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# HARMONIC 20 RESONANCES IN THE CPS

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It has been pointed out [1] that the stop-bands at $3\,Q = 20$ may influence the behaviour of the CPS, because 20 is an important component in the superperiod structure (there are 20 equispaced long straight-sections), and, since we can tune out the $2\,Q = 13$ stop-bands with 13th harmonic quadrupoles, it is possible that the $3\,Q = 20$ ones play some part in limiting the available working region at injection.

On the other hand their importance should not be exagerated, as the $Q_r + Q_z = 13$ skew-gradient sum resonance is at present uncorrected, and the $3\,Q_r = 20$ has been crossed experimentally [2].

This note is a list of the various ways in which harmonic 20 resonances may be excited in the CPS, with a few remarks but no quantitative analysis.

We should consider together the four resonances

$$3\,Q_z = 20$$

$$Q_r + 2\,Q_z = 20$$

$$2\,Q_r + Q_z = 20$$

$$3\,Q_r = 20$$

They are all sum resonances,so may be unstable; they all require a 20th harmonic perturbation; and they are all 3rd order, so require the perturbation to be of sextupole (3rd power in the Hamiltonian) nonlinearity, or of a higher power with the same parity.

The difference resonances

$$4\,Q_r - Q_z = 20$$

etc.

are of higher order.

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1. The obvious way of exciting these resonances is with a 20th harmonic sextupole field:

(a) From random sextupolar stray fields or random sextupolar **field** imperfections in the magnets. It seems unlikely that these are strong enough : statistically the 19th harmonic should be just as strong and make bigger effects.

(b) From sextupolar fields associated with the sets of 10 sextupoles in the machine. It seems unlikely that the odd sextupoles should be blamed, as they are commonly used for ejection and excited with polarities chosen to make mainly 6th harmonic, so that one would expect frequencies like 6, $6 \pm 10$, $6 \pm 20$, ....., in their remanence, but not much 20. The even sextupoles are often used individually, but if many get used in zero harmonic for changing $\frac{\partial Q}{\partial r}$ , this would leave a 20th harmonic remanence, and it might be worth doing a demagnetisation cycle oh the lenses after such a use. But in operation it seems that pulsing sextupoles at high energy has no noticeable effect on injection.

(c) From sextupolar stray fields in the long straight-sections. The cavity ferrite and soft-iron chamber must make a very good shield against high-multipolarity fields. Long straights without cavities are relatively few, and used in such diverse ways that a systematic effect seems unlikely.

(d) Sextupolar stray fields in some other type of straight-section. It is difficult to imagine a systematic sextupole effect from quadrupoles or PUS or "free" short straight-sections. It is possible that the octupoles have a certain sextupole component in their remanent field after use of their dipole windings, but these windings again are not used with the same sign in the whole set, so there is not much tendency to generate a 20th harmonic (no more than the 19th, anyway). The same remark applies to the possibility of octupole 20th harmonic remanence combining with a general $\Delta p$ orbit shift.

Ideally the field of the machine should have "median-plane" symmetry, that is

$$B_z(-z) = +B_z(z)$$

$$B_r(-z) = -B_r(z)$$

and most of the perturbations and corrections respect this symmetry too *. They give only even powers of $z$ in the Hamiltonian, and can only excite resonances

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*) The main exceptions are vertical dipoles and skew quadrupoles.

involving even multiples (including zero) of $Q_z$. Thus the second and fourth of the resonances listed on page 1 are likely to be stronger than the other two. The odd ones are not completely forbidden; among the perturbations mentioned in (a) to (d) above, the field of the tunnel, and the remanence of vertical dipoles, lack median-plane symmetry.

2. Next we may consider the terms arising from a zero-harmonic sextupole field combining with the 20th harmonic periodicity of the basic linear lattice.

For sources of the zero-harmonic sextupole field we enumerate:

(a) The nonlinearity of the PS magnet, including end-effects, low-field effects from remanence and finite $\mu$ , vacuum chamber eddy currents, the self-energised PFW currents. The total is supposed to be rather small. Intensity-independent.

(b) PFW currents from supplies. These are adjustable, but the resulting nonlinearity is not fully under our control because the linear part changes too. Intensity-independent, but may be correlated (PFW influences intensity, intensity can shift the PFW optimum).

(c) Space-charge. This can produce significant nonlinear fields at high intensity, but we must expect the beam to have up/down and left/right symmetry about its own centre, and so only terms of even power in $z$ and in $r$ will arise in the Hamiltonian. But particles with non-zero $\Delta p$ , making their betatron oscillations largely on one side of the beam, may experience a sextupole-type nonlinearity, and this will alternate in sign at the frequency of the synchrotron oscillations, so that the satellites

$$3\,Q_r \qquad + Q_s = 20$$

$$Q_r + 2\,Q_z + Q_s = 20$$

are especially worth looking at. The ones with $-Q_s$ are equally strong resonances, but further away; the ones with $Q_z$ or $3\,Q_z$ have the wrong kind of symmetry about the median plane. Higher multiples of $Q_s$ could enter, especially if odd.

In the CPS $Q_s$ is about 0.06 at injection and decreasing, so this group of satellites is rather narrow and all nearly as far from the working point as the basic $6\frac{2}{3}$ line. So the only special interest in this process is that it enables space-charge to produce a term having the dangerous parity.

There are two sources of 20th harmonic in the linear lattice of the machine

(i) The long straight-sections. There are ten in F and ten in D ; for calculating their contribution to harmonic 20, it may be adequate to treat them all as the same in some smoothed model machine.

(ii) The injection quadrupoles. There are 20 in F and 20 in D (though not completely regularly distributed), and common components of current in each set are used for shifting the two Q's at injection. At typical operating currents they are probably stronger than the long straights.

(iii) In principle the RF in the cavities can produce 20th harmonic effects, as there is an RF defocusing field in each gap, and a saw-tooth component in the particle longitudinal momentum. Both of these are very small.

These linear perturbations will produce a 20th harmonic variation of the $\beta$ functions round the machine. Since these functions determine the local sensitivity of the beam, their 20th harmonic converts a general sextupole (zero harmonic) field into a 20th perturbation. But their effect on the space-charge term is more complicated because one has in addition an envelope "sausaging" which modulates the space-charge force directly, and also a variation of $\alpha_p$ in the synchrotron oscillations.

## Conclusions

It would be good to estimate the amount of 20th harmonic from the injection quadrupoles and from the long straights, in order to have at least a rough idea of their combined effect. With that linear term, the space-charge $Q_s$ satellite looks worth a quantitative study. Next in interest would be the PFW sextupole component.

## References

1) Notes sur la réunion du 11 juillet. MPS/DL-Note 68-20.

2) MCR. PS Operation Book. 19th September 1968.

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