ZGS - Machine Experiment Report
Investigation of Beam Depolarization Due to an Imperfection Resonance

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ABSTRACT

We have investigated the depolarization of the ZGS polarized beam during passage through the γ G = 6 and γ G = 7 imperfection resonances. The resonances were enhanced by reducing the crossing speed by up to a factor of 400 compared to normal operation and by producing an orbit distortion using horizontal magnetic fields. Each resonance was identified, and then could be either compensated by appropriate fields or accentuated to give almost complete spin flip.

INTRODUCTION

Imperfection depolarizing resonances have been a somewhat controversial subject compared to intrinsic resonances which have been observed at the ZGS to be in good agreement with theory. It seems that imperfection resonances had never been detected. Some theoretical models[2] indicate that they do not exist at all, whereas others suggest that they may be very serious in a strong focusing synchrotron. The experiments described here were undertaken to see if such resonances existed, to study their properties and to see if they could be successfully jumped using the proposed techniques.

EXPERIMENTAL PROCEDURE

The ZGS magnetic field cycle used is shown in Figure 1. A reduced $\dot{B}$ window was centered at the expected position of the $\gamma G = 6$ or the $\gamma G = 7$ resonance. Most measurements were done at $\gamma G = 6$ because this resonance is most isolated from neighboring intrinsic resonances as shown in Figure 2. Vertical orbit bumps were produced by pulsing poleface winding magnets (PFW) in octant III of the ZGS. The windings on the upper and lower pole were pulsed with opposite polarities to produce a strong radial field. The resulting orbit distortion is shown in Figure 3. This distortion was measured using a scraper target which detected the beam edge. The amplitude of the orbit distortion (1/2 of the peak to peak value) is shown in Figure 4 as a function of the PFW current.

The PFW excitation current was a bump or a square wave pulse whose length could be varied from about 30 msec to 1 sec. Measurements of the polarization were taken for different crossing speeds, pulse lengths, pulse positions and pulse magnitudes. The polarization was measured by the "high rate" polarimeter constructed at CERN after extracting the beam at 4 GeV/c using targeted extraction. The pulsed quadrupoles were used as in normal operation to pass each intrinsic resonance occurring between injection and 4 GeV/c.

RESULTS

The polarization as a function of horizontal magnetic field is shown in Figures 5, 6, and 7 for $\gamma G = 6$ and Figure 8 for $\gamma G = 7$. The $\dot{B}$ - window with $\dot{B}$ = 200 G/sec has a length ΔB = 100 G and thus a length $\Delta t \approx 0.5$ sec; similarly when $\dot{B}$ = 100 G/sec then ΔB = 100 G, and when $\dot{B} \approx 50$ G/sec then $\Delta B \approx 40$ G.

Note that a PFW-current of ~ 2 A is required to make the 4 GeV/c polarization equal to the injection polarization. This indicates that the 2 A exactly compensated the 6th harmonic of the ZGS orbit distortions which happened to have their maximum close to octant III allowing compensation with one PFW. For larger currents the polarization decreases and eventually changes sign. The degree of spin flip depends on magnitude and length of the magnetic pulse and on the value of $\dot{B}$.

QUALITATIVE EXPLANATION

Neglecting energy oscillation and neighboring resonances, the asymptotic polarization after crossing a constant strength resonance with constant speed is given by Froissart and Storas formula[4)]

$$P = P_o \left(2 e^{-D/2} - 1\right)$$

where

$$D \approx \frac{\pi^2}{2} \left[\frac{1 + \gamma G}{\sqrt{G\dot{\gamma}}\; T_{rev}} \rho_o \left| z''(s) \right|_k \right]^2 \qquad (1)$$

The quantity $|Z''(s)|_k$ is the k-th Fourier amplitude of $Z''(s)$ in the precession plane θ where $Z''(s)$ is defined by

$$Z''(s) = \frac{d^2 Z}{d s^2} \approx \frac{B_r}{p/e} . \qquad (2)$$

The quantity s is the azimuthal position along the beam orbit which is related to the precession angle $\int \gamma G \, d\theta$ around the vertical field B_Z by

$$d\theta = \frac{ds}{\rho(s)} = \frac{B_Z(s)}{p/e} \; ds. \qquad (3)$$

Before reaching its asymptotic value the polarization can oscillate considerably. For a strong resonance or slow crossing, the extreme excursions for individual protons are very large and tend towards $\pm P_o$. Due to energy spread, the phases of these oscillation get smeared out and the average polarization of the entire beam behaves somewhat more smoothly. Further due to energy oscillation of the form

$$\gamma = \dot{\gamma} t + \Delta\gamma \cos(\omega_s t)$$

protons may cross each resonance several times. Qualitatively this becomes important when $\Delta\gamma\, \omega_s \gtrsim \dot{\gamma}$.

Finally if one rapidly reduces the strength of a resonance (trailing edge of the PFW bump) before the polarization has settled near the asymptotic value then the polarization has a tendency of getting frozen at the instantaneous value attained during the fluctuations at the switch point. These qualitative elements may serve to explain the shape of the polarization vs. magnetic bump current curves. With the short magnetic bump, the excitation is changed and hence P frozen in a regime where it tends to oscillate, whereas with the long bump window (Figure 6) repetitive crossing might be important.

Comparison between Figures 5 and 8 suggests that the $\gamma G = 7$ resonance is about 10 times stronger than the $\gamma G = 6$ for the same PFW bump. This is probably because our pulsed orbit distortion has a strong first harmonic (harmonic closest to ν) whereas the focussing fields have an eight-fold periodicity. The combination of the two terms $\cos 8\theta$ and $\cos\theta$ leads to strong 7th and 9th harmonics. (See Appendix).

Without a PFW-pulse, the depolarization due to $\gamma G = 7$ is relatively weak which suggests that there is normally little 7th harmonic perturbation in the machine.

A quantitative comparison of our data with theory will require a very detailed analysis of the ZGS. However, the measured depolarization agrees within a factor of 2 with simple prediction of the theory. (See Appendix).

ACKNOWLEDGEMENTS

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2) Ernst, V., Nuclear Inst. Meth. 60, 52 (1968).

3) PS Machine Studies Team: The Possibility of Acceleration of Polarized Particles in the CERN-PS Proceedings of Summer Study on High Energy Physics with Polarized Beams. Argonne, July, 1974. J. B. Roberts, Editor.

4) Froissart, M. et., Stora, R.: Depolarization d'un Faisceau de Protons Polarises dans un Synchrotron, Nucl. Inst. Meth. 7, 297 (1960).

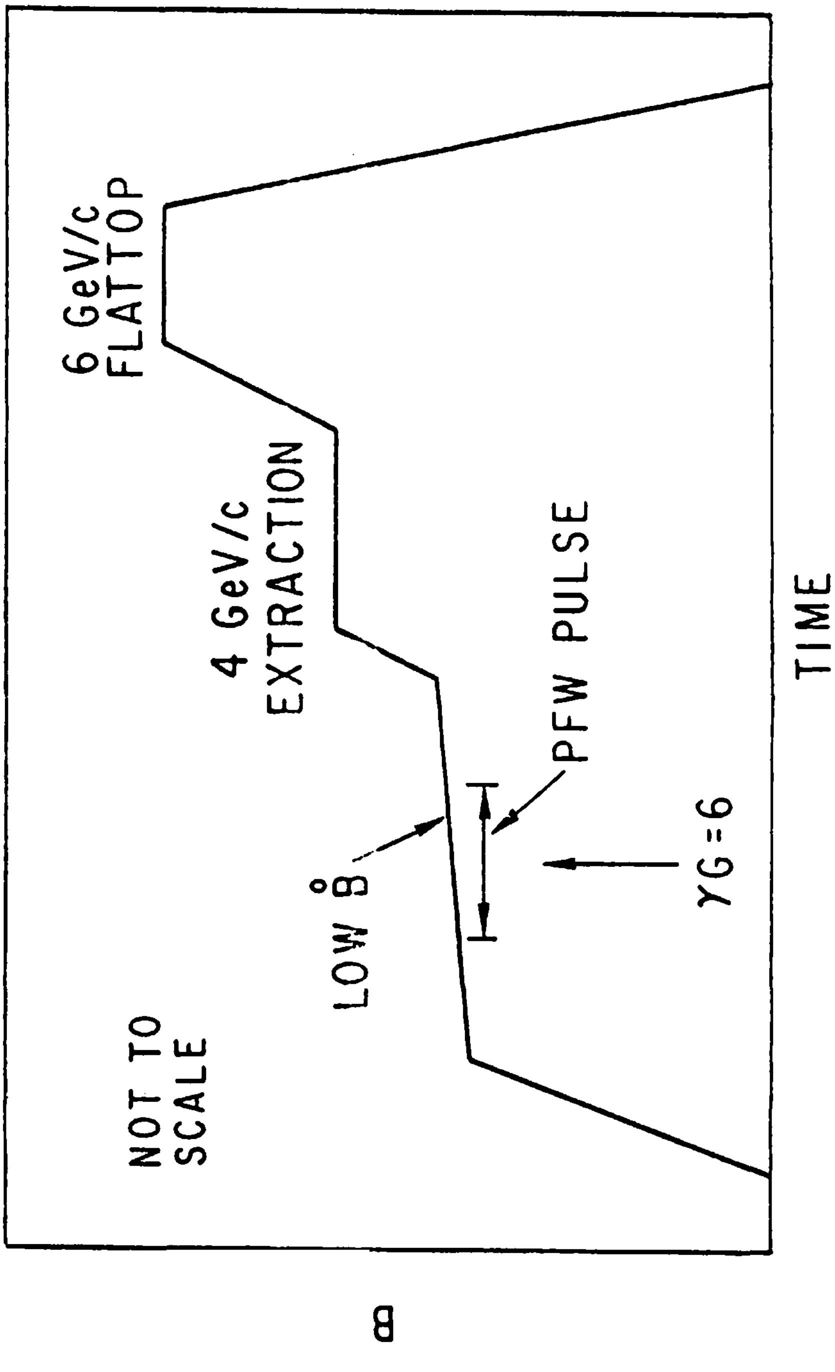

6 GeV/c
FLATTOP
4 GeV/c
EXTRACTION
PFW PULSE
LOW $\dot{B}$
$\gamma G=6$
NOT TO
SCALE
TIME
B

FIG. 1

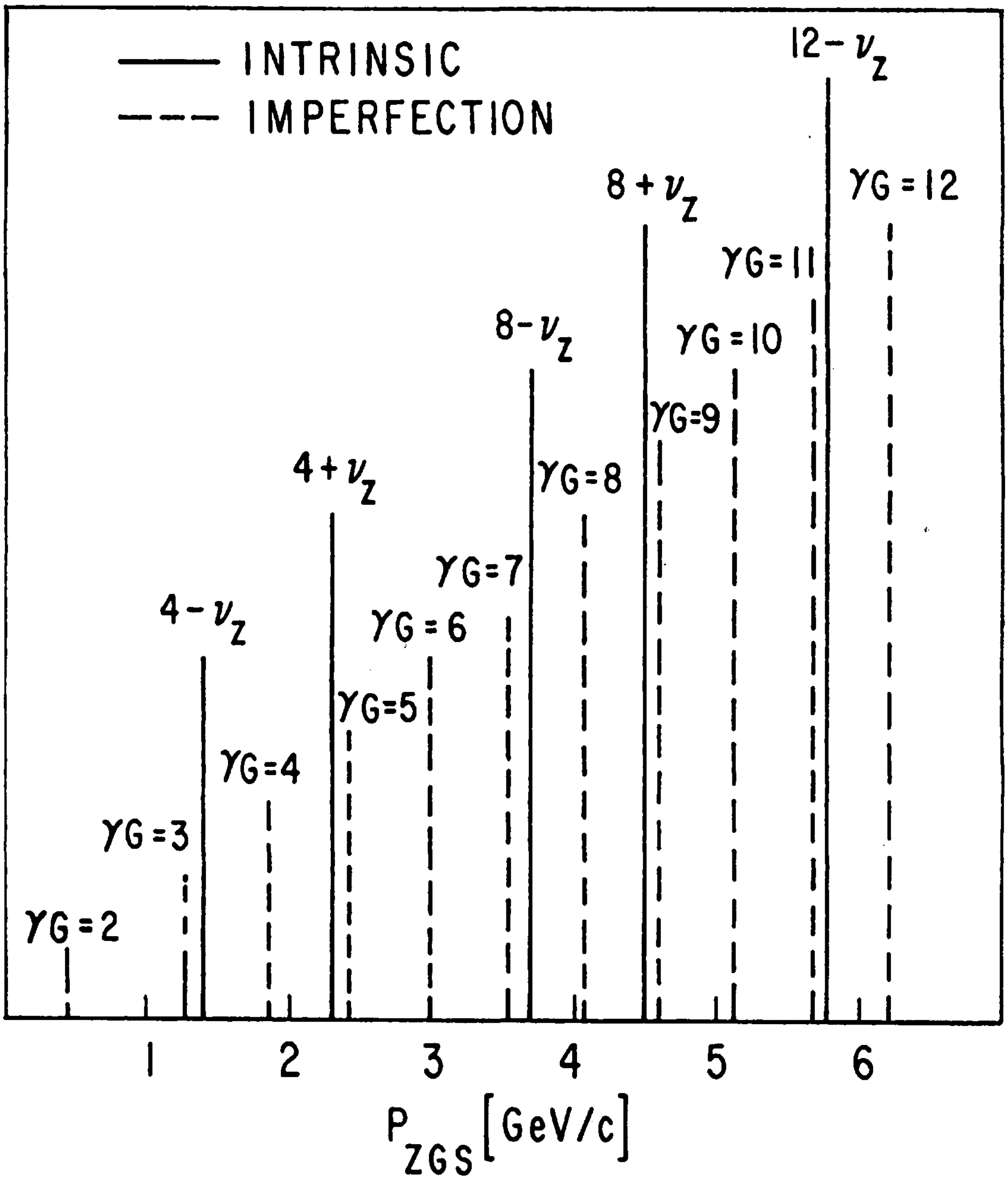

INTRINSIC
IMPERFECTION
12- ν_Z
γG = 12
8 + ν_Z
γG = 11
8- ν_Z
γG = 10
γG=9
4 + ν_Z
γG = 8
γG = 7
4 - ν_Z
γG = 6
γG=5
γG = 4
γG = 3
γG = 2
1
2
3
4
5
6
P_ZGS [GeV/c]

FIG. 2

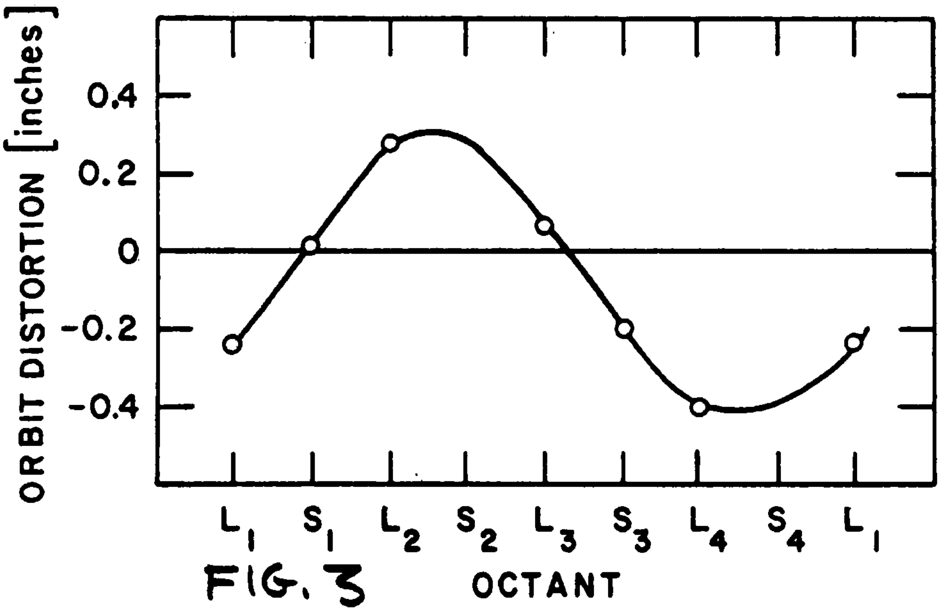

ORBIT DISTORTION [inches]
0.4
0.2
0
-0.2
-0.4
L1 S1 L2 S2 L3 S3 L4 S4 L1
OCTANT

FIG. 3

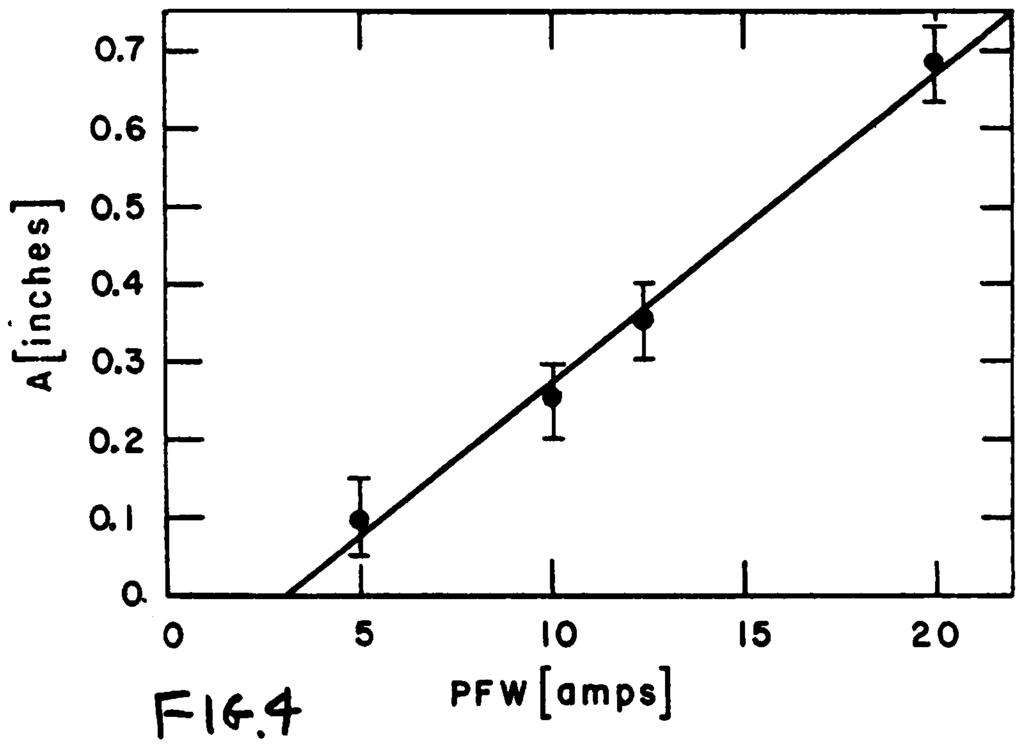
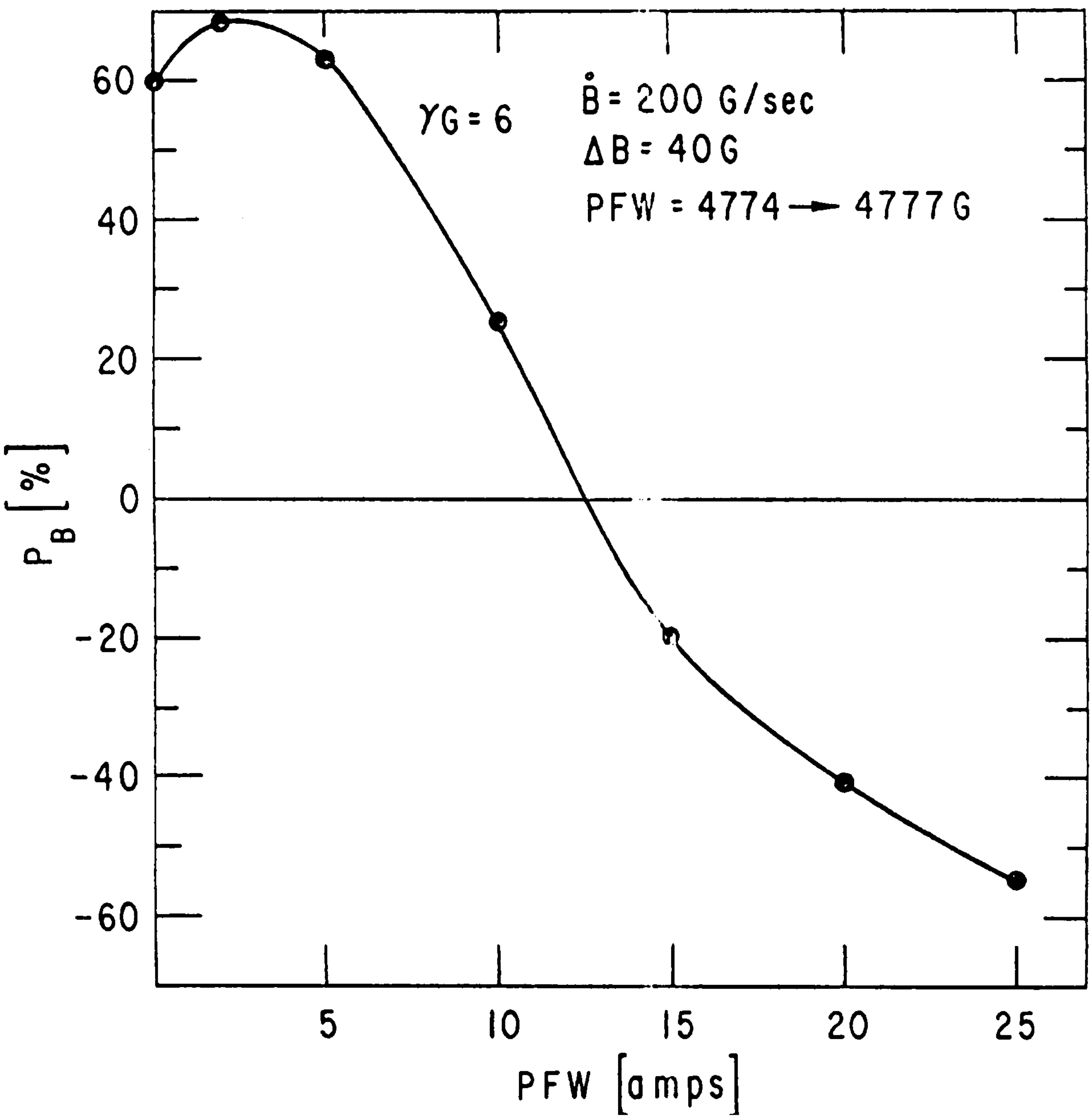
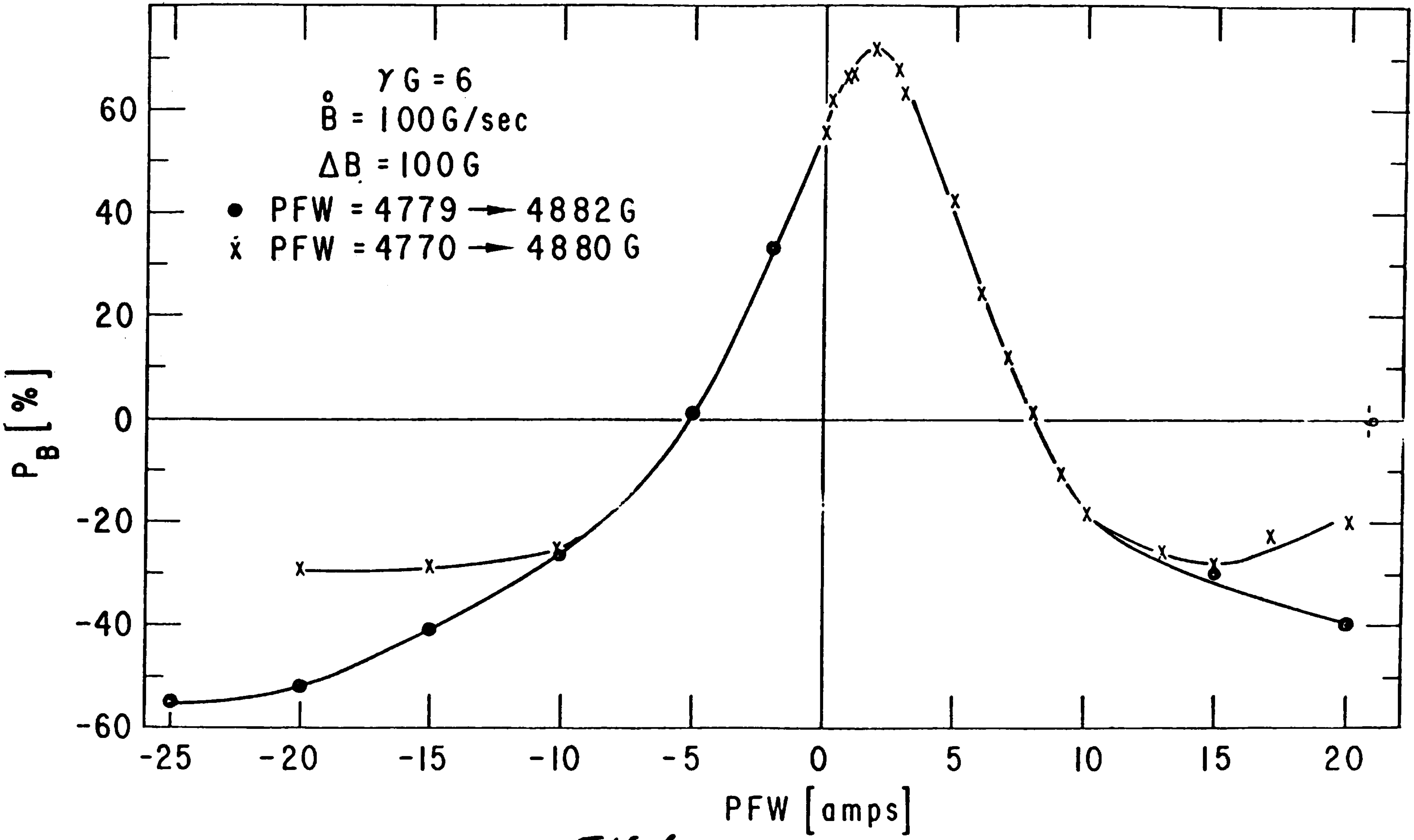

A [inches]
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.
0 5 10 15 20
PFW [amps]

FIG. 4



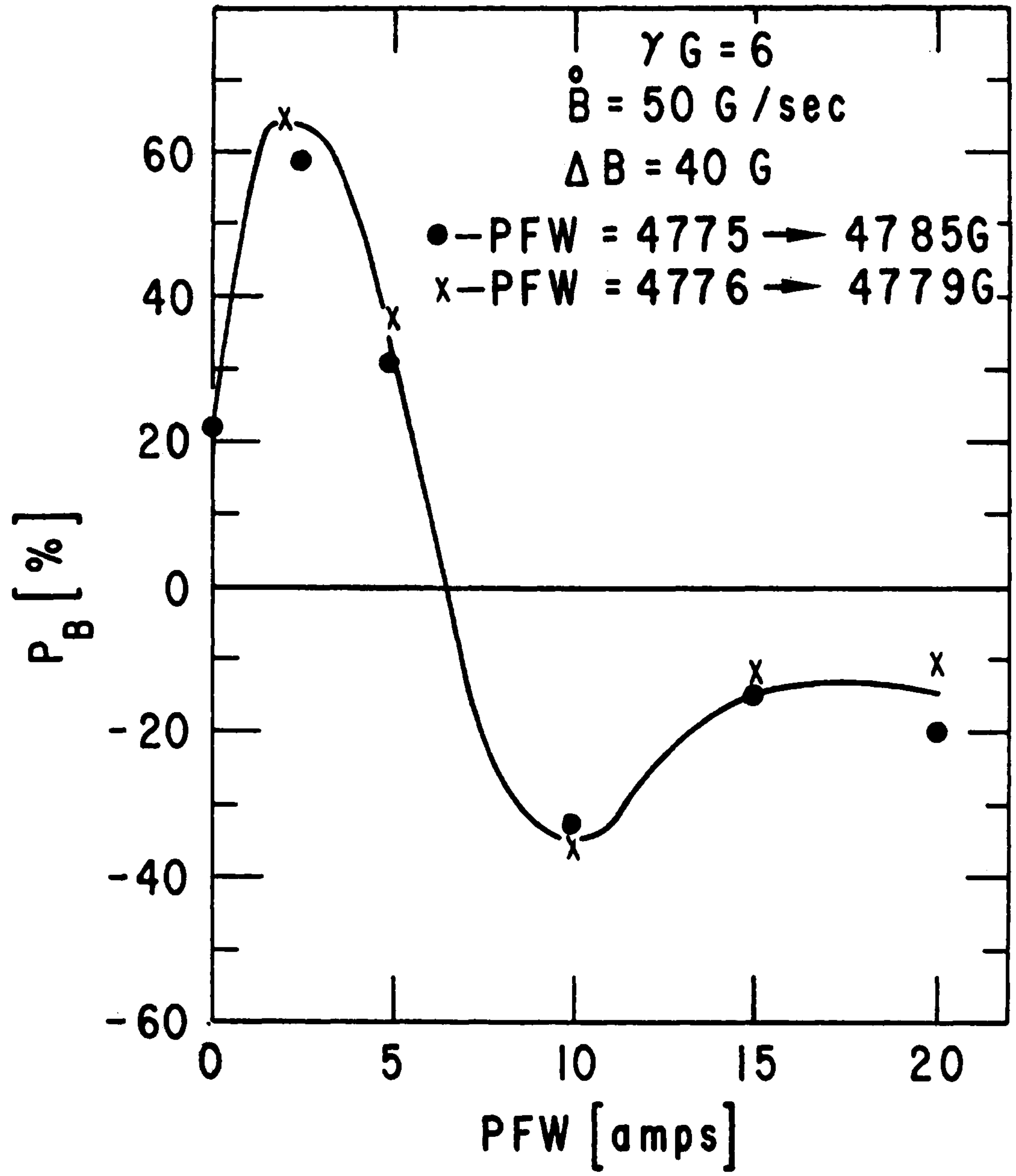
60
40
20
0
-20
-40
-60
P_B [%]
$\gamma G = 6$
$\dot{B}$ = 200 G/sec
ΔB = 40 G
PFW = 4774 → 4777 G
5
10
15
20
25
PFW [amps]

FIG. 5



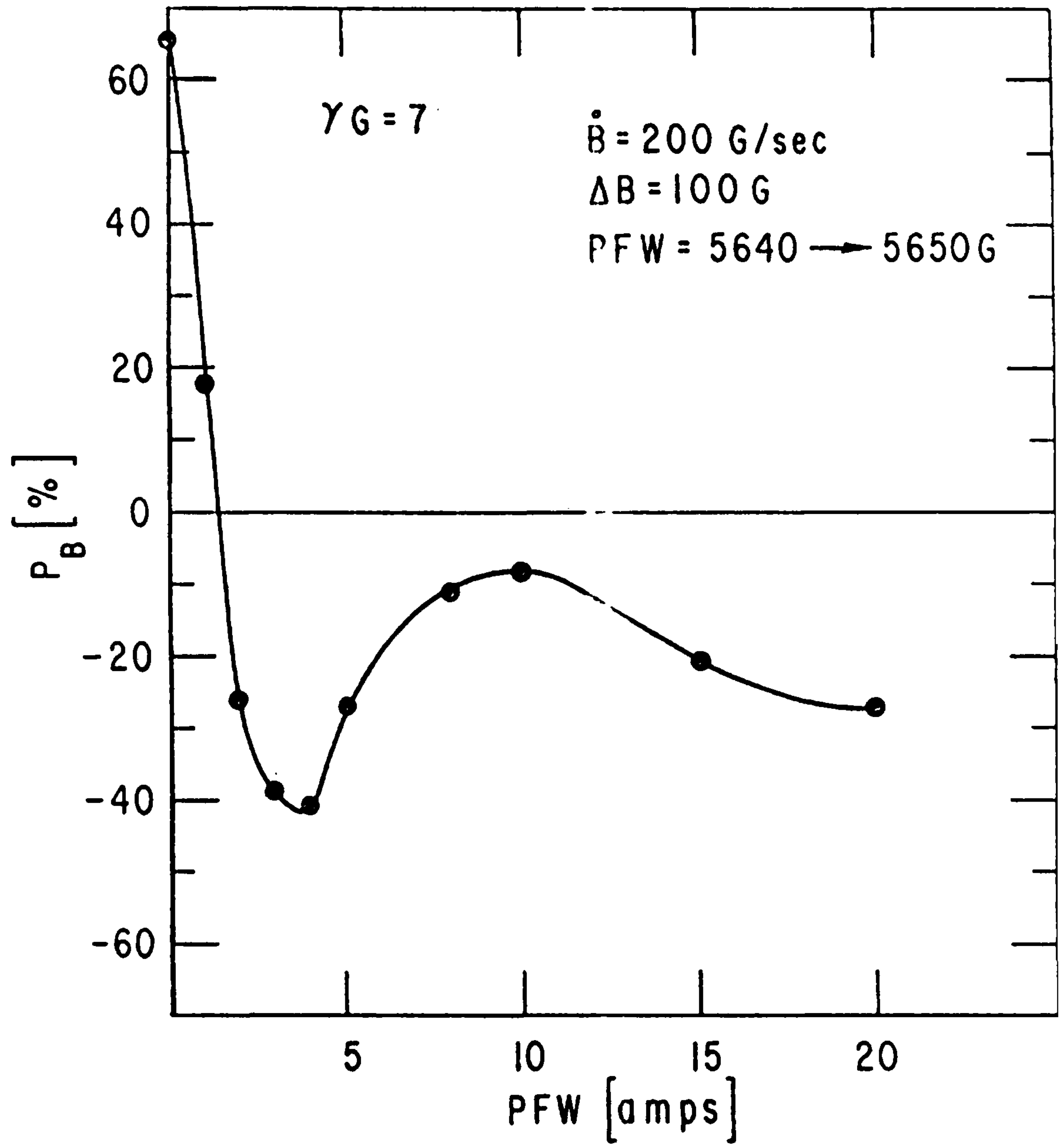
γ G = 6
B̊ = 100 G/sec
ΔB = 100 G
● PFW = 4779 → 4882 G
x PFW = 4770 → 4880 G
60
40
20
0
-20
-40
-60
P_B [%]
-25
-20
-15
-10
-5
0
5
10
15
20
PFW [amps]

F16.6



γG = 6
B̊ = 50 G/sec
ΔB = 40 G
●–PFW = 4775 → 4785G
x–PFW = 4776 → 4779G
60
40
20
0
-20
-40
-60
P_B [%]
0
5
10
15
20
PFW [amps]

FIG.7



$\gamma G = 7$
$\dot{B} = 200$ G/sec
$\Delta B = 100$ G
PFW = 5640 ⟶ 5650G
60
40
20
0
-20
-40
-60
P_B [%]
5
10
15
20
PFW [amps]

FIG. 8

Appendix

Computed and measured resonance strength

To evaluate the depolarization at $\gamma G = k$ from Froissart and Stora's formula (1) we need the "Fourier coefficients",

$$\left| \frac{1}{\pi B_o \rho_o} \int_o^{2\pi R} B_r \, e^{-ik\theta} \, ds \right| \tag{A1}$$

which are to good approximation given by

$$\left| \frac{1}{\pi} \int \frac{d^2 z}{ds^2} \, e^{-ik\theta} \, \rho_o \, d\theta \right| \equiv \rho_o \, Z_k'' . \tag{A2}$$

where we assume that $d^2z/d_s^2 = \Sigma z_k'' \cos(k\theta - \theta_k)$. Here $d\theta$ is given by equ (3) p. 3 above and we have assumed that radial fields B_r are in -or close to the bending magnets where $ds = \rho_o \, d\theta$.

The radial field B_r experienced by the particles has contributions from the perturbing field (the PFW bump and machine imperfections in our case) and from the focusing field of the ideal machine. Focusing fields enter because the perturbed orbit is no longer in the median plane. Both contributions are adequately taken into account by inserting the exact vertical orbit distortions $Z(s)$ into (A2).

A simple approximation is to replace the precession angle $(1 + \gamma G)\, d\theta$ by the geometrical angle $(1+\gamma G)\, ds/R$. This is a reasonable approximation for most harmonics especially in a combined function machine like the PS where the focusing is distributed over a fair fraction of the circumference. However, in the ZGS, where focusing is concentrated at the magnet edges, this approach becomes poor for some of the harmonics like e. g. $k = 7$.

We therefore return to (A1) and use an improved approximation. We replace the wedge focusing by short quadrupoles in the center of the straight sections which for simplicity we take to have equal strength (in reality focusing is slightly different in the long and in the short straight sections of the ZGS). We adjust the lenses to give the correct tune

$$\nu_z^2 \approx - \frac{R}{B_o \rho_o} \int_o^{2\pi R} \frac{\partial B_r}{\partial z} \frac{ds}{2\pi}$$

and obtain (taking for convenience $s = o$ in the center of octant 3):

$$- \frac{\partial B_r}{\partial z} = \frac{B_o \rho_o}{R^2} \nu^2 \frac{\pi}{4} \sum_{\ell = 1}^{8} \delta (S/R + \frac{\pi}{8} - \ell \frac{\pi}{4}) . \qquad \text{(A3)}$$

Next we approximate the PFW field by a radial field B_{PFW} which is constant in the magnet of octant 3 and zero outside. Actually we shall find best agreement with measured data if we assume that the bump extends in azimuth over only ≈ 95% of the magnet (or alternatively for a B_r which falls off towards the magnet edges). Expanding in the θ - plane we have

$$B_{PFW} = \begin{Bmatrix} \Delta B & - \theta_M \leq \theta < \theta_M \\ o & \theta_M < | \theta | \end{Bmatrix} = \Delta B \sum_{k = o}^{\infty} b_k \cos k \theta \qquad \text{(A4)}$$

where $b_k = \frac{2}{\pi k} \sin k \theta_M$, $b_o = \frac{\theta_M}{\pi}$ and $\theta_M = \pi/8$ for a bump over a whole magnet.

We also need an expansion in the geometrical plane ($\phi = S/R$);

$$B_{PFW} = \Delta B \sum_{k=0}^{\infty} \tilde{b}_k \cos k\, S/R$$

$$\tilde{b}_k = \frac{2}{\pi k} \sin k \frac{\rho_o}{R}\, \theta_M, \qquad \tilde{b}_o = \frac{\rho_o}{R}\, \theta_M \tag{A4 a}$$

Now to calculate orbit distortions we use smooth approximation (see e. g. E. Courant, H. Snyder, Ann. Phys. 3, p. 1-48, 1958) and obtain

$$Z = \frac{R^2 \Delta B}{\rho_o B_o} \sum_{k=o}^{\infty} \frac{\tilde{b}_k}{\nu^2 - k^2} \cos k\, S/R\,. \tag{A5}$$

Hence, the focusing field along the orbit is by virtue of (A3) and (A5):

$$\frac{\partial B_r}{\partial Z}\, Z = -\nu^2 \Delta B \frac{\pi}{4} \sum_{k=0}^{\infty} \left(\frac{\tilde{b}_k}{\nu^2 - k^2} \cos k\, S/R \sum_{\ell=1}^{8} \delta\left(\frac{S}{R} + \frac{\pi}{8} - \ell \frac{\pi}{4}\right)\right)$$

Due to the δ-function characteristic of the focusing only the orbit $Z\left(S/R = -\frac{\pi}{8} + \ell \frac{\pi}{4}\right)$ enters. We can now transform into the θ - plane. Expanding the δ-functions we obtain

$$\frac{\partial Br}{\partial Z}\, Z \approx -\frac{R}{\rho_o}\, \nu^2 \Delta B \sum_{k=0}^{\infty} \frac{\tilde{b}_k}{\nu^2 - k^2} \cos k\,\theta\, (1 - 2\cos 8\,\theta + 2\cos 16\,\theta \quad . \; . \; . \;) \tag{A6}$$

Then collecting terms, the harmonics of the total radial field $B_{PFW} + \frac{\partial B}{\partial Z} Z$ may be approximated as

$$\frac{B_k}{\Delta B} \approx b_k - \frac{R}{\rho}\left[\frac{\nu^2}{\nu^2 - k^2}\,\tilde{b}_k + \tilde{b}_{8-k}\,\frac{\nu^2}{\nu^2 - (8-k)^2} + \tilde{b}_{8+k}\,\frac{\nu^2}{\nu^2 - (8+k)^2} \cdots\right] \quad \text{(A7)}$$

To the extent that $R \approx \rho_o$, the first two terms agree with the simple approximation obtained by replacing the precession angle θ by S/R and calculating Z'' using a suitable approximation. The extra terms are a correction due to the strong 8-fold periodicity of the focusing and from (A7) it is clear that this correction becomes important for harmonics k close to $8 \pm \nu$, $16 \pm \nu$, . . . for which the "resonant dominators" become small.

To compare with measurement we take a look at the points where the polarization vs PFW - field curves cross zero (corresponding to a theoretical $D_k \approx 1.3$ from equ.(1/). Weighted averages for $Z/\sqrt{\dot{B}}$ at these points are worked out in table A1. To obtain these values we take into account that a PFW current of about 2 amps appears to eliminate the distortions in the machine contributing to the excitation $\gamma G = 6$ (see figs. 4-7).

Table (A1): PFW - current and $\frac{Z}{\sqrt{\dot{B}}}$ to give P = 0

k = 6

Fig.	$\Delta I = I - 2.0$	$\Delta Z = 0.04 \frac{in.}{A} \times \Delta I$	$\dot{B}$	$\frac{Z}{\sqrt{\dot{B}}}$
5	10 [A]	0.4 [in.]	0.2 [KG/sec]	0.89 [in. / $\sqrt{\text{KG/sec}}$]
6	5 "	0.2 "	0.1 "	0.63 "
7	4 "	0.16 "	0.05 "	0.7 "

average taken $\frac{Z}{\sqrt{\dot{B}}} = 0.75$ in. $/\sqrt{KG/sec} \approx 2 \times 10^{-2}$ m $/\sqrt{KG/sec}$.

k = 7

Fig.	$\Delta I \approx I$	$AZ = 0.04 \frac{in.}{A} \Delta I$	$\dot{B}$	$\frac{Z}{\sqrt{\dot{B}}}$
8	1.1 [A]	0.045 [in.]	0.2[KG/sec]	0.1 [in./$\sqrt{KG/sec}$]

value taken $\frac{Z}{\sqrt{\dot{B}}}$ = 0.1 in. $/\sqrt{KG/sec} \approx 2.5 \times 10^{-3}$ m$/\sqrt{KG/sec}$

To complete comparison with measurement we express the bump strength ΔB by means of the (measured) orbit amplitude. The peak to peak distortion $2\hat{Z}$ can to good approximation be replaced by the dominant first harmonic of (A5). One obtains

$$\frac{\Delta B}{B_o \rho} = \frac{Z_1(\nu^2-1)}{\tilde{b}_1 \; R^2} \approx \frac{\hat{Z}\,(\nu^2-1)}{\tilde{b}_1 \; R^2}$$

We can now rewrite the depolarization equ. (1) p. 3 above for the resonance $\gamma G = k$ as:

$$D_k = \frac{\pi^2}{2}\left[\underbrace{\left(\frac{1}{\sqrt{G\, T_{rev}\, \frac{\dot{\gamma}}{\dot{B}}}} \; \frac{\rho_o}{R} \; \frac{\nu^2-1}{R}\right)}_{\substack{\text{machine}\\ \text{parameters}}} \underbrace{\frac{B_k(1+k)}{\tilde{b}_1\, \Delta B}}_{\substack{\text{calculated}\\ \text{(A4, A7)}}} \; \underbrace{\frac{\hat{Z}}{\sqrt{\dot{B}}}}_{\substack{\text{measured}\\ \text{(Table A1)}}}\right] \qquad (1a)$$

For the parameters $R = 27.4$ m, $R/\rho = 1.3$, $\nu^2 = 0.67$, $\dot{\gamma}/\dot{B} = 0.67$/KG, $T_{rev} = 0.57\ \mu$sec, $G = \frac{g-2}{2} = 1.79$

$$D_k \approx \left[25 \frac{B_k}{\Delta B\, \tilde{b}_1} (1+k) \frac{Z}{\sqrt{\dot{B}}} \right]^2 \quad Z \text{ in m, } \dot{B} \text{ in kG/sec} \qquad (1b)$$

Finally in Table (A2) we compare the D-factors calculated from measured $Z/\sqrt{\dot{B}}$ with the theoretical $D_k = 1.3$ to give P = O. For convenience we have also included some D-values obtained by using the simple approximation [$\theta \approx S/R$ and smooth approximation to obtain Z'' entering into (1)] to unfold the measurements.

Table A2: D-values giving complete depolarization as obtained from the measurements. Values in brackets are obtained by unfolding data using the crude approximation. The others rely on the improved approximation.

Resonance	$k = \gamma G = 6$	$k = \gamma G = 7$
Calculated field harmonic $\frac{B_k}{\Delta B\, b_1}$	-0.24 (+ 0.55)	-2.45 (0.42)
measured $\frac{Z}{\sqrt{\dot{B}}}$ $\left[\frac{m}{\sqrt{kG/sec}}\right]$ c. f. Table (A1)	2×10^{-2}	2.5×10^{-3}
D-value from (1b)	0.7 (3.6)	1.5 (0.04)
Theoretical D-Value	1.3	1.3

One concludes that the improved approximation gives the correct resonance strengths to within a factor of 2. Actually the 6th harmonic depends critically on the length of the bump and one obtains much better agreement assuming a bump over only 95% of the magnet (as might be justified by stray field and edge angle effects). For a bump of a 100%, cancellation between the first two terms and the last term of equ. (A7) tends to give a weaker depolarizing effect than measured.

The simple approximation still gives the correct order of magnitude for $\gamma G = 6$ although this has to be regarded as an incident. For $\gamma G = 7$ this approximation leads to a strong underestimate.

We now apply these results to the PS and to normal ZGS operations. In the ZGS with $\dot{B} = 20$ kG/sec and admitting 1% depolarization per imperfection resonance we obtain from (1) for the Kth harminic of B_r

$$\frac{B_k}{B_Z} \lesssim \frac{1.7 \times 10^{-4}}{(1 + k)} \qquad (k >> 1) \tag{A8}$$

i.e. harmonics of the radial field should be smaller than say 10^{-5} $(k \approx 10)$ of the main field. Using the simple smooth approximation this can be expressed in terms of harmonic distortions of the orbit (Z_k) as:

$$Z_k \lesssim \frac{6 \text{ mm}}{k^2 (1 + k)} \quad . \tag{A9}$$

Actually scaling our measurements for zero PFW-bump to $\dot{B} = 20$ kG/sec we have $D \approx 0.15\%$ for $K = 6$. Hence 6th harmonic distortion is by $\sqrt{\frac{1\%}{0.15\%}} \approx 2.5$ less in the ZGS than values given by (A8) (A9). This is partly explained by the calculations discussed above.

We have to remember that the smooth approximation becomes poor especially for $k = 8 \pm \nu$, $16 \pm \nu$. In the PS we obtained under similar assumptions (taking $\dot{\gamma} = 80\ \mathrm{sec}^{-1}$, $\rho = 70$ m, $R = 100$ m, $T_{rev} = 2.2\ \mu$ sec):

$$\frac{B_K}{B_Z} \lesssim \frac{5 \times 10^{-4}}{1 + k} \quad , \quad Z_k \lesssim \frac{70\ \mathrm{mm}}{k^2\ (1 + k)} \ .$$

Since the PS has a weak super periodicity of 10 and a strong periodicity of 50 the simple smooth approximation may become poor for imperfection resonances $\gamma G = k$ with $k \simeq 10 \pm \gamma$, $20 \pm \gamma$ . . . and especially $50 \pm \gamma$, etc.

From the above figures the PS seems to be less vulnerable to imperfection resonances. This is however a somewhat premature conclusion. In fact, as is discussed in ref. 3 the expectation value of Z_k for a given magnet misalignment is proportional to $\frac{n}{\sqrt{M}} \ \frac{R}{\rho}$ (n: field gradient, M number of magnets), and this factor is about 85 times larger in the PS than in the ZGS. In the ZGS a total depolarization of up to 5% is observed due to imperfection resonances up to 12 GeV. Hence for the same alignment accuracy one expects in the PS about 40% depolarization up to the same energy and about 4 times as much up to 20 GeV. This figure can however only be used as a rough guide due to the differences in magnet design, periodicity and ν-values.