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INSTABILITIES OF THE COHERENT LONGITUDINAL OSCILLATIONS OF THE BUNCH SYSTEM

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1. INTRODUCTION

As is known from the papers of C. Pellegrini[1] and M.M. Karliner[2], a longitudinal instability can arise if the bunch system interacts with long memory equipments like cavities etc. This effect was observed in many electron and proton machines. In particular, it was carefully investigated in the case of the AGS[3] and the CPS[4].

In this paper the effect is studied for the SPS for which limits of the tolerable shunt impedance are calculated.

2. SKETCH OF THE THEORY

The N bunches are treated as charged rigid bodies, their longitudinal motion is described by the time of arrival $\tau_\nu(t)$. Using the linear approximation of free synchrotron oscillations, we get the following system of complete single bunch equations :

$$\begin{aligned} \ddot{\tau}_1 + \Omega_0^2\tau &= \frac{\Omega_0^2}{\cos\psi}\,\frac{V_1}{U_c}\left[I_1(\tau_1),\ldots,I_N(\tau_N)\right] \\ \ddot{\tau}_N + \Omega_0^2\tau &= \frac{\Omega_0^2}{\cos\psi}\,\frac{V_N}{U_c}\left[I_1(\tau_1),\ldots,I_N(\tau_N)\right] \end{aligned} \qquad (1)$$

where Ω_o^2 is the synchrotron frequency of free oscillations and ψ the phase angle which may depend on time adiabatically; U_c is the main peak voltage produced by the acceleration system, whereas the $V_k\left[I_1(\tau_1),\ldots, I_N(\tau_N)\right]$ represent the voltages induced vy the beam depending on the bunch currents $I_\nu(\tau_\nu)$ which in turn depend on the corresponding arrival time τ_ν

The currents $I(\tau)$ can be represented by a Fourier-series in term of the revolution frequency w_o and τ

$$I(\tau) = \sum_n I_n e^{in w_o(t+\tau)} \tag{2}$$

if $\Omega_o \ll w_o$

For stability considerations it is sufficient to study the system at threshold, i.e. for small τ . So we can apply perturbation theory and linearize (1) keeping nly linear terms of (2) :

$$I_\nu = I_{\nu o} + I_{\nu 1} \cdot \tau \tag{3}$$

If there is no gap in the machine and if in addition all bunches are filled equally, the coupled system (1) can be solved in terms of the normal mode vectors

$$\vec{\tau}_m(t) = \vec{\tau}_{om} \cdot e^{iw_m t},\ \vec{\tau}_{om}\begin{pmatrix} \vdots \\ e^{2im\nu\pi/N} \\ \vdots \end{pmatrix} ;\ m = 1 \ldots N \tag{4}$$

where the complex frequency w_m splits as follows :

$$w_m = \Omega_m - i\delta_m .$$

a) <u>If $\Omega_m \approx \Omega_o$</u> the decrement δ_m can be written as

$$\delta_m = I_b \Omega_o \sum_\nu \left\{ \frac{b_{\nu\, N+m}\, R^+_{\nu\, N+m}}{U_c \cos\psi} - \frac{b_{\nu N-m}\, R^-_{\nu N-m}}{U_c \cos\psi} \right\} \tag{5}$$

In this formula I_b denotes the beam current and the $R_\mu^\pm$ are defined through

$$I(t) = \Sigma I_\mu e^{i\mu w_o t},\quad V\left[I(t)\right] = \sum_\mu Z(\mu w_o) I_\mu e^{i\mu w_o t} \tag{6}$$

and

$$R^\pm_\mu = \frac{\mathrm{Re}Z(\mu w_o \pm \Omega_o) + \mathrm{Re}Z(\mu_o w_o)}{2}$$

Finally the coefficient b_μ depends on the longitudinal shape of the bunch and on the frequency of the resonator. Figure 1 gives an idea of its variation.

b) <u>If the frequency shift is large</u> we have to look at the exact equation for the eigen-values, which has the form

$$\Omega_o^2 - w^2 = \frac{\Omega_o^2 \; I_b}{U_c \; \cos\psi} \left\{ i \left[R^+(\Omega) - R^-(\Omega) \right] + \left[X^+(\Omega) + X^-(\Omega) \right] \right\} \tag{7}$$

where we have dropped the indices. The coefficients b have been put to unity. The first term of (7) leads to (5) for $\Omega \approx \Omega_o$, while the second term corresponds to the frequency shift. This can be easily seen if one linearizes (7) :

$$R^+(\Omega) - R^-(\Omega) \sim \frac{\Delta R}{\Omega_o} \cdot \Omega$$

$$X^+(\Omega) + X^-(\Omega) \sim 2X \; .$$

Then one obtains from (7)

$$w \sim -i \, \frac{I_b \Omega_o \Delta R}{U_c \cos\psi} \pm \sqrt{\Omega_o^2 + \frac{2X\Omega_o^2 I_b}{U_c \; \cos\psi} - 0 \; (I_b^2)} \tag{8}$$

So in general, the instability is not governed by the first term only. If one allows for

$$X < 0 \qquad \text{and} \qquad \frac{2|X| I_b}{U_c \cos\psi} > 1, \tag{9}$$

the square root becomes imaginary and we run into an instability. Thus we must have a look on the real part of the shunt impendance and on the imaginary part as well.

If there is a gap in the machine or if the bunches are not equally filled, the simple formula (5) does not apply. In this case the real and imaginary parts of the shunt impedances appear in the corresponding formula. But this does not change the results essentially.

3. NUMERICAL ESTIMATE FOR THE SPS

Putting $1/\delta$ = 500 msec., $I_b \approx 0.1$ Amp, $U_c \cdot \cos\psi$ = 4 MV, b = 1, $\frac{\Omega_o}{2\pi}$ = 200 Hz for the SPS, we obtain assuming that there is one dominating term in (5), which leads to an instability :

$$R < \frac{\delta \cdot U_c \cos}{I_b \Omega_o} \approx 65 \text{ K}\Omega \qquad (10)$$

From equation (9) we get for the imaginary part

$$|X| < \frac{U_c \cos\psi}{2\ I_b} = \frac{\Omega_o}{2\delta} R, \quad \frac{\Omega_o}{2\delta} \approx 600.$$

So the restriction on the imaginary part is weak as compared to the real part in the case of azimuthal symmetry.

If there is a gap or an unequal bunch filling, the X, R get mixed, and we have

$$R,\ |X| < 65 \text{ k}\Omega.$$

4. COMMENTS

a) If bunches are short (0,5 ns) and some structure resonates at about 1 GHz, the tolerable impedance goes down to 22 kΩ (see Fig. 1). This seems to be the worst case.

b) The simple estimate made above is valid for modes m such that $0 \ll m \ll N$, i.e. when the phase shift between bunches is different enough from 0 or π (worst case, $\frac{\pi}{2}$). The modes m = 1,2,... for example, could be excited by the main cavity resonance around 200 MHz. But then R must be the difference between the impedances seen by the beam at frequencies corresponding to modes +m and −m.

c) Landau damping: the U and V terms usually used in dispersion relation analyses are likely here to be of the same order of magnitude. Both calculations and experiments done for the CPS (4) show that the "natural" non linearities, enhanced if necessary by reducing the accelerating voltage, are sufficient to ensure stability if the growth times calculated without Landau damping exceed 50 ms.

d) Higher modes: very high frequency resonators (F > 1.5 GHz), which are not efficient in exciting the dipole (rigid) motion of the bunches, can excite quadrupole-type modes.

Sacherer[5] states that, as a thumb rule, one can take the same formula as for the dipole motion (formula 10) and multiply by 2 (for sextupole modes by 3, etc.).

5. CONCLUSION

Bunched beam instabilities should show up in the SPS at $N = 10^{13}$ p/p if coupling impedances of the order of 20 kΩ (high frequency) to 60 kΩ (∿ RF frequency) are introduced. Nevertheless, ten times these figures can be tolerated if one is able to manipulate at will the accelerating voltage such as to fix the instabilies by Landau-damping.

REFERENCES

1) C. Pellegrini, unpublished.

2) M.M. Karliner, SLAC Translation 132

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4) D. Boussard, J. Gareyte, VIIIth Internat. Conference on High Energy Accelerators, CERN-Geneva, 20-24 September 1971.

5) F. Sacherer, unpublished.

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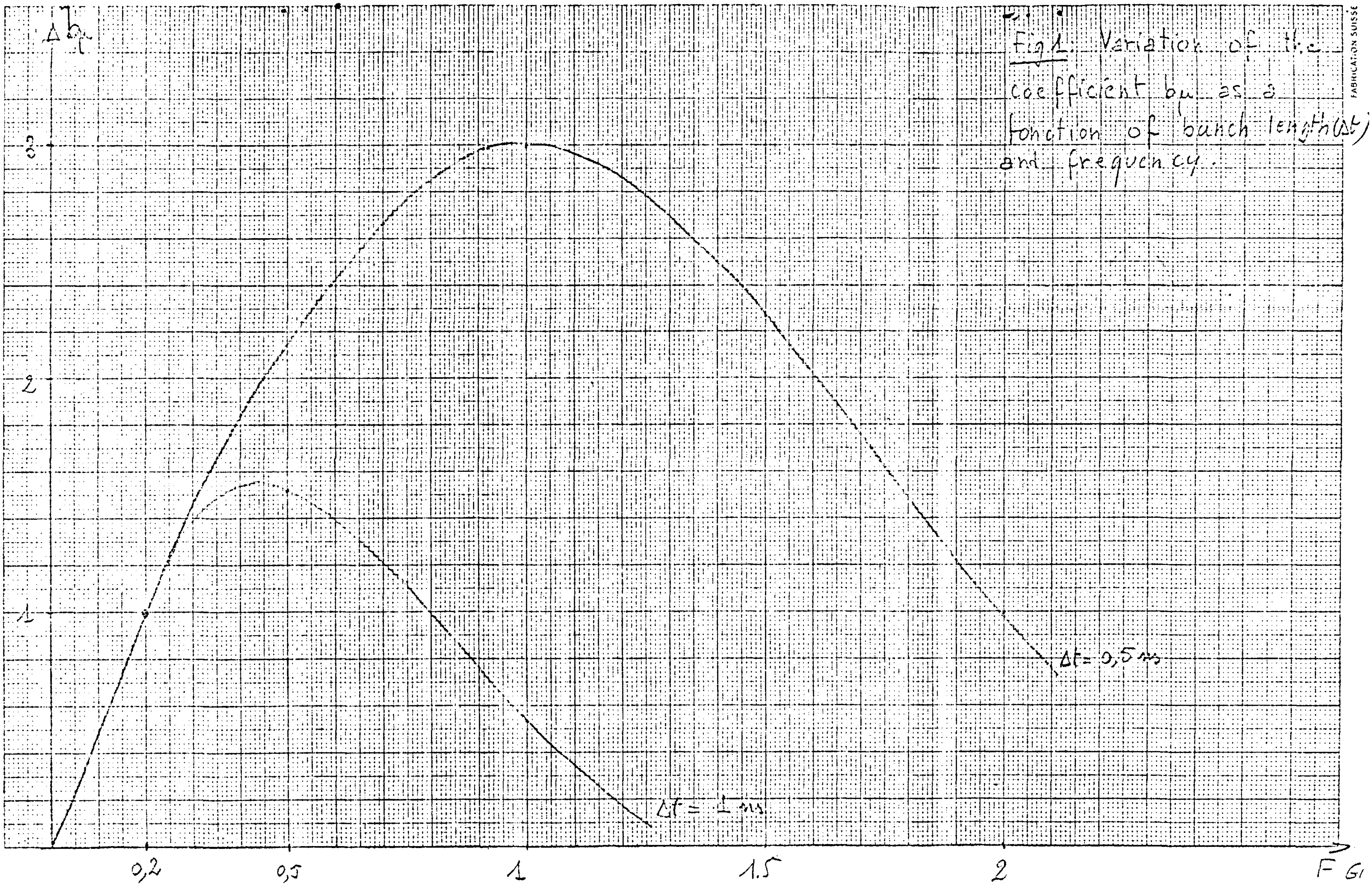

Fig 1. Variation of the coefficient b_μ as a fonction of bunch length (Δt) and frequency.