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A COMPARATIVE STUDY OF THE METHODS OF STABILIZING THE FIELDS OF THE INFLECTOR MAGNETS OF THE CERN P.S.

by

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Abstract

This report describes the basic considerations of the design of systems for the stabilization of magnetic fields to a precision of 5-10 parts in 10^6. This is a preliminary study of such systems for use on the inflector magnets between the Linear Accelerator and the Proton Synchrotron at CERN. The final recommendation of this study is that in the present circumstances the most convenient method is that which utilises the "Hall Effect".

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1. INTRODUCTION

There are several methods by which magnetic fields may be stabilised. These are :

(i) Stabilization of the current which produces the field.

(ii) The detection (and subsequent feedback) of changes in the magnetic field by the phenomenon of "nuclear magnetic resonance".

(iii) A similar system incorporating a "Hall Probe".

The last methods (ii) and (iii) are preferable [1] since they make direct measurements of the magnetic field. Various thermal effects can cause the strength of a magnetic field produced by a constant current to vary.

It should be pointed out that Sommers et al[2], report a current stabilized system in which the variations of the magnetic field strength was about 1 part in 10^6. However the system was complicated electronically and consequently expensive.

For the present purpose the report is confined to the last two methods.

The basic criteria for the device are reliability and the provision of a visual display, this latter point facilitates the detection and pin pointing of faulty systems.

The first section of this report is devoted to a brief description of the magnetic field which is required to be stabilized. In the subsequent sections the relevant theory and a critique of the particular methods are described. Then follows a comparative study of the systems.

2. THE MAGNET[3]

The magnetic fields of the inflector are about 2,000 gauss and are produced by a current of about 10 Amp flowing through coils, on which it is possible to wind on "shimming turns" for control purposes. Also one of the inflector magnets is close to the magnet of the Proton Synchrotron and hence experiences the pulsed field periodically.

Another important factor in the succeeding discussion is the homogeneity of the vertical component of the magnetic field. This quantity may be estimated from graph 1 page 18, which was taken from a CERN report by Septier [3]. It is apparent that a region exists within five centimetres of the vacuum chamber walls, in which the gradient $\frac{dHx}{dx}$ is less than 10 gauss cm^{-1}.

3. NUCLEAR MAGNETIC RESONANCE THEORY[4] [5] [6] [7]. See Appendix I

The elements of the theory of Nuclear Magnetic Resonance are described in Appendix I. It is shown there that the variation of the susceptibility $\lambda = \lambda' + i\lambda''$ which is complex, is given by

$$\lambda' = \frac{1}{2} \lambda_o \, \omega_o \, T_2 \, \frac{(\omega_o - \omega_r) \, T_2}{1 + (\omega_o - \omega_r)^2 \, T_2} \tag{1}$$

$$\lambda'' = \frac{1}{2} \lambda_o \, \omega_o \, T_2 \, \frac{1}{1 + (\omega_o - \omega_r)^2 \, T_2^2} \tag{2}$$

The variation of λ' and λ'' is shown in figures 2 and 3 page 19.

where λ' and λ'' are the real and imaginary parts of the susceptibility, T_2 the relaxation time, associated with the spin-spin interaction, ω_o is the resonant angul. frequ, ω_r the angul. frequ of the radiofrequency and λ_o is the static susceptibility of the sample.

Further it is shown that the change of impedance of a tuned circuit, comprising of a coil containing the sample and a suitable condenser is

$$\Delta Z_o = 4\pi \zeta \, LQ \, (\lambda' + i \lambda'') \tag{3}$$

where ζ is the filling factor of the sample, Q the Q factor of the resonant circuit of resonant impedance Z_o

A useful numerical calculation may be made with relation (3) to determine the depth of modulation of a given radiofrequency current Jo cosωt - we require only the λ' part. λ''_{max} may be determined from formula (14) and is found to be $\frac{1}{2}\lambda_0 \omega_r T_2$. Hence the degree of modulation is $2\pi\lambda_0 \omega_0 T_2 Q \mathcal{E}$. Now $\lambda_0 \sim 3 \times 10^{-15}$ erg gauss^{-2} cm^{-3},

$\omega_0 \sim 2\pi \times 20$ Mc/s for $H \sim 5{,}000$ gauss. Suppose that $\mathcal{E} = 1$ and $Q = 100$, $T_2 \sim 3 \times 10^{-4}$, then the degree of amplitude modulation is $\frac{1}{100}$

So if 10 volts appear across the coil the maximum amplitude modulation will be 0.1 volt. Later it will be shown that modulation within the absorption line produces a useful output. If this is $\frac{1}{20}$ of the line width then a signal to be observed with

$$\sim \frac{0.1}{20} = \frac{0.01}{2} = 0.005 \text{ volt}$$

A similar calculation shows that the expected value of the induced voltage in a coil perpendicular to the exciting coil is $\sim 85 \times 10^{-3}$ volts.

4. STABILIZATION SYSTEMS INCORPORATING NUCLEAR MAGNETIC RESONANCE

All methods of stabilization must consist of a probe which detects the changes in the field and a system which feeds back this information for the control of the field. This signal may cause the current in some "shimming coils" to change, or the actual exciting current of the magnet may be changed.

One obvious way of detecting changes in the field would be the detection of the E.M.F. induced in a coil wound about the sample at right angles to the coil producing the R.F. field. The maximum induced EMF occurs at the resonant conditions. Such a system has been described by Packard [8].

However, the difficulties of manufacturing two coils at right angles eliminate this method from the study. In fact even a small departure, say 1' of arc can cause the desired resonant signal to be swamped by the coupling E.M.F. (This difficulty can be avoided by using a paddle device). Hence this method was abandoned in favour of single coil methods.

In such methods the coil containing the sample forms a part of an oscillating circuit.

The first class of such systems i.e. that of Thomas et al.[9], incorporate a fixed frequency corresponding to that of the resonant frequency and depend on the modulation of the magnetic field for their operation. The effect of large ($\gg$ 0.5 gauss) and small ($\ll$ 0.5 gauss) modulation will now be considered. At fields far from the resonant value Hx, the amplitude of the field frequency signal will be V_o. However as the field modulation proceeds the condition of resonance is approached and power is absorbed from the fixed frequency. This impresses an amplitude and phase modulation on the signal, which is repeated every cycle of the modulating field (50 cps is often used - however a smaller field modulating frequency may be better since the conditions are more unlikely to depart from the equilibrium state. If the amplitude and frequency of the modulating field are too large then transient effects are observed). The modulating envelope may be separated from the carrier frequency by means of a detector, this can then be rectified and applied to the X plates of a cathode ray oscillograph, which is swept by the Y plates at the same frequency as the modulating field. Under these conditions the variation of χ'' is observed and makes an ideal visual record. Usually the shape shows the effect of transients mentioned earlier.

However a slightly different situation is observed when the amplitude of the modulating field is small and its frequency is in the audio frequency range. At resonance the output of the carrier frequency is decreased. Also the component of the audio frequency is zero. At points off the resonance the magnitude of the carrier frequency moves towards its value far from resonance.

However the amplitude of the audio frequency changes, such that **its** amplitude is proportional to the first derivative of the Absorption v.H. curve, i.e. the resonance line. See figure 4 page 20.

It is not necessary to modulate the field at all. In fact the same effect is obtained by modulating the carrier frequency. Any change in the external field causes the amplitude of the audio frequency output to change in the manner shown in figure 5 page 19. Such a system has been described by Knoebel and Hahn(10). A very comprehensive account is given in this paper of the construction of the above system.

In both previous systems the Alternating audio Frequency signal is fed into a phase detector where it is mixed with a 200 cps signal. The rectified output of this detector was the characteristic shape shown on figures 4 and 5 pages 20-19 **respectively.** The primary AC (audio frequency) voltage output may be of the order of 0.05 - 5 millivolts. This depends on the voltage across the resonant circuit which may be 10.0 to 0.1 volts.

The practical details of the latter system are discussed in the next section.

5. THE TRANSITRON N.M.R. STABILIZING SYSTEM(10)

Very full details of this system are to be found in the reference quoted above. Here only a brief discussion of the advantages and disadvantages of the system will be given.

One requirement of any nuclear magnetic resonance system is the provision of a backing system for course control. The N.M.R. systems are not effective outside the line width which is of the order of 0.3 - 0.6 gauss. It is to be expected however that the control within these limits provided the system is well designed should be to a precision of 5 parts in 10^6. This assumes a sensitivity of 1/20 of a line width.

This line width can only be obtained if the steady field is homogeneous throughout the sample ; a $\frac{dH}{dx}$ of 10 gauss cm^{-1} is probably tolerable.

As pointed out earlier, a region in which this condition is satisfied exists in the magnetic field. In fact in this particular case this region is sufficiently large not to seriously affect the size of the probe.

Rigid probes should be used such that the length between the oscillator, in this case the transitron unit, should be small. A reasonably large probe say 3/4" ($\sim$ 2.0 cm) overall diameter constructed from a minimum amount of dissipative insulators results in a high Q factor. Other factors which contribute to this desirable feature is that the sample should fit snugly into the R.F. coil. A satisfactory sample is made by dissolving 1 gram of $Fe\ (NO_3)_3\ 9\ H_2O$ crystals in 100 gram of water.

A long transmission line between the probe and oscillator reduces the Q factor so that it is desirable to put the transitron oscillator close to the probe. Unfortunately, one of the magnets is close to the PS magnet, and the periodic pulsed field can cause trouble in the oscillator circuit.

Thermal and radiation effects are not serious limitations on the method.

A detailed account of the Electronics appears in the reference cited at the beginning of the section [10].

6. THE THEORY OF THE HALL EFFECT

When a conductor is placed in a magnetic field, perpendicular to the direction of current flow, a potential is developed across the specimen in a direction perpendicular to both the current and the magnetic field. It is developed because the moving charges which constitute the current are forced to one side by virtue of their motions in the magnetic field. The charges accumulate on a face of the specimen until the electric field associated with this distribution is sufficient to cancel the effect of the magnetic force.

The Lorentz Force on a charge carrier is (21)

$$F = e \left[E_{em} + \frac{1}{C} V_n \wedge H_z \right]$$

At equilibrium

$$E_y = \frac{V_x H_z}{c} = \frac{J x H_z}{Nec}$$

where Ey is the field, V_x the velocity of the charge carrier of charge e, H_z is the field, N the number of carrier per CE and c is the velocity of light.

Hence the Hall Potential will be given by

$$U_{20} = C_h H_z$$

where C_h is a temperature dependent constant.

Thus any change in H_z should produce a charge in the potential U_{20}.

The driving current of the Hall Generator should be alternating to reduce the effects of galvanic potentials.

The equivalent circuit of the Hall Plate is shown in figure 6 page 21.

7. APPLICATION OF THE HALL PLATE TO FIELD STABILIZATION

As an illustrative example a representative Hall plate is considered. The Hall voltage U_{20} is 80×10^{-3} volts at a field of 10 kG and a driving current of $\sim$ 150 mA. The temperature coefficient $\frac{\Delta V}{V}$ is 0.04 %/$^{\circ}$ C. The Hall constant is thus 80×10^{-7} volts per gauss. This system must detect changes of 0.02 gauss in 2,000 gauss for a precision of 1 in 10^5.

Hence

$$\Delta V = C \Delta H = 80 \times 10^{-7} \times 2 \times 10^{-2}$$

$$= 160 \times 10^{-9} = 1.6 \times 10^{-7} \text{ volts.}$$

Fischer(11) considers such a system quite possible.

Any ambient temperature change should only cause about $\frac{1}{10}$ the variation in the Hall voltage which a change in the field produces i.e. $\frac{\Delta V}{V} < 10^{-6}$

$$10^{-6} \simeq 0.04 \times 10^{-2} \times \Delta t = 4 \times 10^{-4} \Delta t$$

i.e $\Delta t \sim \frac{1}{400} {}^{\circ}C$

This is clearly a difficult condition to satisfy.

A new Hall plate has been developed by Siemens with an improvement in the temperature coefficient by a factor ten so that a temperature control of $\frac{1}{40} {}^{\circ}C$ should be sufficient.

Braunersreuther(12) and Fischer(11) feel confident that such a system could be developed.

The temperature control can be effected by varying the driving current of the Hall plate. This work has been described in a paper by Umstätter(13). The most convenient method of utilizing the Hall effect is to feed the driving current through a standard resistor. The Hall voltage is suitably amplified and compared to the potential on the standard resistor. At the required field there is no difference between the potentials. However if the field changes an output is obtained which may be used to control the field.

A block diagram is given in figure 7 page 21.

8. COMPARISON OF THE SYSTEMS, PAST EXPERIENCE AND CONCLUSION

From sections 4, 5 and 7 it is apparent that the major limitations of the contending systems are limited control (N.M.R.) and temperature effects (H.P.). Recent advances in the development of the Hall plates have virtually solved this temperature problem. However there is no simple method of solving the former problem except of course by the inclusion of a current stabilizer.

Mechanically the Nuclear Magnetic Resonance technique is more complicated than the Hall method. However, when the temperature stabilizer is taken into account there is little difference between the two methods as regards electronic complexity. The changes in Hall voltage which have to be detected are extremely small $\sim 2 \times 10^{-7}$ volt. Consequently the electronics will be reasonably complicated.

Most past experience has been gained with the Nuclear Magnetic Resonance system - some of these systems have been manufactured commercially. Of these Braunersreuther found the Perkin Elmer system unsatisfactory. This was three years ago and it is possible that improvements have been effected. Also it is reported[14] that the AEG system, in a magnetic field of $\sim$ 15 kG failed to produce a trace. This was probably caused by the non uniformity of the field rather than any defective instrument.

A satisfactory trace[14] has been obtained by the Varian system. At present, a N.M.R. system is operating satisfactorily arc visual indicator on the inflector magnet - this is relevant because it shows that the field is sufficiently uniform for such a probe.

It is recommended that this system be retained as a visual display.

Most experience with Hall plates (the old type) has been gained in the stabilization of fields to a precision of 1 part in 10^4 .

Braunersreuther is confident that with the improved plate, this precision can be improved by a factor of ten.

It is the final conclusion of this report that the best and most convenient method for the present application is the Hall method.

APPENDIX I

There are two standpoints from which the phenomenon of Nuclear Magnetic Resonance may be described. The first approach, associated with the names of Bloembergen, Purcell and Pound[6], is essentially microscopic and argues from the Quantum Mechanical standpoint. The second, associated with Block[7] is essentially a macroscopic and classical theory.

In the usual experimental situation a small Radio Frequency Field described by

$$H_r = 2\,H_r \cos \omega_r t = H_r \cos (\omega_r\, t) + H_r \cos (-\omega_r t) \qquad f_r = 2\pi\omega_r$$

is superimposed at right angles to a steady field H_x.

In both approaches it is assumed that the nuclear magnetic moment μ, associated with a given nucleus of angular momentum quantum number I, does not suffer strong interactions with the other nuclei which form the lattice.

This implies, in the quantum mechanical standpoint, that it is meaningful to suppose that the nuclear magnets will occupy one of the (2I + 1) possible states relative to the direction of the applied (steady) field H_x. Such states are known as m substates and a purely classical argument involving the concept of a dipole of moment μ orientated with respect to the field H_x shows that E, the energy difference between successive m substates is given by

$$E = h\,V_o = \frac{\mu\, H_x}{I} = g_n \mu_o\, H_x \qquad (1)$$

where h is Planck's constant, V_o is the resonant frequency, μ is the maximum observable nuclear magnetic moment, μ_o is the Nuclear Magneton and g_n the nuclear g factor.

In the absence of the radiofrequency, the nuclear magnets are in equilibrium at the lattice temperature T_L. The population of an ensemble of a particular m substate must be given by the Maxwell Boltzmann distribution function i.e.

$$N\,(m) = \frac{N}{(2I + 1)} \;\exp\left(-\frac{E_m}{kT}\right)^{*} \qquad (2)$$

where N is the total number of nuclei and E_m the potential energy of the substate considered.

* The factor (2I+1) is introduced so that $\sum N(m)$ gives the correct total population

At this equilibrium temperature T_L n_o is defined as the excess number of nuclei in the m^{th} state in comparison with the $(m + 1)^{th}$ state, i.e.

$$n_o = N(m) - N(m+1) \quad \frac{N \quad H_x}{(2I+1)\ KT_2 I} \tag{3}$$

by approximating the exponentialin (2)

The application of the R.F. field causes upward and downward transitions between the m sublevels. Since there are more nuclei in the lower sublevels, and since the transition probability P, for downward and upward transitions are the same (at these frequencies), a net absorption of energy results. The population of the sublevels change and acquire a distribution corresponding to some higher temperature T_s - see (2). T_s is now called the spin temperature.

However this process cannot proceed indefinitely because a temperature difference exists between the spin and lattice systems.

There is a tendency for the temperatures T_s and T_L to come to some intermediate value. In fact to all intents and purposes this is the lattice temperature T_L because the specific heat of the lattice far exceeds that of the spin system. T_1 is defined as the relaxation time (or decay time) of the process by which the spin system returns to the equilibrium condition.

At equilibrium during the application of the R.F. field, n_s is the difference between the population of the m^{th} and $(m + 1)^{th}$ sublevel. For n_s we have that :

$$n_s = - \frac{N \mu \ H_x}{(2I+1)\ KT_s} \tag{4}$$

Now the rate of change of this excess number, say n where $n_o < n_s < n$ is determined by the relaxation process described above, and the change produced by the radiofrequency, i.e.

$$\frac{dn}{dt} = \frac{n_o - n}{T_1} - 2n\ P \tag{5}$$

At equilibrium $\frac{dn}{dt}$ must be zero. Hence from (5)

$$\frac{n_s}{n_o} = \frac{1}{1 + 2\,PT_1} = Z \qquad (6)$$

where n_s is the excess at the steady state in the presence of the RF field of frequency V_r and pulsatance and ω_r the same in the absence of the RF field. P is given Quantum mechanically by the expression

$$P_m \rightarrow m' = \frac{1}{2}\gamma^2 H_r^{\,2} /\langle m/I/m' \rangle/^2 g(V_r) = \frac{1}{2} g_n^{\,2}\mu_o^{\,2} H_r^{\,2} /\langle m/I/m \rangle/^2 g(V_r)$$

$$= \frac{1}{2}\, g_n^{\,2}\mu_o^{\,2} H_r^{\,2}\ \frac{1}{2}\,(I+m)\ (I-m+1)\ g(V_r) \qquad (7)$$

where $\langle m/I/m \rangle$ is an overlap integral, H_r is the radiofrequency field, $\gamma = g_n \mu_o$ is the gyromagnetic ratio and $g\,(V_r)$ is a function describing the shape of the frequency spectrum such that

$$\int_{-\infty}^{+\infty} g(V_r)\,dV_r = 1$$

$$\frac{n_s}{n_o} = \frac{1}{I + \frac{1}{4}\ g_n^{\,2}\mu_o^{\,2} H_y^{\,2}\ (I+m)\ (I-m+1)\ g(V)\ T_1} \qquad (8)$$

By inspection it is apparent that $n_s = n_o$ if H_r is small and T_1 is less than one (i.e. $Z = 1$ and consequently from (2) (4) and (6) $T_L = T_s$)

It is now possible to evaluate the power absorbed by the nuclear system. By equation (4) the excess number of nuclei in the lower of two states, m and m + 1 is

$$N(m) - N(m-1) = \frac{NMH_x}{(2I+1)\ IKT_L} = \frac{NhV_o}{(2I+1)\ KT_s} \qquad (9)$$

$(T_L = T_s)$

where V_o is the resonant frequency.

Each upward transition requires the absorption of a quantum h_v of energy and since the probability of such a transition is $P_{m \to m_1}$ the Power absorbed between these two levels is

$$= \left\{ \frac{Nh^2 V_o V}{(2I+1)KT_L} \right\} P_{m \to m_1} \qquad (10)$$

Hence the total power is obtained by summing (10) over all levels, this yields

$$A = \frac{1}{4} \gamma^2 H_r^2 g(V) \left\{ \frac{Nh^2 V_o V}{(2I+1) KT_L} \right\} \frac{2}{3} I (I+1) 2(I+1) \qquad (11)$$

From the circuit concept of Power and the assumption of a complex susceptibility $\lambda = \lambda' + i \lambda''$ it can be easily shown that

$$A = 4\pi V \lambda'' H_r$$

$$\lambda'' = \frac{\pi}{2} \left\{ \frac{N\mu^2 (I+1)}{3KT_L I} \right\} V_o g(V) \qquad (12)$$

$$= \frac{\pi}{2} \lambda_o (T_L) V_o g(V)$$

since $\lambda_o(T_L)$ the static susceptibility given by $\lambda_o (T_L) = \frac{N\mu^2(I+1)}{3KT_L}$

The classical theory of Bloch also leads to an expression for λ''. In this theory the rate of change of the components M_x , M_y and M_z are considered under the influence of the radiofrequency field, the relaxation effect and also a third interaction known as the spin-spin interaction. This interaction (which is closely related to the external inhomogeneity of the steady field) is caused by the presence of other dipoles surrounding the reference dipole. These change the value of the local field ($\Delta H \sim 5$ gauss). By comparison of the result of Bloch and equation (12)

$$g(V_r) = \frac{2 T_2}{1 + 4 \pi^2 (V_o - V) T_2^2} \qquad (13)$$

where T_2 is the relaxation time associated with the spin-spin and field inhomogeneity interactions.

Hence (see figures 2, 3 page 19 for plots of λ'' and λ' Vω)

$$\lambda'' = \frac{1}{2}\lambda_o \omega_o T_2 \frac{1}{1 + (\omega_o - \omega)^2 T_2^2} \qquad (14)$$

and by the Kronig-Corter relation

$$\lambda' = \frac{1}{2}\lambda_o \omega_o T_2 \frac{(\omega_o - \omega) T_2}{1 + (\omega_o - \omega)^2 T_2^2} \qquad (15)$$

From these fundamental equations it is possible to determine the changes in inductance of a coil containing a sample of nuclear magnets and also the E.M.F. induced in a coil placed at right angles to the RF coil.

At frequency ranges far from the resonant condition the flux linkage through the coil is

$$(1 + \xi 4\pi \lambda_o) H_r \; nA \; i \qquad (16)$$

where n and A are respectively the number of turns and Area of the coil, and i is the radio frequency current, ξ is the filling factor.

This unit current (10) is the inductance by definition

$$L = (1 + \xi 4\pi \lambda_o) H_r \; nA \; i \qquad (17)$$

At regions close to that of resonance

$$L' = \left\{ 1 + \xi 4\pi (\lambda' + i\lambda'') \right\} nA \qquad (18)$$

whence $\Delta L = 4\pi \left\{ L \left\{ \lambda' + i\lambda'' \right\} \right.$

If this coil forms part of a tuned circuit of resonant impedance $Z = \frac{\omega^2 L^2}{R} = \omega L Q$ where L and R are respectively the inductance and series resistance of the coil, then the change in Z for a given change in L, ΔL, is $\omega Q \Delta L$

Hence the change of potential across such a tuned circuit is given by

$$\Delta V = J_r \Delta Z = J_r \omega \Delta L Q \qquad (19)$$

$$= J_r \; 4 \pi \omega \; L Q \zeta \; (+\chi' \sin \omega t - \chi'' \cos \omega t) \qquad (20)$$

$$LJ_o = 2H_1 A \quad = \gamma \pi H_1 \; \omega A Q \; (+\chi' \sin \omega t - \chi'' \cos \omega t)$$

A very similar expression may be obtained for the signal induced in a coil placed around the sample perpendicularly to the exciting coil. Such a coil detects the changes in the induction through the sample. The only difference is that a phase difference of $\frac{1}{2}\pi$ exists between the results.

9. ACKNOWLEDGMENTS

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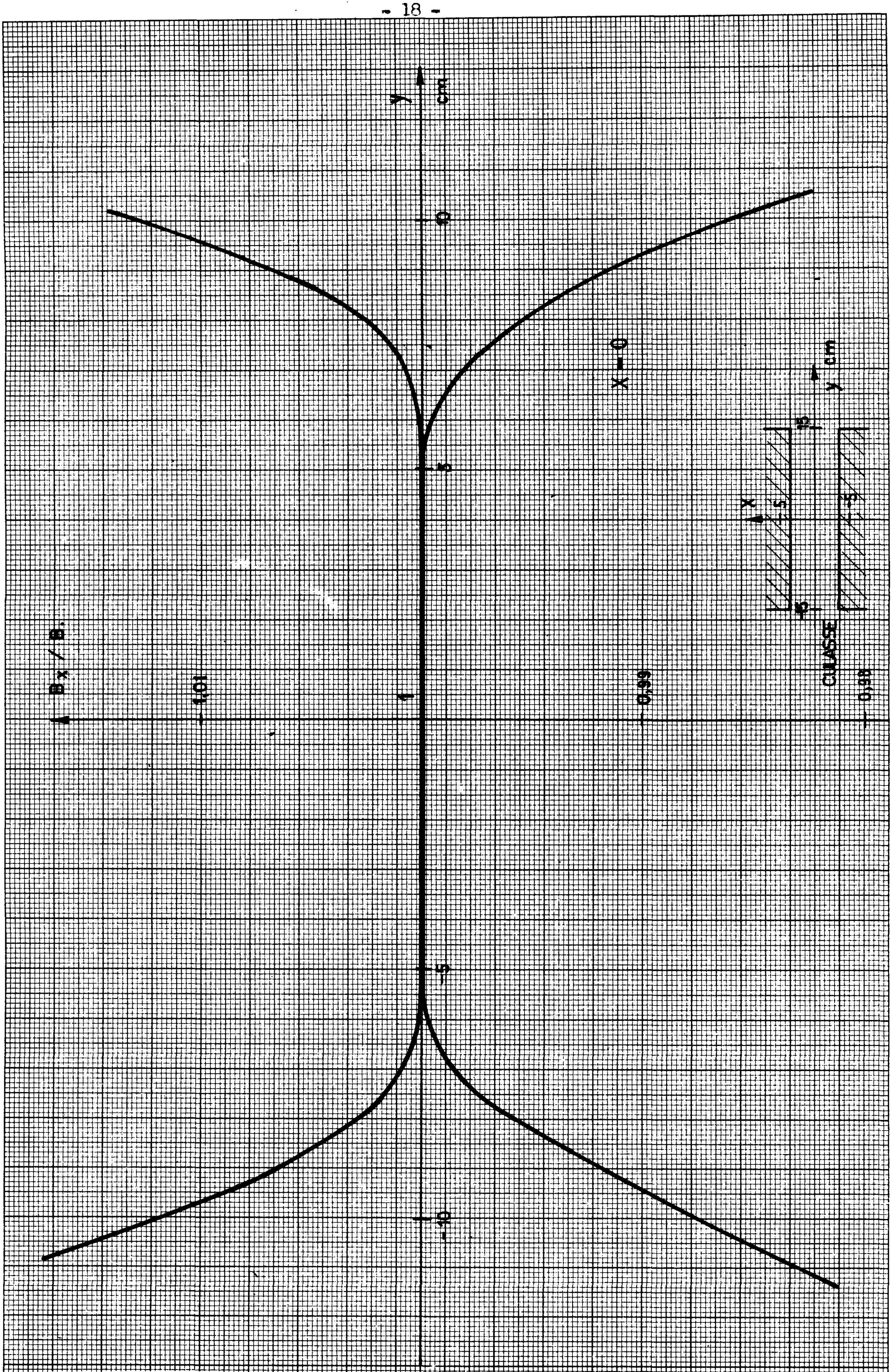

FIG 1 HOMOGENEITY OF MAGNETIC FIELD (SEPTIER CERN 59-1)

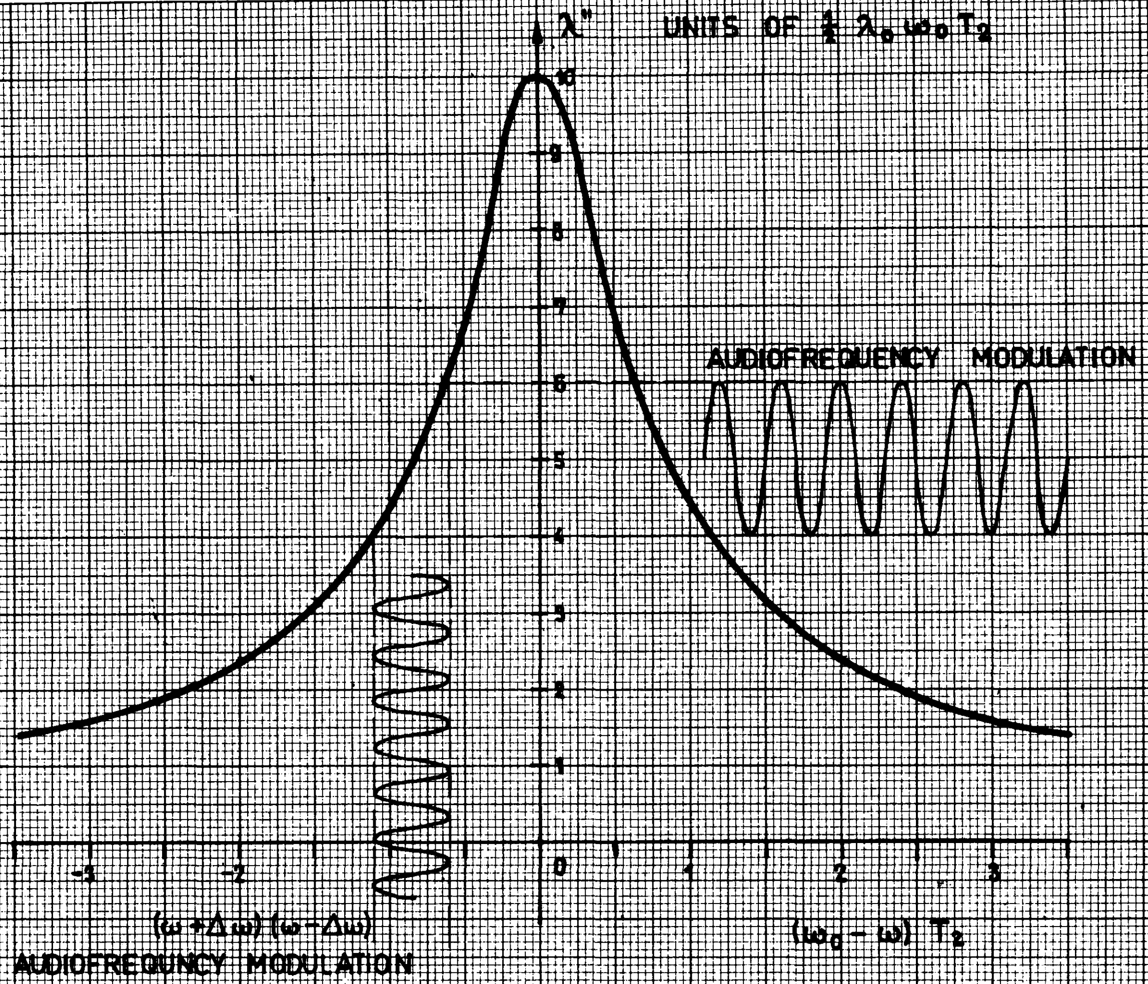

FIG.3 AND 5 VARIATION OF λ" WITH ω (ω_0 IS THE RESONANT ANGULAR FREQ.)

AND EFFECT OF SMALL MODULATION OF ω (FOR DC OUTPUT SEE FIG. 4).

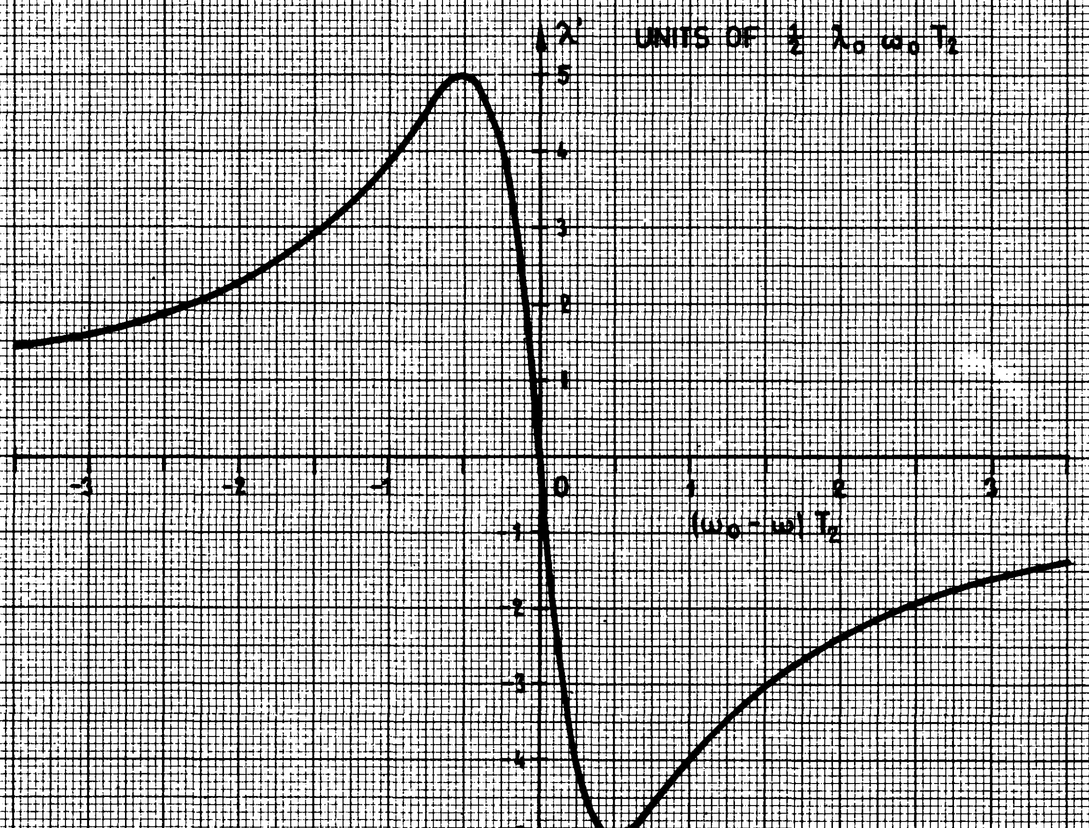

FIG. 2 VARIATION OF λ' WITH ω (ω_0 IS THE RESONANT ANGULAR FREQ.).

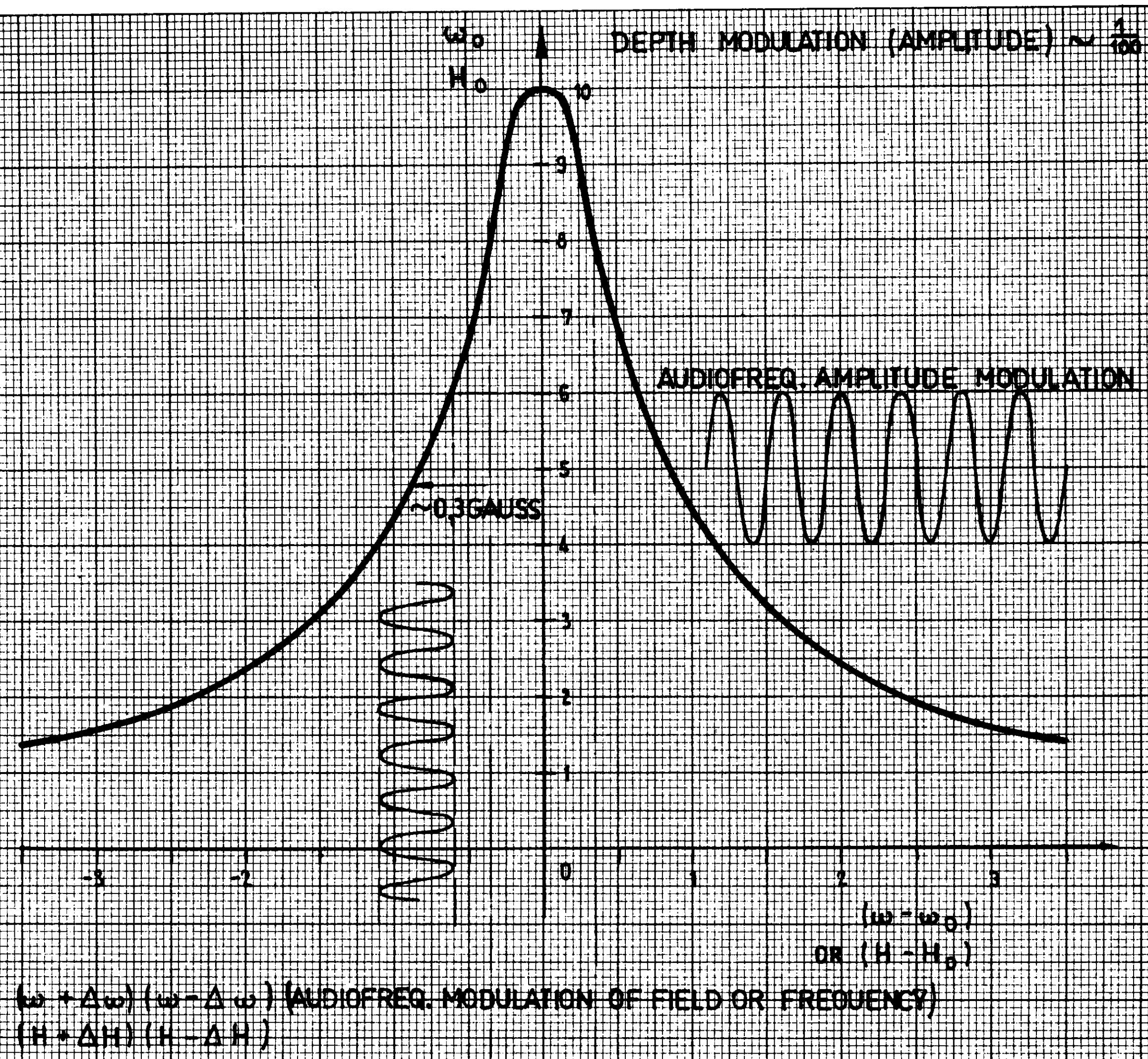

FIG. 4 EFFECT OF FIELD OR FREQUENCY MODULATION

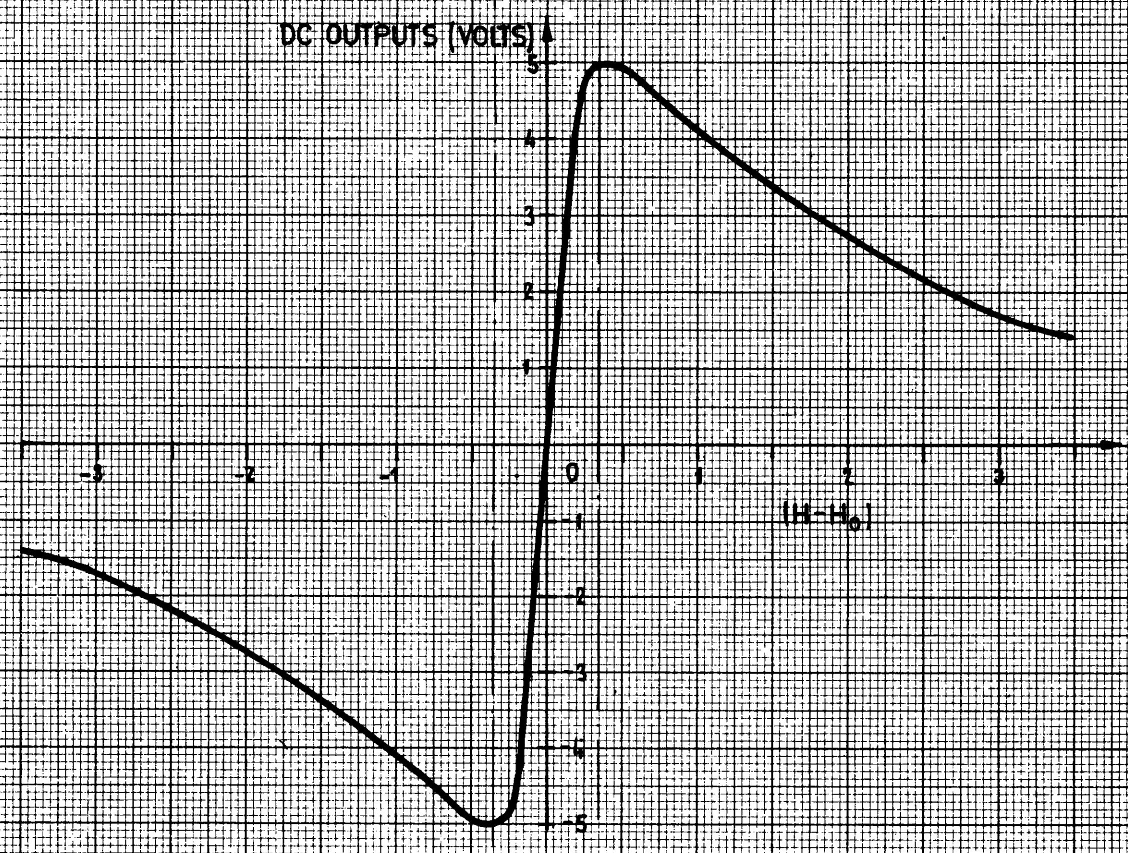

FIG. 4 DC OUTPUT FROM RECTIFIED AND AMPLIFIED AUDIOFREOUENCY OUTPUT

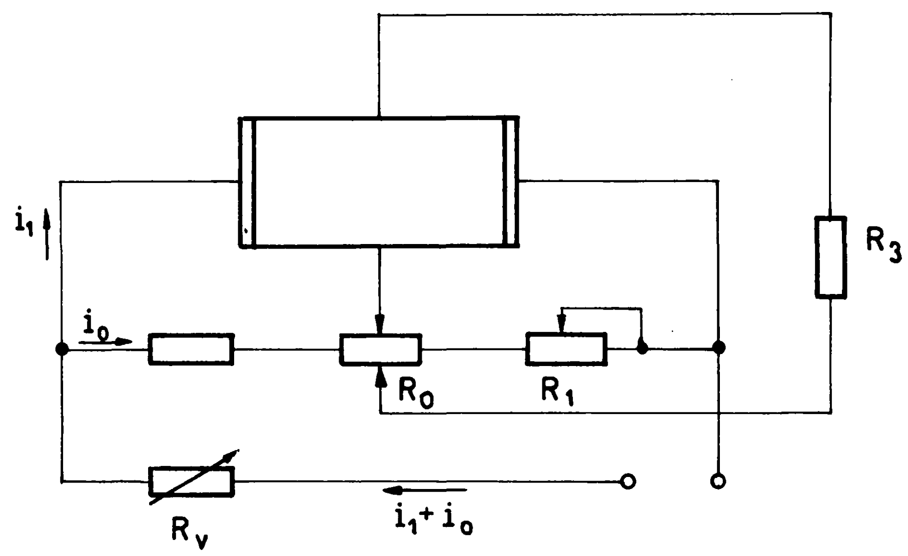

FIG.6 CIRCUIT OF HALL GENERATOR (INCLUDING COMPENSATING CIRCUIT)

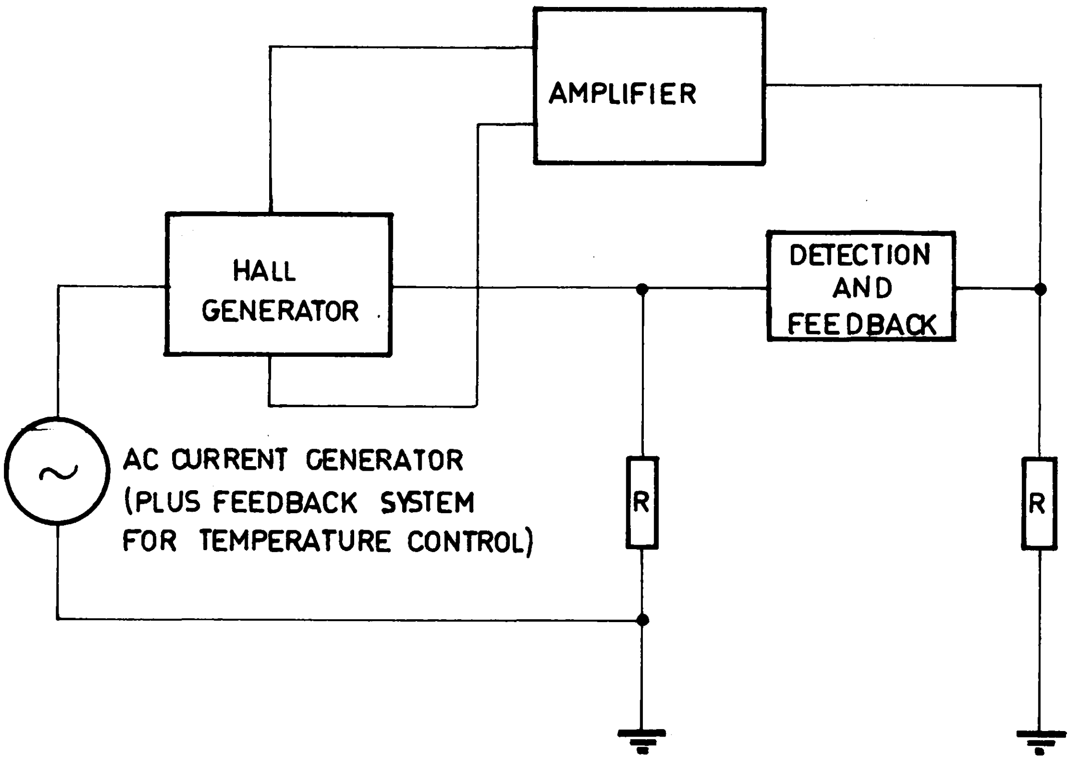

FIG. 7 BLOCK DIAGRAM OF HALL PROBE SYSTEM