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THE "MISSING BUNCH" EXPERIMENT

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1. The experiment.

An experiment was performed during two MD's to test whether the longitudinal instability observed in the PS is due to "low Q coupling impedances".

Up to 13 consecutive bunches were chopped off before injection or were ejected by means of a fast kicker. Thus a gap of up to 13 bunches was created leaving 13 of the 20 RF buckets empty. We expected that the remaining bunches would be stable. This because there had been evidence[1] that the 45 MHz parasitic resonance of the rf cavities is responsible for the instability. This resonance has a relatively low Q-value ($Q \approx 20$) so that the beam induced signal decays practically to zero during the passage time of the gap. It was expected that this would ensure stability because the bunches at the head would not notice the oscillation of the bunches at the tail.

However, strong instability remained even with 13 bunches missing. As an example we reproduce two oscillogrammes (fig.1) taken with 9 bunch present, 11 bunches being chopped off before injection.

The explanation first suggested was that the beam control system "bridges the gap". However the calculations described in ref.1) show that beam control leaves the bunch system unaffected, provided that the (remaining) bunches have equal oscillation frequencies.

It was therefore suspected that a spread in the bunch frequencies together with the action of the beam control system is responsible for the effect. In fact this suspicion led us to the refinement of the theory which is described in ref.2).

The present note is intended to apply this theory to the situation of missing bunches where the remaining bunches are unequally populated, have different frequencies and are subject to the action of the beam control system.

We find that already for very small bunch to bunch frequency spreads ($\pm$ 0.5 o/oo in the PS) the phase lock loop bridges the gap.

2. The explanation.

2.1. Analytical calculations.

We shall use the formalism and the notations used in ref.2 and limit ourselves to the case of 4 bunches remaining in the machine and being coupled via low Q wake fields.

For equal synchrotron frequencies the characteristic matrix of the system, including the effect of the beam control is

$$\begin{pmatrix} \Omega^2 - \omega^2 & 0 & 0 & 0 & -\Omega^2 \\ \beta & \Omega^2 - \omega^2 - \beta & 0 & 0 & -\Omega^2 \\ 0 & \beta & \Omega^2 - \omega^2 - \beta & 0 & -\Omega^2 \\ 0 & 0 & \beta & \Omega^2 - \omega^2 - \beta & -\Omega^2 \\ A & A & A & A & -1 \end{pmatrix} \qquad (1)$$

Note that the first bunch is not coupled to the others (no β term in the first line). Using the results of ref.2 , the determinant $H(\omega)$ of the matrix (1) can be expressed as

$$H(\omega) = \omega(\omega+j\,G\Omega^2)(\omega^2-(\Omega^2-\beta))^3$$
$$= \omega(\omega+j\,G\Omega^2)(\omega^2-\omega_1^2)^3 \qquad (2)$$
$$\omega_1^2 = \Omega^2 - \beta$$

Now in order to find the root displacements due to a perturbation of the Ω's we shall use the same root locus method as in ref.2. In the present case ω_1 is a triple root, therefore we have to expand $H(\omega)$ up to the third order in $d\omega$, because the two first derivatives $\frac{\partial H}{\partial \omega}$ and $\frac{\partial^2 H}{\partial \omega^2}$ vanish.

$$H(\omega,\delta) = H(\omega_1,0) + \frac{1}{3!}\frac{\partial^3 H}{\partial \omega^3}(d\omega)^3 + \frac{\partial H}{\partial \delta}\delta + \ldots\ldots \qquad (3)$$

($\delta \approx 2\Omega d\Omega$ as in ref.2).

Note that the first derivative $\frac{\partial H}{\partial \delta}$ does not vanish because the system is no longer symmetric.

The root displacement $d\omega$ is then given by:

$$(d\omega)^3 = -\frac{\frac{\partial H}{\partial \delta}\delta}{\frac{1}{3!}\frac{\partial^3 H}{\partial \omega^3}} \qquad (4)$$

From equation (2) one finds:

$$\frac{1}{3!}\frac{\partial^3 H}{\partial\omega^3}\Big)_{\omega=\omega_1} = -\omega_1(\omega_1+jG\Omega^2)(2\omega_1)^3 \simeq -8\Omega^5 \qquad (5)$$

($G\Omega$ is assumed negligible compared to unity.)

The quantity $\frac{\partial H}{\partial\delta}$ depends on the modulation pattern. For the sake of simplicity let us assume a "sinusoidal" pattern defined by:

1st bunch	$\Omega \rightarrow \Omega + \delta$
2nd bunch	$\Omega \rightarrow \Omega$
3rd bunch	$\Omega \rightarrow \Omega - \delta$
4th bunch	$\Omega \rightarrow \Omega$

Then the determinant of the matrix (1) decomposes into a sum of determinants and one finds:

$$\frac{\partial H}{\partial\omega}\Big)_{\omega=\omega_1} = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ \hline 0 & \beta & \Omega^2-\omega^2-\beta & 0 & -\Omega^2 \\ \hline A & A & A & A & -1 \end{bmatrix}_{\omega=\omega_1} + \begin{bmatrix} \Omega^2-\omega^2 & 0 & 0 & 0 & -\Omega^2 \\ \hline 0 & 0 & -1 & 0 & -1 \\ \hline A & A & A & A & -1 \end{bmatrix}_{\omega=\omega_1} \qquad (6)$$

$$\frac{\partial H}{\partial\omega}\Big)_{\omega=\omega_1} = \beta^2 A(\Omega^2-\beta^2) \simeq \beta^2 A\Omega^2 \qquad (7)$$

From equations (4) (5) and (7) one has:

$$(d\omega)^3 \simeq \frac{3}{8}\,\frac{\beta^2}{\Omega^2}\,d\Omega$$

This equation has three roots; two of them which are complex conjugate have opposite imaginary components. Therefore the triple root ω_1, which was purely real transforms into three separate roots, one of which will give rise to an instability.

The imaginary component is given by:

$$|\mathrm{Im}\ d\omega| = \frac{\sqrt{3}}{2}\ \frac{\sqrt[3]{3}}{2}\ \left(\frac{\beta^2\ d\Omega}{\Omega^2}\right)^{1/3} \qquad (9)$$

For an order of magnitude estimate we assume a coupling wakefield such that the normal growth time (all the bunches present) is 50 ms and a frequency spread $\pm \frac{d\Omega}{\Omega} = 10^{-2}$.

Under these conditions we find a growth time of the instability, for only 4 bunches left in the machine of 90 ms. We note the surprising fact that this growth time is shorter than the $\frac{50\ \mathrm{msec} \times 20}{4} = 250$ msec, obtained by scaling the growth with the number of remaining bunches. However it should be noted that perturbation technique used may no longer be valid at frequency spreads as large as 1%.

2.3. Numerical results.

The computer programme described in ref.2 was used to simulate the situation in the PS with a gap of 13 bunches. Low Q wake fields are assumed so that every bunch acts only on the subsequent one through induced fields.

The results are in agreement with the conclusion drawn from the simplified model in section 2.

As an example let us again assume a coupling coefficient $\beta = 0.02\,\Omega^2$. With all 20 bunches present this bunch-to-bunch coupling would lead to a growth rate $1/\tau \approx \frac{\Omega}{2}\beta \approx (50\ \text{msec})^{-1}$ in the PS. Thus with only 7 bunches present one expects a growth rate of $\approx (140\ \text{msec})^{-1}$ by just scaling the growth time with the number of bunches.

The maximum growth rates actually found for the example of sinusoidal frequency variation between the remaining 7 bunches $(\Omega_k = \Omega + \Delta\Omega\cos(2\pi\, k/8))$ are listed in table 1

$\pm \frac{\Delta\Omega}{\Omega_s}$	$(\tau\Omega_s)$	τ_{PS} (msec)
5×10^{-8}	1.2×10^{-3}	600
5×10^{-4}	2.4×10^{-2}	120
5×10^{-3}	2.2×10^{-3}	110
5×10^{-2}	stable	--

It will be concluded from this example that already with bunch-to-bunch frequency spreads of o.5 o/oo in the PS $(\pm\,\Delta\Omega \approx 0.006\ \beta/\Omega_s)$ the phaselock system bridges the gap. Again we find that for frequency spreads comparable to the wake field, the growth is faster than one expects from linear scaling with the number of bunches.

It seems very likely that frequency spreads of at least 1 o/oo are created with missing bunches due to a difference in bunch shape and in the beam loading at the fundamental rf frequency.

The computer programme was also used to simulate the situation assuming a "modified beam control system" which takes as a reference the motion of only one bunch instead of the average motion of all bunches. As one expects from a simple consideration the system was found to be completely stable when the first bunch after the gap was taken as the reference.

3. Conclusion

It seems very likely that the results of the experiment are due to the combined effect of the 45 MHz parasitic resonance and the action of the phase-lock loop.

A small spread in the bunch frequencies makes the oscillations apparent to the phase-lock system.

In contrast to our earlier assumptions, this explanation is in agreement with the hypothesis that "low Q wake fields" cause the instability. Measurements made during other MD's support this hypothesis.

Both theoretical and experimental results seem therefore to give a strong indication that the 45 MHz resonance or other low Q impedance cause the instability in the PS.

Acknowledgement

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References

1. Y.Baconnier, D.Boussard, J.Gareyte.
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2. D.Boussard, J.Gareyte, D.Möhl.
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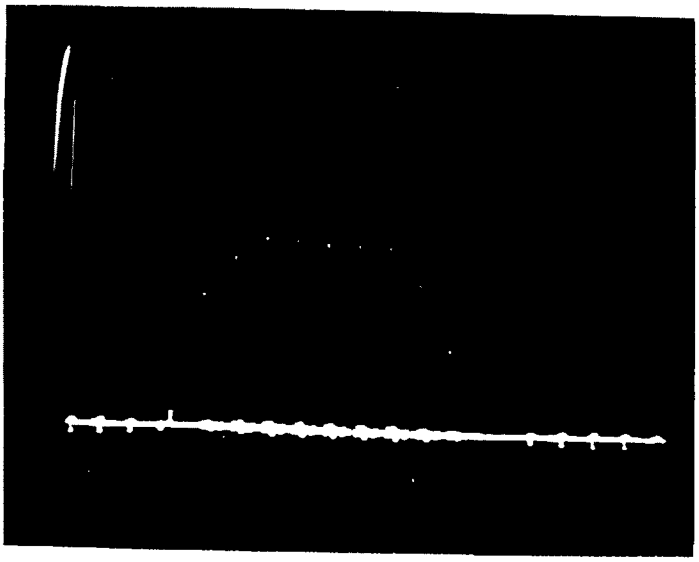
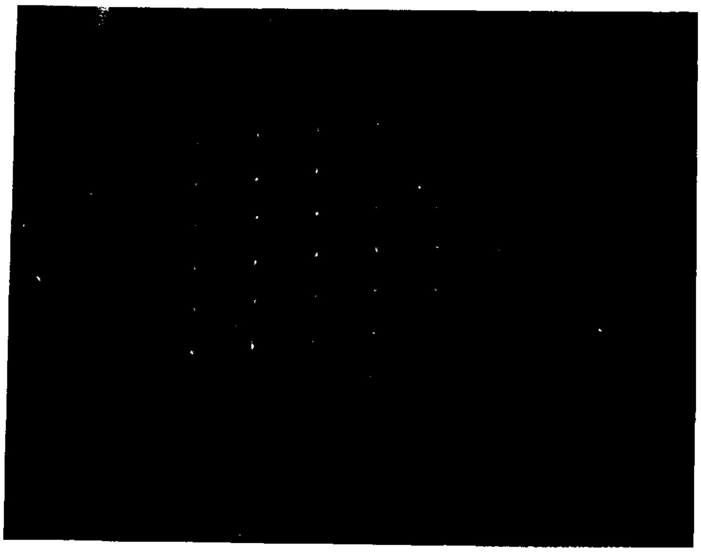
Fig.1



a) "Missing bunch" pattern
200 nsec/cm →



b) Mountain range display (Cappi trigger) of bunch shape around 10 GeV/c.

- Bunch pattern as in a).
- Total intensity: N = 0.4×10^{12} p/p

100 nsec/cm →

3 msc/cm ↑