water collection basin which is then supplied to different clients to meet their cooling requirements.

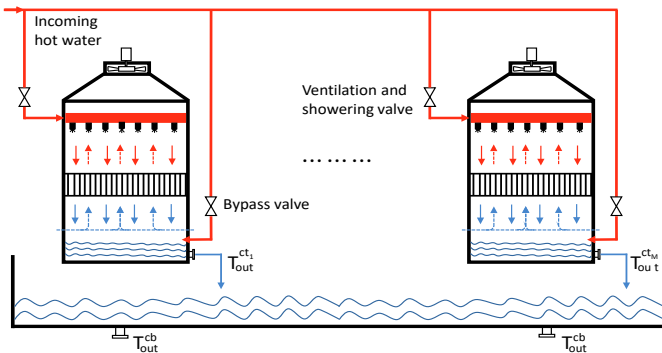


Fig. 1. Induced draft cooling towers with a shared water collection basin.

Different operating modes of a cooling tower can provide different cooling capacities. Hence, the outlet water tem-

perature for an individual cooling tower depends on the chosen operating mode for the cooling tower.

### 2.1 Switched Dynamics of Outlet Water Temperature of the Cooling Tower

Next, we provide an overview of the outlet water temperature dynamics in different operational modes of the cooling tower. The outlet water temperature in the ventilation mode is governed by nonlinear dynamics whereas showering and bypass modes are governed by autonomous affine dynamics.

*Showering and Ventilation Mode:* In showering and ventilation modes, the bypass valve remain closed and the hot water is sprayed downwards through the spray nozzles. The ambient air moves upward through natural (showering) or forced draft (ventilation) depending on the operational mode of the cooling tower. In ventilation mode, the mass flow rate of ambient air can be varied by modulating the fan speed of the cooling tower. However, in showering mode, the fan remains off and the mass flow rate of ambient air is roughly fixed. The cooling of the incoming hot water stream takes place through simultaneous heat and mass transfer occurring throughout the cooling tower (Afshari and Dehghanpour (2019)). Mass transfer occurs due to evaporation whereas convective heat transfer takes place at the air water interface. Under suitable assumptions, mass and energy conservation is used to derive a semi-empirical model to effectively represent the evolution of outlet water temperature of the cooling tower as presented in Jin (2011).

$$\begin{aligned} \dot{T}_{out}^{ct}(t) = & -c_1 \dot{m}_{w,in}^{ct}(t)(T_{out}^{ct}(t) - T_{in}^{ct}(t)) - c_2 (\dot{m}_{w,in}^{ct}(t))^l \\ & \left( \frac{(\dot{m}_a^{ct}(t))^l}{(\dot{m}_a^{ct}(t))^l + c_3 (\dot{m}_{w,in}^{ct}(t))^l} \right) (T_{in}^{ct}(t) - T_{wb}^i(t)) \end{aligned} \quad (1)$$

where  $T_{in}^{ct}(t)$ ,  $T_{out}^{ct}(t)$ , represents the inlet and outlet water temperatures respectively.  $T_{wb}^i(t)$  is the wet bulb temperature of the ambient air entering the cooling tower.  $\dot{m}_{w,in}(t)$  and  $\dot{m}_a(t)$  are the mass flow rate of water and air entering the cooling tower.  $c_1$ ,  $c_2$ ,  $c_3$  and  $l$  are empirical parameters of the model that must be determined from the operational data. The incoming hot water stream can be cooled below the dry bulb temperature of ambient air whereas the wet bulb temperature is the lowest achievable temperature. More details on model derivation and parameter estimation can be found in Jin (2011).

*Bypass Mode:* In the bypass mode the incoming water is routed directly to the water collection basin of an individual cooling tower. The bypass mode is activated when the ventilation and showering modes provide excessive cooling of the incoming water stream. Assuming perfect mixing, the energy balance for the bypass mode gives:

$$\dot{T}_{out}^{ct}(t) = -\frac{1}{m^{ct}} (\dot{m}_{w,out}^{ct}(t) \dot{T}_{out}^{ct}(t) - \dot{m}_{w,in}(t) T_{in}(t)) \quad (2)$$

where  $m^{ct}$  is the mass of the water in the collection basin,  $\dot{m}_{w,in}^{ct}$  and  $\dot{m}_{w,out}^{ct}$  represents the mass flow rate of the water entering and leaving the water collection basin of an individual cooling tower. Similarly, the mass balance for the bypass mode gives:

$$\dot{m}_{w,out}^{ct}(t) = \dot{m}_{w,in}^{ct}(t) \quad (3)$$

(2) and (3) hold under the assumption that loss of the water due to evaporation is negligible. Finally, the outlet water temperature of an individual cooling tower can be compactly represented using switched system representation.

$$\dot{T}_{out}^{ct}(t) = f_{\sigma(t)}(T_{out}^{ct}(t), u(t)) \quad (4)$$

where

$$f_{\sigma(t)} = \begin{cases} f^{vent}(T_{out}^{ct}(t), u(t)), & \sigma(t) = 1 \\ f^{show}(T_{out}^{ct}(t)), & \sigma(t) = 2 \\ f^{byp}(T_{out}^{ct}(t)), & \sigma(t) = 3 \end{cases}$$

$\sigma(t) \in \{1, 2, 3\}$  represents the active operational mode at any time instant.  $f^{vent}(T_{out}^{ct}, u(t))$  represent the nonlinear outlet water temperature dynamics in the ventilation mode given by (1).  $u(t)$  is the percentage fan speed that can be manipulated to control the mass flow rate of air entering the cooling tower.  $f^{show}(T_{out}^{ct})$  and  $f^{byp}(T_{out}^{ct})$  represent the affine autonomous dynamics of outlet water temperature in showering and bypass modes given by (1), (2) respectively.

## 2.2 Shared Water Collection Basin

The outlet water from individual cooling towers goes to the shared water collection basin. The cooled water is then supplied to different clients from the shared water collection basin of the cooling tower. Assuming perfect mixing, the energy balance for the shared water collection basin leads:

$$\begin{aligned} \dot{T}_{out}^{cb}(t) = & -\frac{1}{m^{cb}}(\dot{m}_{w,out}^{cb}(t)T_{out}^{cb}(t) - (\dot{m}_{w,out}^{ct_1}(t)T_{out}^{ct_1}(t) \\ & + \dots + \dot{m}_{w,out}^{ct_n}(t)T_{out}^{ct_M}(t))) \end{aligned} \quad (5)$$

where  $\dot{T}_{out}^{cb}(t)$ ,  $\dot{m}_{w,out}^{cb}(t)$  represents the outlet water temperature and mass flow rate of water leaving the shared water collection basin.  $m^{cb}$  is the mass of the water in the collection basin.  $M$  represents the number of cooling towers sharing a common water collection basin.

The mass balance for the shared water collection basin gives:

$$\dot{m}_{w,out}^{cb}(t) = \dot{m}_{w,out}^{ct_1}(t) + \dots + \dot{m}_{w,out}^{ct_M}(t) \quad (6)$$

Since the loss of water due to evaporation is considered negligible, (5) and (6) do not account for the makeup water entering the shared water collection basin.

## 3. MIXED INTEGER MODEL PREDICTIVE CONTROLLER

In this section, we define the control objectives, highlights the physical and operational constraints and details the development of mixed integer model predictive controller to enable optimal operation of the cooling towers.

### 3.1 Control Objectives

The main objective is to determine the best operational mode and optimal fan speed for each cooling tower that can regulate the outlet water temperature of the shared water collection basin while minimizing the total power consumption of the cooling towers. Moreover, the control strategy must adhere to the physical and operational constraints and must provide robustness against the unavailability of an operational mode of the cooling tower.

### 3.2 Physical and Operational Constraints

Certain physical and operational constraints must be satisfied to ensure the nominal operation and maximize the lifetime of the mechanical equipment.

- To satisfy the cooling and lubrication requirement for the motor and gearbox assembly, the minimum fan speed (when the fan is in operation) must be set to 60% of the rated fan speed.
- The excessive switching between different operational modes of the cooling towers must be prevented to avoid frequent maintenance of the fan motor.
- The operating time of the identical cooling towers must be balanced to maximize the lifetime of the equipment.
- The temperature gradient inside the shared water collection basin must be minimized to ensure uniform supply water temperature to different clients.
- Due to piping layout and hydraulic constraints, the bypass mode of all the cooling towers must be activated together.

### 3.3 Mixed Integer Program

A mixed integer formulation is presented based on the objectives, constraints and switched dynamics of the outlet water temperature of the cooling tower. The interaction between dynamics and logic rules is handled by incorporating continuous and discrete variables in the formulation. For clarity of the presentation, let  $\mathcal{C} = \{1, \dots, M\}$  be the set of cooling towers,  $\mathcal{C}_I(\subset \mathcal{C}) = \{1, \dots, I\}$  be the set of identical cooling towers<sup>1</sup> and  $\mathcal{K} = \{1, \dots, N\}$  be the set of time instants.

*Fan power consumption:* Affinity laws suggests a cubic relation of the power consumption with the fan speed (see Ford (2011)). However, the power consumption of the fan can be roughly approximated by a quadratic polynomial.

$$P_{fan}^{ct} = \alpha_1(u^{ct})^2 + \alpha_2u^{ct} \quad (7)$$

*Switched system dynamics:* The evolution of outlet water temperature of the cooling tower is dependent on *if-then logic*. In the optimization program, the outlet water temperature must satisfy specific dynamics (equality constraint) based on the chosen operational mode. For instance, if the ventilation mode is chosen, the outlet water temperature must satisfy (1), similarly if the bypass is activated the outlet water temperature must satisfy (2). Such requirements can be converted into mixed integer inequalities using the Big-M technique as described in Bemporad and Morari (1999).

$$\begin{aligned} (1 - b_v^{ct_j}[k])m_1^{ct_j} &\leq T_{out}^{ct_j}[k+1] - f_{vent}^{ct_j}(T_{out}^{ct_j}[k], u^{ct_j}[k]) \\ &\leq (1 - b_v^{ct_j}[k])M_1^{ct_j}, \forall k \in \mathcal{K}, \forall j \in \mathcal{C} \\ (1 - b_s^{ct_j}[k])m_2^{ct_j} &\leq T_{out}^{ct_j}[k+1] - f_{show}^{ct_j}(T_{out}^{ct_j}[k]) \\ &\leq (1 - b_s^{ct_j}[k])M_2^{ct_j}, \forall k \in \mathcal{K}, \forall j \in \mathcal{C} \\ (1 - b_b^{ct_j}[k])m_3^{ct_j} &\leq T_{out}^{ct_j}[k+1] - f_{byp}^{ct_j}(T_{out}^{ct_j}[k]) \\ &\leq (1 - b_b^{ct_j}[k])M_3^{ct_j}, \forall k \in \mathcal{K}, \forall j \in \mathcal{C} \end{aligned} \quad (8)$$

where  $b_v^{ct}$ ,  $b_s^{ct}$ ,  $b_b^{ct}$  are the binary variables associated with ventilation, showering and bypass modes of the cooling tower.  $T_{out}^{ct}$  represents the outlet water temperature of the cooling tower.  $f_{vent}^{ct}$ ,  $f_{show}^{ct}$ ,  $f_{byp}^{ct}$  represents the discretized ventilation showering and bypass modes dynamics. The values of  $m_1^{ct}$ ,  $m_2^{ct}$ ,  $m_3^{ct}$  must be sufficiently small whereas  $M_1^{ct}$ ,  $M_2^{ct}$ ,  $M_3^{ct}$  must be sufficiently large. In addition, we also need to ensure that only one operational mode must be turned on at any time instant. The following logical constraint ensures this requirement.

$$b_v^{ct_j}[k] + b_s^{ct_j}[k] + b_b^{ct_j}[k] = 1, \forall k \in \mathcal{K}, \forall j \in \mathcal{C} \quad (9)$$

*Minimum fan speed:* The cooling tower fan is only tuned on in the ventilation mode. The fan speed must adhere to the minimum speed constraint imposed by the cooling requirements of the motor and gearbox assembly. Otherwise, in showering and bypass modes, the fan must remain off. Such *if-then-else* logic constraint can be written as a mixed integer inequality.

$$b_v^{ct_j}[k]u_{min}^{ct_j} \leq u^{ct_j}[k] \leq b_v^{ct_j}[k]u_{max}^{ct_j}, \forall k \in \mathcal{K}, \forall j \in \mathcal{C} \quad (10)$$

*Shared water collection basin:* The dynamics of the outlet water temperature of the shared collection basin is imposed as an equality constraint in the optimization program.

$$T_{out}^{cb}[k+1] - f_{cb}(T_{out}^{cb}[k], T_{out}^{ct_j}[k]) = 0, \forall k \in \mathcal{K}, \forall j \in \mathcal{C} \quad (11)$$

where  $T_{out}^{cb}$  and  $f_{cb}(T_{out}^{cb}[k], T_{out}^{ct_j}[k])$  represents the outlet water temperature and discretized dynamics of outlet water temperature of the shared water collection basin. Depending on the clients' requirements, the allowable outlet temperature range of the shared water collection basin is imposed as a soft constraint in the optimization program.

$$T_{min}^{cb} - \epsilon^{cb} \leq T_{out}^{cb}[k] \leq T_{max}^{cb} + \epsilon^{cb}, \forall k \in \{2, \dots, N+1\} \quad (12)$$

where  $T_{min}^{cb}$  and  $T_{max}^{cb}$  represent the admissible temperature range for the outlet water temperature of the shared collection basin.

Moreover, an admissible temperature range can be imposed on the outlet water temperature of the individual cooling towers to avoid a temperature gradient inside the shared water collection basin. This ensures a uniform water supply temperature to different clients which are fed from different points of the shared water collection basin.

$$T_{min}^{ct_j} - \epsilon^{ct_j} \leq T_{out}^{ct_j}[k] \leq T_{max}^{ct_j} + \epsilon^{ct_j}, \forall k \in \{2, \dots, N+1\}, \forall j \in \mathcal{C} \quad (13)$$

*Piping layout:* Due to the piping layout and hydraulic constraints, the bypass mode for all the cooling towers must be activated together.

$$b_b^{ct_1}[k] = b_b^{ct_i}[k], \forall i \in \{2, \dots, M\}, \forall k \in \mathcal{K} \quad (14)$$

*Dwell time constraint:* The minimum speed requirement imposed to meet the cooling requirement of the motor

<sup>1</sup> We consider cooling towers to be identical if they have the same construction and operational performance effectively parameterized using the same set of identified parameters. Note that, there can be multiple sets of identical cooling towers. However, we consider only one set of identical cooling towers for the simplicity of the presentation.

results in a large forbidden operating zone for the cooling towers. Whenever heat rejection requirement lies in the forbidden operating zone, switching will occur to keep the outlet water temperature around the desired set point. However, frequent switching of the cooling tower fan can cause damage to the motor. To overcome this problem, dwell time constraints (15) and (16) are added to ensure that a change in the operational mode of the cooling tower is retained for a given time interval. (15) ensures to retain a certain operational mode of a cooling tower for the first  $U^{ct}$  time steps.

$$\begin{aligned} b_v^{ct_j}[k] &= b_{v,init}^{ct_j}, b_s^{ct_j}[k] = b_{s,init}^{ct_j}, b_b^{ct_j}[k] = b_{b,init}^{ct_j}, \\ b_{v,init}^{ct_j} + b_{s,init}^{ct_j} + b_{b,init}^{ct_j} &= 1, \forall k \in \{0, \dots, U^{ct_j}\}, \forall j \in \mathcal{C} \end{aligned} \quad (15)$$

(16) ensures that, if a change in the operational mode of the cooling tower takes place, then the new mode would be retained for a given time interval.

$$\begin{aligned} b_v^{ct_j}[i] &\geq b_v^{ct_j}[k] - b_v^{ct_j}[k-1], \forall i \in \{k, \dots, \min(N, k + up_v^j)\} \\ b_s^{ct_j}[i] &\geq b_s^{ct_j}[k] - b_s^{ct_j}[k-1], \forall i \in \{k, \dots, \min(N, k + up_s^j)\} \\ b_b^{ct_j}[i] &\geq b_b^{ct_j}[k] - b_b^{ct_j}[k-1], \forall i \in \{k, \dots, \min(N, k + up_b^j)\} \\ &\forall k \in \{U^{ct_j} + 1, \dots, N\}, \forall j \in \mathcal{C} \end{aligned} \quad (16)$$

where  $up_v^j$ ,  $up_s^j$ ,  $up_b^j$  represent the time interval for which ventilation, showering and bypass operational modes must be retained. Table-1 shows that dwell time constraints are only enabled whenever a change in the operating mode of the cooling tower takes place. Otherwise, the constraints remain redundant.

Table 1. Dwell time constraints

$b[k]$	$b[k-1]$	$b[k] - b[k-1]$	Constraint	
1	0	1	$b[i] \geq 1$	Active
0	1	-1	$b[i] \geq -1$	Redundant
0	0	0	$b[i] \geq 0$	Redundant
1	1	0	$b[i] \geq 0$	Redundant

*Robustness against mode unavailability:* A continuous operation of the cooling towers is required to ensure the reliable operation of a large scale cooling plant. The inherent over actuated structure of the cooling process must be exploited to provide robustness against mode unavailability. For instance, the ventilation mode might become unavailable due to a fault in the motor or gearbox assembly whereas valve malfunctioning can result in the unavailability of both ventilation and showering modes of the cooling tower. In such situations, the optimization program must explore the solution space while taking into account the unavailability of an operational mode of the cooling tower. The information regarding the unavailability of ventilation and showering modes of a cooling tower can be incorporated on the run without changing the structure of the optimization problem.

$$\begin{aligned} b_v^{ct_j}[k] &\leq (1 - \lambda_v^{ct_j}), \forall k \in \mathcal{K}, \forall j \in \mathcal{C} \\ b_s^{ct_j}[k] &\leq (1 - \lambda_s^{ct_j}), \forall k \in \mathcal{K}, \forall j \in \mathcal{C} \end{aligned} \quad (17)$$

where  $\lambda_v^{ct_j}$  and  $\lambda_s^{ct_j}$  denotes the auxiliary variables representing the availability of the ventilation and showering modes of the cooling towers. In case of unavailability of an operational mode, the associated auxiliary variable must

be manually set to 1. Hence, the feasible solutions to the optimization program will not select that particular operational mode. Otherwise, the auxiliary variables are set to 0 and the constraints in (17) remain redundant.

*Cost Function:* Finally we define the cost function of the optimization problem which aims to regulate the outlet water temperature of the shared water collection basin while minimizing the total power consumption of the cooling towers.

$$J = \sum_{j=1}^M \sum_{k=1}^N (r^{ct_j} P_{fan}^{ct_j}(u^{ct_j}[k])) + \sum_{j=1}^M s^{ct_j} \epsilon^{ct_j} + s^{cb} \epsilon^{cb} + \sum_{k=1}^N (q[T_{out}^{cb}[k] - T_{out}^{ref}]^2) + q_f(T_{out}^{cb}[N+1] - T_{out}^{ref})^2 \quad (18)$$

where  $q \geq 0$ ,  $q_f \geq 0$ ,  $r^{ct_j} > 0$ ,  $s^{ct_j} > 0$ ,  $s^{cb} > 0$ .  $P_{fan}^{ct_j}$  represents the power consumption of the cooling tower fan.  $\epsilon^{ct_j}$  and  $\epsilon^{cb}$  are additional variables that are used to impose soft constraints on the admissible outlet water temperature of individual and shared water collection basin.  $T_{out}^{ref}$  is the desired set point for the outlet water temperature of the shared water collection basin. In addition to the dwell time constraints for individual cooling towers, an additional switching cost ( $S^{ct_j}(\mathbf{b}[k], \mathbf{b}[k-1])$ ) can be added to manage the switching frequency of the overall system.

*Run time balance among identical cooling towers:* In many practical cases, multiple identical cooling towers are installed in a cooling facility. The run time among identical cooling towers fan must be balanced to maximize the lifetime of the mechanical equipment. This can be accomplished by turning on the fan with the largest down time and turning off the fan with largest up time. One way to achieve such a requirement is to add an adaptive penalty in the cost function based on the operational time of the cooling tower fans. Let  $\gamma_{up}^{ct_j}$  and  $\gamma_{down}^{ct_j}$  represents the counters which track the up and down time of the cooling tower fans. Then an adaptive penalty can be defined as follows:

$$J_{ad}^{ct_j} = \begin{cases} 0, & U^{ct_j} > 0 \\ w_1^{ct_j} (\gamma_{down}^{ct_j}) b_v^{ct_j}[1] + w_2^{ct_j} (\gamma_{up}^{ct_j}) b_s^{ct_j}[1], & U^{ct_j} = 0 \end{cases}$$

$\forall j \in \mathcal{C}_I$ . Here,  $w_1^{ct_j}(0) = 0$  and  $w_2^{ct_j}(0) = 0$ . For  $\gamma_{show}^{ct_j} > 0$ ,  $\gamma_{vent}^{ct_j} > 0$ ,  $w_1$  and  $w_2$  are a strictly decreasing function. The adaptive penalty term discounts the showering mode (ventilation mode) among cooling towers based on the time spent in ventilation mode (showering mode).

*Initial Conditions:* Initial condition constraints must be added to complete the formulation of the mixed integer optimization program.

$$T_{out}^{ct_j}[1] = T_{out,init}^{ct_j}, \forall j \in \mathcal{C}, \quad T_{out}^{cb}[1] = T_{out,init}^{cb} \quad (19)$$

Finally, the optimization program (20) must be solved to determine the best operational mode and optimal fan speed for each of the cooling towers while respecting the physical and operational constraints.

$$\begin{aligned} & \min_{\mathbf{u}, \mathbf{b}, \mathbf{T}_{out}, \epsilon} J \\ \text{s.t.} & \quad (8), (9), (10), (11), (12), (13), (14), (15), (16), (17), (19) \end{aligned} \quad (20)$$

Note that, (20) is a nonlinear mixed integer program (MINLP) due to the nonlinear dynamics associated with the ventilation mode of the cooling tower. MINLP is hard to solve, however linear tangent approximation of the nonlinear ventilation dynamics around the operating point can be used to simplify the optimization program as mixed integer quadratic program (MIQP). Many off the shelf solvers are available to effectively solve the MIQP. The solution to (20) provides an open loop control action. MIQP must be solved in a receding horizon fashion providing robustness against uncertainties and model mismatch.

Although the control strategy is developed for the cooling towers equipped with variable frequency drives, it is easy to extend the framework to the cooling towers equipped with single or multi-speed fans. The control strategy can also be applied to the cases with a combination of single, multi, and variable speed cooling tower fans.

#### 4. SIMULATION RESULTS

In this section, we show the efficacy of model predictive control strategy on experimentally validated models of the cooling towers. The cooling facility has 5 induced draft cooling towers (4 identical) providing 47.5 MW of cooling capacity. The cooled water is supplied from the shared water collection basin to different clients including chillers, cryogenics, etc. The incoming mass flow rate of water is equally distributed among all five cooling towers. Each cooling tower fan is driven by a 70 KW motor operated with a variable frequency drive. Several sensors are installed which provide the values of relative humidity, dry bulb temperature, mass flow rate of water entering the cooling towers and outlet water temperature of individual cooling towers. Three sensors are installed in the shared water collection basin at different points which provide an estimate of the outlet water temperature of the collection basin. More details about the available data and parameter estimation can be found in Peljo (2018).

The client side of the cooling towers is modeled as a constant load. Table 2 summarizes the values of the parameters used in the simulation study. The values of the parameters are same for all the cooling towers unless stated otherwise. The prediction horizon of the MIQP is set to 16 minutes with a sampling time of 2 minutes. GUROBI (Gurobi (2020)) is used to solve the MIQP with YALMIP (Löfberg (2004)) as high level interface. An average time of 17 seconds is noted to solve the MIQP on the Intel Core i7-10610U CPU @2.3GHz machine with 32GB RAM.

To provide a comparison of the model predictive control with conventional solutions, two heuristics are considered which have been applied to real plants in operation. Both heuristics are mainly based on split range PID with temperature dependent thresholds. The main idea behind the first heuristic is to ramp up the cooling tower fan to the maximum speed before turning on the next cooling tower fan. The second heuristic attempts to utilize maximum number of fans at the lowest possible speeds to meet the heat rejection requirement.

Table 2. Values of the parameters used in the simulation study

Parameter	Value	Parameter	Value
$q$	1	$q_f$	1
$r$	$10^{-2}$	$s^{ct}$	$10^6$
$s^{cb}$	$10^6$	$T_{out}^{ref}$	$23.2^\circ C$
$T_{min}^{cb}$	$20^\circ C$	$T_{max}^{cb}$	$23.5^\circ C$
$T_{min}^{ct}$	$20^\circ C$	$T_{max}^{ct}$	$28^\circ C$
$u_{min}^{ct}$	60 %	$u_{max}^{ct}$	100 %
$\dot{m}_{w,in}^{ct}$	150 kg/s	$\dot{m}_{w,out}^{ct}$	150 kg/s
$\dot{m}_{w,in}^{cb}$	750 kg/s	$\dot{m}_{w,out}^{cb}$	750 kg/s
$\rho_w$	1000 kg/s	$c_{pw}$	4.18 kJ/kg $^\circ C$
$up_v$	6 min	$up_s$	6 min
$up_b$	0 min	$S^{ct}$	50000

Fig.2 shows the performance of different control strategies for regulating the outlet temperature of shared water collection basin to  $23.2^\circ C$  under changing heat load and wet bulb temperature. At low wet bulb temperature, the cooling towers configuration (i.e. operational modes and fan speeds) that can provide the required cooling capacity lies within the forbidden operating zone. Hence, the optimal configuration of the cooling towers provide either excessive or insufficient cooling of the hot water stream. This causes deviation of the outlet water temperature around the set point. It is evident from the Fig 2 that, the model predictive control strategy provides better regulation performance as compared to the heuristics control strategies.

Table 3 summarizes the performance and energy utilization of different control strategies to regulate the outlet water temperature of the common collection basin. It is evident that the model predictive control strategy can significantly improve the performance and energy usage over conventional heuristic solutions. Appendix A, provides more details on the performance comparison of individual cooling towers.

Table 3. Energy usage and performance of different control strategies

	Energy (KWh)	RMSE	Max	Mode Switch
Heuristic-1	3471	0.67753	25.531	47
Heuristic-2	1509.9	0.39929	24.717	69
MPC	1351	0.21567	23.505	77

## 5. CONCLUSION

In this paper, we designed a mixed integer model predictive control strategy to enable the optimal operation of multiple induced draft cooling towers. Features like run time balance among identical cooling towers, robustness against mode unavailability and prevention of excessive mode switching were incorporated to make the framework suitable for practical implementation. A comparison was done with heuristic solutions which showed significant improvement in performance and energy savings. In the future, we plan to investigate plant wide optimization to determine the optimal set points for different interacting elements in a large scale cooling plant.

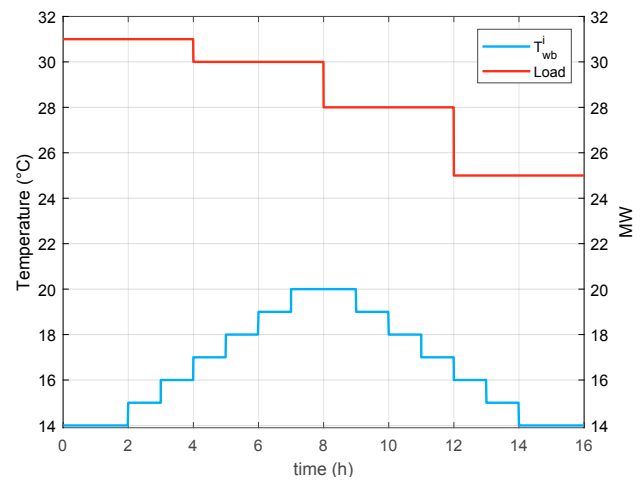
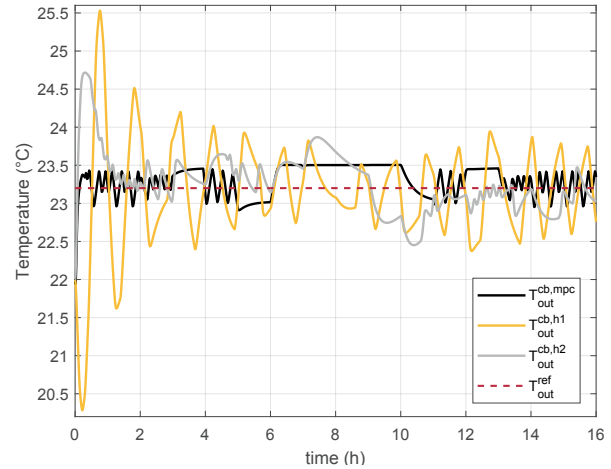


Fig. 2. Comparison of MPC with heuristic strategies for regulating the outlet water temperature to the desired set point under changing heat load and weather conditions.  $T_{out}^{cb,mpc}$ ,  $T_{out}^{cb,h1}$ ,  $T_{out}^{cb,h2}$  represents the evolution of outlet water temperature of the common collection basin under model predictive controller and heuristic control strategies.

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#### Appendix A. PERFORMANCE COMPARISON OF INDIVIDUAL COOLING TOWERS

The cooling facility has four identical cooling towers having similar structure and performance characteristics. The first cooling tower differs from the other four based on the blade construction that results in a higher mass flow rate of air in ventilation mode as compared to other cooling towers. Fig.3 shows that the model predictive control strategy can effectively utilize the higher cooling capacity provided by the first cooling tower by operating the first cooling tower at higher speeds as compared to other cooling towers. Under the model predictive control strategy, the outlet water temperature of each individual cooling tower adheres to the specified admissible temperature and shows relatively smaller variations as compared to heuristic solutions. This ensures a relatively uniform water supply temperature to different clients which are fed from different points of the shared water collection basin.

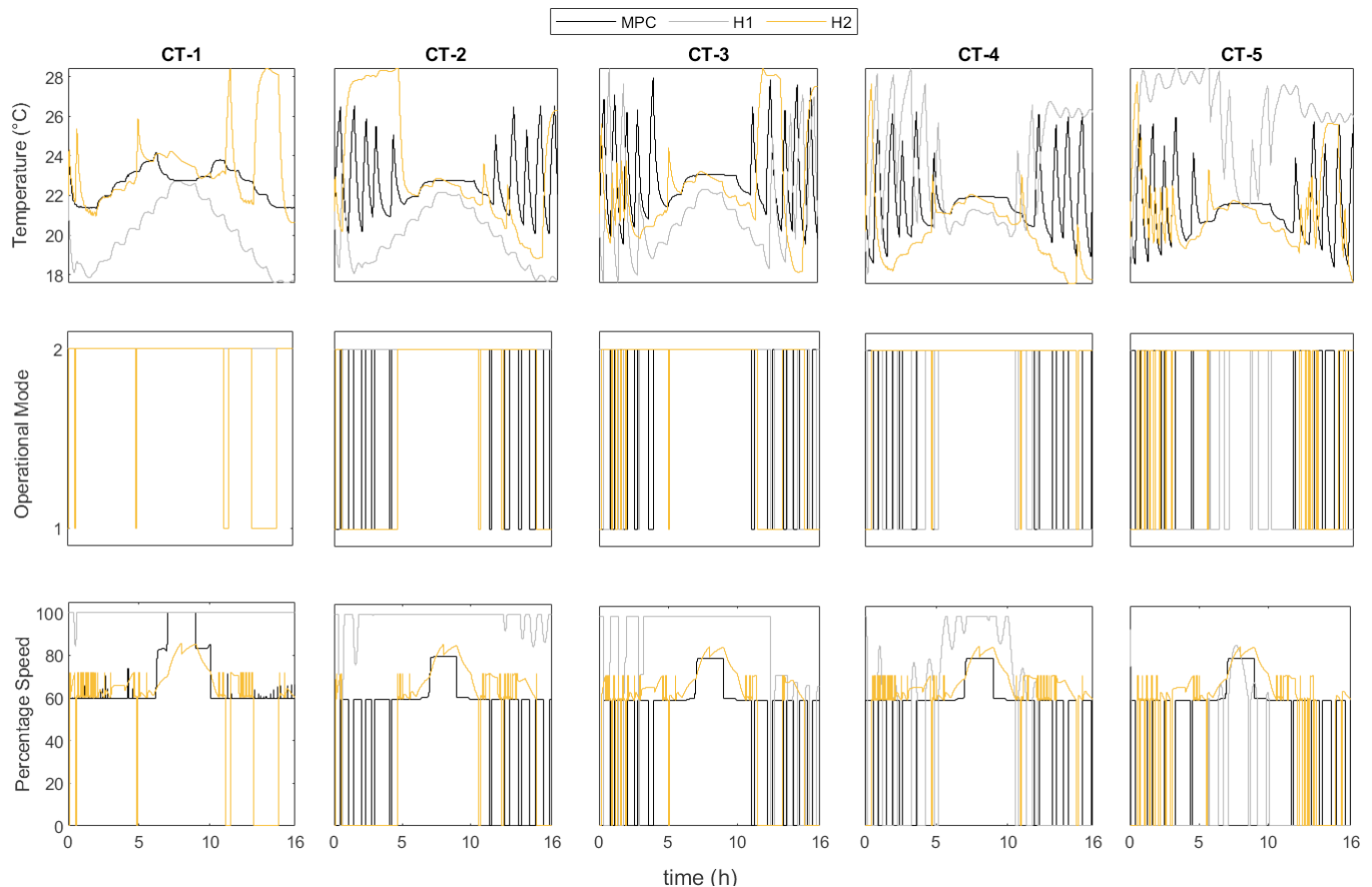


Fig. 3. Performance comparison of individual cooling towers under model predictive controller and heuristic control strategies. Plots in the first row represent the evolution of outlet water temperature of individual cooling towers. The second row represent the operational modes of the cooling towers where 1, 2 corresponds to showering and ventilation modes respectively. The third row represent the fan speed of the individual cooling tower fans.