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Solar parameters in long-baseline accelerator neutrino oscillations

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Abstract: Long-baseline (LBL) accelerator neutrino oscillation experiments, such as NOvA and T2K in the current generation, and DUNE-LBL and HK-LBL in the coming years, will measure the remaining unknown oscillation parameters with excellent precision. These analyses assume external input on the so-called "solar parameters," θ_{12} and Δm^2_{21}, from solar experiments such as SNO, SK, and Borexino, as well as reactor experiments like KamLAND. Here we investigate their role in long-baseline experiments. We show that, without external input on Δm^2_{21} and θ_{12}, the sensitivity to detecting and quantifying CP violation is significantly, but not entirely, reduced. Thus long-baseline accelerator experiments can actually determine Δm^2_{21} and θ_{12}, and thus all six oscillation parameters, without input from *any* other oscillation experiment. In particular, Δm^2_{21} can be determined; thus DUNE-LBL and HK-LBL can measure both the solar and atmospheric mass splittings in their long-baseline analyses alone. While their sensitivities are not competitive with existing constraints, they are very orthogonal probes of solar parameters and provide a key consistency check of a less probed sector of the three-flavor oscillation picture. Furthermore, we also show that the true values of Δm^2_{21} and θ_{12} play an important role in the sensitivity of other oscillation parameters such as the CP violating phase δ.

Keywords: CP Violation, Neutrino Mixing

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Contents

1 Long-baseline accelerator neutrino physics introduction

Determining the six standard three-flavor oscillation parameters has been a top priority in the particle physics community since the discovery that they were physical in 1998 [1]. To date, remarkable progress has been made on several of the parameters. In particular, θ_{13}, $|\Delta m^2_{31}|$, θ_{12}, and Δm^2_{21} have all been determined to good precision. The sign of Δm^2_{31} is still to be determined, whether θ_{23} is in the upper octant, lower octant, or very close to maximal is an open question, and the complex CP violating (CPV) phase δ is largely unconstrained, see [2] for a recent review. The best experiments to probe these remaining unknowns are appearance experiments which are accomplished with long-baseline (LBL) accelerator[1] neutrinos in both neutrino mode and anti-neutrino mode. While there are numerous partial degeneracies among these parameters, the ongoing experiments, currently running NOvA [3] and T2K [4], still have some discriminating capabilities among these parameters. It is well established that the successors to these experiments, DUNE-LBL [5] and HK-LBL [6], will have excellent precision to all three of the remaining unknowns with more than 5σ sensitivity to disfavor $\sin\delta = 0$ for much of the parameter space.

[1] In this paper, LBL will refer only to accelerator experiments and not to long-baseline reactor experiments such as KamLAND or JUNO.

Long-baseline oscillation analyses assume input from other experiments, however, in particular for the so-called "solar parameters":[2] θ_{12} and Δm^2_{21}. Many also include input on θ_{13} from medium baseline reactor experiments such as Daya Bay [7], RENO [8], and Double Chooz [9], although some long-baseline accelerator experiments will have comparable (within a factor of ~ 2) sensitivity to this quantity [10].

While it has been appreciated that a non-trivial three-flavor oscillation scenario is a necessary requirement for CP violation [11–13], a modern study on the impact of each of the other oscillation parameters on the final parameter, δ, does not exist. Specifically, it is important to understand the interplay of all of the oscillation parameters considering their now approximately known sizes together with the fact that the final parameters will be measured in experiments experiencing the matter effect.

In this paper, we will show that input on the solar parameters Δm^2_{21} and θ_{12} from other experiments is absolutely necessary to reach the physics goals of long-baseline accelerator experiments. Then, we will investigate the sensitivity long-baseline accelerator experiments have to both their primary physics parameters, such as δ, without input from Δm^2_{21} and θ_{12}, as well as the ability of long-baseline accelerator experiments to actually determine Δm^2_{21} and θ_{12}. We will show that, without priors on Δm^2_{21} and θ_{12} from solar and reactor experiments, the sensitivity to δ is significantly reduced due to some unusual oscillation scenarios where all the oscillation parameters take values very far from known values in an attempt to find agreement with the simulated data. We will carefully investigate how this sensitivity depends on the precision on Δm^2_{21} and θ_{12} (very little) and on the true value of Δm^2_{21} and θ_{12} (modest dependence). Then, since the sensitivity to determine δ does not go exactly to zero, this means that DUNE-LBL and HK-LBL will have some sensitivity to measure Δm^2_{21} and θ_{12} in their long-baseline channels; we will determine the statistical level at which the long-baseline accelerator experiments can actually determine Δm^2_{21} and θ_{12}.

Other studies exist exploring the impact of Δm^2_{21} and θ_{12} in experiments not traditionally designed to measure these parameters. For example, [14, 15] investigated the ability to probe these parameters at Daya Bay and RENO where the Δm^2_{21} oscillations have only just started to develop. They found that these experiments can constraint $|\Delta m^2_{21}| \lesssim 20 \times 10^{-5}\,\mathrm{eV}^2$. In addition, [16, 17] found that the true values of Δm^2_{21} and θ_{12} within their current uncertainties have a potentially sizable impact on JUNO's sensitivity to the sign of Δm^2_{31}.

Throughout the paper we will show results for characteristic experiments or parameters that best highlights the physics. Other combinations of the results are shown in the appendix A for completeness. We begin the manuscript by providing an analytical understanding of the impact of Δm^2_{21} and θ_{12} on the measurement of CPV in section 2 followed by a description of our numerical analysis in section 3. We present our results in section 4 and then discuss them and conclude in section 5. In appendix B we demonstrate the impact of Δm^2_{21} and θ_{12} on the determination of θ_{23}, θ_{13}, and Δm^2_{31}.

[2]These parameters are referred to as solar parameters as they were first determined from solar neutrino data, but are now partially determined from solar data and partially from reactor neutrino data. In the future the best constraints on these two fundamental parameters will be from reactor neutrino data.

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2 The role of the solar parameters in the CPV measurement

All three mixing angles need to be non-zero to allow for CPV in the neutrino sector [11–13]. Furthermore, their values together with the value of δ dictate the size of CPV in the lepton sector, measured via the Jarlskog invariant $J = s_{12}c_{12}s_{13}c_{13}^2 s_{23}c_{23}\sin\delta$ [12] where we use the common notation $s_{ij} \equiv \sin\theta_{ij}$ and $c_{ij} \equiv \cos\theta_{ij}$. Hence a measurement of all oscillation parameters is required to quantify leptonic CPV. While θ_{13} is already well measured and the interplay between a more precise measurement of θ_{23} and δ at future LBL experiments has been studied before [18–23], the role of the solar parameters, Δm_{21}^2 and θ_{12}, in the leptonic CPV measurement has not been analysed in detail. In the following we will therefore conduct a detailed study of the role of Δm_{21}^2 and θ_{12} at LBL accelerator experiments.

The two solar parameters, θ_{12} and Δm_{21}^2, have been determined in solar experiments such as SNO [24], SK [25], Borexino [26, 27], Homestake [28], GALLEX [29], and SAGE [30]. The values of those parameters have been confirmed in the long-baseline reactor experiment KamLAND [31]. In particular, a combined fit of solar data provides a good measurement of θ_{12} and KamLAND's reactor measurement of θ_{12} is only a bit less constraining. The constraint on the frequency, Δm_{21}^2, is dominated by KamLAND with some additional information from solar data, albeit at significantly lower precision. A small tension briefly existed between solar and reactor determinations of Δm_{21}^2 at the $\sim 2\sigma$ level [32, 33], although this seems to have evaporated with new solar data and analyses from SK [34].

Nevertheless, an ambiguity exists in the definition of the Δm_{21}^2 and θ_{12}, and really in the definition of all three mass states. Multiple viable definitions exist, see e.g. [35, 36]. One possible definition is $m_1 < m_2 < m_3$, although until the atmospheric mass ordering is known, this leads to rather complicated conditional expressions for many oscillation experiments. Another possible definition is

$$|U_{e1}| > |U_{e2}| > |U_{e3}|\,, \tag{2.1}$$

which is this definition that we choose to use in the following. We use this definition since we know the magnitude of all three elements of the electron neutrino row quite well from medium- and long-baseline reactor neutrino experiments, as well as solar experiments. This definition means that $\theta_{12} < 45°$, $\sin\theta_{12} > \tan\theta_{13}$, while Δm_{21}^2 can be positive or negative. We note that the definition in eq. (2.1) differs from another definition that is sometimes used which is: $m_1 < m_2$, $|U_{e3}| < |U_{e1}|$, and $|U_{e3}| < |U_{e2}|$ which means that $\Delta m_{21}^2 > 0$ and $\tan\theta_{13} < \min(\sin\theta_{12}, \cos\theta_{12})$. Thus the practical difference between these two definitions is that the fact that the ^{8}B solar neutrino disappearance probability is $P_{ee}^{^8B} \sim \frac{1}{3}$ tells us that $\Delta m_{21}^2 > 0$ in our definition, while in the other definition it tells us that $\theta_{12} < 45°$.

Solar parameters have some partial degeneracies with the CP phase as well as some other parameters. For example, in vacuum near the first oscillation maximum when $\Delta m_{32}^2 L/4E \simeq \pi/2$ the CP difference is

$$P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e) = -16J\sin\left(\frac{\Delta m_{31}^2 L}{4E}\right)\sin\left(\frac{\Delta m_{32}^2 L}{4E}\right)\sin\left(\frac{\Delta m_{21}^2 L}{4E}\right)\,, \tag{2.2}$$

$$\approx -8\pi J\frac{\Delta m_{21}^2}{\Delta m_{32}^2}\,, \tag{2.3}$$

where J is the Jarlskog invariant. Thus without knowledge of Δm^2_{21} or θ_{12} there is a degeneracy between $\sin\delta$ and the solar parameters, up to the limit from unitarity $|J| \leq \frac{1}{6\sqrt{3}} \approx 0.096$. We note, however, that in vacuum there is no asymmetry if CP is conserved and it is impossible to "dial up" Δm^2_{21} and $\sin 2\theta_{12}$ enough to get something that looks like CP violation. Equations (2.2)–(2.3) also highlight the important role a non-zero value of Δm^2_{21} plays in vacuum oscillations. That is, all three mass states must be different in order to have CP violation. This can be seen in other ways as well in that if $m_1 = m_2$ then the mixing angle θ_{12} is no longer physical which also removes the possibility to detect CP violation.

For DUNE-LBL and HK-LBL, however, the matter effect plays a role in oscillations. Among other things, this leads to an *apparent* CP violating effect [13, 37–46] with the same $(L/E)^3$ dependence in eq. (2.2). The probability does not, however, depend on δ if $\Delta m^2_{21} \to 0$, as we outline here. The matter equivalent version of Δm^2_{21}, denoted with a hat as $\Delta\widehat{m^2}_{21}$, is always non-zero even when $\Delta m^2_{21} = 0$ and is well approximated [47, 48] (see also [49]) as

$$\lim_{\Delta m^2_{21}\to 0} \Delta\widehat{m^2}_{21} \approx a\cos^2\hat{\theta}_{13} + \Delta m^2_{ee}\sin^2(\hat{\theta}_{13} - \theta_{13})\,, \tag{2.4}$$

where $a = 2\sqrt{2}G_F N_e E$ is the contribution from the matter effect, N_e is the electron density, E is the neutrino energy, $\Delta m^2_{ee} = \cos^2\theta_{12}\Delta m^2_{31} + \sin^2\theta_{12}\Delta m^2_{32}$ [50, 51], and

$$\cos 2\hat{\theta}_{13} \approx \frac{\cos 2\theta_{13} - a/\Delta m^2_{ee}}{(\cos 2\theta_{13} - a/\Delta m^2_{ee})^2 + \sin^2 2\theta_{13}}\,. \tag{2.5}$$

Therefore it appears as though this will nonetheless lead to apparent CPV that still depends on δ and has the same $(L/E)^3$ dependence as in eq. (2.2). However, we must account for the behavior of the Jarlskog coefficient in matter. From [52] we have that

$$\hat{J} \approx \frac{J}{\sqrt{(\cos 2\theta_{12} - c^2_{13}a/\Delta m^2_{21})^2 + \sin^2 2\theta_{12}}\sqrt{(\cos 2\theta_{13} - a/\Delta m^2_{ee})^2 + \sin^2 2\theta_{13}}}\,, \tag{2.6}$$

where the corrections to this approximation are proportional to Δm^2_{21}, thus it becomes exact at $\Delta m^2_{21} \to 0$. Thus $\hat{J} \to 0$ as $\Delta m^2_{21} \to 0$ and therefore the triple sine term is zero in this limit in matter as well. In fact, due to the Naumov-Harrison-Scott identity [53, 54],

$$\hat{J}\Delta\widehat{m^2_{32}}\Delta\widehat{m^2_{31}}\Delta\widehat{m^2_{21}} = J\Delta m^2_{32}\Delta m^2_{31}\Delta m^2_{21}\,, \tag{2.7}$$

and the fact that none of the $\Delta\widehat{m^2_{ij}} \to 0$ as $\Delta m^2_{ij} \to 0$, $\hat{J} \to 0$ if any of the three $\Delta m^2_{ij} \to 0$. That is, even though the effective mass squared splitting is always non-zero in matter, there is no impact due to real CPV in neutrino oscillations if $\Delta m^2_{21} = 0$.

In addition to the role the matter effect plays, the simple story shown in eq. (2.3) is further complicated by several additional effects. First, via the presence of a near detector and a careful understanding of the flux and cross sections, neutrino oscillation experiments measure each appearance channel, $P(\nu_\mu \to \nu_e)$ and $P(\bar{\nu}_\mu \to \bar{\nu}_e)$, independently. Second, DUNE-LBL — and to a lesser extent HK-LBL — measure the spectrum around the oscillation maximum in the appearance channels. This provides key shape information.

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Third, it is conceivable that DUNE-LBL and/or HK-LBL might be able to gain some information about the second oscillation maximum which would provide additional important information about CP violation. Finally, the matter effect [55] which is quite important for DUNE-LBL and NOvA, plays a key role as discussed above. In particular it eases the measurement of the atmospheric matter effect which reduces a key degeneracy for DUNE-LBL and, to a lesser extent, HK-LBL.

In figure 1 we illustrate the impact of Δm^2_{21} and θ_{12} on the appearance probability at DUNE-LBL to set the stage for the rest of the paper. We use a matter density of $\rho = 3$ g/cc, a baseline of 1300 km, and as benchmark the oscillation parameters defined in the next section and in table 2 and $\delta = -90°$. We choose Δm^2_{21} and θ_{12} which extremize the probability as a function of the energy to arrive at a possible range of probabilities. To demonstrate the effects of each of Δm^2_{21} and θ_{12}, we vary only one of them and allow $\theta_{12} \in [0, 45°]$ and $\Delta m^2_{21} \in [-\Delta m^2_{31}, \Delta m^2_{31}]$ to ensure that the two mass splittings remain different. This envelop can be compared to the probabilities with $\delta = 0,\ 90°,\ 180°,\ 270°$ and fixing Δm^2_{21} and θ_{12} to the SK+SNO+KamLAND best fit. If these probabilities are enclosed in the envelop of probabilities with extreme values of Δm^2_{21} and θ_{12}, $\delta = -90°$ may not be easily distinguishable from other values of δ without the addition of solar priors. We find as extreme values for θ_{12} at the peak of the DUNE neutrino flux at $E = 3\,\text{GeV}$ $\theta_{12}^{\text{min}} \approx 2.7°$, $\theta_{12}^{\text{max}} \approx 44°$, and for Δm^2_{21} at $E = 3\,\text{GeV}$ $\Delta m^{2,\text{min}}_{21} \approx -5.6 \times 10^{-4}$ eV^2, $\Delta m^{2,\text{min}}_{21} \approx 2.3 \times 10^{-3}$ eV^2. From the upper plot of figure 1 we see that even with extreme values of θ_{12} the changes of the oscillation amplitude are not dramatic and some probability curves with fixed Δm^2_{21} and θ_{12} lay outside of the envelop. On the other hand, from the lower plot of figure 1, we see that the effects of changes in Δm^2_{21} are more pronounced and all probabilities with fixed Δm^2_{21} and θ_{12} are contained in the envelop. Note, however, that figure 1 does not provide any shape information about the behavior at the first oscillation maximum as the parameters are varied, as well as any potential impact at the second oscillation maximum. We therefore conclude that it seems likely that priors on both Δm^2_{21} and θ_{12} are important to obtain sensitivity to δ and to achieve precision on δ.

3 Analysis details

To estimate the sensitivities at various LBL experiments we use the GLoBES software package [56]. We use the publicly available experimental files for NOvA [57, 58], T2K [59–61], DUNE [62], and HK-LBL [59–61] and modify them to ensure agreement to the most recent quoted sensitivities from the experiments for a given set of assumptions about the oscillation parameters [63–65]. For each of the four long-baseline accelerator experiments, we consider both neutrino mode and anti-neutrino mode as well as both disappearance ($P(\nu_\mu \to \nu_\mu)$) and appearance ($P(\nu_\mu \to \nu_e)$) modes. We do not include ν_τ appearance mode which may be relevant for DUNE-LBL [66–68], see also [69]. The experimental details of all four long-baseline accelerator experiments are summarized in table 1 and are set to match the latest experimental sensitivity curves for the assumed oscillation parameters. While these details may change as the upcoming experiments evolve, we have checked that they do

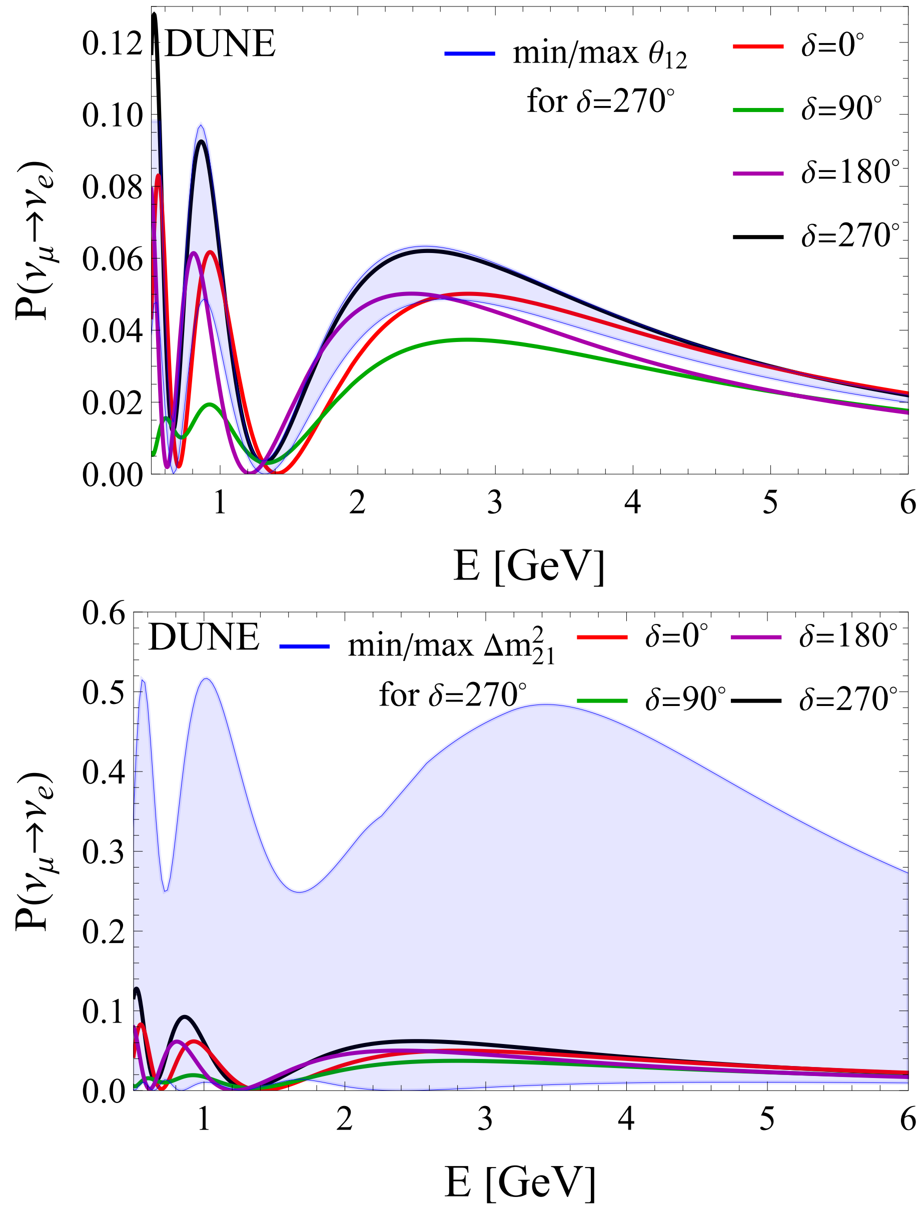

Figure 1. The oscillation probability at DUNE-LBL for $\delta = 0,\ 90^\circ,\ 180^\circ,\ 270^\circ$ and fixing the oscillation parameters to the benchmark scenario defined in table 2 and in the text as a function of energy are shown as colored lines. The blue regions show the extreme values of the probability assuming $\delta = -90^\circ$ varying either θ_{12} between 0 and 45° (top) or Δm^2_{21} between $-\Delta m^2_{31}$ and Δm^2_{31} (bottom). If the colored lines are contained in the blue regions $\delta = -90^\circ$ cannot be easily distinguished from other values of δ without a prior on the solar parameter.

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Experiment	Technology	Fiducial Volume	Total POT (ν+$\bar{\nu}$)	ν:$\bar{\nu}$
NOvA	Scintillator	25 kT	7.2×10^{21}	1:1
T2K	Water Cherenkov	22.5 kT	10×10^{21}	1:1
DUNE-LBL	LArTPC	40 kT	14×10^{21}	1:1
HK-LBL	Water Cherenkov	190 kT	27×10^{21}	1:3

Table 1. A summary of the relevant experimental details assumed for each experiment where POT is the total accumulated protons on target and ν:$\bar{\nu}$ is the ratio of neutrino to anti-neutrino mode.

capture the relevant features and that changes in the exposure do not significantly modify the results.

To study the sensitivity to the oscillation parameters we make various assumptions on the priors of Δm^2_{21} and θ_{12}. The case of no priors provides a testament to what an experiment can do entirely on their own. Our current knowledge of Δm^2_{21} and θ_{12} comes from solar data, KamLAND, and a combined analysis of both; the last of these is the closest approximation to the fiducial analyses that most experiments run. This is also the benchmark scenario we will use in the following, unless otherwise stated. In the future our knowledge of Δm^2_{21} and θ_{12} will increase with information from solar neutrinos at HK-LBL or DUNE-LBL and information from reactor neutrinos at JUNO.[3] An overview of our current knowledge on Δm^2_{21} and θ_{12} can be found in tables 2, 3 which also includes the global fit results from [71–73]. We see that the best fit values of Δm^2_{21} and θ_{12} vary among the different determinations by a $\sim 1\sigma$ spread among the global fits. This is illustrated in figure 2 where we also include global fit results on the remaining oscillation parameters (not including δ). This figure shows that, in fact, there is a $\sim 1\sigma$ difference among the global fits for many of the parameters including Δm^2_{21} and θ_{12}.

In our study, for the remaining parameters, when we use priors on them, we assume

$$\begin{aligned} \sin^2 2\theta_{13} &= 0.0853\ (\pm 2.8\%)\ \text{from [74]}, \\ \Delta m^2_{32} &= 2.454 \times 10^{-3}\ \text{eV}^2\ (\pm 2.3\%)\ \text{from [74]}, \\ \sin^2 \theta_{23} &= 0.57\ (\pm 7.0\%)\ \text{from [75]} \end{aligned} \tag{3.1}$$

where the first two parameters were determined from the most recent Daya Bay results [74, 76][4] and we use the current results from NOvA on $\sin^2\theta_{23}$ [75]. The choice of priors on θ_{13}, Δm^2_{32}, and $\sin^2\theta_{23}$ will not strongly affect our results as the future LBL experiments are able to determine these parameters on their own with good precision as indirectly shown in figure 3 below.

Finally, we assume the true mass ordering to be normal but we test both orderings in our analysis, i.e. we do not fix the mass ordering in our analysis

[3]JUNO will also have sensitivity to Δm^2_{21} and θ_{12} via solar neutrinos [70], but will not be competitive with DUNE-solar.

[4]The latest Daya Bay results [76] are $< 1\sigma$ different from the numbers mentioned here.

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Data	Δm^2_{21} [10^{-5} eV^2]	$\sin^2\theta_{12}$	Ref.
SK+SNO	6.10	0.305	[34]
KamLAND	±7.54	0.316	[31]
SK+SNO+KamLAND	**7.49**	**0.305**	[34]
Global fit	7.42	0.304	[72]
	7.5	0.318	[71]
	7.36	0.303	[73]

Table 2. The current best fit values for Δm^2_{21} and θ_{12} including different data sets. Unless otherwise specified, the bolded values are the default values taken.

Generation	Data	$\delta x/x$: Δm^2_{21}	$\delta x/x$: $\sin^2\theta_{12}$	Ref.
Current	SK+SNO	15%	4.6%	[34]
	KamLAND	2.5%	9.5%	[31]
	SK+SNO+KamLAND	**2.4%**	**4.3%**	[34]
	Global fit	2.8%	4.3%	[72]
		2.9%	5.0%	[71]
		2.2%	4.3%	[73]
Future	DUNE-solar	5.9%	3.0%	[78]
	JUNO	0.3%	0.5%	[79]

Table 3. The precision on Δm^2_{21} and θ_{12} used in different cases and the associated reference for the precise input used. Unless otherwise specified, the bolded values are the default values taken. HK will also measure solar neutrinos [6] but with a precision comparable to the total current solar data [77] of 14% and 5.6% for Δm^2_{21} and $\sin^2\theta_{12}$ respectively.

4 Results

In this section we present various numerical results to support our claims on the importance of solar neutrino parameters in long-baseline experiments. We present the results for DUNE-LBL while similar conclusions can be reached for HK-LBL as well. We also calculate the same results for NOvA and T2K, however their sensitivities are usually much less competitive such that we omitted them from plots. Additional results are shown in appendix A.

4.1 Sensitivity to the complex phase

We first investigate the sensitivity to disfavor $\sin\delta = 0$ for DUNE-LBL and HK-LBL with priors on all five oscillation parameters, θ_{12}, θ_{13}, θ_{23}, Δm^2_{21}, and Δm^2_{31}, as described above. The sensitivity, shown in black in both panels of figure 3 is in excellent agreement with DUNE-LBL's quoted sensitivity [62] and HK-LBL's sensitivity from [63]. We then remove each of the five priors, one at a time to see which, if any, affects the sensitivity. We see that removing the priors on θ_{23} and Δm^2_{31} have little effect as expected since DUNE-LBL can provide an excellent measurement of these parameters without further input. Removing the prior on θ_{13} has a small effect on the sensitivity to δ. Removing the priors on one of Δm^2_{21} or

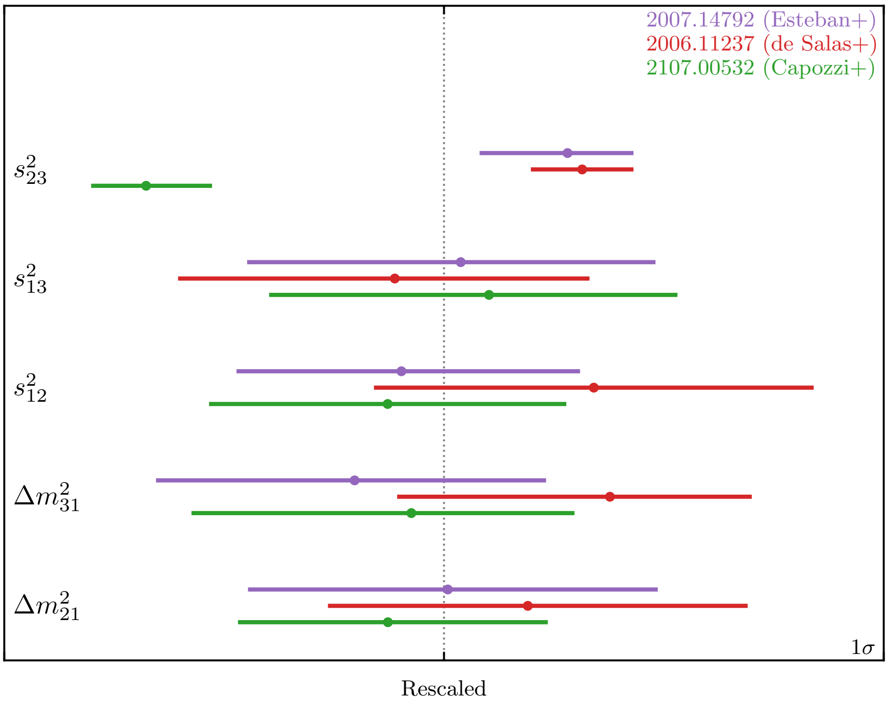

Figure 2. The comparison on the preferred values of the five oscillation parameters (not including δ) from the three primary global fits [71–73]. The quoted 1σ uncertainties are shown and the normal ordering is assumed. The vertical dashed line is at the weighted average and the spread is rescaled to fit on the same scale; s^2_{23} is rescaled $2.5\times$ as much as the other parameters.

θ_{12} is comparable in effect to removing the prior on θ_{13}, while removing *both* priors on Δm^2_{21} and θ_{12} dramatically reduces the sensitivity to δ, in particular for $\delta \in [0, 180°]$ where the sensitivity is at the $\sim 2\sigma$ level at best; for $\delta \in [-180°, 0]$ the sensitivity is only at 5σ for δ very close to $-90°$. Finally, with no priors from other experiments, the sensitivity to δ is at best $\sim 3.5\sigma$ at $\delta \simeq -90°$. This dramatic reduction in sensitivity comes at fairly unusual oscillation parameters, known to be dramatically inconsistent with other oscillation measurements, most notably Δm^2_{21} up to $\sim 60 \times 10^{-5}\,\text{eV}^2$ and θ_{12} taking any value from 0 to 45°.

Similar conclusions about the importance of Δm^2_{21} and θ_{12} can be reached for HK-LBL as well. Unlike DUNE-LBL, HK-LBL cannot determine the mass ordering with high sensitivity, therefore the sensitivity to CPV in the range $\delta \in [0, 180°]$ is below 3σ even when including priors on all oscillation parameters. However also for HK-LBL a drop in the sensitivity arises from removing priors on both Δm^2_{21} and θ_{12}, in particular for $\delta \approx 45°$, 135° where the sensitivity falls below 1σ. If we fix the mass ordering, then the reduction in sensitivity to δ without the inclusion of solar priors is less drastic but still appreciable as shown in figure 9.

The current generation of LBL experiments, NOvA and T2K, have a CPV sensitivity of $\lesssim 2\sigma$ even when including all priors, see figure 9. Nevertheless, also for them we find a reduced CPV sensitivity in the absence of solar priors. Similar to HK-LBL, T2K's CPV sensitivity for $\delta \in [0, 180°]$ is improved when fixing the mass ordering.

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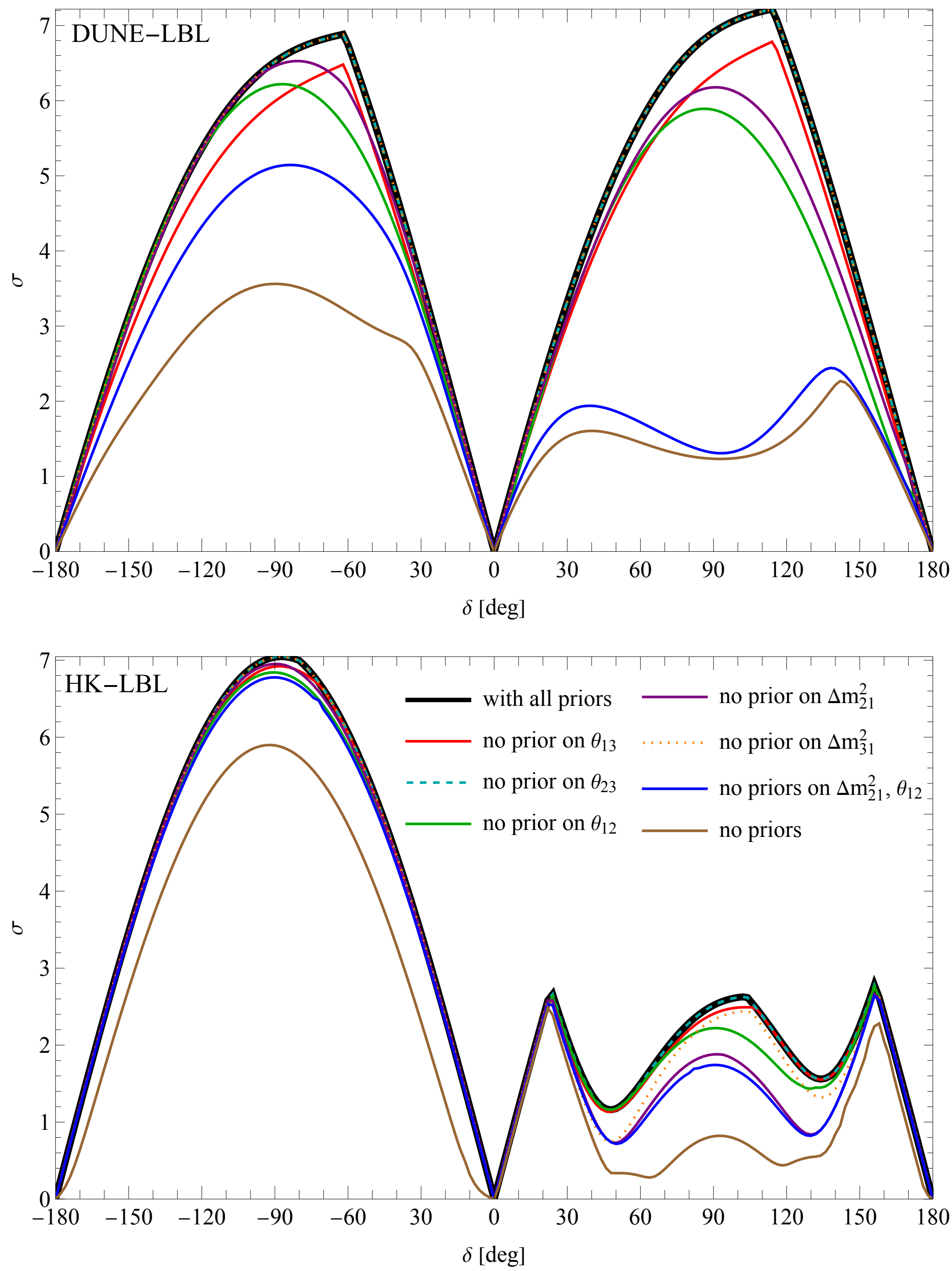

Figure 3. The sensitivity of DUNE-LBL (top) and HK-LBL (bottom) to disfavor $\sin\delta = 0$ in the NO as a function of the true value of δ using the benchmark scenario defined in table 3 and in the text. In black is the sensitivity with a prior on all five constrained oscillation parameters while the other colors are the sensitivity with priors on all but one of the parameters. For DUNE-LBL, the curves without a prior on θ_{23} or Δm^2_{31} coincide with the curve assuming priors on all oscillation parameters due to the excellent sensitivity of DUNE-LBL to these parameters; the same is true for HK-LBL, but only for the θ_{23} curve. The mass ordering is free.

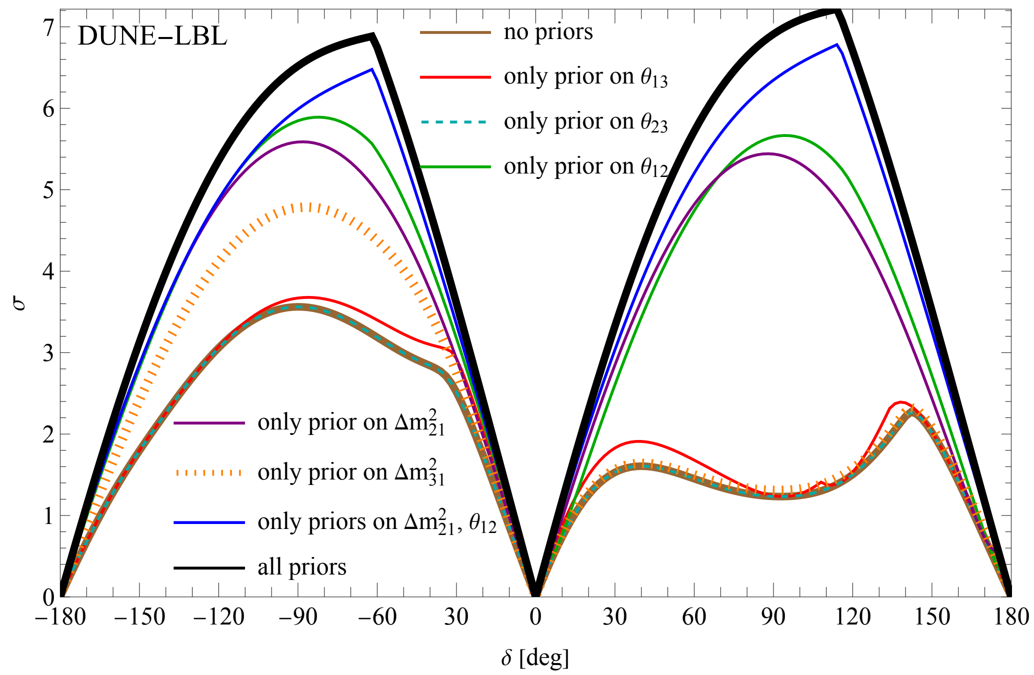

Figure 4. The same as figure 3 at DUNE-LBL but now starting with no prior in brown and the other curves are with one prior on one parameter. The curve with only a prior on θ_{23} lays on top of the curve without priors.

Next we show in figure 4 again the sensitivity at DUNE-LBL without priors. We then include priors one at a time. We see, consistent with the above text, the impact of priors on θ_{23} and Δm^2_{31} are negligible and that including the prior on θ_{13} provides only a marginal improvement. Including either of the solar priors significantly enhances the sensitivity to $> 5\sigma$ for some values near $\delta \simeq \pm 90°$ and including both solar priors increases the sensitivity even more.

While determining if $\sin\delta = 0$ can be disfavored or not is a crucial part of the neutrino physics program, it is also important to measure the value of δ precisely regardless of whether it is close to CP conserving values or not to have a chance of solving the flavor puzzle [80]. We therefore show in figure 5 the 1σ precision with which DUNE-LBL will be able to determine the value of δ as a function of the true value of δ with the different priors on solar experiments defined in table 2. For the remaining parameters we used the benchmark priors provided in the text. We use as measure of the precision $\delta(\delta)$ defined as the maximum of the two 1σ uncertainties as they are often asymmetric. This figure shows that with priors on Δm^2_{21} and θ_{12} from current or future experiments (including those DUNE is expected to get from measuring solar neutrinos itself), DUNE-LBL can determine δ to within $\sim 10° - 15°$ precision. Without priors on Δm^2_{21} and θ_{12} but with priors on θ_{13} the precision is much worse, reaching $\sim 35° - 40°$ precision around $\delta \sim \pm 60°$. Finally, with no external priors at all, DUNE-LBL may only be able to determine δ to $\sim 60° - 70°$ precision. The qualitative results for HK-LBL are similar, note however that the precision at HK-LBL worsens if the mass ordering is not fixed.

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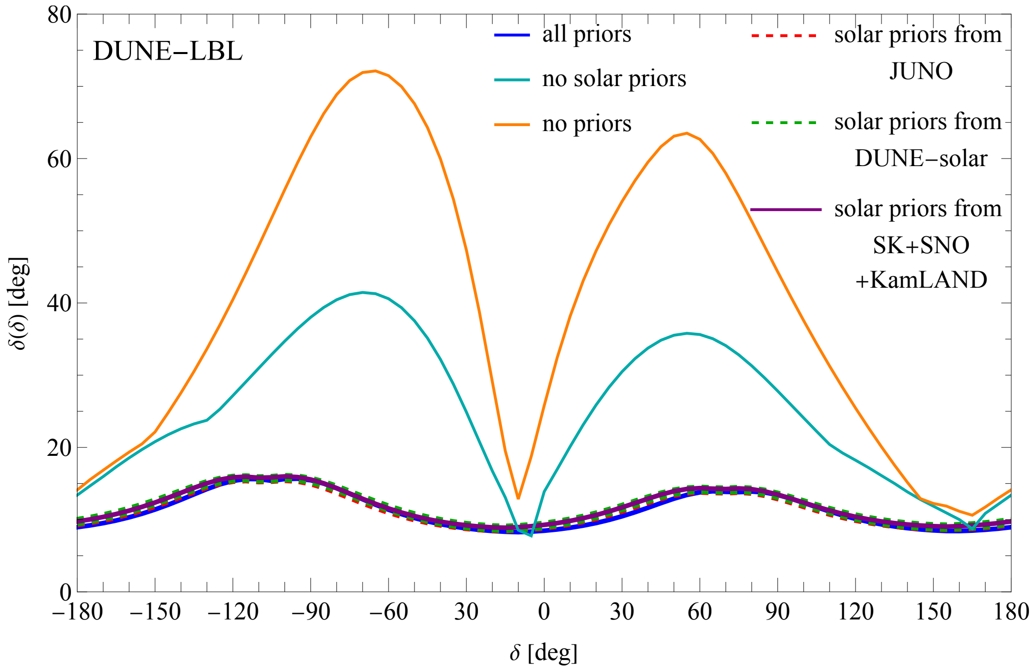

Figure 5. The precision on δ at DUNE-LBL assuming the benchmark scenario defined in table 3 and in the text, and different solar priors from table 2 (red, green, purple lines) and NO. We define the maximum of the two 1σ uncertainties as the precision as they are asymmetric. The blue curve shows the precision using priors on all parameters, the cyan curve shows the precision without solar priors, and for the orange curve we do not assume any priors at all. The red, green and purple curves lay basically on top of each other and on top of the blue curve which assumes priors from our current knowledge of the solar parameters.

Now that it is clear that our knowledge of Δm^2_{21} and θ_{12} plays a key role on our ability to measure the complex phase δ, we investigate the sensitivity to δ as a function of both the precision of those priors as well as the central values. We show in figure 6 how the sensitivity DUNE-LBL and HK-LBL to discover CPV at $\delta = -90°$ depends on the true central values while keeping the absolute uncertainty δx fixed to the one from SK+SNO+KamLAND. For the remaining parameters we used the best fit from our benchmark scenario but assumed no uncertainty on them. We also show the currently preferred values for Δm^2_{21} and θ_{12} from solar data only and from KamLAND as useful benchmarks. We see that changing the true value of Δm^2_{21} and θ_{12} from the best fit from KamLAND to that from solar data reduces the peak sensitivity at DUNE-LBL and HK-LBL to CP violation by $> 1\sigma$. We also see that, consistent with expectations based on the discussion in 2, smaller values of Δm^2_{21} and θ_{12} lead to lower peak sensitivities to CP violation. Similar conclusions also apply when using $\delta = -90°$ as true value, see figure 10. At HK-LBL negative true values of Δm^2_{21} and $\delta = -90°$ lead to a lower sensitivity to discover $\delta = -90°$ compared to positive values of Δm^2_{21} as a change of sign of Δm^2_{21} is equivalent to a change of sign of Δm^2_{31} where for HK degeneracies between the mass ordering and δ appear. This degeneracy is not present in DUNE as it can measure the MO at high significance.

Finally, we also test the impact of the uncertainty of Δm^2_{21} and θ_{12} on the CPV sensitivity. Unlike the impact of the central values we find that the uncertainties only plays a minor role for the CP sensitivity. Also the impact on the precision of δ is marginal, which can also be seen from figure 5 where the results using various priors, which differ by their uncertainty on Δm^2_{21} and θ_{12} according to table 3, are very comparable. We conclude that the true value of Δm^2_{21} and θ_{12} plays a bigger role in the CPV sensitivity and precision for future experiments making a reliable knowledge of the true values highly desirable.

4.2 Sensitivity to the solar parameters

Next we investigate the relative role of Δm^2_{21} and θ_{12} information in figure 3. In particular, we see that with a prior on either Δm^2_{21} or θ_{12} the sensitivity to δ is only partially degraded, but with a prior on neither solar parameter the sensitivity to δ is considerably degraded. This demonstrates that LBL experiments are sensitive to Δm^2_{21} and θ_{12} using accelerator neutrinos, a fact previously not discussed in the literature that we are aware of.

For this reason we study the sensitivity of current and future LBL experiments using the benchmark scenario defined in section 3. In figure 7 we show the sensitivity of long-baseline accelerator neutrino experiments to measure Δm^2_{21} and θ_{12}. While current generation experiments cannot disfavor $\theta_{12} = 0$ and prefer a very wide-range of Δm^2_{21} values and are therefore omitted from this figure, DUNE-LBL and HK-LBL can measure these parameters with some precision. In fact, current LBL experiments only have very weak sensitivity to both Δm^2_{21} and θ_{12} and allow $\Delta m^2_{21} = 0$ and $\theta_{12} = 0$ at the 2σ level, but future LBL experiments can exclude zero values of both Δm^2_{21} and θ_{12} at high significance matching the ability to discover CP violation with no priors on Δm^2_{21} and θ_{12} since if either of these parameters goes to zero, then CP is conserved. By a similar argument, at different values of δ, the precision on Δm^2_{21} and θ_{12} will worsen. However they cannot determine the sign of Δm^2_{21} at high significance such that there are two disjoint preferred regions.

It may seem somewhat unexpected that DUNE-LBL does slightly better on measuring Δm^2_{21} and θ_{12}, while removing those parameters has a bigger impact on its ability to measure CPV than for HK as shown in figure 3. This can be understood from figure 1 which shows that the variation in the probability due to Δm^2_{21} and θ_{12} is generally as big or larger than that due to δ. Thus the remaining sensitivity to CPV or similarly to measure Δm^2_{21} and θ_{12} must come from a combination of shape effects and neutrino/antineutrino modes.

We also point out that if we take a reasonable prior on θ_{12}, we see that DUNE-LBL and, to a lesser extent, HK-LBL, can determine Δm^2_{21} with precision comparable to within a factor of ~ 2 of current solar measurements. Thus, given current data and a future LBL measurement alone, the LBL measurement would provide a relevant constraint on Δm^2_{21}. The previously mentioned caveat about the true value of δ still applies, however. We demonstrate this result in figure 8 where we show the one-dimensional constraints from long-baseline accelerator neutrino oscillation experiments on both Δm^2_{21} and θ_{12}. We assume as true value $\delta = -90°$. As is already discussed for figure 7 the current generation of LBL accelerator experiments is not very sensitive to Δm^2_{21} and θ_{12}. NOvA and T2K can exclude $\sin^2\theta_{12} = 0$ at 1.5–2σ and allow maximal θ_{12} at less than 0.5σ and are therefore omitted from figure 8. Future LBL experiments however will provide better constraints,

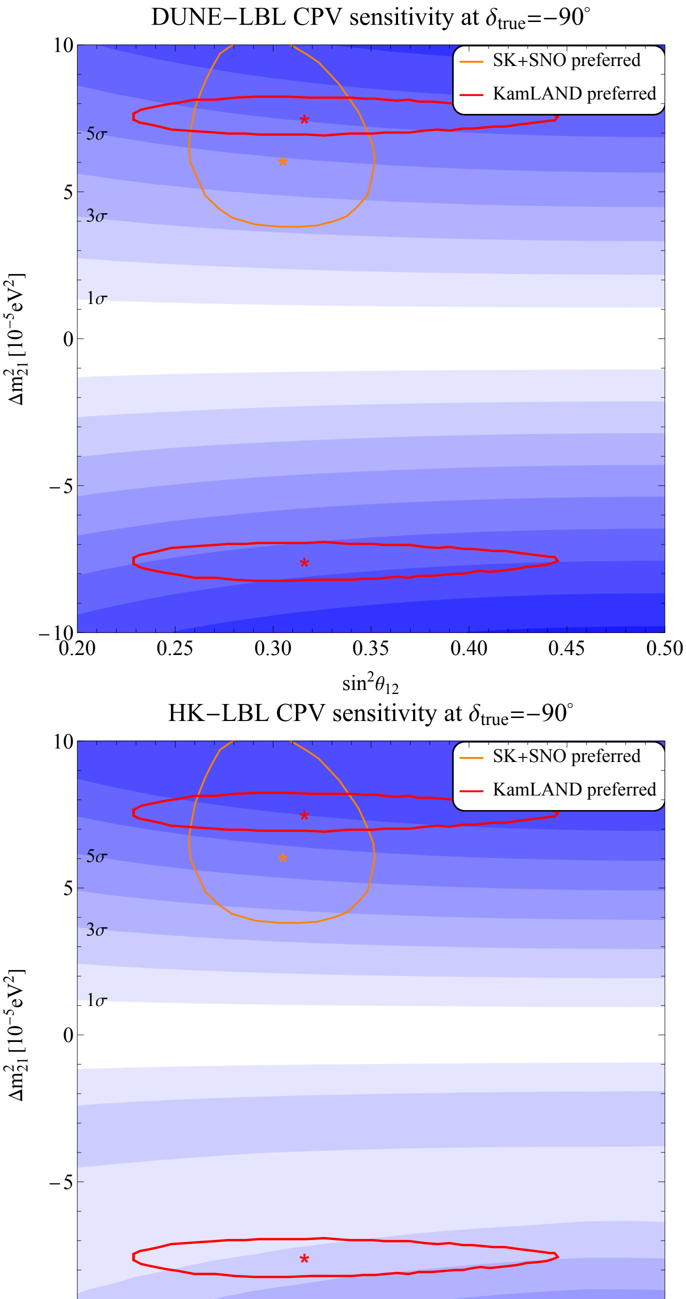

Figure 6. The sensitivity to discover CPV at $\delta = -90°$ at DUNE-LBL (top) and HK-LBL (bottom) in NO while varying the true values of Δm^2_{21} and θ_{12} but keeping their absolute uncertainty δx fixed to the latest combined fit of SK+SNO+KamLAND, see table 3. For the remaining parameters we use the best fit values from our benchmark case but we do not assume any priors on them. For comparison we show the current experimental preferred regions at 3σ for Δm^2_{21} and θ_{12} from SK+SNO, and KamLAND using priors from reactor experiments on θ_{13}. The results assuming the true value is $\delta = 90°$ are shown in figure 10.

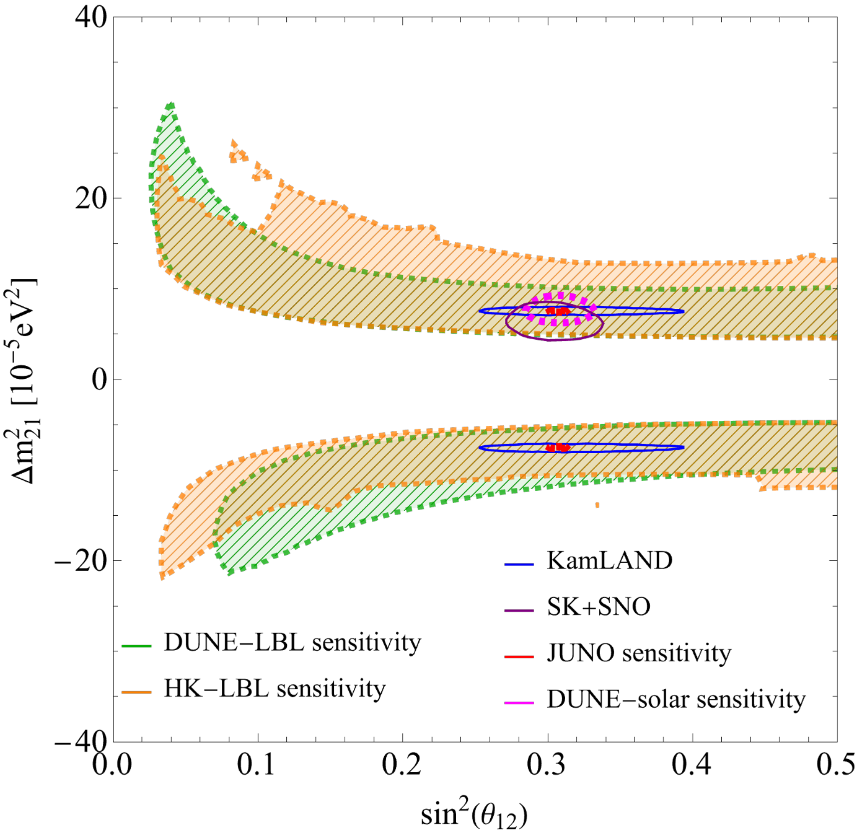

Figure 7. The sensitivity to the solar oscillation parameters at upcoming long-baseline accelerator neutrino oscillation experiments along with existing constraints from KamLAND [31], solar data [34] using priors on θ_{13} from the reactor data, and the expected future sensitivities from DUNE-solar [10, 78], and JUNO [79]. All curves are drawn at 2σ. The JUNO curve is very small due to its excellent sensitivity to Δm^2_{21} and θ_{12}. For the other oscillation parameters we assumed priors and the benchmark values from the text with $\delta_{\text{true}} = -90°$ in NO. The corresponding contours for T2K and NOvA extend over nearly the whole region and have been omitted for clarity.

DUNE-LBL and HK-LBL exclude $\sin^2\theta_{12} = 0$ at 7σ but allow maximal solar mixing at less then 0.5σ. For $\delta = 0$ the sensitivity decreases such that current LBL experiments allow all values of θ_{12}, including $\theta_{12} = 0$ at $\lesssim 0.5\sigma$ while future experiments can exclude $\theta_{12} = 0$ at $1 - 2\sigma$ while they can only distinguish non-zero values with $0.5 - 1\sigma$ sensitivities and prefer a wide range of values for $\sin^2\theta_{12}$.

Regarding the solar mass splitting, current LBL experiments can provide only very mild bounds; T2K and NOvA have sensitivity to constrain $|\Delta m^2_{21}| \lesssim 45$ eV^2 at 3σ and both allow $\Delta m^2_{21} = 0$ at 2σ. However future LBL experiments will provide slightly stronger constraints with $|\Delta m^2_{21}| \lesssim 35$ eV^2 at 3σ at HK-LBL and a slightly narrower constraint at DUNE-LBL due to its ability to measure the octant of θ_{23} with higher sensitivity. Both experiments present two minima at $\pm|\Delta m^2_{21}|$ where the minimum for negative Δm^2_{21} is lifted at the $\lesssim 1\sigma$ level. This means that both future LBL experiments can also determine the sign of the solar mass splitting with some significance. The presence of two disparate minima also demonstrates that both experiments can exclude $\Delta m^2_{21} = 0$ at a high significance ($\gtrsim 7\sigma$).

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For $\delta = 0$, on the other hand, the exclusion of $\Delta m_{21}^2 = 0$ persists due to the shape information which is stronger for DUNE than for HK but it shrinks to 4σ (DUNE-LBL) and 2σ (HK-LBL) and for current experiments to below 1σ. However the negative solution remains lifted at 1σ for the future experiments.

Furthermore, DUNE-LBL's and HK-LBL's sensitivity to Δm_{21}^2 demonstrates in particular that they can measure both dominant frequencies. That is, they can determine the atmospheric Δm^2 whose information comes dominantly from their disappearance channel, and is thus well described by [50]

$$\Delta m_{\mu\mu}^2 = s_{12}^2 \Delta m_{31}^2 + c_{12}^2 \Delta m_{32}^2 + \cos\delta s_{13} \sin 2\theta_{12} \tan\theta_{23} \Delta m_{21}^2 \,, \tag{4.1}$$

$$\approx s_{12}^2 \Delta m_{31}^2 + c_{12}^2 \Delta m_{32}^2 \,. \tag{4.2}$$

The ability to measure both dominant frequencies means that these experiments can measure all 6 oscillation parameters on their own, without any further input which provides an important test of the three-flavor oscillation picture. There is some sensitivity to the solar mass ordering (the sign of Δm_{21}^2) due to several phenomena including the second oscillation maxima.

In comparison to current and future constraints on Δm_{21}^2 from KamLAND and SK+SNO and JUNO and DUNE-solar future LBL experiments are unlikely to improve the constraints, nevertheless they provide an important complementarity to other measurements using neutrinos from different sources and energies and therefore a crucial consistency check of the three-flavor picture. This is very important given the modest spread in the preferred values of Δm_{21}^2 and θ_{12} from the global fits as shown in figure 2.

5 Discussion

The quest for leptonic CP violation is one of the main targets of current and upcoming LBL experiments. However, typically all studies and analyses rely on the input of external parameters which come with their own unique systematic uncertainties which may be correlated with a LBL experiment's systematic uncertainties or may be completely unrelated. In this context, the role of the input of external data on Δm_{21}^2 and θ_{12} from solar and long-baseline reactor experiments has not been carefully discussed in the literature yet. In this manuscript we have studied for the first time the effect of solar priors on the CPV sensitivity and precision of δ at current and upcoming LBL experiments. We have shown that priors on Δm_{21}^2 and θ_{12} are quite relevant for the sensitivity to CP violation but, quite interestingly, LBL accelerator experiments still have some sensitivity to CP violation even with no information from the solar parameter experiments. This then implies that LBL accelerator experiments have sensitivity to Δm_{21}^2 and θ_{12} at a certain level.

To better understand the impact of these priors on the measurement of δ we note that in order to reduce the sensitivity to δ for large $|\sin\delta|$ one needs solar parameters highly inconsistent with existing data. In particular, we find that the data would be most comparable to a CP conserving $\sin\delta = 0$ scenario for $\Delta m_{21}^2 \simeq 60 \times 10^{-5}\,\text{eV}^2$ instead of the usual $7.5 \times 10^{-5}\,\text{eV}^2$. This alone is not enough, one also needs $\theta_{12} \sim 10^\circ$ for δ not too

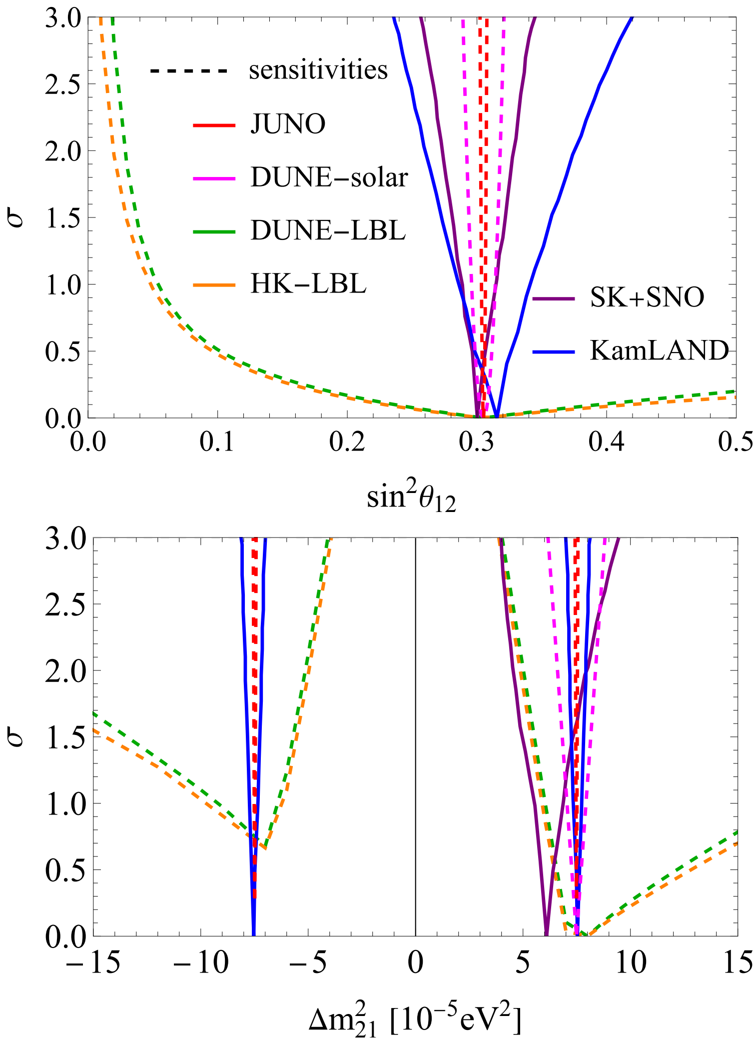

Figure 8. The sensitivity of various different experiments for Δm^2_{21} and θ_{12}. The lines use priors on θ_{23}, θ_{13}, Δm^2_{31} and the benchmark values from section 2 and $\delta_{\rm true} = -90°$ in NO. The top plot shows the sensitivities to θ_{12} and the bottom plot shows the sensitivities to the solar Δm^2. We also show the current constraints from KamLAND [31], solar data [34], and the expected future sensitivities from DUNE-solar [10, 78], and JUNO [79]. We do not show the sensitivities of NOvA and T2K as they only provide weak constraints on these parameters.

near $0, \pi$. It turns out that the significant change to either solar parameter alone does not do a good job; only both of them together are able to approximately mimic a CP conserving scenario. This is why relaxing the solar input to only one of the parameters does not significantly reduce the sensitivity to CP violation.

We also notice that the matter effect and the ability to measure the atmospheric mass ordering plays a key role. For example, DUNE-LBL will have excellent sensitivity to the atmospheric mass ordering, even without priors on Δm^2_{21} and θ_{12}, because DUNE-LBL is the only experiment that will measure the mass ordering at high significance via the matter effect in Δm^2_{31} oscillations (all measurements of the atmospheric mass ordering require a measurement of the matter effect somewhere), DUNE-LBL is also able to determine the solar mass ordering (that is, that $\Delta m^2_{21} > 0$) at $\sim 1\sigma$. HK-LBL has comparable sensitivity to the solar mass ordering since it will determine the atmospheric mass ordering at $> 1\sigma$ and knowledge of the atmospheric mass ordering is a prerequisite to determining the solar mass ordering at LBL experiments.

Finally, for completeness, we briefly comment on the role priors on Δm^2_{21} and θ_{12} as well as θ_{13} will have on the determination of the other oscillation parameters, Δm^2_{31}, θ_{23}, and θ_{13}. We show several plots in appendix B and discuss the results here. Some versions of these questions have been asked before, see e.g. [81–83], but not in terms of the role of Δm^2_{21} and θ_{12}. In the appendix we show the sensitivities of current and future LBL experiments to θ_{23}, Δm^2_{31}, θ_{13} with all priors and without priors on key combinations of the Δm^2_{21}, θ_{12}, and θ_{13}. We find the impact of the solar priors on the sensitivity to θ_{23} is minor and most pronounced for T2K. The lack of a prior on Δm^2_{21} is responsible for a significant portion of this. Even more important than solar priors is the prior on θ_{13} which we show affects the octant sensitivity more that the priors on Δm^2_{21} and θ_{12}.

Upcoming experiments will not improve Daya Bay's measurement of θ_{13} but without solar priors, the sensitivity is further reduced such that considerably smaller values of θ_{13} are allowed. We show that without a prior on Δm^2_{21}, the sensitivity declines making this the most important prior for the determination of θ_{13} at LBL experiments. Without priors on both Δm^2_{21} and θ_{12}, the allowed ranges extend to smaller values of θ_{13}, in the case of T2K $\theta_{13} = 0$ is allowed at 3σ.

LBL experiments also have good sensitivity to Δm^2_{32} in the disappearance channel. In order to derive the constraint on Δm^2_{31}, however knowledge on Δm^2_{21} is required since the measurement depends on a weighted combination of Δm^2_{31} and Δm^2_{32}, therefore the sensitivity to Δm^2_{31} is substantially reduced without solar priors. In particular, without a prior on Δm^2_{21} the effect is considerable for current LBL experiments but the effect is also significant for future experiments. In fact, a second nearly degenerate minimum appears around $\Delta m^2_{31} \approx 2.42 \times 10^{-3}$ eV^2 which corresponds to the resulting value of Δm^2_{31} from the measurement of Δm^2_{32} but with $-\Delta m^2_{21}$. Finally without priors on solar data HK-LBL will not be able to measure the atmospheric mass ordering at more than $\sim 1\sigma$ and HK-LBL's precision on Δm^2_{31} gets considerably worse; the same effect is present, and even more dramatic, for the current LBL experiments.

6 Conclusion

We have demonstrated that solar oscillation parameters, θ_{12} and Δm^2_{21}, play an important and largely unrecognized role in long-baseline (LBL) neutrino oscillations. In particular, without external knowledge on Δm^2_{21} and θ_{12}, LBL experiments have significantly limited sensitivity to discover CP violation. Moreover, Δm^2_{21} and θ_{12} can actually be determined at LBL experiments providing a valuable cross check, especially given that there a spread of preferred values from the experiments and the global fits. In addition, the true values for the solar parameters, in particular Δm^2_{21}, plays an important role in the ability to measure

CPV with LBL neutrinos affecting the peak sensitivity to CPV by $> 1\sigma$. Thus having precise measurements of Δm^2_{21} and θ_{12} from e.g. JUNO is important for determining CPV.

We also found that Δm^2_{21} and θ_{12} have an impact on the determination of other oscillation parameters like θ_{13} and Δm^2_{31} and the octant of θ_{23}, making precise knowledge of Δm^2_{21} and θ_{12} fundamental for the success of the next generation of LBL experiments. In turn the sensitivity of LBL accelerator experiments to Δm^2_{21} and θ_{12} allows us to probe the three-flavor paradigm in one experiment only and provides a consistency check of our understanding of neutrino oscillations in a different energy and baseline regime than usually used to determine Δm^2_{21} and θ_{12}.

Acknowledgments

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A Additional figures

To complement the figures in the main text, we performed various additional parameter scans to further elucidate the interplay of Δm^2_{21} and θ_{12} in current and next-generation long-baseline neutrino oscillation experiments.

Figure 9 shows the CPV sensitivities of HK-LBL, NOvA, and T2K under the assumption of different priors and fixed or free mass ordering. Note that the for HK-LBL and T2K without external information on the mass ordering, in the true NO it is very hard to discover CP violation for $\delta \in [0°, 180°]$ as Δm^2_{21} and θ_{12} can radically change. Note that we do not show the corresponding plot for DUNE due to its excellent sensitivity to the MO, thus fixing the MO does not have an impact on the sensitivity to disfavor $\sin\delta = 0$.

Finally, in figure 10 we show the DUNE-LBL and HK-LBL sensitivities at $\delta = 90°$ for different central values of Δm^2_{21} and θ_{12} keeping their absolute uncertainty δx fixed to the one from the latest combined fit of SK+SNO+KamLAND from table 3.

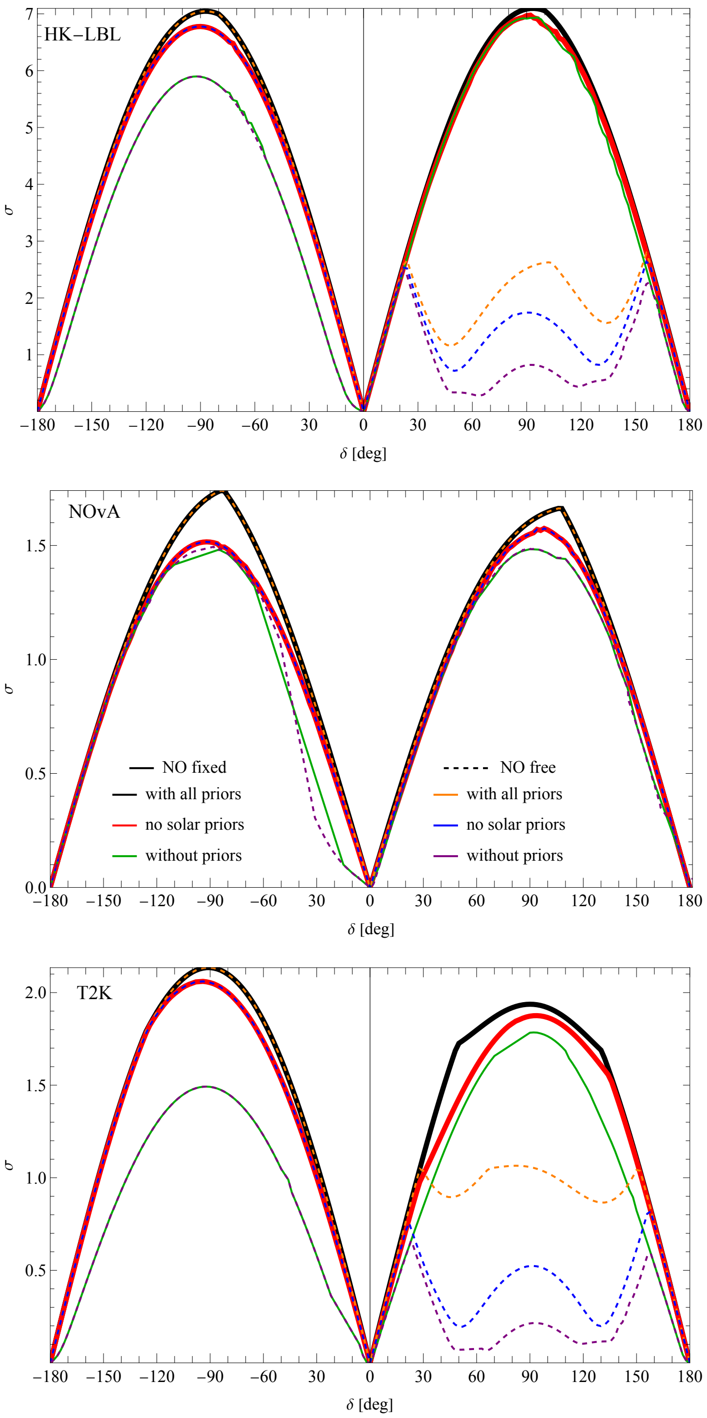

Figure 9. The sensitivities to disfavor $\sin\delta = 0$ of HK-LBL, NOvA, and T2K corresponding to the exposures from table 1 with fixed normal ordering or free mass ordering. The black and orange lines shows the results using all priors, red and blue lines are the results without solar priors while the green and purple lines show the results without any priors. DUNE is not shown here due to its excellent sensitivity to the MO, thus fixing the MO does not have an impact on the sensitivity to disfavor $\sin\delta = 0$.

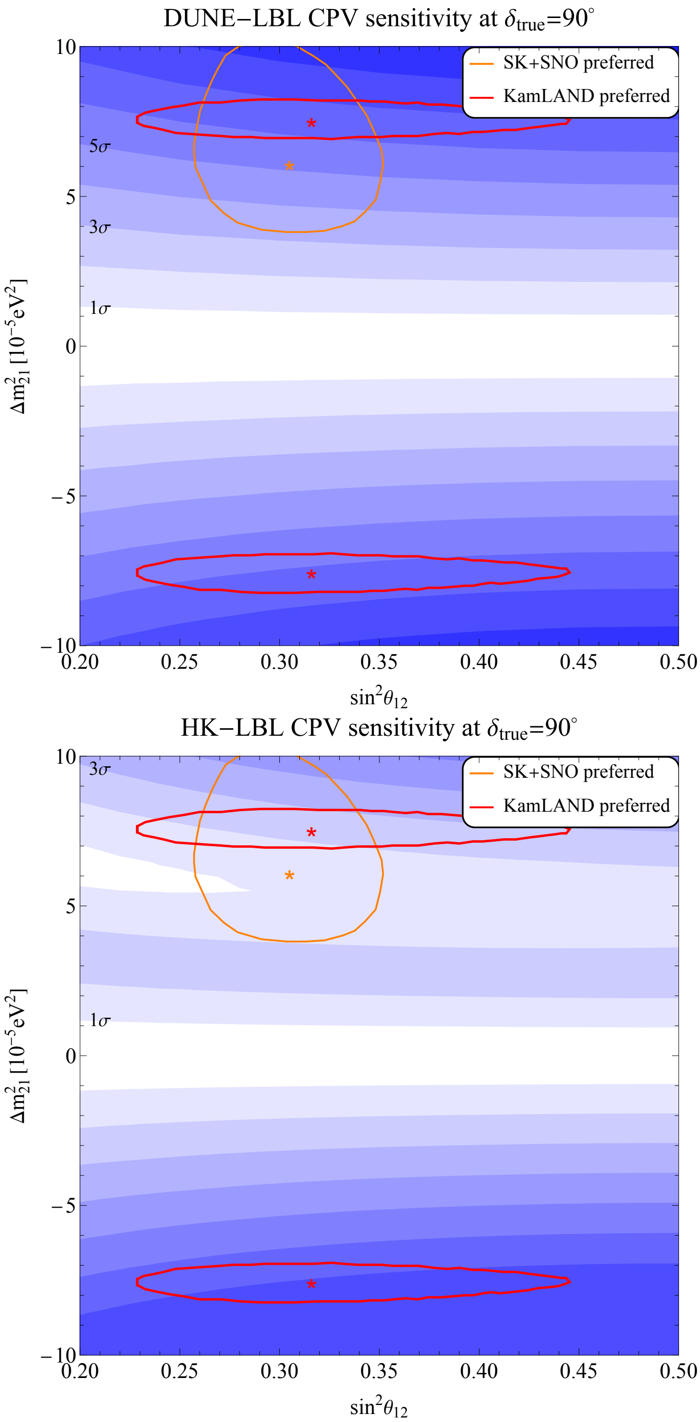

Figure 10. The sensitivity to discover CPV at $\delta = 90°$ at DUNE-LBL and HK while varying the true values of Δm^2_{21} and θ_{12} but keeping their absolute uncertainty δx fixed to the latest combined fit of SK+SNO+KamLAND, see table 3. For the remaining parameters we use the best fit values from our benchmark case but we do not assume any priors on them, we assume NO. For comparison we show the current experimental preferred regions at 3σ for Δm^2_{21} and θ_{12} from SK+SNO, and KamLAND.

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B The impact of solar parameters on the determination of the atmospheric parameters

The solar parameters also affect the measurement of the other parameters, apart from δ, which we demonstrate in figures 11, 12, 13.

As DUNE-LBL and NOvA have good sensitivity to θ_{23} on their own, the impact of the absence of solar priors is very small. This is different for T2K and HK-LBL where the absence of solar priors affects their sensitivity to resolve the octant. The dominant source for the reduction of the sensitivity is the absence of the prior in Δm^2_{21}. Furthermore, we show that the prior on θ_{13} is even more important to resolve the octant. This is because the octant information comes from presence of the s^2_{23} term in the $\nu_\mu \to \nu_e$ appearance probability which is paired up with s^2_{13}, see e.g. [47, 83–85].

Also for the θ_{13} sensitivity the absence of solar priors affects DUNE-LBL's sensitivity only marginally whereas T2K's, NOvA's and HK-LBL's sensitivities worsen and the allowed ranges extend to smaller values of $\sin^2 2\theta_{13}$. Without solar priors T2K cannot exclude $\theta_{13} = 0$ at more than 3σ. Also in this case the prior on Δm^2_{21} is important. Finally, the sensitivity to Δm^2_{31} gets severely affected without solar priors, in particular Δm^2_{21}, at T2K, NOvA and HK-LBL, while the effect at DUNE-LBL is smaller. The reduction of sensitivity can be understood from the fact that LBL experiments are somewhat more sensitive to Δm^2_{32} in ν_μ disappearance and Δm^2_{31} is then derived from the sum rule $\Delta m^2_{31} = \Delta m^2_{32} + \Delta m^2_{21}$. As the LBL experiments' sensitivity to Δm^2_{21} leads to less severe constraints than our current knowledge of this parameter the derived sensitivities of Δm^2_{31} worsen as well. In fact, two nearly degenerate minima appear which correspond to the different signs of Δm^2_{21} to which LBL experiments are not very sensitive. However as we have shown in figure 8 the LBL accelerator experiments have some sensitivity to Δm^2_{21} and θ_{12}, in particular future experiments disfavor $\Delta m^2_{21} = 0$ at a high significance. This leads to $\Delta m^2_{31} \approx 2.45 \times 10^{-3}$ eV^2 to be excluded as in this case $\Delta m^2_{31} = \Delta m^2_{32}$ and Δm^2_{21} is required to be zero.

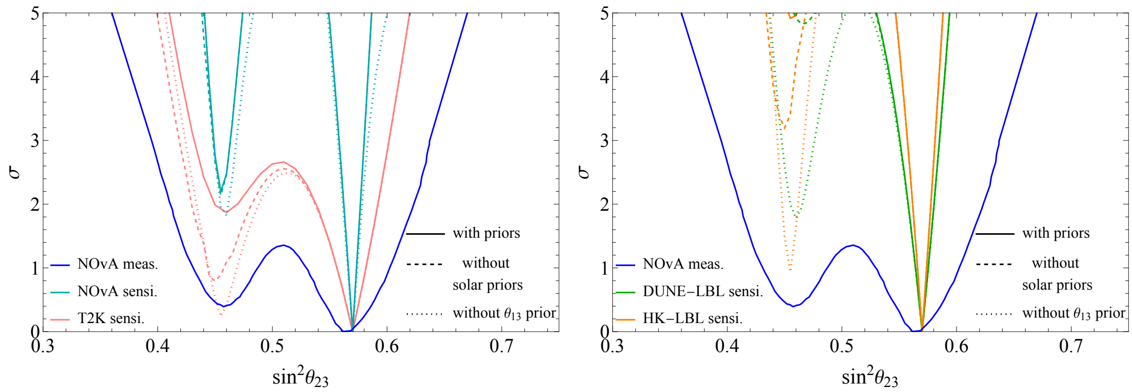

Figure 11. Sensitivity to θ_{23} of current (left) and future LBL experiments (right) compared to the current best measurement from NOvA [75] in NO. The solid lines show the sensitivity using priors on all parameters but δ, the dashed lines do not use solar priors. The dotted curve shows the results without a prior on θ_{13}. The solar parameters have an impact on the sensitivity to the sensitivity to the octant. For the other parameters we assumed the benchmark values from above and $\delta_{\text{true}} = -90°$.

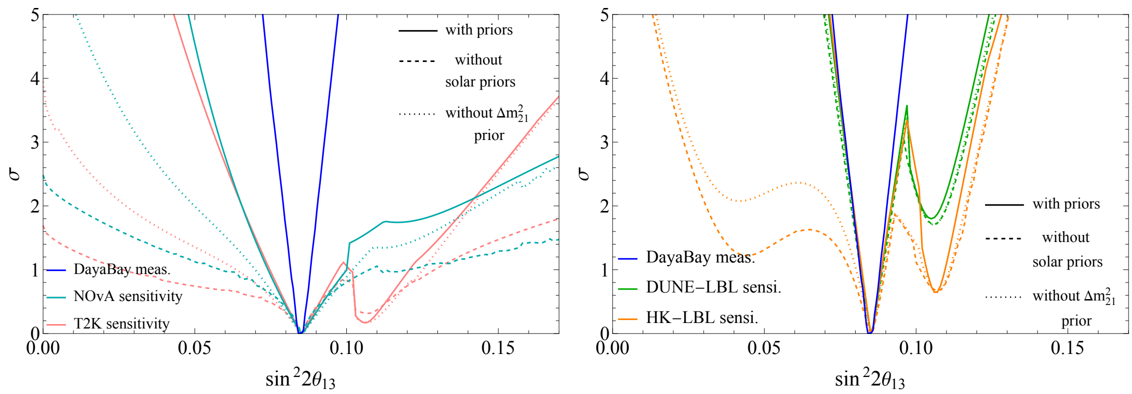

Figure 12. Sensitivity to θ_{13} of current (left) and future LBL experiments (right) compared to the current best measurement from Daya Bay [74] in NO. The solid lines show the sensitivity using priors on all parameters but δ, the dashed lines do not use solar priors. The dotted curve shows the results without a prior on Δm^2_{21}. The second higher local minimum comes from the other octant of θ_{23}. For the other parameters we assumed the benchmark values from above and $\delta_{\text{true}} = -90°$.

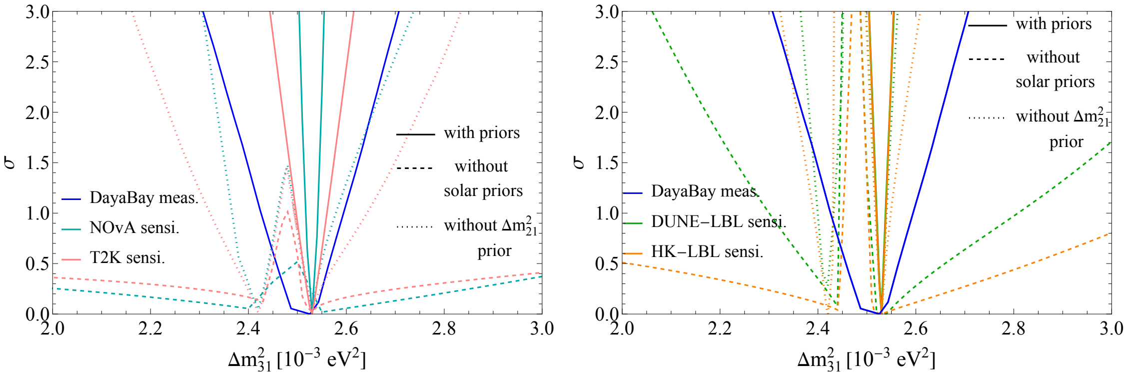

Figure 13. Sensitivity to Δm^2_{31} of current (left) and future LBL experiments (right) compared to the current best measurement from Daya Bay [74] in NO. We show the sensitivity using priors on all parameters but δ, no solar priors, and without a prior on Δm^2_{21}. As true values for the parameters we assumed the benchmark values from above and $\delta_{\text{true}} = -90°$. LBL experiments are chiefly sensitive to Δm^2_{32} in muon disappearance and in a three-flavor scenario Δm^2_{31} is then derived from $\Delta m^2_{31} = \Delta m^2_{32} + \Delta m^2_{21}$. The two minima in the case without prior on Δm^2_{21} correspond to the different signs of Δm_{21} while the value $\Delta m^2_{31} \approx 2.45$ eV^2 is excluded at 4–5σ at HK-LBL and DUNE-LBL as in this case $\Delta m^2_{31} = \Delta m^2_{32}$ and Δm^2_{21} is required to be zero which is disfavored at HK-LBL and DUNE-LBL, see figure 8.

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