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C O N T E N T S

P R E F A C E

The 1963 International Conference on High Energy Physics and Nuclear Structure was organized jointly by CERN and the Weizmann Institute. The intention was to bring the diverging fields of high energy and nuclear physics together once more. It was felt that there should be a large number of interesting, though presently neglected, lines of investigation on the borderline between these fields. An informal discussion was thought to be the best way of arousing interest and clarifying possibilities. Consequently, every half day was opened by a one-hour invited talk followed by several hours of discussion. All formal contributions were banned, and there was initially no intention to issue proceedings. Even before the conference many requests for some kind of written report were received. It was therefore at a late stage decided to issue those of the invited talks, which were available. Since the purpose of the invited talks was to serve as a basis for the discussion to follow, the Committee had specifically asked the speakers to be provocative, and even speculative, in their statements so as to obtain a response from the audience. They were furthermore instructed to pay less attention to past work than to future possibilities and consequently they were not to occupy themselves with giving exact credit to past work in the field. In short, the speakers were asked to present their material in a way opposite to the one normally used in publications. The Committee takes entire responsability for this choice of presentation. All criticisms on these grounds should be directed to the Committee and not to the individual speaker. We are extremely grateful to the invited speakers for their agreement to publish their talks in a form so close to the one actually presented.

We further wish to thank the many persons at CERN who, in different ways, have participated in the production of these proceedings. Particular thanks are due to the secretaries of the Theory Division for their patient and careful typing, and to Miss Braun and Mrs. Trentini for having drawn a large number of the figures.

T.E.O. Ericson

Geneva, July, 1963.

SESSION I

COMPLEX NUCLEI AND STRANGE PARTICLES

Speaker :

D.H. WILKINSON

COMPLEX NUCLEI AND STRANGE PARTICLES

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INTRODUCTION

As general themes I want to talk chiefly about ways in which we can use strange particles for finding out about the structure of complex nuclei and secondarily about some ways in which complex nuclei can bring information about strange particles. Much of the following material is speculative in the sense that the reactions discussed may never, in practice, be measurable in sufficient detail to be useful or may prove so complicated when examined closely that the desired information, although contained within the experimental data, cannot be extracted from them. If these notes have a useful purpose it is to encourage people to think of strange particles as ordinary particles, capable of interacting with complex nuclei in much the same way as more familiar nuclear constituents. I am convinced that out of this interaction much amusement is to be had, and profit too, even though at the moment we are largely in the stage of making our investment without any very clear idea about the form the dividends might take.

HYPERNUCLEI

Ordinary hypernuclei

The study of hypernuclei, both experimental and theoretical, is already very detailed and I shall not go into it. Shell-model and other-model type calculations are now being done in which the properties of the complex nuclei that form the cores of the hypernuclei are involved in detail. All that need be said is that these calculations should reveal more and more about the Λ^o-N interaction as different states are brought into play - for example by detailed work on p shell cores as well as the s shell cores that have been chiefly studied so far. At the moment we are largely in the state where we pretend we know all about the ordinary complex nuclei and use the hypernuclear properties to tell us something about the Λ^o-N interaction. This has by no means gone as far as it can. Many of the exciting possibilities of gaining more Λ^o-N information from hypernuclei have been outlined by others elsewhere and I will not repeat them. It is quite difficult to see how to gain certain Λ^o-N information other than from complex nuclei. Obvious examples come in the non-mesonic decay modes. These are sensitive to the relative phases of the $\Lambda^o \rightarrow p+\pi^-$ and $n+\pi^o$ matrix elements which is difficult to find out by studying the free decay. Equally the question of the relative strengths of the reactions $\Lambda^o+n \rightarrow 2n$ and $\Lambda^o+p \rightarrow n+p$ requires complex nuclei for its elucidation.

Double hypernuclei

A further and very exciting possibility that has been discussed is that of forming hypernuclei such as $^6He_{\Lambda\Lambda}$ most probably following the absorption of stopped Ξ^--hyperons and the conversion reaction :

$$\Xi^- + p \rightarrow 2\Lambda^o$$

This reaction releases only about 25 MeV so there is good likelihood that both Λ^o-hyperons may stick in the same fragment. The chief point about this is the possibility of getting information on the Λ^o-Λ^o interaction which is obviously difficult to do otherwise.

Three-body forces of the Λ^o-hyperon

Another important question about the interaction of the Λ^o-hyperon with nucleons concerns the possible importance of the three-body forces $\Lambda^o NN$ that may be expected because the Λ^o cannot interact with a nucleon by single pion change. It must interact by K meson exchange or by double pion exchange through an intermediate virtual Σ state. If the latter process is important we might expect to find genuine three-body forces due to the exchange of the two pions with two different nucleons. Three-body forces, that may also play a role in ordinary nuclear interaction, are notoriously difficult to detect. Study of the non-mesonic hypernucleus decay offers a chance. Consider a light hypernucleus where we might hope that final state interactions may be not too important. As the extreme case think of $^3H_\Lambda$, but the argument is more general. When the non-mesonic decay takes place we are left with one proton and two neutrons. If only two-body forces are significant then, because all the binding energies are low, the usual mode will be into two fast nucleons and a spectator. If three-body forces are important they will favour close simultaneous approach of all three particles and so a more uniform sharing of the energy in the final state. A study of this process over several species of hypernuclei may throw important light on the three-body force. Nuclei in which $\Lambda^o pp$ states can be involved will be easier to handle technically. Such work as this, and also the $\Lambda^o n$ vs $\Lambda^o p$ conversion studies mentioned before obviously must be accompanied by careful Monte-Carlo/optical model treatment of the final state interactions.

Nuclear parentage through hypernucleus decay

But can we also reasonably look to hypernuclei for information about complex nuclei ? It may indeed be possible to use hypernuclei, assuming their interactions with nucleons to be known, as a valuable new window into the wave-functions of ordinary nuclei. Consider ordinary ^{12}C. The ground state has as its parents the ground ($J^{\pi}=3/2^{-}$) and first excited ($J^{\pi}=1/2^{-}$) states of ^{11}B and from the fractional parentage coefficients, experimentally determined, we draw conclusions about the model operative in this region of the periodic table. If we commit ourselves to the independent particle model (IPM) in intermediate coupling we learn the intermediate coupling parameter a/K. If we adopt a Nilsson-type description for ^{11}B we learn the deformation parameter and so on. We then check this against the prediction the same model makes for the excited state structure of these nuclei and so on. It would be most valuable if we could now introduce into these ordinary nuclei a foreign particle, with known interactions with the nucleons to make a new object. The properties of this new object would involve the wave functions of the original nucleons in new ways and so the comparison of its calculated with experimental properties would afford a new check on our ideas about the behaviour of the nucleons, the ordinary nuclear wave function. Such a new object is the hypernucleus. Consider $^{12}B_{\Lambda}$ instead of ^{12}C. Talk in terms of the IPM and assume, for illustration, that the spin of the ground state of $^{12}B_{\Lambda}$ is J=1. The parents of $^{12}B_{\Lambda}$ are then the same two states of ^{11}B that are parents of ^{12}C and the same type of calculation will reveal the fractional parentage coefficients; the calculation now uses the Λ°-N interaction as well as the N-N. The fractional parentage coefficients are nicely revealed by the decay :

$$^{12}B_{\Lambda} \rightarrow {}^{11}B + p + \pi^{-}$$

A succession of such studies throughout the p shell will be useful in testing the internal consistency of the IPM account of light nuclei and in comparing the IPM with other accounts (and also, of course, in giving details of the Λ°-N interaction; the two studies must in practice go side by side).

We may notice in passing that this type of study will be useful in determining spins of hypernuclei. If, for example, the spin of the ground state of $^{12}B_\Lambda$ were $J=2$ then the first excited state would not be a parent and so would not be formed to first order in the decay process. On the other hand the second excited state $(J^\pi=5/2^-)$ would become a parent and so would be formed. Obviously in all these studies careful attention to final state interactions is necessary.

Charge independence

Very interesting questions of charge-independence, both for ordinary nuclei and for the Λ°-N interaction can arise in this work. Consider, for example, $^7Li_\Lambda$ and suppose it has $J=1/2$ with the ground state $(J^\pi=1^+)$ of ^6Li as its chief parent as we should expect. This makes $^7Li_\Lambda$ of $T=0$. But from the point of view of spin and parity the second excited state of ^6Li $(J^\pi=0^+ ; T=1)$ could be a parent of $^7Li_\Lambda$. Charge independence says no. But now isotopic spin is not a good quantum number so a little of the $T=1$ parent will be let in anyway. And what of the Λ°-N interaction ? The degree of charge-independence there has not been very well checked. It will be most interesting to study such decays as :

$$^7Li_\Lambda \rightarrow {}^6Li+\pi^-+p$$

with this consideration in view. (Notice that a final-state interaction can bring in the $T=1$ state of ^6Li so once again we are warned that we shall have to devote a lot of attention to final state problems.) The same information could, of course, come from inelastic scattering studies such as $^6Li(\Lambda^\circ,\Lambda^{\circ\prime})^6Li$ in this case.

Λ^o-decay versus radiation

Occasionally hypernuclear properties may reflect on more specialized aspects of nuclear structure. A possible illustration to hand at the moment concerns ${}^{7}He_{\Lambda}$ and ${}^{7}Be_{\Lambda}$. If charge independence holds in the Λ^o-N interaction the Λ^o binding energy should be the same in both species since the cores are the flanking members of the isobaric triplet ${}^{6}He$-${}^{6}Li$-${}^{6}Be$ where the ${}^{6}Li$ state is the first T=1 state just referred to. There seems to be, however, a clear experimental discrepancy in binding energy of about 2 MeV, the Λ^o-hyperon apparently being more strongly bound in ${}^{7}Be_{\Lambda}$ than in ${}^{7}He_{\Lambda}$. How can we understand this without giving up charge-independence of the Λ^o-N interaction ? One possible explanation lies in the different characters of ${}^{6}He$ and ${}^{6}Be$. The former is a particle-stable nucleus; the latter is unstable by 1.6 MeV against ${}^{6}Be \rightarrow {}^{4}He+2p$, is only defined as a short-lived state seen, for example, in ${}^{6}Li(p,n){}^{6}Be$ and, presumably, has a rather expanded openwork structure. The binding energies of ${}^{7}He_{\Lambda}$ and ${}^{7}Be_{\Lambda}$ are defined relative to the experimentally-defined nuclei ${}^{6}He$ and ${}^{6}Be$, the one, presumably, of a regular structure, the other unstable and "openwork". The addition of the Λ^o-hyperon stabilizes ${}^{6}Be$ against heavy particle break-up, and so the ${}^{6}Be$ core in the ${}^{7}Be_{\Lambda}$ is presumably more like the ${}^{6}He$ core of the ${}^{7}He_{\Lambda}$ than ${}^{6}Be$ is like ${}^{6}He$. In this case it is perfectly reasonable for the binding energy in ${}^{7}Be_{\Lambda}$ to be greater than that in ${}^{7}He_{\Lambda}$. One way to see this possibility is as follows. When we discuss say, the binding energy of the last neutron in ${}^{12}C$, we mean the energy that has to be given to the system to produce a free neutron leaving ${}^{11}C$ in its ground state. This situation may be thought of as reached in two stages : in the first we pay the "removal energy" to remove the neutron from ${}^{12}C$ leaving a hole, the other 11 nucleons occupying the same volume that they did when they were part of ${}^{12}C$, i.e., a greater volume than they do in the ground state of free ${}^{11}C$; as the second step the 11 nucleons shrink to the proper size of the ${}^{11}C$ ground state. The process of shrinking releases some "shrinkage energy", so the binding energy equals the removal energy minus the shrinkage energy. In the $^7He_\Lambda$, $^7Be_\Lambda$ case it may well be that the ^6Be core of $^7Be_\Lambda$ is about as big as the openwork ^6Be ground state whereas the ^6He core of the $^7He_\Lambda$ is bigger than the ^6He ground state as in the example of ^{12}C. In this case the same removal energy is required for the Λ°-hyperon from both hypernuclei but in the case of $^7He_\Lambda$ some energy is paid back as shrinkage energy so that the binding energy is less than the removal energy whereas in ^7Be no shrinkage energy is available so the binding energy is the same as the removal energy, i.e., greater than the binding energy for $^7He_\Lambda$. However, there is another possibility to understand the apparent discrepancy in binding energies. Suppose the cases of $^7He_\Lambda$ so far investigated were mostly of a Λ°-hyperon bound not to the ground state of ^6He but to the $J^\pi=2^+$ excited state at 1.71 MeV, in other words excited hypernuclei. If, then, this excitation energy were taken away in the decay of the hypernucleus the binding energy would apparently be too small, presumably by about 2 MeV. In order for this theory to be tenable we must suppose that the lifetime of the excited $^7He_\Lambda$ against radiative de-excitation to its ground state, i.e., the transition of the ^6He core to its $J^\pi=0^+$ ground state, is several times longer than the decay time of the Λ°-hyperon in $^7He_\Lambda$, say several times 10^{-10} sec. Is this reasonable ? The core de-excitation is basically an E2 process and, if the excited $^7He_\Lambda$ were of J=5/2, which we have no difficulty in hypothesizing, we should not be worried about possible M1 admixtures. If the excited state is at about 1.7 MeV, as in the ^6He itself, the Weisskopf estimate of the lifetime would be a few times 10^{-11} sec and so the lifetime against radiation would be considerably too short to allow Λ°-decay from the excited state. But the "radiating particles" in ^6He are both neutrons so if the shell model were literally true the radiative lifetime would be much longer than the Weisskopf estimate, indeed, with harmonic oscillator wave functions, infinite. What do we expect ? We may represent the situation phenomenologically by awarding to each neutron an effective charge x and then calculating the expected rate using the usual IPM wave functions. This is the standard procedure, the effective charge so defined parameterizing the effect of configuration mixing, collective motion, polarization of the core or whatever you like to call it. It would be exceedingly interesting from the nuclear structure point of view to know the speed of this transition, the value of x. To the lightest nuclei investigated accurately, namely A=10, x retains a remarkably constant value of about 0.5. This would not be small enough to allow Λ^o-decay from the excited state. There are some signs, not yet confirmed, that roughly the same figure may hold in ^6Li for the E2 transition to ground from the first excited state. It would be most interesting to know the situation in ^6He, the lightest nucleus in which an identifiable E2 transition is available out of a pure neutron configuration. We do not see how to measure this by conventional means since ^6He is not available as a target nucleus and the 2^+ state is unstable by 0.77 MeV against break-up into α+2n. It may be that here, by using the lifetime of the Λ^o-hyperon suitably adjusted to allow for non-mesonic decay as indicated by experiment, as the time scale, we can measure the transition within $^7He_\Lambda$. [In passing we may make a remark about the decay of the excited $^7He_\Lambda$. Decays such as $^7He_\Lambda \rightarrow {}^3H + {}^4He + \pi^-$ which break up the core must take away the excitation energy that we hypothesize as 2 MeV or so, because the residual nuclei have no excited states. But the decay $^7He_\Lambda \rightarrow {}^6He + \pi^- + p$ will have a strong tendency to leave ^6He in the excited 1.71 MeV parent state which will then break up : $^6He \rightarrow {}^4He + 2n$. Such decays will be established uniquely only with great difficulty. If ^6He is formed in its ground state, identified by electron emission, then, of course the excitation energy must have got involved in the decay although this is a second order process. There may be some indication that the $^7He_\Lambda$ decay, leading to ^6He in the ground state, shows a larger binding energy than decays in which no excitation can be left behind. Events that could be $^7He_\Lambda \rightarrow {}^7Li + \pi^-$, for which the maximum excitation of the ^7Li can be 0.48 MeV, also tend to show the larger binding energy. We can see that the ^6He producing events will tend to come from the ground state of $^7He_\Lambda$ as explained and so should tend to show the larger binding energy. It is not so easy to see why the decay of excited

$^{7}He_{\Lambda}$ should not produce the ground ($J^{\pi}=3/2^{-}$) and 0.48 MeV ($J^{\pi}=1/2^{-}$) states of ^{7}Li. The fractional parentage coefficient linking the 2^{+} excited state of ^{6}He with the ground state doublet of ^{7}Li is $-\sqrt{2/3}$ while that linking the ground state of ^{6}He with the ground state doublet is $-\sqrt{5}/(3\sqrt{2})$ which is almost the same, so excited $^{7}He_{\Lambda}$ should be as ready to form the low-lying states of ^{7}Li in its decay as is the ground state of $^{7}He_{\Lambda}$. Again, the fractional parentage coefficient linking the excited ^{6}He state with the unstable $J^{\pi}=7/2^{-}$ and $5/2^{-}$ states of ^{7}Li is $-1/\sqrt{2}$ so there is no strong intrinsic preference for decay to these excited states rather than the low-lying states. (These parentage coefficients are in LS coupling which is a good approximation in these very light nuclei.) But if the low-lying states of ^{7}Li are formed from the decay of the excited state of $^{7}He_{\Lambda}$ we should find the lower binding energy showing up in this mode. So to accept the excited hypernucleus explanation for the $^{7}He_{\Lambda}$-$^{7}Be_{\Lambda}$ discrepancy we must understand why the lower binding energy is not found in decays to the bound ^{7}Li states and also why the decay of the $^{7}He_{\Lambda}$ ground state does not go through the mode $^{3}H+^{4}He+\pi^{-}$ which seems to show only the lower binding energy. Of course, statistics are very poor so far and these worries may resolve themselves naturally.]

This competition between Λ^{0}-decay and radiation is an amusing and exciting possibility for gaining important nuclear structure information from hypernuclear properties. Are there other places where the trick may be played ? It would be interesting to look at $^{11}B_{\Lambda}$ where the binding to the first excited state of ^{10}B should show up since the E2 lifetime is about 10^{-9} sec but the technique is likely to remain pretty limited.

One interesting possibility is raised in $^{17}O_{\Lambda}$. There will presumably be an excited state based on the first excited $J^{\pi}=0^{+}$ state of ^{16}O at 6.05 MeV. Both states of $^{17}O_{\Lambda}$ will have $J=1/2$. Will the hypernuclear excited state decay by pair emission like the core state and so show a comparable lifetime (about 10^{-10} sec) or will the hypernucleus contrive an M1 transition, forbidden on the simple model ? We may be sure it will but how fast will it be ? Perhaps competition with Λ^{o}-decay will tell.

Production and decay of hypernuclei

The whole question of the production mechanisms of hyperfragments by various means and of their decay by various modes is full of fascinating problems in hypernuclear reaction mechanisms. They are too numerous to write down and many seem to be problems in their own right, i.e., do not promise clear information either on ordinary complex nuclei or on elementary particle properties, but there is obviously a great deal to be understood here.

Hypernuclei as labels

Hypernuclei, identified by their decay, are self-labelling and so may be of interest in studying spallation and similar reactions. They may, in particular, be useful in giving information about Coulomb barriers in highly excited nuclei.

Strangeness-exchange studies

The formation of hyperfragments both in direct collisions such as K^- + nucleus and by Λ^{o}-hyperon and $\Sigma^{\pm}$-hyperon bombardments may give information about strangeness-exchanging reactions. The best chance for a Λ^{o}-hyperon to remain bound into a hyperfragment probably comes when a K-meson is exchanged in a peripheral collision, the resulting Λ^{o}-hyperon having low energy. Study of hyperfragment production may then give information on such peripheral processes.

K^--MESON ABSORPTION

Peripheral absorption of stopped K^--mesons

We consider the absorption of stopped K^--mesons. The argument that this absorption is predominantly peripheral, in the "nuclear stratosphere", has been put forward many times and will not be repeated in detail. Basically it is that the K^--mesonic atomic orbits become circular in the course of the K^--meson's progression towards the nucleus because of the greater statistical weight of the higher angular momenta. This means that the K^--meson's wave function as we penetrate into the nucleus from outside is a rapidly-falling function of position. Furthermore the K^--N interaction is so strong that absorption, by $K^- + N \rightarrow Y + \pi$ (real or virtual pion), takes place in the region effectively outside the nuclear surface where, as we go inwards, the rapidly-falling K^--meson wave function first overlaps significantly with the rapidly-rising nuclear density distribution. If this is so then the absorption of stopped K^--mesons brings us news of dilute nuclear matter and, in particular, the high probability of non-mesonic absorption : $K^- + NN \rightarrow Y + N$ found in practice shows that nuclear correlations are as strong in the nuclear stratosphere as they are in the alpha-particle. Several objections have been raised to this picture; the chief being :

i) the K^--meson orbits are circular all right but the first interaction may be a scatter not an absorption so that although it may take place outside the nucleus it merely knocks the K^--meson into the body of the nucleus and the actual final absorption may happen inside the nucleus;

ii) the processes of Auger transitions followed by X-ray transitions by which the K^--meson works its way in towards the nucleus are not adequate to achieve pure circular orbits so that absorption takes place inside the nucleus from orbits of lower ℓ-value than the maximum, $\ell < n-1$;

iii) we must treat the K^--mesonic atom as a single system, nucleus plus meson, and allow that the electrostatic interaction between the two components may give total states in which combination of K^--meson orbits are added to combinations of virtually-excited nuclear states together with the nuclear ground state. This has two effects : the first is that lower ℓ-values get involved for the K^--meson so the meson's wave function is brought closer in to the nucleus; the second is that the excited nuclear states may be considerably deformed and so stick out to meet the K^--meson, the absorption taking place in the meaty part of the protuberances rather than in the dilute region of the simple model;

iv) the absorption may go in two stages, the first being the formation of a Y^*-hyperon which then wanders into the nucleus, the second being the absorption of the Y^*, $Y^*+N \rightarrow Y+N$ in the nuclear interior.

These various objections can be answered to some degree theoretically. Rook has treated the first and shown that, although the scattering and absorption cross-sections are comparable, the "scattering in" effect is negligibly small, chiefly owing to the poor overlap between the initial and final state K^--meson wave function, but with other effects contributing significantly. Rook has also considered the second objection and shown that, although lower ℓ-values than n-1 are still significant by the time absorption takes place, the absorption is still overwhelmingly peripheral. Martin has made an extended version of this calculation using Coulomb functions for the Auger electrons and finds a more peripheral capture than Rook. The third objection is an interesting one. The calculations of Fowler and Crossland used an approximation in which the coupling of the K^--meson orbit to the nucleus to produce the virtual excited states enjoyed the same collective enhancement as the free real de-excitation of those states and so was preferentially concerned with the low-lying collective quadrupole states. In this approximation an appreciable non-mesonic absorption probability was found without having to hypothesize correlations in the stratosphere. But there is no reason for taking this approximation and, by using a different but equally-plausible approximation, Rook has obtained results, by the same method, that give no significant non-mesonic absorption due to this effect, leaving the bulk still to be accounted for by peripheral correlations. The fourth objection is difficult to discuss because it is difficult to make it quantitative. It certainly seems likely that at least one species of Y* is made in considerable abundance in K^--meson absorption process (see later).

It seems to me that, at this stage, we ought to stop arguing about this problem of peripheral absorption or otherwise and, if it is interesting enough, do something about it. What can we do about it ? There are several tests than can be made :

i) K-mesonic X-rays. The calculations, for an assumed optical model K-nucleus potential, tell us what X-rays should come from the stopping of K^--mesons, at which member of the chain the series should break off if everything is going according to plan. This should be investigated throughout the periodic table. If the cascade and absorption mechanisms are once understood and the interaction turns out indeed to be peripheral, K-mesonic X-rays become in themselves a valuable tool for studying details of the radial distribution of the nuclear density and have an obvious importance for nuclear structure in their sensitivity to shape effects, shell effects and so on. Agreement of the X-ray spectrum with that calculated for circular orbits with peripheral interaction, while a necessary condition, would not, of course, be sufficient to show that absorption is peripheral because objections i) and iv) would still be in play;

ii) details of the absorption process. We know that some absorption is peripheral because, if we choose absorption events of the $\Sigma - \pi$ type with no other particles showing, or only a short recoil, there is a strong anti-correlation in the directions of the Σ and π. This needs working out in more detail. We should calculate, using Monte-Carlo and/or optical model methods, what value we should expect for the fraction of all absorptions that give just $\Sigma + \pi$ in this way if the absorption is peripheral and what fraction should lead to more-complicated stars, and compare it with what we see in practice. For those cases where we see just $\Sigma + \pi$ in the final state what should be their angular correlation, assuming some reasonable nuclear wave function or momentum distributions and again handling the final state distortions by some means ? (Of course, if the whole thing were done by Monte-Carlo methods with allowance for the real part of the potentials everything, clean events plus complicated stars, would come out in one go.) Then do the whole calculation again for volume absorption. If the experimental situation agrees substantially with one picture of the absorption and not with the other we have evidence. This would not answer all questions. In particular, it would not answer the objections that allow one-nucleon, i.e., $Y + \pi$ producing, absorption to be peripheral but want the two-nucleon, non-mesonic absorption to be volume, by Y^*, knock-in, or some other means. Here we need experiment as well as calculation. If the non-mesonic, $Y+N$, absorption is peripheral, then in the final state the fast hyperon and nucleon should be anti-correlated - probably more sharply so than in the one nucleon, $Y+\pi$, case because the momentum of the product particles is considerably greater for $Y+N$. Experimental data have not been reported here but should be available;

in particular, K^--meson capture on an np pair should give a Σ^-+p final state susceptible of study in nuclear emulsions.

Another interesting way of doing many of these studies throughout this K^--meson absorption field, one that would bring a great deal of most valuable supplementary information, would be to use a xenon bubble chamber instead of nuclear emulsion. This would enable neutral particles such as the Λ^0-hyperon to be studied and also indicate the production of π^0-mesons (Σ^0-hyperon production could similarly be distinguished at sight from Λ^0-hyperon production). Note that a freon chamber, while collecting the gamma rays, would not be nearly so satisfactory because the elements are not so heavy, and so the surface problem is not so well-defined, and we would not know in which nuclear species absorption had taken place. Use of a xenon chamber would greatly speed and greatly facilitate and extend this work.

To return to the immediate point, if we wish to maintain that the non-mesonic absorption is peripheral and so indicates nucleon clusters in the nuclear surface then the Y-N anti-correlation must be there and to a degree that could be computed as for the Y-π case. Is it ?

iii) in-flight absorption. If we absorb K^--mesons in flight, either slow, (ℓ=0), or fast, then the absorption will be in the interior of the nucleus or at least below the stratospheric region possibly operative for absorption at rest. In this case we should expect the spectrum of absorption products to be different from that due to absorption at rest. But if absorption at rest is itself a volume process the two sets of products will be similar - the more nearly the same the lower we make the K^--meson energy in the in-flight case. If the correlations in the stratosphere are the same as those in the depths we should get more fast non-pion-accompanied hyperons from the at-rest absorption than for the in-flight because of the added difficulty that the hyperon has in getting out of the nucleus in the in-flight case. Such experiments could be most interesting and should be done at as low a K^--meson energy as possible.

We may note in passing that it may be convenient to use K_2^o-mesons for this purpose as well as, or rather than, K^--mesons. Absorption of K_2^o-mesons of around 100 MeV, which energy is higher than one would like, suggests that 2-nucleon absorption is not strong in the nuclear interior. It would indeed be fascinating if the correlations turned out to be stronger in the stratosphere than inside the nucleus.

vi) objection iii) depends on the virtual excitation of strongly collective quadrupole states. We should look at the non-mesonic 2-nucleon absorption probability as a function of position in the periodic table; if this objection is valid it should have a maximum in regions where collective oscillation is strong.

These tests and calculations together should establish whether both types of absorption, mesonic and non-mesonic, are largely stratospheric or not. Let us assume that they indeed are and examine the kinds of work that then become possible. Much of this has already been discussed elsewhere and will not be dwelt on here.

Texture of the nuclear surface

The abundant non-mesonic absorption shows that high momentum states are common in the stratosphere. It will be important to make a quantitative statement as to their abundance relative to the low momentum states that make up the bulk of the wave function even in dense nuclear matter. This can be done by comparison with results from absorption of stopped K^--mesons in deuterium for which we know the momentum distribution reasonably accurately - or at least much better than we do in complex nuclei. It has already been shown that the calculated non-mesonic probability for absorption in ^4He based on the experimental figure for deuterium comes out right. The fact that the non-mesonic probability in complex nuclei is about the same as in ^4He may be suggestive. It is certainly consistent with the idea that the stratospheric high momentum states are contained in alpha-particle-like structures of presumably very short lifetime that constantly form and redissolve into the body of the nucleus. We now have a lot of data about the products of K^--meson absorption in helium. It would be most interesting to take these as the input data of an optical model/ Monte-Carlo calculation to estimate what happens if such absorption takes place in stratospheric "alpha-particles", and compare this with what is found in practice. One rather clear test that could be made would be to look for evidence of peripheral $K^- + "^4He" \rightarrow {}^4He_\Lambda + \pi^-$ in complex nuclei. With peripheral "alpha-particles" this should happen quite often and not infrequently the products should get clean away. That a heavy nucleus was involved might be picked up in the usual way by finding Auger electrons. The distortion and "compound-nucleus-forming" effects in the final state here should not be terribly difficult to evaluate for this simple case. (Note that "getting clean away" does not imply that other heavy particles may not emerge from the star; a fair measure of excitation in the parent body of the nucleus may have accompanied the launching of the "alpha-particle" into the stratosphere. This point and related points must always be borne in mind in discussing the consequences of peripheral absorption : the rest of the nucleus is not necessarily anywhere near its ground state.) If the calculated probability of this process should agree with expectation based on the known probability for forming $^4He_\Lambda$ in helium and the assumed stratospheric absorption then we have useful evidence that the absorption is stratospheric and that there are "alpha-particles" out there. Conversely if the answer is very different from what we expect we are not in a position to claim the simultaneous validity of both starting points of the computation.

Questions of parentage

The fact that many K^--meson absorption events at rest consist of clean $\Sigma + \pi$ production, as revealed by the good Σ-π anti-correlation, means that such processes are a fine tool for investigating nuclear parentage. If, as is likely, such events are chiefly peripheral, final state excitations, which would distort the spectrum of residual states from the parentage spectrum will not be strong and can, to a fair degree, be calculated. It may appear that we can, in principle, sort out this question of secondary excitation to some degree by comparison with experiments using slow K-mesons in flight where the final-state excitations will be stronger, but this begs an important question to be touched on later, and good calculations, aided by empirical data on nuclear and pion interactions, are needed.

What is the point of such work, which will certainly be more difficult by far than the carrying out of processes such as A(p,d)B that seem to give just the same information ? It may be that the K^--meson absorption process is a cleaner one. We argue naively that if we choose Σ-π events with a large angle between the two particles then the final state interaction has not been strong and so the residual excitation spectrum will be the true parentage spectrum. This is a dangerous argument and needs theoretical investigation to which it is susceptible. The analogous argument in, say, the (p,d) case might be that if we confine attention to the main pick-up peak (for a particular ℓ-value) we are looking at events that satisfy the classical kinematical criterion for clean pick-up and so again will reveal the true parentage spectrum. But we know that this is wrong and that, although the peak may be at its "classical" position in some cases, this is an accident due to essentially chance cancellation of considerable Coulomb and nuclear distortions with which excitation and so a change in the parentage spectrum may well be associated. Do we have the same phenomenon in the $\Sigma+\pi$ case ? Good distorted wave calculations would be very interesting here. But, whatever the situation in respect of distortions, there are problems of such importance for nuclear structure to be tackled by nucleon-removal methods that any new approach should be pursued.

An obvious problem is testing for excited configurations in ground states. How close is ^{16}O to $(p)^{12}$? If it contains some, say $(d_{5/2})^2$, $(2s_{1/2})^2$ and $(d_{3/2})^2$ then the reaction :

$$K^- + {}^{16}O \rightarrow {}^{15}N + \Sigma^+ + \pi^-$$

will excite $J^\pi=5/2^+$, $1/2^+$, $3/2^+$ states of ^{15}N as well as the expected $J^\pi=1/2^-$, $3/2^-$ states. This we can investigate experimentally by Σ-π measurements. It may be experimentally most advantageous to use the Σ-π correlation without specially good energy resolution (permitting the use of targets of workable thickness) to establish the "clean" character of the absorption and use coincidence gamma-ray measurements to determine the residual ^{15}N states. This type of work can be carried out in many regions of the periodic table and may give valuable information on the structure of the nuclear ground state. Such measurements should obviously be combined with conventional stripping studies, $^{14}N(d,p)^{15}N$ in the present example. Obvious cases for study abound. Will the reaction $^{14}N(K^-,\Sigma^+\pi^-)^{13}C$ excite the 3.09 MeV $J^\pi=1/2^+$ and 3.85 MeV $J^\pi=5/2^+$ states of ^{13}C, showing up the $(2s_{1/2})^2$ and $(d_{5/2})^2$ components of ^{14}N, and if so in what intensity relative to the 3.68 MeV $J^\pi=3/2^-$ state that comes from $(p)^{10}$? Will such pairing states of the A ground state, broken up by K^--meson capture, tend to have as parents the states of "wrong" parity in A-1 that we are familiar with as being formed by adding a loose nucleon to the parent state of A-2 ? In the case of $^{16}O \rightarrow {}^{15}N$ will the ^{15}N states formed by $^{16}O(K^-,\Sigma^+\pi^-)^{15}N$ be those at 7.16, 7.31, 7.58 and 8.32 MeV strongly formed with $\ell=2$, 0, 2 and 0 respectively in $^{14}N(d,p)^{15}N$?

The states in ^{13}C quoted as possibly reached from ^{14}N have the ground state of ^{12}C as a fairly strong parent.

On a less ambitious level it will be interesting to investigate parentage within the dominant configuration, for example to determine the spectrum of odd parity excited states in $^{12}C(K^-,\Sigma^+\pi^-)^{11}B$ and compare it with what we think we know from independent particle model calculations and conventional experiments.

It may be that K^--meson absorption brings some advantages in studying residual states of rather high excitation. For example the fact that no momentum is brought in by the absorption reaction may make it easier to study residual states unstable by a few MeV to proton or neutron emission. Such states could also be studied in reactions such as (p,2p) but there the kinematics do not favour accurate work on residual states not far above the particle thresholds.

Perhaps we should not expect parentage as revealed by K^--meson absorption at rest to be the same as that revealed by methods, theoretical or experimental, that probe the nuclear volume because the K^--meson absorption at rest may well be stratospheric and the parentage in respect of nucleons in the tail of the matter distribution may well be different from that of the bulk of the nucleus. This is a most interesting point and if we can assure ourselves that absorption at rest is indeed peripheral we have a tool for probing a different aspect of the nuclear wave function from conventional pick-up and stripping. A comparison of at-rest and slow in-flight absorption is of interest here and should show the difference between surface and volume parentage, provided the final state processes can satisfactorily be coped with somehow, as mentioned earlier.

Another whole class of interesting parentage questions concern the multi-nucleon processes. What is the nuclear parentage in respect of the high momentum NN states that we are tentatively assuming to be found by K^--meson absorption at rest in the surface ? How does this parentage compare with that found from π^--meson absorption or the high energy quasi-deuteron photo-effect both of which more nearly probe the nuclear volume ? What is the parentage of surface or stratospheric "alpha-particles" as revealed say by $K^-+A \rightarrow B+{}^4He_\Lambda+\pi^-$? How does this compare with the parentage revealed by, for example, $(p,p\alpha)$ reactions ?

An advantage that the K^--meson methods that produce pions have in many of these investigations is that they lay no special momentum requirements relative to the centre of mass of the nucleus on the absorbing body, nucleon or cluster. This is not in general true of the other methods, particularly pick-up, nor is it true of a knock-out reaction such as $(p,2p)$ if definite angular or energy discrimination is imposed on the outgoing particles. Of course, if we experimentally impose a strong anti-correlation on the $\Sigma-\pi$ system in the case of K^--meson absorption we are imposing a momentum condition here also. It would be interesting to investigate the parentage as seen through K^--meson absorption as a function of the $\Sigma-\pi$ angular condition.

Momentum distributions

In principle, the products of K^--meson absorption carry information about the momentum distribution of the capturing nucleons (for the mesonic absorptions) or clusters (for the non-mesonic). In practice, the theoretical primitive distributions (such as $Y-\pi$ or $Y-N$ opening angles, Y energy distributions, etc.) are not very sensitive to the underlying momentum distribution, and, in any case, will be gravely affected by final state interactions. As had already been implied, it will be very interesting to check the experimental distributions against the theoretical distributions with inclusion of final state effects but it seems likely that the chief value of this will be in testing ideas about the absorption process rather than finding anything meaningful about the momentum distributions, although it will obviously be possible to rule out extreme forms for them. These remarks apply only to the momentum distributions relative to the rest of the nucleus of the nucleons or clusters and do not reflect on the point discussed earlier that the relative probability, without regard to detail, of non-mesonic versus mesonic absorption tells us the relative probability, without regard to detail, of high and low momentum states - the former now being chiefly within a cluster presumably.

This last remark opens up a whole new question however : are the high momentum states in the nuclear wave functions associated solely with the close approach of nucleons to each other or do they belong in part to "isolated" nucleons ? It is extremely difficult to gain reliable information about the nuclear momentum distribution and all methods are bedevilled by final state interaction effects and so on. In particular we have no present answer to the question as to whether high momentum states ($p >> \hbar/R$) exist for nucleons not in intimate interaction with others or whether it is only such interaction that gives them. To be sure, some non-strange reactions offer an approach to this problem. For example, we can look for single fast nucleons emerging from the interaction of high energy photons or stopped π^--mesons with nuclei. If the high momentum states come from the close interaction of two nucleons the final state originally contains two fast nucleons. The emergence from the nucleus of such anti-correlated fast nucleons is well known for both photon and π^--meson absorption. But one of the nucleons may get stuck in the nucleus and this is common for all but the very lightest nuclei. In this case we may find a single energetic nucleon emerging such as we should expect if the absorption had been upon an isolated high-momentum nucleon but probably accompanied by several (undetected) low energy particles. The inevitable centre-of-mass motion of the high-momentum pair gives a considerable spread in energy of such single emergent nucleons. It is clear that most high momentum states are associated with close nucleon-nucleon interactions because, for example, the spectrum of energetic photo-protons from high energy bombardment shows a break at $E_p \simeq E_\gamma/2$ but we have little information from such work about the one-nucleon component of the high momentum states. Such information would, however, be forthcoming if we could detect, for example, the production of the ground state of ^{11}B in the reaction :

$$^{12}C+\gamma \rightarrow {}^{11}B+p$$

with photons of, say, 200 MeV. (In order to leave ^{11}B in its ground state either the ^{12}C at the time of absorption contained a single proton in a high momentum state which departed leaving ^{11}B or the proton in ^{12}C owed its high momentum to a close interaction with (presumably) a neutron in which case the residual ^{11}B contains a single nucleon in a high momentum state. We cannot escape the conclusion that one or other nucleus would reveal in this way a high momentum state of a single nucleon; if the initial high-momentum proton were part of a close three-nucleon cluster for example, then after its departure the other two nucleons would be in high but unbalanced momentum states so that after separation they would represent single nucleons in high momentum states. And so on.)

Similarly such a reaction as :

$$^{12}C+\pi^- \rightarrow {}^{11}B+n \qquad \text{or} \qquad \rightarrow {}^{11}Be+p$$

using stopped π^--mesons would do the trick. Of these two types of reaction the former probably cannot be done with adequate precision of energy and momentum determination since sufficiently narrowly monochromatized gamma-rays of high energy are not available. The second reaction could be used for this purpose although the neutron-producing branch would be difficult to determine precisely. The more-easily dealt with proton-producing branch cannot refer to single high momentum nucleons in the initial state owing to the two units change of charge.

The absorption of K^--mesons at rest gives another avenue to this problem. Consider for example :

$$^{12}C+K^- \rightarrow {}^{11}C+\Sigma^- + 219 \text{ MeV}$$

This reaction, following the argument just made, would demonstrate one-nucleon high momentum states in one or other of the two nuclear ground states.

Surface composition

The problem studied by Jones in his original discussion of the surface absorption of stopped K^--mesons, namely the neutron versus proton composition of the nuclear surface, has never been settled but is important. There are many unknowns : final state interactions, here capable of $\Sigma^0 \rightarrow \Sigma^+$ conversion or $\Sigma^- \rightarrow \Sigma^0$ for example, are important and so also are the basic production data, still not fully available. This problem will repay study. It may, perhaps, be pushed forward more quickly by imposing conditions on the absorption and by more complete observation. If, for example, we used a xenon bubble chamber we would, firstly, require that the absorption was peripheral by the usual tests and, secondly, pick up the neutral particles (the π^0-decay products give tolerable indication of the π^0-meson's initial direction so that the usual anti-correlation could be demanded even in $K^-+p \rightarrow \Sigma^0+\pi^0$ for example).

Three-body forces

We are astonishingly ignorant about three-body forces among ordinary nucleons. A possibility of getting some information presents itself through K^--meson capture. This is illustrated by considering :

$$K^- + {}^3He \rightarrow \Sigma^- + p + p$$

If three-body forces are important, close triangular configurations will occur between the nucleons in ^3He, and this absorption process manifests such a situation in a more-or-less equal sharing of the available energy among the three final particles. This method will be better than the alternative $\gamma + {}^3He \rightarrow 2p+n$ or $\pi^- + {}^3He \rightarrow p+2n$ because it does not have the unwanted charge-selectivity of the former or the neutral particles of either.

Y*-phenomena

The observation that the reaction :

$$K^{-}+{}^{12}C \rightarrow {}^{11}B+Y^{*}_{1405} + 11 \text{ MeV}$$

producing the 1405 MeV Y* excited hyperon and the ^{11}B ground state goes very readily raises many interesting questions. The first is the mechanism of the reaction. The final state momentum is low and so an impulse-approximation treatment in which a proton of the normal ^{12}C ground state "shell-model" momentum distribution simply absorbs the K^{-}-meson and leaves the nucleus is probably adequate. (Y* production is a non-mesonic absorption process but the low momentum in the final state means that, unlike the direct non-mesonic hyperon-producing reaction, it is not necessary to invoke a correlated nucleon pair in the absorption process.) In this case we should expect the ^{11}B to be left behind in the parent states of the ^{12}C ground state, i.e., the ground state and first excited state at 2.13 MeV, $J^{\pi}=3/2^{-}$ and $1/2^{-}$ respectively. (Absorption on a $1s_{1/2}$ proton may also lead to Y* emission but not abundantly since not much energy is available and the $(1s_{1/2})^{-1}$ configuration, although very broad, also rather deep below the Fermi-surface - about 18 MeV according to data from (p,2p).) We know from the interpretation of the level schemes of these and neighbouring nuclei given by the independent particle model in intermediate coupling, that the squares of the fractional parentage coefficients connecting the ground and first excited states of ^{11}B with the ground state of ^{12}C with respect to a p proton stand in the approximate ratio 10:1. This is in agreement with the relative population of the two states found in the reactions $^{12}C(p,2p)^{11}B$ and $^{12}C(p,pn)^{11}C$ at proton energies of about 150 MeV, reactions closely analogous to Y* production in that they simply involve the removal of a nucleon without disturbing the rest of the nucleus. What is the situation in $^{12}C(K^-,Y^*)^{11}B$? It is that the ground state to first excited state production ratio is certainly not much less than 10:1 which is in agreement with the other two lines of attack which are of a very different type. This is pleasing.

What about other nuclei ? All data so far come from nuclear emulsions. Capture in oxygen should be roughly as likely as in carbon. What about $^{16}O(K^-,Y^*)^{15}N$? It seems to be rare to the ^{15}N ground state which has Q=14 MeV, considerably less abundant than the corresponding reaction to the ^{11}B ground state. Why ? Naively we may argue that the ^{12}C reaction involves a $1p_{3/2}$ proton of which there are 4 while the ^{16}O case involves a $1p_{1/2}$ proton of which there are only 2. We may further say that, owing to the greater energy release in the ^{16}O reaction, the impulse approximation mechanism calls for the proton to be in a less-richly populated region of the momentum spectrum. However, the empirical data on momentum distributions in light nuclei, unsure as they are, suggest that something like a Gaussian of width about 12 MeV may be not far off the mark and this would scarcely give a large discrimination against oxygen. But if these explanations are, together, the full explanation what about the $J^{\pi}=3/2^-$ state of ^{15}N at 6.33 MeV ? As we understand it this state is $(1p_{3/2})^{-1}$. The Q value for reaching it is 8 MeV so not only do we have as many protons available for getting to it as in the ^{12}C case but the momentum requirement is lower so the reaction should go more easily. Does it ? Apparently not. Here we have a bit of a puzzle, the resolution of which obviously involves nuclear structure considerations and so contains nuclear structure information. We have to remember that the Y^* production is in competition with many other processes while the considerations here presented apply only to the partial width for forming the boron and nitrogen states. The resolution of the problem may lie in finding why the competing processes such as straightforward $K^-+N \rightarrow Y+\pi$ are strong in ^{16}O rather than why the Y^* production is there weak. But whatever the explanation it clearly must be nuclear-structure independent.

Y^* production may become a tool for parentage studies in the way we have discussed earlier for other products of K^--meson absorption. We see that it is also sensitive to the nucleon momentum spectrum, and in a cleaner way than the other processes already discussed.

K^+-MESON SCATTERING

Knock-out reactions

K^+-mesons have no interesting at-rest properties for nuclear structure nor are interactions in flight at low energies likely to be useful, unlike the K^--meson, the K^+-meson has no hyperon-producing reaction at low energies. There may, however, be some scope for K^+-nucleus interactions in the 100 MeV range and above. It is always interesting to compare neutron and proton phenomena to get some idea about the similarities and differences between neutron and proton distributions and motions. For example no unbiased comparisons are yet available for differential cross-sections of (p,2p) and (p,pn) processes in the "knock-out" region above 100 MeV. This is largely for technical reasons - it is very difficult or impossible to measure the neutron production in the second reaction without imposing any angular constraint and such a constraint immediately introduces a bias into the experiment. Use of K^+-mesons may overcome this difficulty because we can now study, as the analogue of the first reaction, (K^+,K^+p) and, as the analogue of the second, $(K^+,K^\circ p)$, the knock-out neutron being converted into a measurable proton by a charge-exchange process and the resultant K°-meson itself being accurately measurable (in half the cases) through its decay. We clearly need to know quite a bit about K^+N interactions before interpreting such work quantitatively.

Isobaric states

Another interesting possibility, the excitation of isobaric levels, for example $^{12}C(K^+,K^{+'})^{12}C_{15.1}$ and $^{12}C(K^+,K^o)^{12}N_{g.s.}$, is not so exciting because here the situation is kinematically much simpler and the same comparison could be made much more simply using (p,p') and (p,n) although the K-meson check would be interesting. A possible valuable feature of the K^+-meson situation is that the K^+-nucleus potential is repulsive so resonance distorted wave effects would not give the structure to the cross-sections that sometimes confuses the issue when nucleons are used.

ABSORPTION OF Σ^--HYPERONS

Σ-hypernuclei

Σ^--hyperons can be made quite abundantly and it is quite realistic to contemplate studying their absorption in a variety of nuclei. We may take it that their own properties of first interest, namely their spin (J=1/2) and parity relative to the Λ^o-hyperon (most likely even), are known so that we are thinking of their interactions with complex nuclei as possibly bringing information of nuclear structure interest rather than of interest in relation to the Σ-hyperon itself. An exception to this could arise if the interaction with complex nuclei could bring quantitative information about the Σ-N coupling. Such would be the case, for example, if Σ-hypernuclei existed for a length of time that permitted their definition and identification. The only Σ-hypernuclei stable against the conversion reaction :

$$\Sigma + N \rightarrow \Lambda^o + N$$

are Σ^+p and Σ^-n. Present indications are that these species are not bound. Heavier Σ-hypernuclei are very unlikely to live for a time long compared with the imaginary part of the Σ-nucleus optical model potential which is necessary to define a hypernucleus reproducibly. However, we may gain kinematical evidence for such intermediate states as, for example :

$$K^- + {}^{12}C \rightarrow {}^{12}C_{\Sigma} + \pi^- \rightarrow {}^{8}Be_{\Lambda} + \alpha + \pi^-$$

and this would contain information on the Σ-nucleus interaction. Of course, such information will come more obviously from the free-state scattering of Σ-hyperons.

Single-nucleon high momentum states

Turning to the absorption of Σ^--hyperons by complex nuclei, what might we learn about complex nuclei ? One possibility concerns the presence in the nuclear wave function of high momentum states of individual as opposed to paired nucleons. The argument follows that already made in connection with the production of fast hyperons in K^--meson absorption. Here we absorb stopped Σ^--hyperons and look, for example, at the reaction :

$$^{12}C + \Sigma^- \rightarrow {}^{12}B + \Lambda^0 + 67 \text{ MeV}$$

This reaction can use a single high-momentum proton in the initial state. The momentum transfer here is different from that in the case of K^--meson absorption and so the information brought by the two processes is complementary. (It may be noted in passing that the nucleon-ejecting reactions such as :

$$^{12}C + \Sigma^- \rightarrow {}^{12}Be_{\Lambda} + p$$

may be very interesting in revealing the excited state structure of a limited class of hypernuclei.)

Three-body Λ^0-hyperon forces

Another possible sort of information forthcoming from the capture of Σ^--hyperons in complex nuclei concerns the importance of three-body forces in the interaction of the Λ^0-hyperon with nucleons such as have already been mentioned. Consider now the capture of a stopped Σ^--hyperon in a light nucleus such as ^4He in which we might hope final state interactions are not too important. Capture on single nucleons cannot yield fast protons; these can come only from capture on pp pairs. The spectrum of fast protons and, in particular, its correlation with the Λ^0-hyperon spectrum will then depend on whether in the interaction as the proton, neutron and Λ^0-hyperon separate, two-body or three-body forces are significant. Qualitatively, three-body forces will make for a more equal momentum sharing among the three particles than two-body. Clearly, it will be very difficult to gain any clear-cut answer, particularly in view of the inevitable final state interactions between the product particles and the remainder of the absorbing nucleus, but it is very difficult to get a line on this three-body force by any method.

Super-neutron-rich nuclei

An amusing possibility is that of getting some information about super-neutron-rich light nuclei using Σ^--hyperon absorption. As an example suppose we could identify the reaction :

$$\Sigma^- + {}^{11}B \rightarrow {}^{10}Li + p + \Lambda^0$$

that could go by $\Sigma^- + p \rightarrow \Lambda^0 + n$ followed by charge-exchange scattering of the neutron. The ^{10}Li will quickly break up : $^{10}Li \rightarrow {}^9Li + n$ but it may be long enough lived to enable the Σ^--absorption to look like an effective three-body reaction. In this way we might find the effective ground state mass of ^{10}Li which would be very interesting. It is very difficult to make such super-neutron-rich nuclei by conventional means under circumstances where all energies are well-defined. The advantage of the Σ^--hyperon here is its negative charge and high and well defined energy release.

ABSORPTION OF Ξ^--HYPERONS

Double Λ^o-hyperon production in ^{4}He

It may become possible to study the absorption of Ξ^--hyperons in complex nuclei. This will happen first in bubble chambers so we restrict attention to absorption by complex nuclei that are found in bubble chambers namely ^{4}He and ^{12}C. From what orbital states will stopped Ξ^--hyperons absorb into these nuclei ? We do not know. The situation is more complicated than for stopped K^--mesons. The Ξ^--hyperon is heavier so its radiative transitions are faster but its orbits are smaller so it reaches the nuclear surface in higher angular momentum states for a given principal quantum number. There is no reason to suppose the Ξ^--nucleus interaction is weak but the ultimate conversion reaction :

$$\Xi^- + p \rightarrow 2\Lambda^o$$

is relatively slow because selection rules are very restrictive and the K^--meson vertices may be rather feeble. If we want to base arguments on the dominance of initial s states the only thing to do in the absence of a detailed theoretical treatment of absorption at rest is to study the in-flight absorption of slow (few MeV) particles which will favour s state interactions. This is obviously a very difficult business but let us see if anything can come of it.

We ask whether the s wave absorption reactions can give any information about the Ξ-N parity. Consider :

$$\Xi^- + {}^4He \rightarrow {}^3H + 2\Lambda^0 + 6\ MeV \qquad (1)$$

$$\text{or} \rightarrow {}^4H_\Lambda + \Lambda^0 + 8\ MeV \qquad (2)$$

Take $J_\Xi = 1/2$ for illustration. Also assume that, in emerging from the absorption, the Λ^0s do not have a large probability of flipping their spins. $J=1/2$ in the initial state. If the Ξ-N parity is even all particles will emerge in relative s states from reaction (1) owing to the small energy release, the $\Lambda^0\Lambda^0$ will be a singlet and this will be revealed, statistically, in the spin-spin correlation as measured by the decay asymmetries, the Λ^0-hyperon being an excellent self-indicator of its own spin state through the parity violating decay : the two decay pions will tend to go off into opposite hemispheres. The obvious way to tackle the Ξ-N parity problem is to study Ξ^--hyperon capture in hydrogen, using the decay of the Λ^0s as described here to indicate the relative spin states : if, then, we are confident of s state capture the relative parity is directly indicated. It may be, however, that the complex nuclei phenomena such as are discussed in this section are found "accidentally" in helium or hydrocarbon bubble chambers before they are in hydrogen with its lower stopping power. We should at least be ready for them. If the Ξ-N parity is odd the situation is more complicated. We must now contain one unit of orbital angular momentum somewhere in the final state. This is unlikely to be in the relative ${}^3H$-$\Lambda^0\Lambda^0$ motion because the Ξ-p encounter that leads to conversion is with both particles in s states relative to the centre-of-mass of the whole nucleus. (Encounters with protons in higher relative angular momentum states lead to the reaction under discussion only very inefficiently since they usually imply considerable excitation in the residual nucleus; this would not be ${}^3H$ and in any case there is no energy to spare for excitation.) The one unit of angular momentum necessary to change the parity could appear in the $\Lambda^0\Lambda^0$ system and so imply a triplet spin state, detected in the decay correlation by a tendency for the decay pions to go into the same hemisphere. However, the $\Lambda^0\Lambda^0$ system is unlikely to be formed in a p state at such low energies since the Ξ-N conversion reaction is of short range ($\simeq\hbar/m_K c$) and so a more likely thing to happen is reaction (2) following "compound nucleus" formation. Reaction (2) can also take place, of course, for even Ξ-N parity but there is then no reason that favours it especially. A model of the process is needed to compare it with experiment. Tentatively we may say that if process (1) is found abundant and indicating a singlet $\Lambda^o\Lambda^o$ state then even Ξ-N parity is strongly suggested while if the triplet $\Lambda^o\Lambda^o$ state is found and/or reaction (2) dominates odd Ξ-N parity is suggested.

Reaction (2) is not directly useful for our present purpose because of the J=0 character of the $^4H_\Lambda$ which means that the captured Λ^o-hyperon does not remember its spin direction. While thinking about this reaction I came upon what appeared to be a jolly paradox and that I pass on for the sake of the moment or two's pedagogical exercise that its resolution brings. If the Ξ-N parity is even and the capture is from an s state then all particles in the final state are in relative s states, in particular the free Λ^o-hyperon relative to that bound into the J=0 ^4H hypernucleus. So the two Λ^os must apparently remain in a singlet relative spin state, despite the J=0 character of $^4H_\Lambda$. Now let them decay. If that in the $^4H_\Lambda$ decays first there is nothing to change the spin state of the remaining, free, one so its eventual decay will be anti-correlated with that from the $^4H_\Lambda$ because its spin was oppositely directed to that of the bound Λ^o. It then seems that directional information has been extracted, through the decay of the bound Λ^o, from a system of J=0. If the free Λ^o-hyperon decays first the bound Λ^o in the $^4H_\Lambda$ forgets its spin direction because there is no longer an identical fermion in the system to keep it locked in place. There is then no correlation in the decays. (Par is 1 minute.) This reaction will be useful to check the expected J=0 for the $^4H_\Lambda$. If J=1 for $^4H_\Lambda$ this will be shown in the decay asymmetry correlation between the hypernucleus and the Λ^o-hyperon but not if J=0.

It may be that, following the usual Day, Snow and Sucher arguments about captures in hydrogen and helium, we can assure ourselves that capture from rest in helium will be predominantly from s states. In this case this investigation becomes distinctly more feasible.

<u>Double Λ°-hyperon production in ^{12}C</u>

We turn to ^{12}C for corroboration of these results. Consider

$$\Xi^- + {}^{12}C \rightarrow {}^{11}B + 2\Lambda^\circ + 10 \text{ MeV}$$

and make the same assumption as before. ^{11}B has two available states, both parents of the ^{12}C ground state. The ^{11}B ground state $J^\pi=3/2^-$ is the stronger parent, as revealed in $^{12}C(p,2p)^{11}B$ for example, but the first excited state at 2.14 MeV, $J^\pi=1/2^-$, is also a working parent. We anticipate that the reaction will go strongly if only s waves need be involved in the final state, and relatively weakly otherwise. If the Ξ-N parity is even we have to have a p wave in the final state so both ^{11}B states are equally accessible : we should expect to find them in the relative intensity that can be estimated from their known fractional parentage coefficients, phase-space and so on. If the Ξ-N parity is odd no orbital angular momentum is needed in the final state if we go to the first excited state of ^{11}B but two units to reach the ground state. We therefore expect some relative favouring of the first excited state. If the first excited state is indeed favoured it is likely confirmation of s state absorption and odd Ξ-N parity since we should not expect this for the higher initial spins associated with capture from higher orbital angular momentum states.

(Note that, because of the different parities of the last protons in ^4He and ^{12}C, the easy process, s wave emission of singlet $\Lambda^\circ\Lambda^\circ$, takes place for even Ξ-N parity in ^4He and odd Ξ-N parity in ^{12}C. We therefore expect a prolific $2\Lambda^\circ$-producing absorption of a clear-cut character from one or other reaction, but not from both, and so a clear-cut answer whichever Ξ-N parity obtains, provided we indeed have s wave Ξ-capture in both cases. If we have higher orbital angular momenta involved we do not expect this. If, for example, Ξ-capture is from s states in ^{4}He but from p states in ^{12}C then, for even Ξ-N parity, the $2\Lambda^\circ$-producing reaction goes strongly in ^{4}He while it also goes strongly in ^{12}C but populates the two states in much the same ratio expected for s wave absorption with odd Ξ-N parity. If the Ξ-N parity is odd the He reaction is weak in this example while the ^{12}C reaction does not favour the first excited state as it would if the capture were from s states. In this way we may hope to get some insight into the operative absorption states.)

Other methods for Ξ-N parity

Finally we note two obvious ways in which the Ξ-N parity might be determined using complex nuclei. If we could be sure the s state capture is indeed involved observation of the reaction :

$$\Xi^- + {}^7\mathrm{Li} \rightarrow 2\,{}^4\mathrm{H}_\Lambda + 7\ \mathrm{MeV}$$

would prove odd Ξ-N parity. Similarly if the short-lived Ξ-hyper-nucleus $^8\mathrm{He}_\Xi$ exists and can be identified, for example kinematically its decay products :

$$^8\mathrm{He}_\Xi \rightarrow 2\,{}^4\mathrm{H}_\Lambda$$

in a reaction such as :

$$K^- + {}^{12}\mathrm{C} \rightarrow {}^8\mathrm{He}_\Xi + \alpha + K^+$$

then the existence of the decay into two $^4\mathrm{H}_\Lambda$ hypernuclei (assumed energetically possible) shows odd Ξ-N parity. This is because one may be rather confident that the lifetime of this hypothetical hyper-nucleus is long compared to the rearrangement times within a nucleus, so that Ξ-hyperon shuffles into an s state before conversion, while it is also likely that any even parity state of ^{7}Li as core would yield a particle-unstable $^{8}He_{\Xi}$ (^{7}Li breaks up into $^{3}H+^{4}He$ above 2.47 MeV) unless the binding energy of the Ξ^{-}-hyperon to ^{7}Li is considerably greater than to ^{3}H or ^{4}He. (It appears from recent work that there is not, in fact, an even parity state of ^{7}Li at about 6.5 MeV as was once thought. This means that the highest even parity state is more than 5 MeV above the break-up energy of 2.47 MeV and so there would have to be a difference of more than this amount between the binding energies of the Ξ^{-}-hyperon into ^{3}H or ^{4}He to stabilize $^{8}He_{\Xi}$ on an even parity ^{7}Li core.)

RELATIVE PARITIES OF STRANGE PARTICLES

Exchange scattering

From time to time the question arises as to whether supposed families of particles, for example the K-mesons or the Σ-hyperons, are indeed members of isobaric multiplets, i.e., belong to each other. If they do then they must all have the same intrinsic parity. The relative parities within pairs of strange particles, not both being members of a supposed isobaric multiplet is a fundamental question. In principle we can gain information on this point by studying exchange scattering on complex nuclei due to direct interaction mechanisms since the angular distributions of the outgoing particles in a reaction such as, for example :

$$^{11}B+K^{+} \rightarrow {}^{11}C+K^{0}$$

will depend on the relative K^{+}, K^{0} parity. The situation where there is a considerable mass difference between the ingoing and outgoing strange particles, e.g., $\Sigma^{+} \rightarrow \Lambda^{0}$, is not so easy to interpret but may also yield information.

The use of complex nuclei through the constraints their spins can place on certain reactions involving strange particles is well known and will not be detailed.

SESSION II

NUCLEAR STRUCTURE INFORMATION FROM HIGH ENERGY ELECTRON SCATTERING

Speaker :

H.W. KENDALL

NUCLEAR STRUCTURE INFORMATION FROM HIGH ENERGY ELECTRON SCATTERING

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I. INTRODUCTION

The purpose of this paper is to summarize the types of nuclear information that can be gained by high energy electron scattering studies and to indicate what new information will become available with the next generation of high intensity electron accelerators.

Essentially all the studies of nuclear structure using electron scattering techniques have been made with beams from electron linacs. The characteristics of electron linacs have so far prevented the utilization of coincidence techniques in the study of electron-induced inelastic reactions. All the results we will discuss involve the detection only of momentum-analyzed electrons scattered from the primary electron beam by various target nuclei. These scattering experiments employ electrons of primary energies from about 45 MeV up to 600 or 700 MeV. They may be separated into three groups depending on the nature of the nuclear transitions induced during the scattering process. The groups are characterized by :

i) elastic scattering;

ii) scattering leaving the nucleus in a discrete excited state, and

iii) continuum scattering in which the nucleus undergoes disintegration.

II. ELASTIC SCATTERING

Elastic scattering through angles less than about 135° is dominated by the interaction of the electron with the charge of the target nucleus. The elastic cross-sections are, in general, ten to many thousand of times larger than cross-sections for excitation of low-lying nuclear states and absolute values can be determined with typical uncertainties of about 5 to 8%. For values of the momentum transfer q in the range from 0.2 to 1.8 $(\text{fermi})^{-1}$ the observed cross-sections depend strongly on the spatial structure of the nuclear ground state charge distribution and can be used to determine within narrow limits the functional forms of the distributions. The uncertainties are smallest for the regions just within, and at the edge of the nucleus.

For nuclei of A greater than about thirty, absolute cross-section studies [1] of the elastic scattering of 183 MeV electrons have shown that the nuclear charge distributions are fitted better by a Fermi distribution than by the Ford "Family II" model. The Fermi distribution is given by :

$$\rho(r) = \frac{\rho_0}{1+\exp[(r-c)/Z_\circ]}$$

where c is the radius at which the charge density falls to one half its maximum value, $Z_\circ$ is the surface thickness parameter, and r is the radial variable measured from the centre of the nucleus. The quantity c may be determined to within uncertainties of one to three percent, and $Z_\circ$ to within five to ten percent. The quantity $c/A^{1/3}$ has an approximately constant value 1.07 Fermi varying no more than ± 2%. This reflects the approximate uniform density of nuclear matter within the heavier nuclei. The parameters are determined by comparison of the measurements with predictions based on phase shift analyses [2] of scattering, assuming one or more functional forms for the nuclear charge distributions. Elastic charge scattering cross-sections are the quantities most precisely determined in electron scattering measurements and are interpreted with what is the most detailed theory.

It should be noted that the most recent study [1] of the $\rho(r)$ for the heavy nuclei Bi-209 and Pb-208 shows that $\rho(r)$ for these nuclei cannot be approximated by any choice of the parameters of the Fermi model nor of the Ford "Family II". The Fermi model predictions fit the scattering data better than those of the Ford model but theoretical and experimental cross-sections differ by 35% at the highest momentum transfers. Work presently under way [3] is expected to resolve this situation.

There are a number of Born approximation treatments of elastic scattering which are somewhat more ambiguous and hence far less useful than the more complex phase shift analyses, especially for high Z targets. Nevertheless, they can be used to determine radius and edge parameters which are in fairly close agreement with those found from the more complex theoretical analyses [4]. The predicted cross-sections differ from the observed ones in that the former have zeros at the diffraction minima whereas the latter, while having minima, fail always to have true zeros. Theory and experiment are in adequate agreement for values of q away from those determining the diffraction dips. The Born elastic scattering results are useful in the analysis of the inelastic scattering as described in Part III. Within the framework of this theory, the cross-sections for charge scattering (elastic as well as inelastic) depend only on q as does the square of the charge form factor, F_e^2, defined by $\sigma_{observed} = \sigma_{point} \times F_e^2$.

In Born appoximation, cross-sections for the elastic scattering by the magnetic moment distribution in the nucleus are not simply q dependent but have, in addition, an explicit dependence on $\tan^2\vartheta/2$, where ϑ is the scattering angle. In principle, the cross-sections for magnetic elastic scattering can be determined by programming a series of measurements at constant q. In practice, a combination of the greatly decreased scattering cross-section for large angles [σ_{point} varies as $(\cos^2\vartheta/2)/\sin^4\vartheta/2$], the smallness of F_m^2, and the small values of the nuclear magnetic moments (of order one nuclear magneton rather than Z times this) have so far prevented measurements of the distribution of magnetization from being carried out for elements heavier than ^{7}Li. Magnetic moment distributions have been studied in the proton [5], the neutron [5], the deuteron [6], ^{6}Li and ^{7}Li [7]. High current linear electron accelerators are expected to allow successful studies of the elastic F_m^2 for heavier nuclei.

III. INELASTIC SCATTERING : DISCRETE STATES

Studies of discrete nuclear transitions excited by inelastic scattering of high energy electrons are beset by experimental and theoretical difficulties considerably more severe than those associated with elastic scattering. The inelastic peaks as observed experimentally ride on the bremsstrahlung tail of the elastic scattering peak [8] and, except for the strongest inelastic transitions, the necessary background subtractions arising in the process of making radiative corrections correspond to "signal-to-noise" ratios in the range from 2 to 0.05. Many known transitions are simply buried in this background. Excepting the lightest nuclei whose levels may be MeV apart, one finds that the uncertainties in inelastic cross-sections are rarely less than 10% and are typically 20 to 30%. Phase shift predictions for the cross-sections are far more complicated than for elastic scattering and, moreover, it is in general not as appropriate to substitute a semi-classical transition charge density for the square of the nuclear transition matrix element as in the elastic case, where such a substitution correctly leads to the static nuclear charge density. That is to say, a quantum-mechanical description of the nuclear transition appears to be necessary and it is difficult to insert in an already difficult theory. Only one inelastic phase shift analysis [9] has so far been published, the bulk of the analysis of the experiment data [10),11] having been accomplished with the aid of an ambiguous Born approximation method employing a semi-classical description of the nuclear transition.

More detailed Born treatments [12] have recently become available for lighter nuclei for which the Born approximation is expected to be adequate.

For heavier nuclei the Born results allow one to determine the multipolarity of an observed transition and the decay rate of the corresponding de-excitation gamma ray. It is surprising that results from such analyses are almost always in agreement with determinations by other methods. (For example, see Ref. [8]).

As in the case of elastic scattering the electron nuclear-charge interaction gives rise to the largest cross-sections so that electric transitions are observed predominantly. An explicit angle dependence in the expressions for magnetic transition cross-sections allows, in principle, the identification and separation of such transitions from the more easily excited electric ones. The same difficulties as for elastic magnetic scattering are encountered in these inelastic studies. Bishop and others at Orsay and Barber and his group at Stanford have succeeded in observing magnetic transitions in some nuclei lighter than ^{32}S. For these transitions the momentum transfers were so low that no structure information was available. In the light nuclei and for one or two isolated transitions among heavier nuclei ($A \gtrsim 40$) electric transitions of about single particle speed have been observed. In general, however, most of the induced nuclear excitations are those corresponding to "collective" states, i.e., those states which γ decay to the ground state with rates ten to forty times single particle speed.

Information from the limited inelastic phase shift studies and also from the corrected Born approximation analysis indicate that there should be some diffraction structure apparent in the q dependence (or angular dependence) of the observed inelastic form factors. This prediction appears to be valid in spite of the known uncertainties in the different treatments. No diffraction structure is apparent (outside of uncertainties in the measurements) in any of the inelastic F^2 measured in any nuclei, to date. Prominent structure is evident in inelastic nucleon scattering cross-sections for excitation of the same states employing reactions such as (p,p'), (d,d') and (α,α'). The contrasting situation in electron excitation is not at present understood.

IV. INELASTIC SCATTERING TO THE CONTINUUM

The inelastic scattering of electrons from nuclei in which energies larger than the nucleon separation energy are communicated to the nucleus is more difficult to study experimentally than most of the processes we have thus far discussed. Such inelastic scattering is often referred to as "quasi-elastic" as it can be qualitatively understood as the scattering from the quasi-free nucleons in the target nucleus. It differs from scattering from a free nucleon through the nucleon's momentum distribution that arises from its initial binding. The spectrum of electrons scattered quasi-elastically is characterized by a broad peak approximately centered at the kinematic position associated with scattering from a free nucleon. For large momentum transfers the area under the peak is roughly equal to $(Z\sigma_p + N\sigma_n)$ where Z and N are the numbers of protons and neutrons, respectively, and σ_p and σ_n are the free electron-proton and electron-neutron scattering cross-sections. The principal experimental difficulty encountered in studying these spectra arises from radiation of photons by the electrons before, during, and after the actual scattering. These radiative processes degrade the observed spectra badly and corrections are very difficult to make. The corrections to the data approach 100% in the low-energy wings of the quasi-elastic peaks and although there exist theoretical studies of the radiative processes [13),14)] and of the unfolding techniques [14)] necessary to correct the data, there is general agreement that the necessary correction formulae are uncertain to within as much as 20%. The principal part of the experimentally determined quasi-elastic peaks may be corrected with moderate resulting uncertainties at the cost of rather extended programmes of data taking [14)]. No such programme has yet been completed.

The shape and magnitude of the quasi-elastic peaks contain information on nucleon momentum distributions and on the mean nucleon binding energy. There have been remarkably few attempts to extract this information experimentally or to provide a firm theoretical basis for interpretation of measurements.

Of all theoretical studies of inelastic continuum scattering the most reliable are the recent sum rules developed by Van Hove and McVoy [15]. The sums extend over the quasi-elastic peak but do not include meson production processes. The results are related to the properties of the nuclear ground state including both Pauli correlations and, of more interest, those correlations consequent on the two-nucleon force for small nucleon-nucleon separation. The alterations of the sum rule predictions resulting from these latter correlations is unhappily so small as to be undetectable with present experimental techniques : recent studies by Bishop and Isabelle [16] at Orsay have shown that evidence for the Pauli correlations is all that can be extracted from sums measured to within 5% uncertainties.

For high Z targets the wing of the quasi-elastic peak (which corresponds to greater energy loss by the incident electron than the centre of the peak) receives its dominant contributions from two-particle correlations in the nucleus. A theoretical study [17] of large-energy-loss electron scattering from a dilute Fermi sea of nucleons has indicated that experimental investigations employing heavy nuclei as targets may be able to extract meaningful information concerning these two-nucleon correlations. The interpretation of all earlier measurements of these elusive correlations by other techniques have been troubled by such serious theoretical difficulties that no unambiguous results have emerged [18].

When the incident electron has lost more than 136 MeV to the target nucleus meson production becomes important and, indeed, dominates the inelastic scattering for such energy losses. Studies of electroproduction of pions from free protons have been employed gy Panofsky and Allton [19] and by Hand [20] to study the neutron's structure form factor but similar studies in heavier nuclei cannot be used to extract information of equivalent interest as the smearing of the kinematics by the nucleon's momentum distributions clouds the interpretation of the measurements.

V. CONCLUSION

Electron scattering has as perhaps its most important property the ability to determine spatial variations of nuclear structure for elastic and inelastic scattering and for electric and magnetic nuclear transitions. It can also shed light on momentum distributions including those that arise from the nucleon-nucleon force. In all these processes the nature of the electron's interaction with the nucleus can be determined with sufficiently small uncertainties so that the reliability of theories employed to interpret experimental measurements may become extremely high. On the experimental side, although there are difficulties arising from radiative smearing and from low cross-sections, there is no reason to believe that these cannot be overcome increasingly well as experimental as well as theoretical techniques improve.

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SESSION III

INTERACTIONS OF PIONS WITH COMPLEX NUCLEI

Speaker :

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INTERACTIONS OF PIONS WITH COMPLEX NUCLEI

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Pion beams suitable for studies of nuclear properties have become available at several places in the last few years. Typical intensities may be exemplified by some figures from the CERN Synchro-Cyclotron : 10^5 pions in flight/sec with an energy definition of 7-8% or 10^4 slow or stopped pions/sec. In the foreseeable future we will presumably have pion factories with intensities of 10^9-10^{10} pions/sec. With these intensities it is no longer appropriate to consider the pion an exotic tool for nuclear investigations. We should rather see it as a nuclear probe on the same footing as the nucleons, the electron or the alpha particle. In so far as pion experiments are mere repetitions of experiments with standard probes the interest in the pion is very limited. Its radically different properties, as compared to those of electrons and nucleons, make it a potentially powerful addition to the arsenal we dispose of for nuclear study.

The following simple examples clearly illustrate this difference.

First, the pion is a boson like the photon. In contrast to the photon, which is well known for poor energy definition, pion kinematics are easily controllable since π^+ and π^- are charged particles. We therefore have at our disposal a probe which can easily be absorbed or created in nuclei provided the necessary energy-momentum balance is furnished. In particular, the absorption of stopped pions in nuclei provides us with a source of 139 MeV of kinetic energy inside the nucleus (the pion rest mass). This energy is known with high precision and is easily obtainable since stopped pions are abundant. From a purely kinematic viewpoint the pion is a strongly interacting photon, since it brings along much energy and little momentum owing to its low mass.

Second, the pion distinguishes itself by three charge states, π^+, π^0 and π^-. This means that we can use double charge exchange pion reactions to switch the nuclear charge by two units. Since this implies $\pi^\pm \to \pi^\mp$, both the projectile and the emerging particle are charged and easily observed. Studies based on this principle can hardly be duplicated by the usual nuclear probes.

Third, a slow pion can be captured in a Bohr orbit around the nucleus, as can a μ^-. Consequently, many details about the nuclear interaction of very slow pions can be inferred from the energy shifts and transition rates of the π mesic X rays, somewhat in analogy with the techniques used in the analysis of μ mesic X rays.

These three rather trivial properties alone indicate that it should be quite interesting to look at nuclei with pions. In preparing this talk it therefore came rather as a surprise to me to see how small the effort has been in this respect, both experimentally and theoretically. Thus we still lack many simple facts about pion interactions with nuclei. This certainly prevents us from perceiving some of the applications to nuclear structure at present. It is indeed curious to notice that on the whole, as much is known about strange particle interactions in nuclei as about pion ones, in spite of the longer time pions have been available. This is at least partly due to the fact that the exploratory work with weak pion beams and nuclear targets was performed at a time when detection techniques were not as advanced as now. Present elementary particle experiments with pions do no longer use complex nuclei as targets, but hydrogen or deuterium. Hence, nuclear information is no longer a by-product of such experiments.
I do not think that these limitations in our present knowledge about pions in nuclei are reasons for pessimism. If we only can get the wagon rolling, theory and experiment will stimulate each other as so

many times before, suggesting new experiments which will permit us to exploit the special properties of the pion in a more profound way.

Let us now turn to what we can hope to get out of pion interactions. I will limit the discussion to pions of energies not higher than several hundreds of MeV, since problems of energy resolution and of meson production will make detailed interpretation more difficult for higher energies.

Let us consider first the capture of a very slow pion in a nucleus. The characteristic feature of the process is the liberation of the 139 MeV pion rest mass as kinetic energy. The concentration of all this kinetic energy on a single nucleon means that it moves with over 500 MeV/c momentum. Since the pion momentum is negligible, this must be the initial momentum of the nucleon by momentum conservation. It is, however, hard to find such high-momentum components in the nuclear momentum distribution, since the Fermi momentum is only of the order of 200 MeV/c. A single nucleon in an average nuclear potential is essentially incapable of pion absorption. Pion capture is mainly a several-particle process, which exploits the high relative momentum components in nucleon interactions at short distance. Absorption on nucleon pairs is particularly important. In this case the capture of the pion by a pair of low total momentum gives two final nucleons running away from each other back to back owing to momentum conservation. They share the available energy almost equally between themselves which means a relative momentum of about 750 MeV/c. From this we can immediately conclude that the capture process involves relative distances of the order of 0.5 F, i.e., it is quite a short range process. The kinematic analogy to photo-absorption is obvious, but the pion-absorption is basically of a shorter range due to the short range nature of the nuclear forces.

There are many indications of pion capture by pairs in the experiments. The point is best illustrated by Ozaki et al. by the observation of the angular correlation of (nn) and (np) pairs emitted in pion capture in C. The expectation is of course a predominant emission of the nucleons at 180° to each other.

Target	Rel. (np) Coincidences		Rel. (nn) Coincidences	
	90°	180°	90°	180°
C	0.8 ± 1.1	4.8 ± 0.7	2 ± 7	25 ± 3

TABLE 1

Other experiments and circumstantial evidence all indicate two-nucleon capture to be dominant. A clear, quantitative experimental demonstration is, however, still lacking, though it would be valuable. Thus the experiment of Ozaki et al. gives no indication of the fraction of captures which have the behaviour indicated by Table 1. It is also clear that there must be cases in which the capture occurs on three or more correlated nucleons. Such events may be studied by the observation of correlations between a fast nucleon and a deuteron etc. I think it is appropriate to mention that the energy spectrum of nucleons emitted in capture can give no clear demonstration of pair capture. For a free pair we would have two final nucleons of precisely the same kinetic energy and hence a sharp peak in the nucleon energy distribution. The nuclear Fermi motion smears this spectrum very badly, giving it a width of about 40 MeV, i.e., spreading the distribution over almost all possible energies. This is in a sense a pity, since the occasional single nucleons of maximum energy might have given otherwise some clues of the tail of the nuclear momentum distribution. It seems likely that the occurrence of such events will rather reflect a pair capture in which one of the components stays in the nucleus. It will be assumed in the following that pion capture is dominated by pair absorption.

In the last years we have seen the striking demonstration of the existence of the deep shell model orbits in the quasi-elastic collisions, i.e., in reactions which remove the nucleon from an unhurt nucleus. It is clear that we cannot expect to see simple shell model states forever as we go on peeling off the nucleons from the nucleus, since the nucleons themselves are responsible for the average potential in a self-consistent way. The question is now : where is the limit of the usefulness of the shell model ? Will we still see shell-model states with the right energies and the right properties when two nucleons are removed ? On a very naïve shell model picture these states should show up with the energy corresponding to the sum of the energies of the two holes. The different angular momentum states formed out of two holes should be degenerate. The two-hole state is, however, an unstable object which has a short lifetime due to residual interactions. The energies of these states are broadened by this effect, and the deeper of these states may collapse so rapidly as to smear out to a general background. The no-show of the deep two-hole states may in this way put a limit to the use of the shell model concept.

When the states can be seen, the spin splitting and the deviation of the energy from that of the simple shell model gives information on the residual interactions.

The study of two-hole excitations by quasi-elastic scattering is difficult. Thus, if two protons are knocked out of the nucleus, the final state will contain three particles which is quite hard to observe. The pion provides an excellent surgical knife for cutting two nucleons out of an otherwise intact nucleus. Since the pion is absorbed, we can straightforwardly measure the binding energy of the emitted pair from the energy sum of the two coincident final nucleons. M. Jean has proposed to absorb π^+ in flight on a nuclear (np) pair. The two final protons can then easily be measured. By an experiment of this type on ^{6}Li it should be possible to find out to what extent it is ^{4}He+d. Furthermore, we should see the (T=1) state of ^{4}He corresponding to ^{4}H and also the excited ^{4}He state $(1p,1s)_p$, $(1p,1s)_n$

due to the removal of two nucleons from the 1s orbit. Similar experiments can be performed with stopped negative pions by observation of coincident (np) pair. The experimental advantage of stopped pions is their greater availability, the large number of captures per pion since they spend a long time close to the nucleus in a Bohr orbit and last, but not least, an energy definition in the keV region, since Bohr orbit energies are extremely well known. The drawback is that one of the nucleons is necessarily a neutron. This is, however, no real difficulty since present time-of-flight techniques permit an energy resolution of something like 5 MeV for 60 MeV neutrons; furthermore, the proton provides an excellent zero-point time. The pair coincidence studies with stopped negative pions is thus quite feasible. It is possible to compare capture on (pp) pairs with those on (np) pairs. A specific example is the comparison

$$^{6}\mathrm{Li}+\pi^{-} \rightarrow {}^{4}\mathrm{H} + n + p$$

$$^{6}\mathrm{Li}+\pi^{+} \rightarrow {}^{4}\mathrm{He}^{*} + 2p .$$

The ground state of ^{4}H has recently been seen. The direct comparison of the transition to the corresponding state $^{4}\mathrm{He}^{*}$ in a basically identical process, as well as the comparison of the excited states corresponding to the removal of two 1s particles would be of interest for testing the purity of these states.

There is a large number of experiments of this kind which permits very detailed comparisons within isospin multiplets. A few examples :

$$^{7}\mathrm{Li}+\pi^{-} \rightarrow {}^{5}\mathrm{H} + n + p \quad ; \quad {}^{9}\mathrm{Be}+\pi^{-} \rightarrow {}^{7}\mathrm{He} + n + p$$

$$^{7}\mathrm{Li}+\pi^{+} \rightarrow {}^{5}\mathrm{He}^{*} + 2p \quad ; \quad {}^{9}\mathrm{Be}+\pi^{+} \rightarrow {}^{7}\mathrm{Li}^{*} + 2p, \qquad \text{etc.}$$

The kinematics of the free capture of a pion on a nucleon pair is changed by the Fermi motion, i.e., the momentum distribution of the two-hole state. The details of the angular correlation of the nucleon pair therefore shed light on the intrinsic structure of the state, provided we can analyse the process reasonably well. The pion production in nucleon-nucleon collisions which is the inverse process to the capture, has been extensively studied. By detailed balance we have therefore considerable information on the basic capture matrix elements. Let us not go into details but simply note that the complicated short character of the absorption process is described by form factors and zero interaction range which incidentally includes the final state interaction of the nucleons. In accordance with pion production data the two initial nucleons are taken to be in a relative s state. On a shell-model picture the transition rate to a two-hole state will then depend on the Fourier transform of two shell-model wave functions. The momentum in the Fourier transform is the centre-of-mass momentum of the pair. The surprisingly high tritium production in capture in ^{4}He has recently been explained by Eckstein along these lines.

In my opinion the two-hole state pion experiments are those which at present will yield most information for the least effort. In addition to their information on nuclear states they should also very clearly demonstrate the extent to which pion absorption is a pure two-nucleon process.

In view of the short range of the pion absorption one might believe that some information on nuclear pair correlations may be obtained. This seems unfortunately difficult due to the problem of separating the final state interaction from the remainder of the form factors.

Directly connected to the question of absorption into individual final states is the question of total absorption into all final states and of the relative yield of correlated (nn), (np) and (pp) pairs. Experiments of this type yield information on the imaginary parts of the pion effective mass and potential as well as some indications on the validity of the pair absorption picture. The pion imaginary parameters must be expected to be energy-independent as long as the incident kinetic energy is much smaller than the energy release of 139 MeV, since the final phase space is essentially unchanged. In particular, they are non-zero also at zero pion kinetic energy. This is in striking contrast to the nucleon imaginary potential which vanishes at zero energy since the inelastic channels are closed by the Pauli principle. The low-energy values of these parameters are given very instructively by measurements of level broadening and intensity attenuation of π mesic X rays. Since the pion in a Bohr orbit of a light element spends nearly all its time outside the nucleus the pion-nucleus interaction may be treated as a perturbation. The velocity dependent pion-nuclear interaction is

$$V' = -\frac{\hbar^2}{2m}\vec{\nabla}\,\alpha(\vec{r})\vec{\nabla} + V(\vec{r})$$

where $\alpha(\vec{r})$ represents the term due to the complex effective mass, $V(\vec{r})$ the complex potential. The level shift $\Delta E = \mathrm{Re}\Delta E - i\Gamma/2$ induced by this interaction is just its expectation value to first order :

$$\Delta E = \int \Phi^* \left(-\frac{\hbar^2}{2m}\vec{\nabla}\cdot\alpha(\vec{r})\vec{\nabla} + V(\vec{r})\right)\Phi\, d\vec{r} = \int \left(\frac{\hbar^2}{2m}\alpha(\vec{r})|\vec{\nabla}\Phi|^2 + V(\vec{r})|\Phi|^2\right) d\vec{r}$$

where we have made a partial integration of the first term. The unperturbed wave function Φ of the pion in the nuclear Coulomb field is well known. The 1s wave function is nearly constant over the nuclear volume for low Z and is proportional to $a_Z^{-3/2} \propto Z^{3/2}$ (a_Z= Bohr radius). For this orbit the gradient term is negligible. The 2p wave function is proportional to $a_Z^{-5/2}\cdot r$ inside the nucleus etc. Both the effective mass term $\alpha(\vec{r})$ and $V(\vec{r})$ have a range of nuclear dimensions. Replacing them by their central values α_0 and V_0 we obtain level shifts in terms of the nuclear volume $\mathcal{V}$ as

$$\Delta E_{1s} \simeq V_0 |\Phi_{1s}(0)|^2 \quad \propto Z^3 A V_0$$

$$\Delta E_{2p} \simeq |\vec{\nabla}\Phi_{2p}(0)|^2 \left\{ \frac{\hbar^2}{2m} \alpha_0 + V_0 \frac{\langle r^2 \rangle}{3} \right\} \propto Z^5 A \alpha_0 + \text{const } Z^5 A^{5/3} V_0 \ .$$

When we deal with heavy elements the finite size effects have to be taken into account, since the pion wave function can then be changed appreciably inside the nucleus. This poses no essential problems, however. The measured absorption rates are seen in Fig. 1. The width of the 1s level (measured by West) is known only for the case of ^{9}Be. It fixes the imaginary part of the potential to be $W_0 = (4 \pm 1)$ MeV If we attempt to fit the 2p and 3d absorption rates (from the old pioneering measurements by Stearns and Stearns) with this number, they come out four times too small. This determines the imaginary effective mass parameter β_0 to be (0.15 ± 0.05). It should be noticed, however, that the determination of the two numbers W_0 and β_0 hinges on the shift measurement in one single nucleus. In order to fix them more definitely it would be good to see width measurements for the 1s orbit in a few additional nuclei.

The interest in the imaginary potential and effective mass term stems from the relations between them, expected on the basis of the pair absorption picture. We try at present to calculate the ratio W_0/β_0 directly from pion production data. While crude estimates look encouraging not only for this ratio, but also for the absolute values of these two quantities separately we have no results as yet. I will therefore only indicate the expected magnitude and the origin of the expected relation.

The basic interaction responsible for pion absorption on a single nucleon is non-relativistically the gradient coupling

$$H' \propto \vec{\sigma}_N \cdot (\vec{v}_\pi - \vec{v}_N)(\vec{\tau}_N \cdot \vec{\Phi}) \propto \vec{\sigma}_N \cdot (\vec{p}_\pi - \frac{m}{M}\vec{p}_N)(\vec{\tau}_N \cdot \vec{\Phi})$$

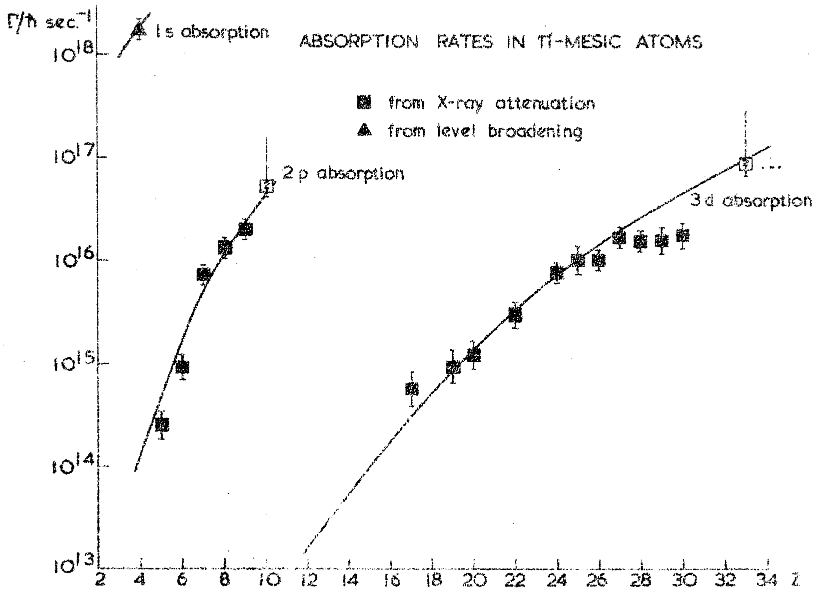

Fig. 1 Absorption rates in π mesic atoms.

where $\vec{\Phi}$ is the pion wave function, $\vec{\tau}_N$ the nucleon isospin operator and $\vec{v}$ and $\vec{p}$ velocities and momenta. The absorption depends on the relative pion-nucleon velocity because of translational invariance. When we now write down a transition probability, we have to replace $\vec{p}_\pi$ by $-i\hbar\vec{\nabla}_\pi$. This gives us a transition rate per unit volume for pair absorption proportional to

$$\rho^2(\vec{r})\left(|\vec{\nabla}\phi|^2 + \left(\frac{m}{M}\right)^2 p_N^2 \, |\phi|^2\right)$$

where $\rho(\vec{r})$ is the one-nucleon density. The cross-term linear in the nucleon velocity averages to zero. The magnitude of the nucleon momentum p_N is seen from the kinetic energy of nucleon in the final state, approximately half the pion mass. Thus

$$\frac{p_N^2}{2M} \simeq \frac{mc^2}{2} \approx 70 \text{ MeV} .$$

Therefore, the magnitude of $W_o/\beta_o \simeq \frac{m}{M}\frac{mc^2}{2} \simeq 10$ MeV neglecting all other factors. Including the statistical weight factors for absorption from relative s states of the nucleons (but taking all form factors equal) we find 23 MeV. The experimental ratio from Fig. 1 is $W_o/\beta_o = 26^{+13}_{-9}$ MeV This is quite encouraging, though we will have to back it up by a more rigorous calculation.

The relative number of (nn) and (np) pairs emitted in π capture is also relevant for the quantitative description of pair absorption. While we do not yet have the results using exact amplitudes, the branching ratio can be estimated from the weight factors associated with the isospin coupling of a π^- to original (pp) and (np) pairs. The experimental values for the ratio (nn)/(np) as obtained by Ozaki et al. is compared below to the statistical factors for absorption through W_o and through β_o.

Target	(nn)/(np)exp.	(nn)/(np) for W_o	(nn)/(np) for β_o
C	5.0 ± 1.5	6.0	3.6
Al	3.9 ± 1.2	5.4	3.4

TABLE 2

From the known absorption rates the gradient absorption (β_o) is expected to dominate in both cases. The agreement between the experiments and this oversimplified argument is again an indication of the general validity of the pair absorption picture.

As far as the absolute value of W_o is concerned estimates using the dominating production amplitudes give values of a few MeV, which is promising in view of the experimental value of 4 MeV. Thus, the different aspects of the total absorption rates offer definite possibilities for a quantitative verification of pair absorption, though any definite conclusion must await the end of the present more accurate calculation.

While the absorption of pions is of interest because of the possibility of describing it quantitatively in terms of pair absorption it might be argued that the refraction of pions is of little interest, since it would only duplicate the nucleon optical model description for a new particle with some new parameters. This argument is not tenable.

The pion optical model is both much simpler in its connection to the basic scattering on individual nucleons in the nucleus and much richer in its manifestations than the nucleon optical model. We will see that a central feature in it is the importance of the effective mass; furthermore, double charge exchange scattering is largely due to the isospin dependence of the optical model. In addition, the optical model parameters are needed for a proper understanding and interpretation of inelastic scattering, as is well known for nucleons.

Let me just remind you of the ordinary impulse approximation to the optical potential in terms of an average over the individual pion-nucleon scattering amplitudes. The correlations between nucleons are neglected.

$$V_{opt} = < \Sigma_i \; t_i >$$

where t_i is the pion-nucleon scattering operator on the i^{th} nucleon. A very great simplification as compared to nucleons occurs in the next step, since it is well known that pion-nucleon scattering is dominated by s and p waves only up to energies of several hundreds of MeV. Furthermore, the low energy description is not veiled by the occurrence of any bound or nearly bound states of the pion-nucleon system, as is the case for nucleons.

The s and p wave character of the scattering operator is expressed by writing it out in terms of momenta with initial and final wave vectors $\vec{k}_i$ and $\vec{k}_f$

$$t_i = s + t \; \vec{k}_f \cdot \vec{k}_i \quad .$$

The constants s and t depend on the <u>incident</u> pion energy only in so far as nuclear Fermi motion can be neglected. Let us now average this over the local nucleon density $\rho(\vec{r})$ and observe that $\vec{k}$ must be replaced by the gradient operator $\vec{k}=-i\vec{\nabla}$. Then

$$V_{opt} = -\frac{4\pi}{2m}\left(s\rho(\vec{r}) + t\,\vec{k}_f\,\rho(\vec{r})\cdot\vec{k}_i\right) = -\frac{4\pi}{2m}\left(s\rho(\vec{r}) - t\,\vec{\nabla}\rho(\vec{r})\cdot\vec{\nabla}\right) \equiv$$
$$\equiv V(\vec{r}) - \frac{\hbar^2}{2m}\vec{\nabla}\,\alpha(\vec{r})\,\vec{\nabla}\,.$$

The p wave pion-nucleon scattering thus introduces a non-locality in the potential. The momentum dependence occurs, however, in the simplest fashion possible as an effective mass. There are no higher derivatives in the potential. This type of pion-nucleon optical potential was first derived by Kisslinger.

At low pion energies this result is all right, though we should more generally have derived a Klein-Gordon equation. The wave equation becomes then

$$\left[\vec{\nabla}(1+\alpha(\vec{r}))\,\vec{\nabla} + k^2\right]\Phi(\vec{r}) = K(\vec{r})^2\,\Phi(\vec{r})$$

where $\alpha(\vec{r})$ and $K(\vec{r})$ can be simply related to the non-relativistic equation. From the pion-nucleon phase shifts at low and moderate energies we obtain the following central values α_o and V_o in the nucleus

$$\alpha_o = -1.8 - 1.8\,\frac{\vec{t}\cdot\vec{T}}{A} \quad (\text{mainly from } \delta_{33}/k^3)$$

$$V_o = \left(7^{+3}_{-2} - 73\,\frac{\vec{t}\cdot\vec{T}}{A}\right)\text{ MeV} \quad (\text{from } \delta_3/k \text{ and } \delta_1/k)\,.$$

The large error indicated in V_o is due to a cancellation between the contributions of the s wave phase shifts

$$\left[\frac{2\delta_3 + \delta_1}{|2\delta_3| + |\delta_1|} \simeq -1/7\right]$$

and reflects the phase shift uncertainty. Due to this cancellation the potential term V_o has a very strong isospin dependence. The potential is weakly repulsive. Since there is such a large cancellation one might worry about contributions to the potential from p wave scattering due to nuclear Fermi motion. This is, however, only a

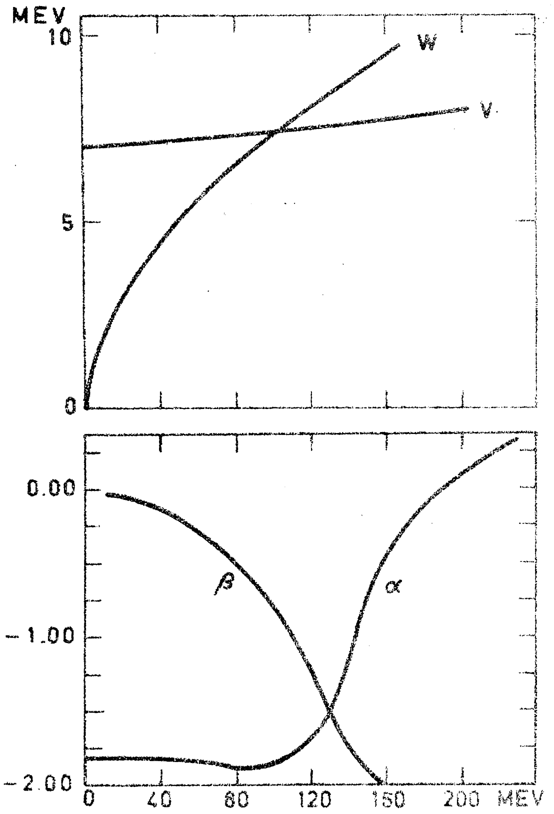

Fig. 2 Energy variation of optical parameters from phase shifts.

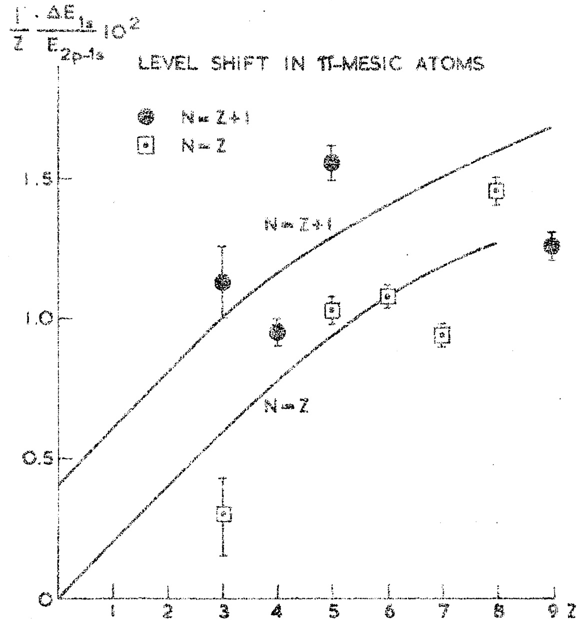

Fig. 3 Energy shifts for the 1s orbit in π mesic atoms.

1 MeV correction and is as yet outside observation. The effective mass term corresponds to an attractive potential. The coefficient is so large that the effective mass comes out negative ! The above parameters are of course energy dependent. Fig. 2 shows the expected energy variation using phase shifts from the Chew-Low theory. We see that α_0 and V_0 are pretty constant up to about 80 MeV, at which energy the influence of of the (3/2,3/2) pion-nucleon resonance becomes important. The imaginary parts β_0 and W_0 are due to incoherent scattering only, not to bona fide absorption which has to be added separately. At low energy the imaginary parts are reduced by the exclusion principle and they should not be taken very seriously.

There are presently practically no low energy scattering experiments which give information on the optical model constants. The mesic atom comes in handy here as it did for the imaginary parameters. We obtain the real optical parameters from the level shifts as we obtained the imaginary ones from the level widths. The ratio of the shifts to the energy as measured about seven years ago is shown in Fig. 3. The precision according to error bars is better than 1/2% which would put these measurements on a level with the best present μ mesonic X ray measurements. The inclusion of systematic errors may double or even triple the indicated errors, so that the deviation of certain nuclei from the general trend in Fig. 3 is a most uncertain phenomenon. We observe, however, that there is a definitely different shift for nuclei of $N=Z$ and $N=Z+1$ as expected due to the isospin dependence of the potential. From the simple perturbation expression for the energy shift derived in connection with the absorption rate and observing that the E_{2p-1s} energy is proportional to Z^2 we have

$$\frac{1}{Z}\frac{\Delta E_{1s}}{E_{2p-1s}} \propto V_o A \quad .$$

This gives us families of straight lines for the different neutron excesses with the one for $N=Z$ going through the origin. The other curves are displaced by constant amounts with respect to this one due to the A^{-1} term in the isospin potential. The slope thus gives the main potential, while the displacement gives the isospin dependence. From Fig. 3 we obtain

$$V_o = (9.4\pm2) - (75\pm20)\,\frac{\vec{t}\cdot\vec{T}}{A}\ \text{MeV} \quad .$$

The agreement between these values and those derived from the pion-nucleon phase shifts is amazingly good for both parts of the potential. The result would be more convincing, however, if it was based on somewhat more precise measurements with less spread of points.

What about the effective mass term α_o ? First of all, we cannot expect theoretically that the simple impulse approximation for independent scatterers will work in this case, in spite of its apparent success for the potential. The reason is that the term α_o is due to the p wave, i.e., the dipole scattering of the pion from individual nucleons. Dipole scattering in a medium is familiar from the ordinary theory of electromagnetic waves in materials, and it is well known there that the wave reacts back on itself via the medium. In other words, the effective field at the position of the scatterer is not just the average field. This gives rise to the famous Lorentz-Lorenz effect which relates the dielectric constant ϵ to the polarizability α as

$$\alpha \propto \frac{\epsilon - 1}{\epsilon + 2} \qquad \text{and not as} \qquad \alpha \propto (\epsilon - 1) \quad .$$

The electric field is related to a potential V by $\vec{E} = -\vec{\nabla} V$. Taking the pion field Φ as the potential we see that $\vec{\nabla}\Phi$ corresponds formally to $\vec{E}$. As pointed out by Kroll the Lorentz-Lorenz effect should thus occur qualitatively also in nuclei.

This leads us to expect the replacement of the previous impulse approximation expression for α_o by

$$(\alpha_o)_{imp} \rightarrow \alpha_o = \frac{(\alpha_o)_{imp}}{1 - \frac{1}{3}(\alpha_o)_{imp}} \simeq -1.1 \; .$$

The isospin part is of course also affected but I have not written it out.

The expression just given applies to an infinite, isotropic medium of independent scatterers. In reality nucleons are densely packed and the medium is finite. We have just seen, however, that corrections of this type seem to be unimportant for the potential, so maybe they are also unimportant for α_o. It would be quite interesting to see if the nucleus exhibits quantitatively this further optical analogy which is virtually impossible to observe in nucleon scattering. From a theoretical standpoint this also raises the question of the leading corrections of a Lorentz-Lorenz effect for small and dense media. Unfortunately, there is not much experimental information at low energies. In 1957 Stearns and Stearns measured the 2p level shift in Ca. This should yield α_o directly as seen from perturbation theory. A shift

$\Delta E_{2p}/E_{3d-2p}$ of $(-1\pm1)\%$ was found. The expected shift with the modified α_o is $+2\%$. However, as previously remarked, the indicated error is likely to be somewhat to the optimistic side, so that the measurement is probably just barely consistent with $+2\%$ with a weak indication of a smaller value for α_o. A negative value of α_o would be most surprising, since it would mean a totally repulsive pion-nuclear potential. The possibility that there is a Lorentz-Lorenz effect in the pion-nucleus interaction is a strong motivation for a measurement of the 2p energy shift in π mesic atoms. Since $\Delta E_{2p}/E_{3d-2p}$ varies like Z^4 the experiment should nowadays be relatively simple in the region Ca-Zn. The shift in Zn is expected to be in the 5-10% region which is large. The experiment is thus well within the range of present possibilities.

Collecting the present information on the zero energy optical parameters as obtained from π mesic atoms, we have

$$\alpha_o - i\beta_o = ? \quad i(0.15\pm0.05) + ? \ \frac{\vec{t}\cdot\vec{T}}{A} \ \text{MeV}$$

$$V_o - iW_o = (9.4\pm2) - i(4\pm1) + (75\pm20)\ \frac{\vec{t}\cdot\vec{T}}{A} \ \text{MeV} \quad .$$

These numbers are all from a preliminary analysis. They assume a square well potential with $r_o = 1.2$ Fm.

At this point I want to comment on the practical implication of the similar magnitude of the real and imaginary part of the potential, which imply ΔE_{1s} and Γ_{1s} to be very close. In discussing the absorption rates we remarked on the need for measurements of Γ_{1s} in several nuclei since the present value hinges on one single measurement. We need this quantity with certainty not only for its own sake but also because it is the major source of uncertainty in β_o. Since the real level shifts can be seen clearly and with reasonable relative error limits, it is hard to see why a level broadening of the same magnitude should be hard to measure. The measurements of shifts and broadening rather seem to go together. I therefore urge the experimentalists to look into this question. After all the width in ^{16}O is expected to about 10% of the 2p-1s transition energy. The direct observation of widths in the 2p orbit is much more difficult, being of the order of 1/5 of the as yet unobserved real shift. In Zn it should, however, be about 1-2% which may be observable. In this fashion an independent measurement of β_0 may be obtained.

As you see, the π mesic atom is an important, though as yet not very exploited, source of information about pion-nucleus interactions at very low energy. I want particularly to emphasize the extremely instructive manner in which the optical parameters are obtained. By six separate measurements we can determine one by one the central values of the real and imaginary parts of the potential and the effective mass as well as their isospin dependence. Just contrast this to the way this information is usually obtained from scattering experiments, where all these quantities enter simultaneously in a complicated fashion, and for which the disentanglement must be made by computer. So far the influence of the nuclear individuality is not clearly manifest in the π mesic experiments. Its effects will presumably turn up in the next more accurate measurements. I will not enter into this, however, but just remark that it is a shame that the field of π mesic atoms has been left untouched for so long since the first exploratory studies.

Let us now turn to pion scattering and, in particular, to elastic scattering. Experiments are essentially missing up to around 60-70 MeV. An exception is a 5-15 MeV π^+ and π^- scattering experiment on C by Demidov et al. done in a propane bubble chamber. The statistics limit the conclusions that can be drawn. One sees, however, definitely that π^+ interferes constructively, π^- destructively with the Coulomb scattering. This indicates a repulsive potential, since it has the same net effect as the Coulomb potential on π^+. This is consistent with π mesic data since at this energy s wave scattering dominates. The data indicate also that the effective mass parameter

α_o is negative by a weak rise of the cross-sections at backward angles. No firm conclusions can be drawn from the present data, however.

The Rainwater group has studied elastic scattering on Li, C, Al and Cu at 70-80 MeV energy during several years. The precision of the measurements has strongly improved through this period. In an early comparison between π^+ and π^- scattering on Cu at 80 MeV the constructive π interference with the Coulomb amplitude at 20°-30° is apparent. The low energy net repulsion has therefore now become a net attraction. Since we have reasons to believe that the potential still is repulsive at these energies, this therefore indicates a negative, i.e., attractive, value for α_o. Figure 4 shows the typical accuracy this group has reached lately in π^- scattering on carbon at 69.5 MeV. Notice in particular that the inelastic scattering can be resolved.

The body of data that has been obtained in this way has been analysed in the relativistic version of the optical model we have considered. It turns out to be absolutely impossible to fit the data without an effective mass term, whatever is tried for the potential. The fit they typically obtain including the effective mass term $\alpha(\vec{r})$ is illustrated by the solid curve in Fig. 4. I have converted the parameters they use into those we have discussed previously so as to make a comparison possible

$$\alpha_o - i\beta_o = -(0.58\pm0.04) - i(0.015\pm0.002)$$

$$V_o - iW_o = (50\pm0.3) - i(17\pm2)\ \text{MeV}$$

$$r_o = (1.05\pm0.02)\text{Fm}$$

$$t = (1.16\pm0.07)\text{Fm} \quad (10\%\ \text{density spacing})\ .$$

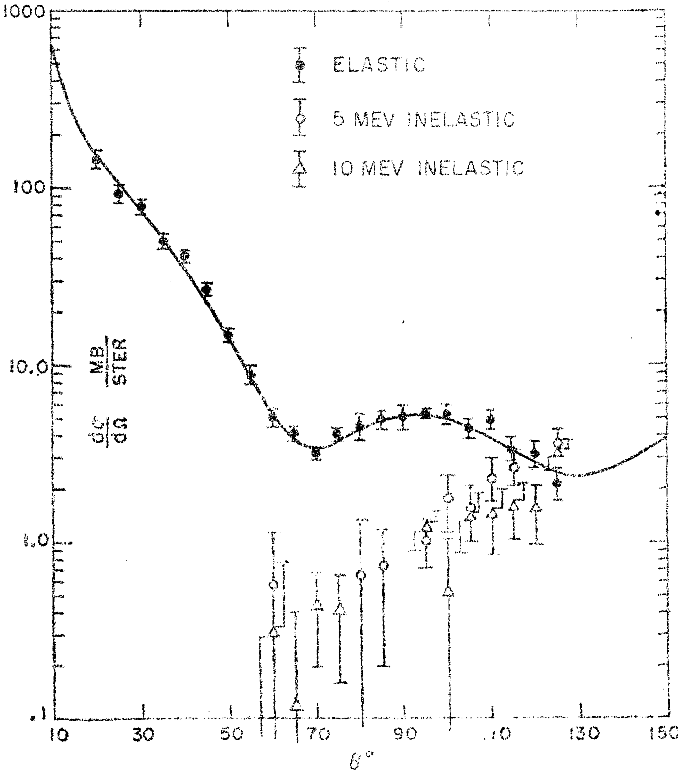

Fig. 4 Differential cross-section for π^- on C at 69.5 MeV.

A look at the previous π mesic values for these quantities reveals large differences : the potential V_o is 5 times larger here, while the imaginary effective mass parameter β_o is reduced by a magnitude in absolute value. Since the energy is vastly different this may be due to a violent energy variation of parameters. A look at the expected energy variation in Fig. 2 indicates, however, that this is unlikely to be the reason, at least within the framework we have used for the optical model. Furthermore, the small value of β_o is particularly hard to understand, since pair absorption is expected to contribute importantly to this term. It is rather more likely that there exists a second set of more reasonable parameters which give as good a solution. This situation is not unreasonable, since there does not exist any information on the scattering in the intermediate energy region by which continuity conditions can be imposed on the parameters and since absorption cross-sections have not been used in the analysis.

Whatever is the origin of this discrepancy, the general question of the energy variation of the pion optical parameters is of undeniable interest. As the pion energy is increased into and beyond the energy region of 100 MeV, the pion-nucleon (3/2,3/2) resonance at 180 MeV becomes important. The influence of resonant phenomena on the optical refractive indices is well known. In that case we find anomalous dispersion. The refractive index as a function of frequency is described

(T+1) amplitude. If we have pure isospin the positive charge exchange should go to an isobaric state with the same amplitude but for some trivial Clebsch-Gordan factors. And similarly, the (T+2) state in the original nucleus has also a predictable amplitude. Within isospin conservation this statement is perfectly general. Thus, insofar as mass differences, binding energies and electromagnetic effects can be neglected (as they can for pions of energy $\gtrsim$ 100 MeV) all cross-sections for these transitions are essentially identical. The validity or non-validity of relations of this type, which do not depend on any dynamics, may be useful both as labels for identifying analogue states and for testing the purity of their wave function. You should note that the isospin tests using this relation are more demanding than usual, since the charge span for these nuclei is four units.

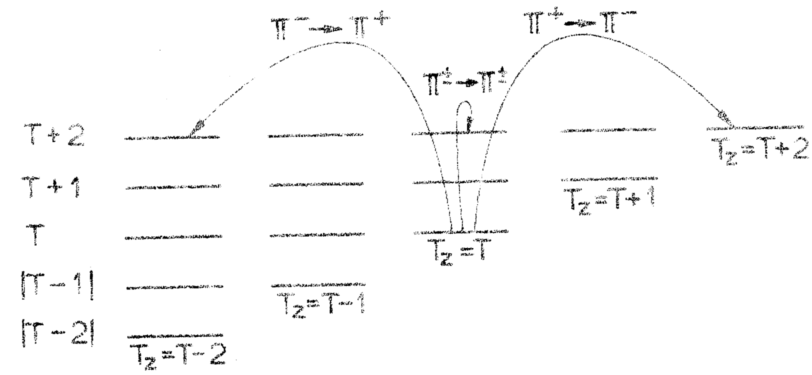

Fig. 5 Picture of corresponding observable transitions to (T+2) states.

The double charge exchange reactions give some unusal possibilities to study nuclei off the ordinary stability line. As an example of this : from ${}^{16}_{8}O$ you may obtain on the one hand ${}^{16}_{6}C$ and on the other ${}^{16}_{10}Ne$. Such experiments have their greatest interest in the very light elements however. There have been recent reports that ${}^{5}H$ and also ${}^{4}H$ have been seen. In view of this the ${}^{4}_{0}n$ system and other neutron systems may be more stable than expected. We can reach two such neutron systems by double charge exchange, the latter being the most interesting

$$ {}^{3}He + \pi^{-} \to {}^{3}_{0}n + \pi^{+} $$

$$ {}^{4}He + \pi^{-} \to {}^{4}_{0}n + \pi^{+} \; . $$

The binding energies of these neutron systems (and possibly of their excited states) are directly reflected in the energy of the final pion. The charged counterparts to these neutron systems are also easily formed :

$$\pi^+ + T \rightarrow {}^3_3Li + \pi^- \rightarrow 3p + \pi^-$$

$$\pi^+ + {}^4He \rightarrow {}^4_4Be + \pi^- \rightarrow 4p + \pi^- \quad .$$

These last reactions are extremely easily detected from their spectacular stars. The 5 prong stars of the last one in a helium bubble chamber can hardly be missed even if the cross-section is low. The interest of experiments of this type in these light elements, and in particular in the neutron and hydrogen systems, comes from the possibility of further information on nuclear forces. It is in the nucleon systems of this kind that we have the hope to learn about three-body nuclear forces, for example. Other experiments of similar type are

$${}^7Li + \pi^- \rightarrow {}^7H + \pi^+$$

$${}^7Li + \pi^+ \rightarrow {}^7B + \pi^- \quad .$$

The possibility of a nearly stable ${}^7H$ has greatly increased after the discovery of ${}^5H$.

The double charge exchange reactions and the capture of pions together should thus give good opportunities for a direct study of nearly the whole range of possible isobars for low A values.

In the absence of any experimental information on double charge exchange processes I will not enter much into the question of dynamics. The transition to the analogue state of the target ground state is the simplest charge exchange process. We deal here essentially with the isotopic spin effects of the elastic channel neglecting Coulomb effects (incident energies $\gtrsim$ 100 MeV say). On the previous optical model picture the charge exchange can hence be achieved by twice exploiting the term $\vec{t}\cdot\vec{T}/A$ in the potential, since terms like t^+T^- switch one charge between the pion and the nucleus. In this case we would deal with a mechanism producing amplitudes proportional to A^{-2}, which is unfavourable in heavy elements. Other mechanisms for charge exchange which have a more favourable A dependence may then become important. The most interesting one is the direct double charge exchange on a nuclear pair, for example $\pi^- + 2p \rightarrow \pi^+ + 2n$. If this mechanism turns out to be important the amplitude will go like A^{-1}. Furthermore, this process is intimately related to the absorption of a pion on a nuclear pair. It is thus not inconceivable that there will be theoretical relations between the double charge exchange reactions and the imaginary optical model parameters.

We should not lose track of the fact that pion creation may be a very useful tool for certain purposes, though incident pions are more flexible in their use and more readily available. A Frascati group has recently been able to measure the ^4H binding energy by a pretty reconstruction of the kinematics of reaction $\gamma + {}^4He \rightarrow {}^4H + \pi^+$, as seen in a cloud chamber. The interest of pion production experiments lies mainly in the possibility they offer to remove one positive or negative charge from the initial nucleus. The residual nuclei will hence be $(Z\pm1, A)$. These reactions therefore complement the charge exchange reactions which yield $(Z\pm2, A)$. It is probably not possible to use γ rays in general for these purposes due to poor energy resolution of the γ ray. The possibility of reconstructing kinematics applies only to a few elements like He and is hence exceptional. In order to study binding energies or excited states in this way we need experiments in which the incident energy is well controlled. One should therefore consider the possibilities of virtual γ rays, i.e., pion electroproduction. We could then measure electron and pion in coincidence with free production kinematics. The sum of the electron and pion energies would then yield the information. It is also easy to see that much further information could be extracted. Of course, such experiments are almost impossible on linear electron accelerators due to the huge number of random events. The problems are exactly those of (e,e'N) quasi-elastic scattering. One should look into ways to go around this difficulty. Storage rings may offer a possibility for example.

The cases I have discussed are but a sample of those in which the pion gives qualitative new information about nuclei. I have only selected these examples since they are well within the range of present experimental techniques and hence ripe for study. We have thus seen that pion capture allows a detailed quantitative study of nuclear pair absorption, which has never been done, though there is a qualitative evidence that capture is understood on these lines. The capture process has also the important and special property of preferentially selecting nuclear two-hole states which thus can be excited and studied. The pion optical model turned out to have a number of interesting features. This is both due to the clearness with which the optical parameters can be associated with pion-nucleon scattering, to the many interrelations between the different optical parameters and to the probable occurrence of a nuclear Lorentz-Lorenz effect. We saw that the coherent scattering by nucleon pairs may be of great importance in charge exchange scattering, reflecting pair absorption by the imaginary parameters. This would generalize the ordinary description of the optical model. The multiple scattering problem for pions is thus of greater interest than the corresponding one for nucleons. Double charge exchange scattering has an interesting potential for picking out correspondence states in nuclei and has in addition properties which make it useful for investigating the isospin purity of nuclear wave functions. A by-product of both pion capture and double charge exchange is the possibility of producing nuclei with unusual properties, particularly in the very light elements. The difficulty of studying any of these general type problems with other means than pions shows that it is time that the pion takes its natural place as one of the tools for nuclear structure studies.

The preparation of this talk has been greatly helped by Mrs. M. Ericson who has kindly provided much material from a current investigation and analysis of the optical parameters from π mesic atoms.

SESSION IV

INFORMATION ON NUCLEAR STRUCTURE FROM NUCLEON AND ELECTRON REACTIONS

Speaker :

A.K. KERMAN

SESSION V

REACTIONS OF HIGH ENERGY NUCLEONS, DEUTERONS AND ALPHA PARTICLES WITH NUCLEI

Speaker :

A.B. CLEGG

REACTIONS OF HIGH ENERGY NUCLEONS, DEUTERONS AND ALPHA PARTICLES WITH NUCLEI

A.B. Clegg
Nuclear Physics Laboratory, Oxford.

I intend to talk mainly about proton induced reactions as most work has been done with protons, so there are most leads into the future. I will mostly talk about work with protons of 145-185 MeV from the cyclotrons at Harvard, Harwell, Orsay and Uppsala. At these energies and higher the nucleus is not too black and the wavelength of the incident proton is comparable with and shorter than the nucleon-nucleon spacing in the nucleus so that it can see and interact with individual nucleons in the nucleus. It is also usual to use the impulse approximation : that the interaction of the incident proton with a bound nucleon is the same as that with a free nucleon at the same momentum transfer. It is perhaps surprising that this is so and that the properties of the bound nucleon are not altered : that its pion cloud is not affected as its diameter is so close to the nucleon's average separation from its nearest neighbours. However, there is support for the impulse approximation at small momentum transfers from nucleon-nucleus elastic scattering and we shall see that there is further support from inelastic scattering, though with some evidence for departures from the impulse approximation at the largest momentum transfers investigated.

I intend to spend about half my time discussing inelastic scattering. The inelastic scattering mechanism seems to be understood in some detail so it is a useful tool for nuclear spectroscopy but there are also some niggling discrepancies which are interesting.

I will first describe the mechanism in some detail so as to have some formulas to point at. I take the cross-section as determined by the distorted wave Born approximation matrix element :

$$\int \chi_f^* \varphi_f^* M(q) \varphi_i \chi_i d\tau \ .$$

χ_i, χ_f are the initial and final proton wave functions, describing its scattering from the other nucleons in the nucleus, φ_i, φ_f are the initial and final nuclear wave functions, q is the momentum transfer, and $M(q)$ is the proton-nucleon scattering amplitude :

$$M(q) = A+B(\vec{\sigma}_1 \cdot \hat{n})(\vec{\sigma}_2 \cdot \hat{n})+C(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\hat{n}+E(\vec{\sigma}_1 \cdot \hat{q})(\vec{\sigma}_2 \cdot \hat{q})+F(\vec{\sigma}_1 \cdot \hat{p})(\vec{\sigma}_2 \cdot \hat{p})$$

where $\hat{q}, \hat{n}, \hat{p}$ are the usual set of orthogonal unit vectors, $\hat{q}$ along the momentum transfer and $\hat{n}$ perpendicular to the scattering plane.

It is usual, and I follow the tradition, to drop terms out of the calculation which are zero in the adiabiatic approximation, an approximation in which the magnitudes of the initial and final momenta are equal, which is when the energy transfer to the nucleus is zero. This is close to being true in the experiments described. The matrix element then becomes :

$$\sum_{\ell} \int \varphi_f^* \left[\rho_\ell^0(r) Y_\ell^0 + \rho_\ell^2(r) \ \{Y_\ell^2 + Y_\ell^{-2}\} \right] M(q) \varphi_i d\tau \, .$$

Hooton and Allcock have calculated this in the WKB-approximation (note that I only wish to use the WKB-approximation as a guide) and find, for the example of the excitation of a J=2+ state from a J=0+ ground state :

$$\rho_\ell^0(r) = f_0(r) j_2(qr) - \left[\tfrac{1}{2} f_2(r) j_0(qr) - \tfrac{5}{7} f_2(r) j_2(qr) + \dots \right]$$

$$\rho_\ell^0(r) = (\tfrac{3}{2})^{1/2} \left[\tfrac{1}{2} f_2(r) j_0(qr) + \tfrac{5}{7} f_2(r) j_2(qr) + \dots \right] .$$

Typically f_0 is fairly constant throughout the nucleus while f_2 and similar functions in higher terms are small inside the nucleus and rise rapidly at the nuclear surface. If we neglect scattering from the other nucleons in the nucleus, χ_i, χ_f are plane waves : then

$f_0 = 1$, $f_2 = 0$. For a proper distorted wave calculation $f_0 < 1$ due to absorption of the proton wave due to other reactions. Thus the cross-section from a distorted wave calculation is distinctly smaller than from a plane-wave calculation, by a factor of three in a typical case. However, the change in calculated polarisations, either of outgoing proton or of nuclear state, on going from plane waves to distorted waves is much less, as they are determined by ratios of amplitudes and so the change in f_0 tends to divide out. There is a large peak in the angular distribution of the scattered protons, which is produced by the $f_0 j_2$ term in our example, so we expect that for scattering angles around the peak this term is dominant and the $\rho^2_\ell(r) \ll \rho^0_\ell(r)$.

I shall now discuss the polarisation of the nuclear state : it is complicated which means that there is a lot of information to be obtained by measuring it. This polarisation is indicated by the angular correlation of the decay gamma radiation in coincidence with protons scattered with some definite momentum transfer. Experimentally we have studied the excitation of the J=2+ first excited state of ^{12}C from the J=0+ ground state and I will discuss this as an example. The calculated gamma-ray angular correlation is

$$
\begin{aligned}
W(\vartheta,\varphi) = {} & \left[|A|^2+|C|^2\right]\left[|N^0_2|^2 - \sqrt{8/3}\,\mathrm{Re}(N^0_2 N^{2*}_2)\cos 2\varphi\right]\sin^2 2\vartheta \\
& + 2/3\left[|C|^2+|B|^2\right]\left|Q^0_2 + \sqrt{2/3}\,Q^2_2\right|^2(\cos^2 2\vartheta\,\sin^2\varphi + \cos^2\vartheta\,\cos^2\varphi) \\
& + 2/3\,|F|^2\left|Q^0_2 - \sqrt{2/3}\,Q^2_2\right|^2(\cos^2 2\vartheta\,\cos^2\varphi + \cos^2\vartheta\,\sin^2\varphi) \\
& + \sqrt{2/3}\,\mathrm{Im}\Big\{(A^*C+C^*B)\,(Q^0_2 + \sqrt{2/3}\,Q^2_2) \\
& \times\left[N^{0*}_2\sin 4\vartheta\,\sin\varphi - \sqrt{2/3}\,N^{2*}_2(\sin 4\vartheta\,\sin\varphi\,\cos 2\varphi - 2\sin^2 2\vartheta\,\cos\varphi\,\sin 2\varphi)\right]\Big\} \\
& + 2/3\left[|A|^2+|C|^2\right]|N^2_2|^2(\sin^2 2\vartheta\,\cos^2 2\varphi + 4\sin^2\vartheta\,\sin^2 2\varphi) \\
& + 4/9\,|E|^2\,|Q^2_2|^2(\sin^2 2\vartheta\,\sin^2 2\varphi + 4\sin^2\vartheta\,\cos^2 2\varphi)
\end{aligned}
$$

where ϑ , φ are spherical polar co-ordinates measured from the recoil direction. It depends on four nuclear matrix elements :

a) the no-spin-flip matrix elements :

$$N_2^m = \langle 2 || \rho_2^m Y_2 || 0 \rangle ,$$

b) the spin-flip matrix elements :

$$Q_2^m = \langle 2 || \rho_2^m T_2 || 0 \rangle$$

(where the T_2^K are irreducible tensors made up of the $Y_2^m \vec{\sigma} \cdot \hat{a}$'s) and the proton-nucleon scattering amplitudes A,B,C,E,F. So if for a range of scattering angles $\rho_2^2 << \rho_2^o$, as we expect, then $N_2^2 << N_2^o$ and $Q_2^2 << Q_2^o$. The first term in the correlation function is much larger than the others as, experimentally, spin-flip is much less probable than no-spin-flip. So by making measurements at $\varphi = 0$ and $90°$ we can find the relative magnitudes of the two contributions to the first term and thus determine $Re(N_2^2/N_2^o)$ and so find out if we are right in thinking this ratio should be small. From the second, third and sixth terms we find the magnitudes of the spin-flip matrix elements while the fourth term is due to interference between the spin-flip and no-spin-flip amplitudes.

Let me now sketch some experimental results and describe the information we obtain. First let me show the proton angular distribution (Fig. 1); these are Orsay and Oxford results. We have made angular correlation measurements at scattering angles of 15, 25, 35, 45°. We obtain good measurements of $Re(N_2^2/N_2^o)$, shown in Fig. 2 and they are small as expected. We then extract from the data values of $[|A|^2 + |C|^2] |N_2^o|^2$ as a function of scattering angle. This is the principal contribution to the differential cross-section, so it has a peak at the same angle as does the angular distribution, at $\sim 19°$. One may think that this is simply due to a peak in $|N_2^o|^2$ at this angle. As

$$N_2^o \sim \int j_2(qr) f_o(r) \psi_f^*(r) \psi_i(r) r^2 dr$$

one would expect the peak in $|N_2^o|^2$ to be at a value of q corresponding to the maximum of $j_2(qR)$, R being where $f_o(r)\phi_f^*(r)\psi_i(r)r^2$ peaks. A peak at $\theta_p = 19°$ corresponds to $R = 3.8\times10^{-13}$ cm = $= 1.65A^{1/3}\times10^{-13}$ cm, which seems disagreeably large. So we do not like this explanation. They way out is to assume $[|A|^2+|C|^2]$ falls off rapidly with increasing angle so that the peak in $|N_2^o|^2$ is at a larger angle. Such behaviour of $[|A|^2+|C|^2]$ is just what is found for scattering from a free nucleon. Using Yale or Harwell phase shifts we find the peak in $|N_2^o|^2$ is at 25° corresponding to $R = 2.9\times10^{-13}$ cm = $= 1.25A^{1/3}\times10^{-13}$ cm which seems more respectable. Thus we have evidence that $[|A|^2+|C|^2]$ for scattering from a bound nucleon decreases rapidly with increasing scattering angle just as it does for scattering from a free nucleon. This thus provides some support for the impulse approximation.

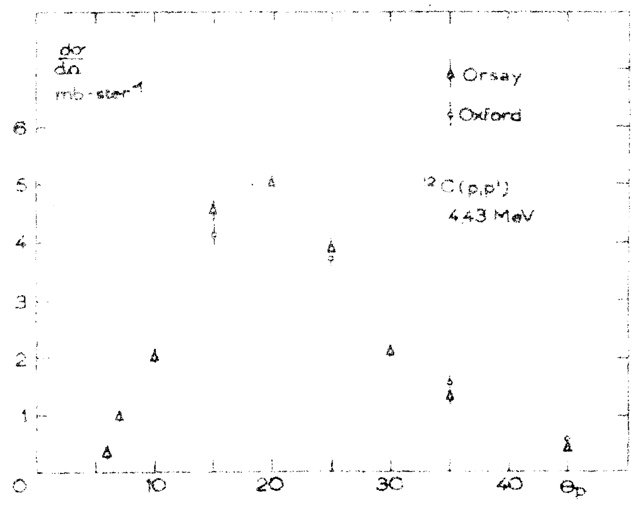

Fig. 1

We find that the most probable values of $|Q_2^o/N_2^o|^2$ vary with scattering angle as shown on Fig. 3. If the nuclear wave functions factor into one function of radius and one function of angle and spin, as do 1p-shell wave functions in a typical shell-model calculation, this ratio should not vary with angle. It does vary so we must look further. We should take into

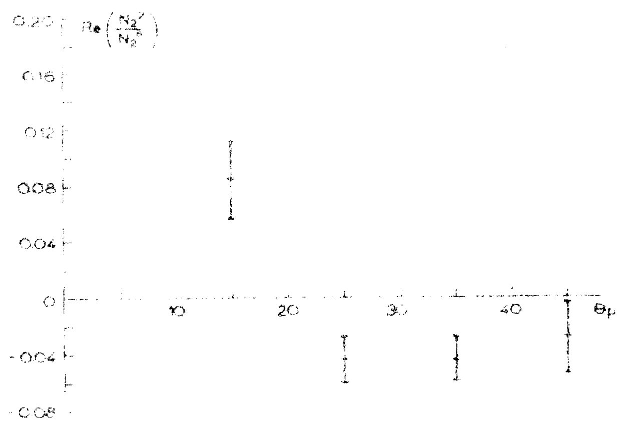

Fig. 2

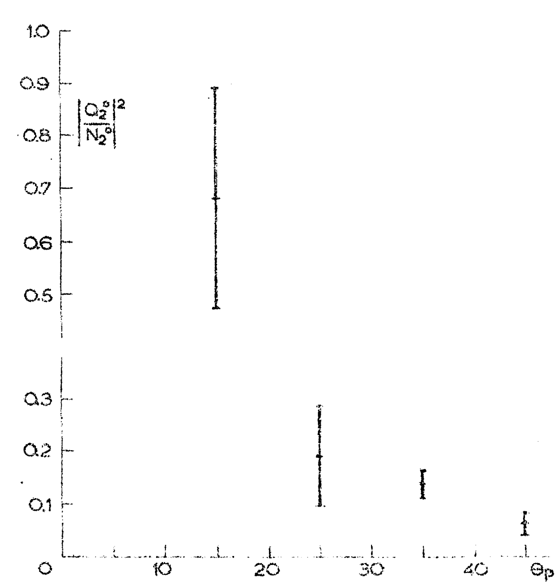

Fig. 3

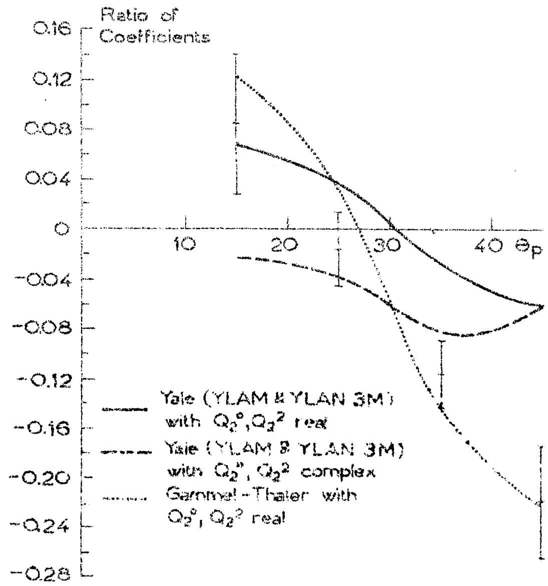

Fig. 4

account such admixture of higher configurations as will enhance the cross-section over the simplest shell-model value to the observed result. Pinkston and Satchler have shown how this would make this ratio decrease with decreasing scattering angles, being ~0.12 at large angles and decreasing to ~0.08 at more forward angles. Again we see that the experimental results differ from these expected values, though expected and experimental values are close at ϑ_p=35, 45°. A possible interpretation goes like this : the $|Q_2^0|^2$ expected from the bulk of the nucleus is small so that any further contribution to the spin-flip matrix element could have a relatively large effect. Such a further contribution is apparently largest at small momentum transfers which would correspond to the matrix element from large radii. Thus we suggest that these results may imply that a proton in the fringe of the nucleus can more easily have its spin flipped in this inelastic scattering than one in the interior of the nucleus. There is some support for this supposition from the coarse information we have about $|Q_2^2|^2$, and it also provides a qualitative explanation of a discrepancy between these results and similar experiments at lower energies. Anyway this seems curious and worth exploring, probably first by criticising the idea. Typically one should investigate the effects of some things we have left out, such as a spin-orbit term in the distorting potential. However, it may present the possibility of obtaining interesting new information about the fringe of the nucleus.

Finally there is the interference term between spin-flip and no-spin-flip amplitudes. On Fig. 4 we show the measured ratio of the coefficient of this term to the coefficient of the $\sin^2 2\vartheta$ term. This ratio should be :

$$\mathrm{Im}\left[(A^*C+C^*B)\ \left(Q_2^0+(2/3)^{1/2}Q_2^2\right)/N_2^0\right] \Big/ \left[|A|^2+|C|^2\right]$$

and from the rest of the experiment we almost have enough information to calculate this quantity : there is some uncertainty about $\mathrm{Im}[(Q_2^0+(\frac{2}{3})^{1/2}Q_2^2)/N_2^0]$ though we know the real part of this quantity. So we try and see how well it works. Taking A, B and C as calculated from Yale phase shifts (here Harwell phase shifts give the same answer) and setting this imaginary part first zero and then minus two-thirds of the real part we get these two curves. We see from these curves that the experimental points for ϑ_p=15-35° could be fitted by a suitable choice of this imaginary part but that by no amount of fudging can we hope to fit the experimental point at ϑ_p=45°. Also shown is a curve calculated using the Gammel-Thaler potential (which is an older and less good fit to nucleon-nucleon scattering) and setting $\mathrm{Im}[(Q_2^0+(\frac{2}{3})^{1/2}Q_2^2)/N_2^0]$ zero which fits the data surprisingly well. This may be a departure from the impulse approximation, that the scattering of a proton from a bound nucleon with this momentum transfer differs from the corresponding scattering from a free nucleon. The fact that we can fit the data so well using the Gammel-Thaler potential shows that only small changes in A, B and C may be necessary. It is perhaps significant that we observe such a departure at the largest momentum transfer studied, where the proton-nucleon scattering involved is furthest from the energy shell. We find a similar result in analysis of the polarisation of the outgoing proton from this inelastic scattering, which is shown on Fig. 5. This is published Uppsala data. For the calculated curves we have set $Q_2^0=Q_2^2=N_2^2=0$ as we find them to be small experimentally; putting in any possible non-zero values

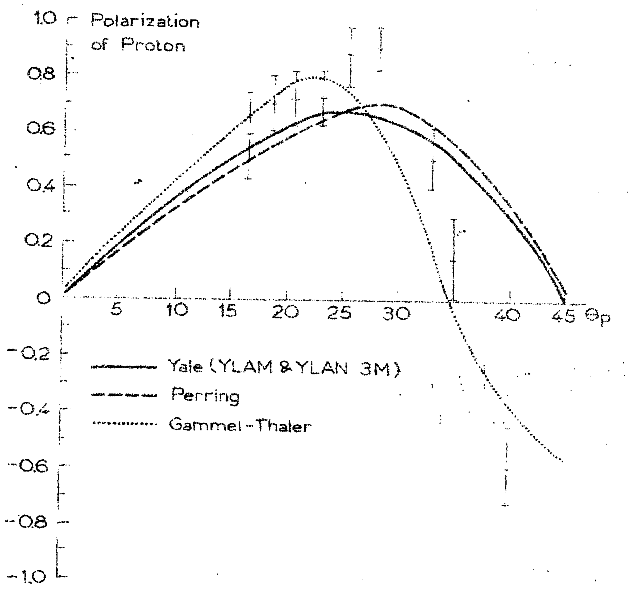

Fig. 5 Proton polarization at 185 MeV in transition to first excited state of ^{12}C.

makes no difference. Then the polarisation is just

$$2Re(A^*C)/[|A|^2+|C|^2] .$$

This is plotted for Yale and for Harwell phase shifts and for the Gammel-Thaler potential. We again find that the good fits to nucleon-nucleon scattering fit the data well to $\theta_p=30°$ but things go wrong at larger angles. The failure of the impulse approximation is here also at the largest momentum transfers studied, and we only need a small change in A,B and C to fit the data as is shown by the fit with the Gammel-Thaler potential. These measurements thus seem to provide the possibility of obtaining information about nucleon-nucleon scattering off the energy shell, which would be very new information about the nucleon-nucleon interaction. It is obvious that they should be extended to larger scattering angles.

I have gone through this case in tedious detail as I think it shows that we have quite a good understanding of the inelastic scattering reaction. There are certainly some loose ends to tied up but these do present interesting possibilities of obtaining further information. I have also shown that there is much more information you can obtain beside the theoretical cross-section with which to confront theoretical ideas. This is a good thing as is shown by the distorted wave analyses of these reactions by Elton and Jackson who find they are unable to tie down their fitting parameters with differential cross-sections alone, though they are able to demonstrate the inadequacy of the simplest shell-model wave functions.

It would seem possible and useful to extend these studies of the inelastic scattering reaction mechanism to other proton energies, up to meson threshold, at which the form of the nucleon-nucleon interaction is well known. The distorted wave effects should be different at these proton energies and so enable us to check our understanding of the reaction mechanism and the distorted wave effects. As the pion-nucleon interaction is so well known it could also be interesting to study inelastic scattering of pions. Observation of gamma radiation in coincidence with the outgoing pions could provide the necessary energy resolution to resolve the separate excited states : at the same time one could measure angular correlations to determine the polarization of the nuclear excited state.

Now I want to demonstrate the use of inelastic scattering as a tool in nuclear spectroscopy without using all the complication of my previous discussion. One can simplify because the states we see excited in inelastic scattering are those which are strongly excited and they are strongly excited because they are collective excitations of the ground state, either with or without spin-flip. With spin-flip $|Q_2^0|^2$ is enhanced and so provides the dominant contribution to the differential cross-section over the main peak in the angular distribution. On the other hand with no-spin-flip $|N_2^0|^2$ dominates similarly. (This latter case applies to the first excited state of ^{12}C we have discussed already.) We can then neglect the other matrix elements to a good approximation. We now explore systematic features of the experimental data to try to establish useful ways of comparing one excitation with another, in such a way that the complications due to distorted wave effects cancel out in the comparison.

Let us look at N_2^0. In a plane-wave model this is the matrix element of $\Sigma_\ell \, j_\ell(qr) Y_\ell^0(\vartheta, \varphi)$. Frequently selection rules will make all except one term in this sum zero; even if more than one term contributes one of them, determined by what sort of collective excitation it is, will be distinctly larger than the others. For small q this is $q^\ell r^\ell Y_\ell^0$ so that the matrix element is q^ℓ times the corresponding electric multipole radiative matrix element, M. We know that the radiative transition rate is $\sim(\frac{1}{\lambda})^{2\ell+1} |M|^2$. Thanks to the factor $(\frac{1}{\lambda})^{2\ell+1}$ electric quadrupole transitions are slower than electric dipole and so on. We avoid this in inelastic scattering as, instead of $(\frac{1}{\lambda})^{2\ell+1}$ we have $q^{2\ell}$, so we can make an electric quadrupole cross-section comparable to an electric dipole cross-section by going to larger q, larger scattering angle. For large enough momentum transfers the approximation for the spherical Bessel function breaks down : then N_2^o rises no further and we have a peak in the angular distribution. For higher multipolarities the peaks are at larger angles as is shown on Fig. 6 which shows differential cross-sections in the $^{40}Ca(p,p')$ reaction for exciting two states, from the J=0+ ground state, with respectively J=3- and J=5-. All these properties of the plane-wave model persist in the proper distorted-wave picture, as this introduces no more rapidly varying functions of radius into the integrand, so that over the peak in the angular distribution the inelastic scattering matrix elements are still only dependent on the general properties of the nucleus and not in any important way on any short-range correlations. Thus we would expect to find $\frac{d\sigma}{d\Omega} \sim (\text{gamma-ray rate})/E_\gamma^{2\ell+1}$. The proportionality constant here is hard to estimate as it depends so on the distorted wave effects. However, one would expect it to be much the same for neighbouring nuclei. We have found that it is so in a comparison between electric quadrupole transitions in $^{12}C(0 \rightarrow 4.43$ MeV) and in $^{11}B(0 \rightarrow 4.46$ MeV).

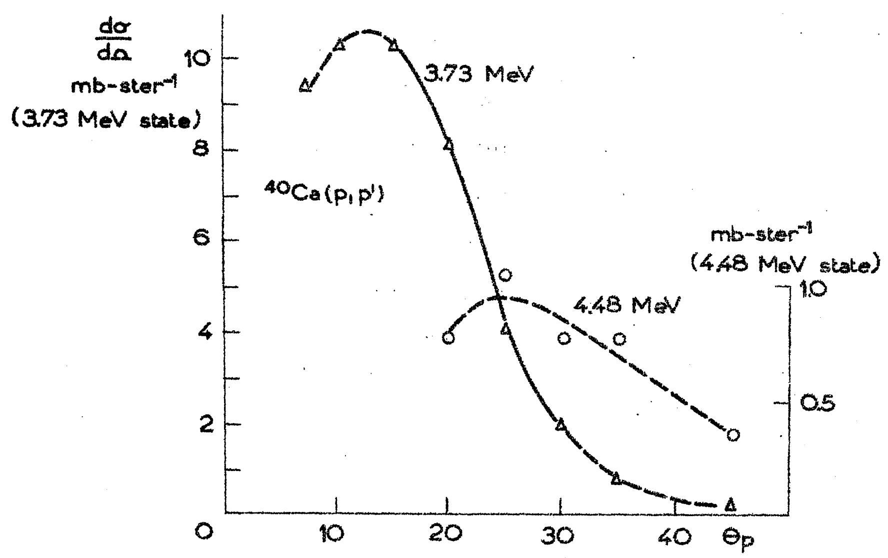

Fig. 6 Excitation of the 3^- and 5^- states in ^{40}Ca by inelastic proton scattering.

The first is J=0+ to J=2+ and so is purely electric quadrupole while the second is J=3/2- to J=5/2- and the gamma-ray transition is found to be 96% magnetic dipole and 4% electric quadrupole. It is, however, the electric quadrupole matrix element that is large : it is only the kinematic factor which makes the magnetic dipole part more rapid. It is therefore the electric quadrupole matrix element that determines the magnitude of the peak in the ^{11}B inelastic scattering and the magnitude is just what we expect from this comparison procedure; the magnetic dipole matrix element must only make a small contribution to the cross-section at forward angles. We tried to extend this comparison between inelastic scattering cross-sections and gamma-ray rates from ^{12}C to ^{24}Mg. We had a cross-section for exciting the first excited state of ^{24}Mg determined by observing production of gamma-rays of the right energy. It seemed improbable that these gamma-rays could be due to any other reaction. The ratio of this cross-section to the gamma-ray rate divided by E_γ^5 was close to the ratios for ^{12}C and ^{11}B. However, it seems that our identification of these gamma-rays was wrong as a more direct measurement at Orsay gives a lower $^{24}Mg(p,p')$ cross-section and thus a lower ratio, about a factor of five lower than for ^{12}C. One would expect the ratio to be lower for ^{24}Mg than for ^{12}C due to the increased absorption in a larger nucleus, but I am surprised that it is this much lower. A similar comparison between electric octopole transitions shows a decrease in the corresponding ratio of not much more than a factor of two on going from ^{16}O to ^{40}Ca. Perhaps someone will do some distorted wave calculations to explain this. There are also other states, typically in ^{28}Si and ^{32}S, whose lifetimes are known. When the cross-sections for their excitation in inelastic scattering are measured it will be possible to extend these comparisons further.

The matrix elements for spin-flip excitations have similar properties. For example, for excitation from a J=0+ to a J=1+ state there would be contributions from matrix elements of $\rho_0(r)Y_0^0\,\vec{\sigma}\cdot\hat{a}$ and of $\rho_2(r)Y_2^m\vec{\sigma}\cdot\hat{a}$. Again usually only one of these matrix elements will be enhanced and make a large contribution to the cross-section.

Again either matrix element will produce a peak in the angular distribution, the peak due to the second would be at a larger angle than that due to the first. Note as well that the first of these matrix elements is related to the magnetic dipole matrix element.

An example of the use of these ideas is given by the $^{7}Li(p,p')$ reaction. For this I try to apply the unified model of Nilsson to ^{7}Li, and to almost every other nucleus in the 1p shell and the 2s-1d shell, except for the closed shell nuclei. Primarily this is because I am then able to do some calculations myself and so have some fun, but one may be able to justify this model as a coarse approximation to the generating procedure of Hill, Wheeler, Peierls, Yoccoz, Elliot, Kurath and Pičman, which itself is believed to be a good approximation to the intermediate coupling shell model. The first three states of ^{7}Li (J=3/2-,1/2-,7/2-) look very like the first three states of the K=1/2 rotational band expected in the unified model, with the fourth state of the rotational band, J=5/2, predicted to be just where it is predicted to be in the intermediate coupling shell model. This J=5/2 state is not known in ^{7}Li but its mirror state in ^{7}Be has been identified in the last year 0.7 MeV above the predicted energy. If this is a rotational band we then have a prediction for the relative cross-sections for exciting the three excited states. We have measured the excitation of the first excited state at Harwell while the excitation of the second has been measured by Riou et al. at Orsay. Here Orsay and we use different techniques which, in each case, preclude the observation of the excitation of the other state. However, when these different techniques are applied to the excitation of the same state we and Orsay find we get the same cross-sections as was shown in Fig. 1. This gives us confidence that we can compare these different measurements to get the ratio of the cross-sections for excitation of the first excited state to that for the second. We thus find an experimental ratio of 0.40 to be compared with the 0.39 expected from the unified model. This striking agreement strongly supports our interpretation of the energy level scheme as a rotational band. One then expects the ratio of cross-section for exciting the third excited state to that for the second to be 0.17. Riou et al. observe the excitation of a state at the right sort of energy and with the right angular distribution and the cross-section for its excitation is $\simeq 0.2$ times that for the second excited state. This agreement with predicted numbers seems wonderful to me so I suggest that this is the observation of the missing $J=5/2$ state of the rotational band.

Thus we apparently have a useable systematic procedure for searching for states which make rapid radiative decays to the ground state of a nucleus, the collective excitations of the ground state. This is very like Coulomb excitation, except that for Coulomb excitation the higher the energy of the state the smaller the cross-section for its excitation, for equal matrix elements. For our inelastic scattering the only such kinematic factor is the final momentum divided by the initial momentum, which ratio is close to one for excitation of all states up to quite high energies. Thus our systematic procedure observes all states with appreciable matrix elements for their excitation, so that one gets more information but the data is more complicated and takes longer to disentangle. A lot of data, such as I have described for the $^{7}Li(p,p')$ reaction, is coming in for nuclei in the 1p and 2s-1d shells. A number of these nuclei are probably distorted : however only ^{7}Li and ^{9}Be seem simple enough to be described by one rotational band. The rest will require more analysis. It would be perhaps interesting to extend the comparison between inelastic scattering cross-sections and radiative decay rates to other proton energies where the distorted wave effects would be different and to extend these procedures to nuclei heavier than ^{40}Ca.

I want now to draw attention to one more trick one can do with inelastic scattering. A good example of this trick is in the $^{12}C(p,p')$ reaction. In ^{12}C there are two $J=1+$ states at 12.71 and 15.11 MeV with $T=0$ and 1 respectively. They are both quite strongly excited in inelastic scattering; in each case it is the matrix element analogous to the magnetic dipole matrix element which is large. One would expect the angular distributions to be proportional to $[|C|^2+|B|^2+|F|^2]|Q_2^o|^2$. The experimental angular distributions differ. However, one would expect the angular distributions of $|Q_2^o|^2$ for these two magnetic dipole excitations to be of much the same form only differing by a numerical constant. Thus the difference observed in the angular distributions should be due to the difference in $[|C|^2+|B|^2+|F|^2]$ for excitations with $\Delta T=0$ and 1. The ratio of this quantity for $\Delta T=0$ and 1 excitations respectively, as calculated from Harwell phase shifts, is shown on Fig. 7, together with the ratio of the two cross-sections multiplied by a suitable numerical factor. These two ratios, experimental and theoretical agree very well over the range of angles for which the cross-sections are well-known. Thus we seem to have a further check on the impulse approximation : this ratio of squared nucleon-nucleon scattering amplitudes is much the same for scattering from a bound nucleon as it is from a free nucleon. You will remember that Morpurgo showed that magnetic dipole radiative transitions with $\Delta T=0$ should be weak. You can have a large matrix element $\langle f|\vec{\sigma}|i\rangle$ for $\Delta T=0$ but in the radiative transition rate its square is multiplied by $(\mu_p+\mu_n-\frac{1}{2})^2$ which is a very small number so that the transition rate is very slow. Now in the cross-section for inelastic scattering the same squared matrix element, $|\langle f|\vec{\sigma}|i\rangle|^2$, is multiplied by $[|C|^2+|B|^2+|F|^2]$ which is relatively large, in place of the small $(\mu_p+\mu_n-\frac{1}{2})^2$. Thus in inelastic scattering magnetic dipole transitions with $\Delta T=0$ are not much weaker than those with $\Delta T=1$. We can therefore search for these comparatively strong $\Delta T=0$ magnetic dipole transitions in inelastic scattering. It would be very nice to have systematic information about them in self-conjugate nuclei as their sum rule is

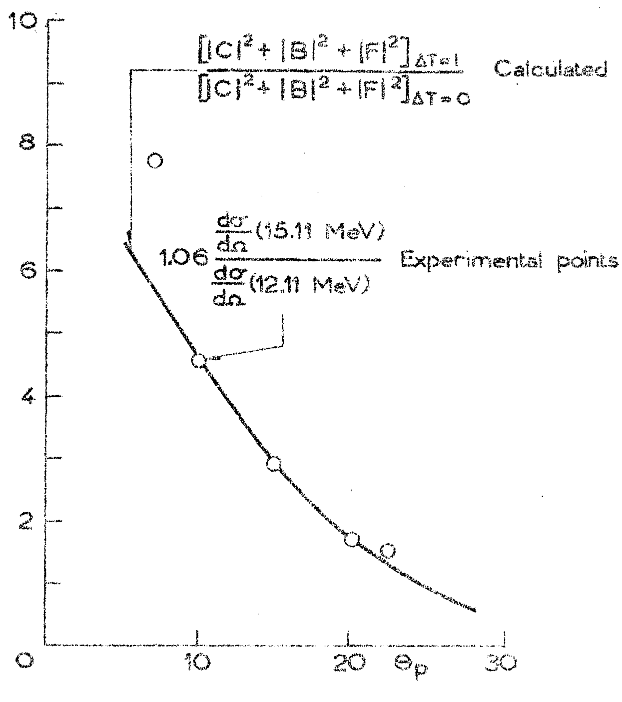

Fig. 7

very dependent on the structure of the ground state. For example, the sum rule gives zero if the ground state is a pure LS-coupling state $^1S_0$, so one would expect almost no $\Delta T=0$ magnetic dipole excitation from the ground states of the closed-shell nuclei ^{16}O and ^{40}Ca.

For no-spin-flip excitations the cross-section should be proportional to $[|A|^2+|C|^2]\,|F_2^0|^2$. For $\Delta T=1$ $[|A|^2+|C|^2]$ is small at 150 MeV, about 1/20 of what it is for $\Delta T=0$, so for $\Delta T=1$ we would expect the no-spin-flip excitation cross-sections to be small at this proton energy. This is rather similar to the Morpurgo rule on magnetic dipole radiative transitions, in that the nuclear matrix element can be quite large but the corresponding transition rate be small as the matrix element is multiplied by another number which is small. The only possible test I have found is the excitation of the 2+, T=1 state of ^{12}C at 16.11 MeV. Unfortunately there are two different measurements of its radiative decay rate : 1.6 eV and 0.22 eV. Taking the larger radiative width we would expect the ratio of its excitation cross-section to that for the 2+, T=0 state at 4.43 MeV to be 0.02, while for the smaller the expected ratio would be 0.003. The measured ratio is around 0.09. This larger result is not unreasonable as the spin-flip matrix elements will be contributing appreciably to this quite small cross-section. Thus there is no evidence against our conclusion from the impulse approximation that $[|A|^2+|C|^2]$ is small for $\Delta T=1$. It would seem that proton inelastic scattering at energies around 150 MeV is not a good way to search for no-spin-flip excitations with $\Delta T=1$. I have not investigated how this would change at other proton energies.

I now want to turn to reactions in which one nucleon is knocked out of the nucleus : (p,2p), (n,pn), (n,np) and such reactions. The classical measurements are those of Tyrén, Hillmann and Maris at Uppsala and they have been followed up at several laboratories. One measures the energies of the incident proton, E, and of the outgoing protons, E_1 and E_2, determining the binding energy of the ejected proton $E-E_1-E_2$, so that one measures a spectrum of the number of

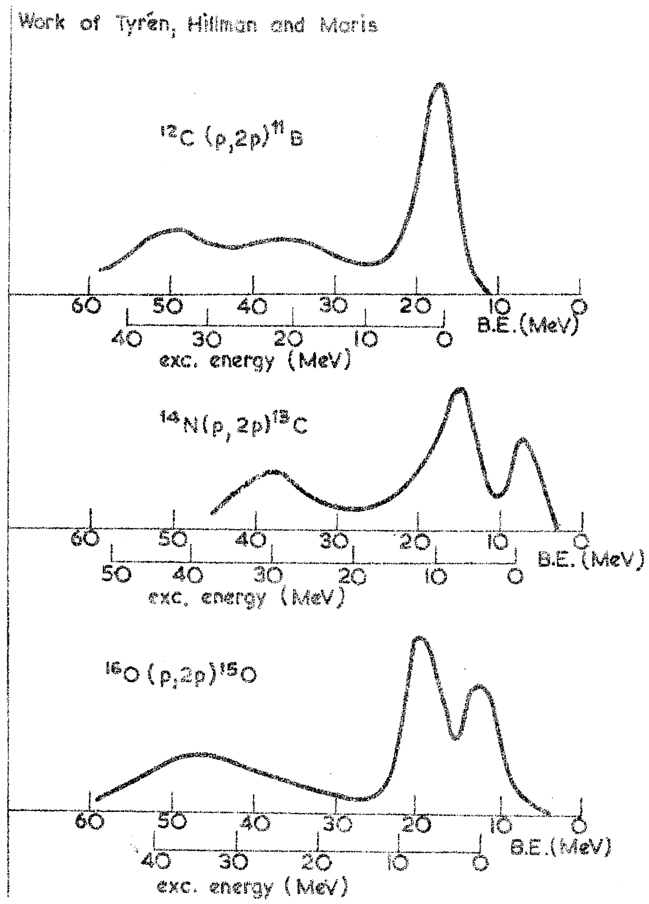

Fig. 8

events as a function of this binding energy. This is just the relative population of the states of the residual nucleus. Some typical binding energy spectra, from the original Uppsala work, are shown on Fig. 8. These show the peaking in the binding energy spectra, which is found quite generally for all nuclei up to ^{40}Ca : only a certain few states of the residual nucleus are produced strongly in a (p,2p) reaction. Such simple results imply a simple mechanism : the struck proton is knocked straight out, the remaining A-1 nucleons just sitting there as spectators and so preserving the structure they had before the collision. The states in which the residual nucleus is left are the parent states of the target ground state. At first sight the idea that this structure is not disturbed by the two protons cruising through it is just a little surprising but after a little thought we see that it is not. If one of the outgoing protons does interact the result will almost always be the ejection of at least one further nucleon. So if we detect two outgoing protons which leave the residual nucleus of mass A-1 in a comparatively lowlying state we are selecting an event in which the outgoing protons have undergone no further interaction. Further, such events will be largely determined by the independent particle shell model wave functions, which describe most of the ground state. The departures from such wave functions are where two nucleons are close together interacting strongly. Thus the incident proton would not interact with just one of these nucleons but with the two together, usually ejecting both of them. This means that these parts of the wave function would not contribute to a (p,2p) reaction which leaves the residual nucleus with comparatively low excitation.

So now to describe the parent states that have been found. The striking result is that there are found to be very few important parents of ground states of stable nuclei. It is found that they can be understood very well in terms of simple jj-coupling wave functions. Typically on Fig. 8 the single peak for ^{12}C corresponds to the ejection of a $1p_{3/2}$ proton and the two peaks for ^{14}N and ^{16}O corresponds to the emission of a $1p_{3/2}$ and a $1p_{1/2}$ proton. The binding energy of a proton from a given subshell is found to vary uniformly with A through the 1p shell. This simple jj-coupling interpretation looks very pretty, but we really ought to be horrified by it as it conflicts with so much work in low energy nuclear physics which shows that the wave functions of these nuclei are far from jj-coupling : a state of intermediate coupling midway between jj-coupling and LS-coupling prevails in the 1p shell. However, these measurements have comparatively coarse energy resolution. They observe the parent states which are produced strongly but they do not provide any good measurement of the weaker production of the other states of the residual nuclei. In jj-coupling the production of these other states would be zero. Experimentally it is non-zero, as for example we have found in detail in studies of the knock-out of a nucleon from ^{12}C. For jj-coupling in this case a $1p_{3/2}$ nucleon would be knocked out leaving ^{11}B or ^{11}C purely in its ground state. We are able to measure production of excited states of these residual nuclei by observing the gamma-radiation from their decay and find results which are in good agreement with intermediate coupling calculations. Of the states of ^{11}B or ^{11}C with excitations up to about 9 MeV, roughly ten states, we find that the ground state is produced 85% of the time, to be compared with the 100% expected for jj-coupling, while the remaining 15% of the cross-section is shared between the production of several states. Similar conclusions are reached in measurements of the $^{12}C(p,d)$ reaction at Orsay. Thus intermediate coupling is probably saved but we have to face the experimental fact that, in intermediate coupling, there are only a few strong parents of nuclear ground states, and that these strong parents are just those one would expect for jj-coupling. These states do not absorb quite 100% of the parentage, as they would for jj-coupling, but perhaps round 85% as we find in ^{12}C. This is an example of a curious phenomenon that for intermediate coupling some property may be consistently close to what it is either for the jj-coupling extreme or for the LS-coupling extreme : one can say that such a property persists from the extreme coupling into the intermediate coupling region. We have just seen that the parentage of many nuclear ground states is close to that expected for the jj-coupling extreme. An example of a property which persists from the LS-coupling extreme is the ratio of spin-flip intensity to no-spin-flip intensity in inelastic scattering. For excitation of lowlying states in, typically, ^{12}C and ^{16}O this ratio is very small, close to the value of zero calculated for LS-coupling wave functions. As a numerical example of this persistence of properties from both extreme couplings for the same nuclear states I show this ratio of spin-flip to no-spin-flip for the excitation of the first excited state of ^{12}C in inelastic scattering, (Fig. 9), the value of which ratio persists from the LS-coupling extreme and, the ratio of the parentage of the ground state of ^{12}C which is contained in the first excited state of ^{11}B to that which is contained in the ground state : the value of this ratio persists from the jj-coupling extreme.

These persistence phenomena, one example of which has been displayed so systematically by the nucleon knock-out reactions, presumably reflect some simplicity of intermediate coupling wave functions which is not directly obvious. A model which does display this simplicity is the unified model, which I have previously argued is perhaps an approximation to intermediate coupling. It has been worked out in detail for ^{11}B, in a way which is interesting as it also displays the complementary information which can be obtained from nucleon knock-out and from inelastic scattering. There are two lowlying rotational bands with respectively K=1/2 and 3/2. States of the same J mix strongly, when one calculates the rotation-particle coupling matrix elements from Nilsson's wave functions, to produce an energy level scheme which shows interesting features. There are only two free parameters left for fiddling according to one's fancy : the separation of the two bands before mixing which somewhat determines the form of the resulting energy spectrum and the moment of inertia which only determines the over-all energy scale. The calculated energy level scheme as a function of η, Nilsson's ratio of spheriodal distortion to strength of the spin-orbit coupling, is shown on Fig. 10. The wave functions of these states can now be calculated : a convenient description is as combinations of ^{12}C states each coupled to a single hole. For $\eta=0$, the jj-coupling extreme, we find the ground state is just the ground state of ^{12}C coupled to a $1p_{3/2}$ hole; the separation of the two bands before mixing was

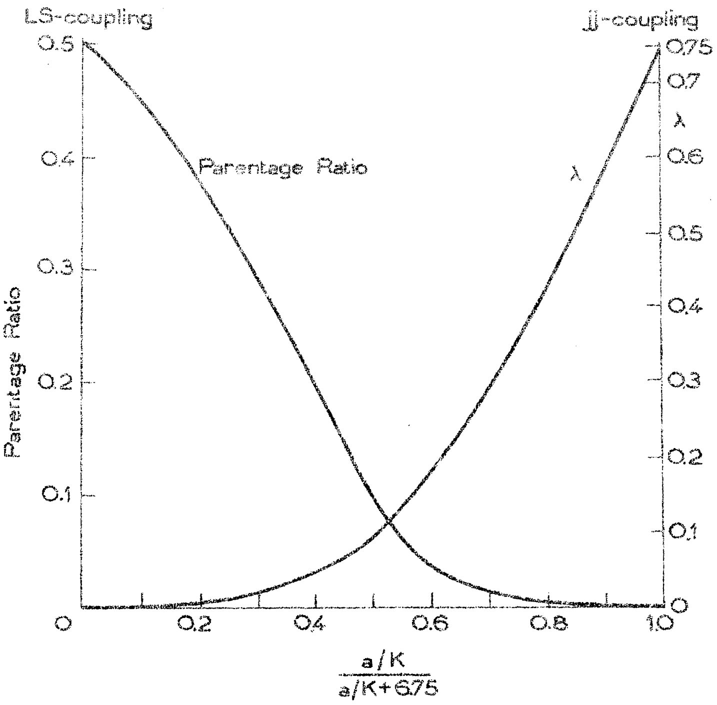

Fig. 9

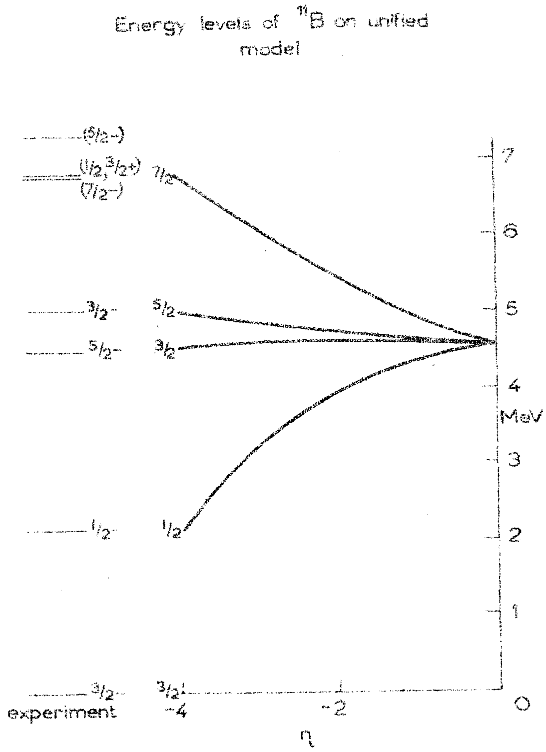

Fig. 10

chosen to make this so as it is the correct jj-coupling result. This degenerate first excited cluster of states is formed by a $1p_{3/2}$ hole coupled to the $J=2$ first excited state of ^{12}C. Thus from the unified model we have produced a weak-coupling spectrum; the Coriolis force can apparently have a decoupling effect in the mixing of two bands just as it does in a $K=1/2$ rotational band. So at $\eta=0$ the ground state of ^{11}B is the sole parent of the ground state of ^{12}C, while this first excited cluster are the only states produced in electric quadrupole inelastic scattering from the ground state. On going from $\eta=0$ to $\eta=-4$ the energies of the states spread out to fit very well with the experimental energies, as is shown in Fig. 10, but the wave functions do not change very much when expressed as combinations of holes and ^{12}C states, the admixtures of other combinations not being greater than 10-20%. (Of course, if one were also to take into account the change in the wave function of the ^{12}C states on changing η, one would see large changes in the ^{11}B wave functions.) So on going from $\eta=0$ to $\eta=-4$, these simple features of the parentage of the ^{12}C ground state and of the ^{11}B inelastic scattering persist, in agreement with experiment. Similar calculations also work very well in the 2s-1d shell, noting that here again mixing of states from different rotation bands is very important. In particular, they explain the experimental fact that the peaks in the binding energy spectra are broader on bombarding (4n-1)- nuclei than they are on bombarding 4n- nuclei.

A further example of this persistence of properties from one extreme or the other, which is particularly illuminating, is provided by the octopole vibration of ^{16}O. Take the ^{16}O ground state to be a filled 1p shell; it is then also an LS-coupling state $^1S_0$. One makes three possible $J=3-$ states, which are thus possible contributors to the octopole vibration, by promoting one particle to the 2s-1d shell and thus leaving a hole in the 1p shell. In the jj-coupling extreme these three are $(1p_{1/2}^{-1}1d_{5/2})$, $(1p_{3/2}^{-1}1d_{5/2})$, $(1p_{3/2}^{-1}1d_{3/2})$ and in the LS-coupling extreme they are $^1F_3$, $^3D_3$, $^3F_3$. As an electric octopole radiative transition cannot, to a good approximation, do anything to spins, such an electric octopole transition from the singlet ground state can only be to the $^1F_3$ state. Thus the $^1F_3$ state would exhaust any electric octopole sum rule (for $1\hbar\omega$ excitations). So we see that the LS-coupling wave functions must display the coherent motions of a collective excitation while the jj-coupling wave functions seem to display more explicitly the single-particle character. Intermediate coupling calculations of Elliot and Flowers produce a lowest J=3- state of ^{16}O which when written in terms of LS-coupling wave functions is ≃80% (in intensity) $^1F_3$ with small contributions from the triplet spin wave functions, and, at the same time, when written in terms of jj-coupling wave functions is ≃80% $(1p_{1/2}^{-1}1d_{5/2})$. This happens even though an expansion of the $(1p_{1/2}^{-1}1d_{5/2})$ wave function in terms of LS-coupling wave functions does not contain an inordinately large proportion of $^1F_3$, and vice versa. So one sees this J=3- state almost exhausts the electric octopole sum rule from the ground state (that for $1\hbar\omega$ excitations described by Brown, Evans and Thouless) and, at the same time has a simple single-particle character : it should be an important parent of the ground state of ^{17}O. It does not really seem surprising that the lowest intermediate coupling state should be that which contains large components of the wave functions corresponding both to the lowest LS-coupling state and to the lowest jj-coupling state. One can thus speculate that one will often find combined in the same lowlying state both such collective properties and such simple single-particle properties, in the sense that a large proportion, though not quite all, of these properties is present in the state, as we have shown for the octopole vibration of ^{16}O.

No such measurements of (p,2p) reactions have been made on nuclei heavier than ^{40}Ca. It would be interesting to see if the ground states of heavier nuclei have such simple parentage.

Angular correlations of the outgoing protons from (p,2p) reactions also provide useful information. The form of the angular correlation depends on the struck proton's momentum distribution : the broader the momentum distribution the broader is the range of angles between the two protons. Measurements have typically been made with the two protons detected in the same plane at equal angle ϑ, on either side of the incident beam, corresponding to the struck proton having its momentum parallel to the incident momentum. The coincidence rate is measured as a function of this angle ϑ for struck protons of a definite binding energy. The momentum distribution for any state except an s state is zero at zero momentum so that the corresponding angular distribution should have a dip in the middle at the angle corresponding to a stationary target proton. For an s state the momentum distribution peaks at zero and so the corresponding angular correlation peaks in the middle. This dip will be smeared out by the distortion of the proton waves going in and out of the nucleus, but is still visible as is shown in Fig. 11 which shows some Orsay measurements of the ^{7}Li(p,2p) reaction. These angular correlations are useful for finding out something about the angular momentum state of the target proton, whether it is s wave or not. The results are throughout consistent with the identifications made from the systematic features of the binding energies.

One can also attempt to extract information about the momentum distributions in detail, but to do this one has to deal with the distorted wave problem. Distorted wave calculations have been made using momentum distributions derived from likely nuclear wave functions. These fit the experimental data quite well but there are some troubles in detail, particularly in fitting measurements of the ^{6}Li(p,2p) reaction. The difficulty now lies in deciding whether the inadequacy of the fits is due to the wave functions assumed or to the calculations which use the WKB-approximation.

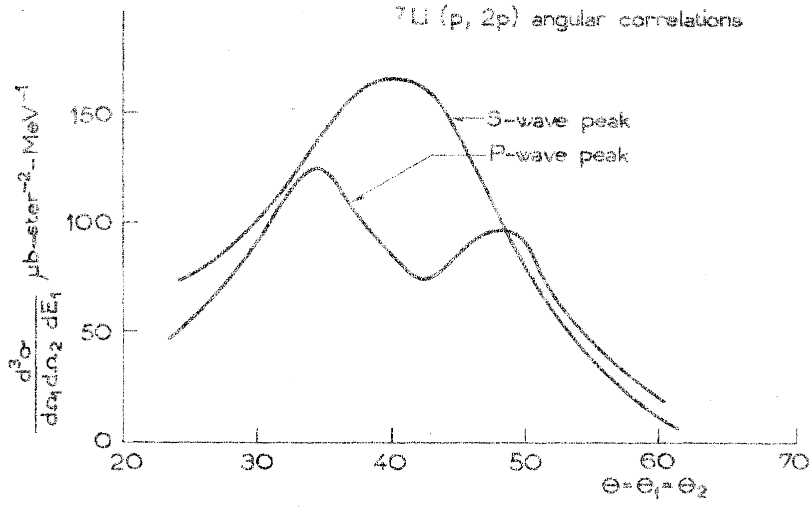

Fig. 11

In these distorted wave calculations the hardest thing to calculate is the absolute magnitude of the cross-section. However, you might think that relative magnitudes of peaks in the binding energy spectrum would tell us the relative contributions to the parentage of the target ground state. That this is not so is seen in the $^{40}Ca(p,2p)$ reaction, in the recent measurements of Tibell and others at Uppsala. They find peaks corresponding to binding energies of 8.3 MeV (this is the ground state of ^{39}K), 10.8 MeV and 15.0 MeV. They assign these to ejection of protons of respectively $d_{3/2}$, $s_{1/2}$ and $d_{5/2}$ from this doubly-closed shell nucleus. The angular correlations shown in Fig. 12 support these assignments. However, the relative magnitudes are not what one might expect. As there are six $d_{5/2}$ protons available to be ejected, four $d_{3/2}$ protons and only two $s_{1/2}$ protons, the fact that there are relatively so many $s_{1/2}$ protons ejected would seem to show that the probability of the incident proton interacting with an $s_{1/2}$ proton is higher than for it interacting with $d_{5/2}$ or $d_{3/2}$ protons. This, however, is not really surprising as the cross-section is

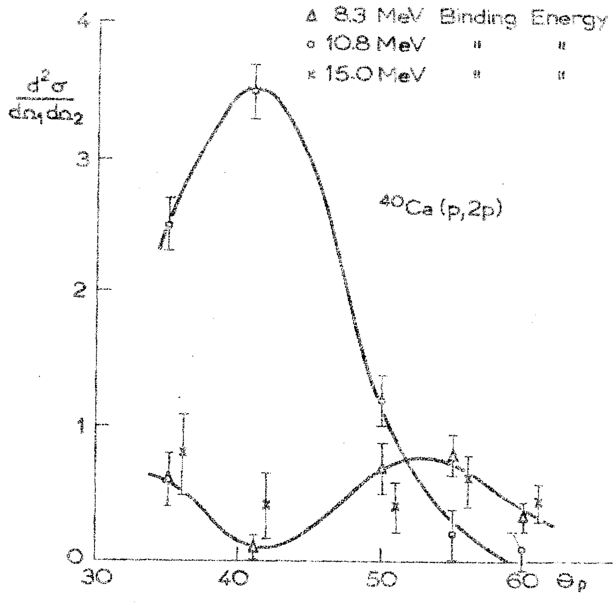

Fig. 12

determined by a matrix element $< \chi_2 \chi_3 \, M(q) | \chi_1 \varphi_0 >$ where χ_2, χ_3 are the wave functions of the outgoing protons, χ_1 of the ingoing proton, φ_0 of the struck proton and $M(q)$ the nucleon-nucleon scattering matrix element. Thus this matrix element and so the magnitude of the cross-section will depend on φ_0, in particular on the relative weight given to different parts of $M(q)$ by the angular momentum involved in φ_0. I will remark that a calculation of a simple model of the scattering of a spinless particle from a heavy spinless particle does show that the scattering cross-section is reduced by the target particle having angular momentum greater than zero, the reduction being greater the higher the angular momentum which is qualitatively what is observed in the $^{40}Ca(p,2p)$ reaction. It would therefore be interesting to study this, to test the impulse approximation further and so obtain further information about the scattering interaction between a proton and a bound nucleon. As well as cross-section measurements it might also be rewarding to measure asymmetries in (p,2p) reactions with polarised protons.

A further measurement which could show such a change in the nucleon-nucleon scattering cross-sections is the ratio of cross-sections for (p,2p) and (p,pn) reactions on self-conjugate nuclei leading to mirror states. For free nucleon-nucleon scattering at 150 MeV this ratio is 0.56 while for the $^{12}C(p,2p)$ and (p,pn) reactions leading to lowlying states of ^{11}B and ^{11}C the ratio is 0.40 ± 0.06.

We thus see that nucleon knock-out reactions do not determine the parentage of the target ground state directly. In our analysis of our measurements of the knock-out of a nucleon from ^{12}C we assumed that the cross-sections for ejecting $1p_{3/2}$ and $1p_{1/2}$ nucleons were equal. Thus some doubt can be cast on our conclusion that there is agreement with the predictions of intermediate coupling calculations. However, we can say that it would need an unbelievably large difference in these two cross-sections to upset this conclusion.

I now want to talk about reactions involving the ejection of clusters from the nucleus. First let me say what I think I mean by clusters. For definiteness I will discuss the example of the alpha-particle model of ^{8}Be. Lowlying states of ^{8}Be can apparently be described in terms of two cluster, each of four nucleons. In each cluster the angle-spin wave functions with respect to the centre-of-mass of the cluster are the same s-wave wave functions as in an alpha-particle, but the radial dimensions of the clusters are probably larger than for real alpha-particles. Nevertheless I shall call these clusters alpha-particles : it is a picturesque description which has the weight of tradition behind it. However, the s wave motion of the nucleons about the centre-of-mass of the cluster is not their only motion : the alpha-particles are in motion about their common centre-of-mass. In the ground state of ^{8}Be, for example, this relative motion is s wave. We further have to allow for exchange of nucleons from one alpha-particle to the other, so that really our alpha-particles are constantly dissolving and reforming. When this is done it is found that four of the eight nucleons are in an s state with respect to the centre-of-mass of the nucleus as a whole and the other four are in a p state just as they would be in the shell model. If you use harmonic oscillator potentials the wave functions of the alpha-particle model are the same as LS-coupling wave functions. For other not too dissimilar potentials this is presumably still true to a good approximation. There is plenty of evidence for such clustering in light nuclei : for example, the lowlying states of ^{8}Be have large reduced widths for break-up into two alpha-particles. This is not spoiled by a difference in size between the clusters and real alpha-particles as a wave function of a cluster shaped like an alpha-particle but larger can presumably be expanded in terms of the wave function of an alpha-particle plus other wave functions of four nucleons with the alpha-particle making a major contribution. It remains to remark that the real ^{8}Be is not in a state of LS-coupling but of intermediate coupling. I assume, without proof, that the experimental results imply that this property of clustering is one that persists from the LS-coupling extreme into the jj-coupling region.

There has been some confusion about this. For example a remark has been made that the $^{16}O(p,2p)$ measurements, which show that most of the ejected protons come from the p shell, provide evidence against the alpha-particle model of ^{16}O. The argument is that in the alpha-particle model all sixteen nucleons are in s states. However, each nucleon is in an s state with respect to the centre-of-mass of its cluster, but this does not mean that it is in an s state with respect to the centre-of-mass of the nucleus. Indeed three-quarters of the nucleons are in p states with respect to the centre of the nucleus and there is no discrepancy.

Several reactions in which such clusters are ejected from the nucleus have been studied experimentally. Typical of these is the $^{6}Li(p,pd)$ experiment at Orsay. The ^{6}Li ground state is believed to be made up of two clusters : an alpha-particle and a deuteron in s state relative motion. The experiment observed the outgoing proton and deuteron in coincidence, with energies corresponding to leaving the alpha-particle in its ground state. The angular correlation of proton and deuteron is shown in Fig. 13 : it peaks strongly at the angle corresponding to scattering of a proton from a stationary deuteron, so giving good support to the cluster picture of ^{6}Li. Similar measurements were also made of the $^{7}Li(p,pd)$ reaction where the yield was much lower : they are also shown in Fig. 13. This is in accord with the belief that the ^{7}Li ground state is made

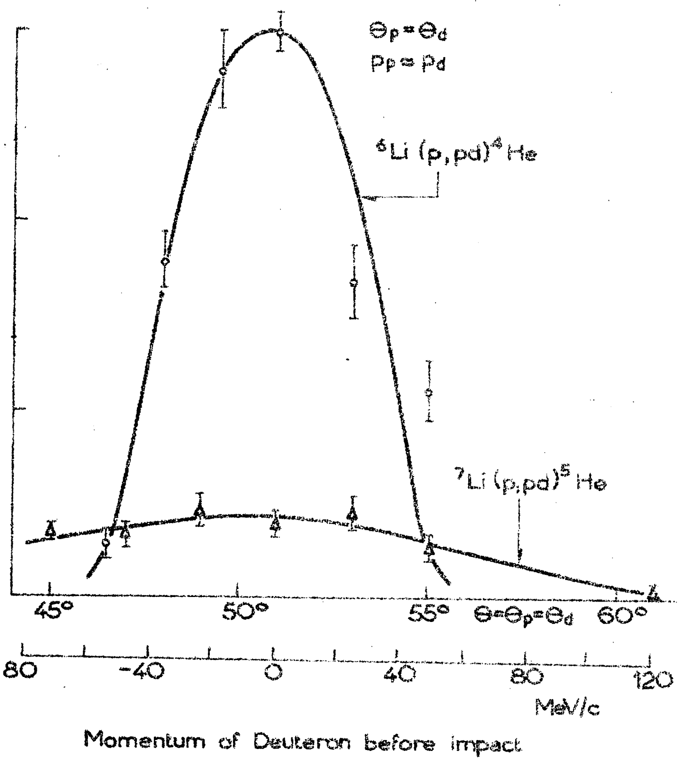

Fig. 13

up of alpha-particles and triton clusters; there are very few deuterons in either of these. A similar angular correlation has been measured in the $^{12}C(p,p\alpha)$ reaction at Harwell which indicates the presence of alpha-particle clusters in ^{12}C as expected. Such clusters in ^{12}C are also indicated in a $^{12}C(\alpha,2\alpha)$ experiment at Berkeley.

A further such indication of clustering has been found in the observation of gamma-rays produced in 150 MeV proton bombardment of a number of nuclei in the 2s-1d shell. For (4n-1) and 4n nuclei as targets the strongest gamma-ray produced almost always has the energy corresponding to the first excited state of the next lightest 4n nucleus : in each case studied this first excited state is the collective J=2+ state. Here is a tabulation of such cross-sections.

Residual Nucleus	p,pt Cross-sections	p,pα Cross-sections
^{12}C	-	8.3 ± 1.7mb
^{20}Ne	4.4 ± 3.1mb	15 ± 3 mb
^{24}Mg	31 ± 4 mb	28.5 ± 4 mb
^{28}Si	32 ± 4.5mb	25 ± 5 mb
^{32}S	~ 40 mb	-
^{36}A	37 ± 5 mb	31.5 ± 5 mb

Production of any higher states of these nuclei must be much weaker as no such gamma-rays are observed, while we can of course say nothing about production of the ground states. Such systematic cross-sections suggest the presence of triton clusters in the

(4n-1) nuclei and alpha-particle clusters in the 4n nuclei; in each case the other cluster is the first excited state and perhaps the ground state of the next lowest 4n nucleus. At the same time there is some slight evidence for proton induced fission of ^{23}Na leaving ^{12}C in its first excited state. Such fission is not quite inconceivable as, in the alpha-particle model, ^{23}Na can be made up of ^{12}C and ^{11}B.

These reactions at present provide very nice qualitative information but I think the trouble will be to determine numerical magnitudes. For example, what part of the wave function of the ground state of ^{16}O is made up of alpha-particles : how close is it to 100% ? Distorted wave calculations will be difficult and it will be even more difficult, initially, to believe in the certainty of any numerical magnitudes deduced from these calculations, without some way of testing them experimentally. I suppose the best prospect is to study a lot of these reactions and so find systematic features, such as has already been done somewhat for both inelastic scattering and for nucleon knock-out reactions.

While on the subject of clustering it is perhaps amusing to mention that one can apparently make rather a pretty picture of the ^{6}Li(p,p') reaction, the states excited being those reached by changing the state of relative motion of the deuteron and alpha-particle clusters from s wave to d wave. Such a picture seems to have some connection with experiment.

There is a further curious result of the experiment in which the gamma-radiation produced by 150 MeV proton bombardment of nuclei up to ^{40}Ca was observed. There were remarkably few gamma-rays produced strongly; indeed, this is why this experiment was possible using a sodium iodide crystal. A typical pulse-height spectrum is seen in Fig. 14. We see the peaks, due to the strong gamma-rays, superimposed on a background. As you see this background remains on inserting lead between target and counter so it is probably largely due to neutrons produced in the target. Certainly very little of it can be due to a large number of individually low intensity gamma-rays such as one might expect to be produced. Thus there is not much gamma-radiation produced besides the few intense ones. The total cross-section for producing these few intense gamma-rays is typically about 100 mb, which is a large proportion, about a quarter, of the total reaction cross-section, typically about 400 mb. We see that only a comparatively small part of the remaining 300 mb can be due to the production of other gamma-rays so there must also be strong production of nuclear ground states. So most of the reaction cross-section in these comparatively light nuclei is due to production of either ground states or a small number of low-lying excited states, each produced with comparatively large cross-sections. One can understand somewhat how this might happen. In the initial direct interaction one or more nucleons or clusters of nucleons are ejected from a ground state with a wave function of high spatial symmetry, leaving the remaining nucleons as spectators with the structure they had before the interaction. This structure will be simple and so often a lowlying state. Sometimes, however, it will be a fairly highly excited state and then, in the evaporation phase of the reaction, one might expect many excited states of the residual nuclei to be produced and thus many gamma-rays each of individually low intensity. However, if this high excited state has a high symmetry it would probably break up most of the time to produce states of high symmetry, so that the final result would again be only a comparatively small number of final states at low excitation energies. This bias towards producing lowlying states of high symmetry in the evaporation phase differs from the ususal statistical description of this phase in spallation reactions. However, our experimental results

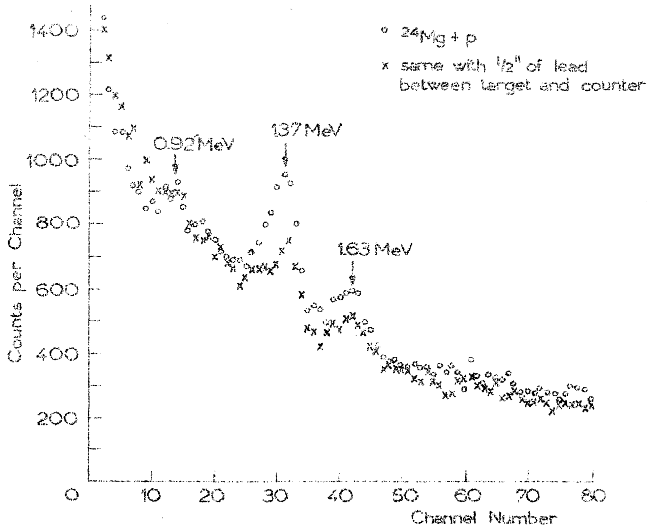

Fig. 14

apply to comparatively light nuclei for which $N \simeq Z$. Most spallation measurements are for heavier nuclei far from the $N=Z$ line, often with protons and neutrons filling different shells. Thus they do not have the high spatial symmetry and general simplicity of the nuclei we have concerned ourselves with. It might be interesting to see if the spectrum of many gamma-rays each individually of low intensity is stronger in the heavier, messier, nuclei.

In all I have said up to now the results have really been determined by the shell-model structure of nuclei, and so we have obtained information about this shell-model structure. I now want to discuss the possibility of obtaining information about the parts of nuclear wave functions where we expect the shell model to fail, wherever two nucleons come close together : less than 0.5 fermi or thereabouts. Such information could be measurements of such things as the nucleon correlation function at small separations or the observation of relatively strong high momentum components in the momentum distribution. On the whole this is a very difficult problem as in most conceivable experiments the effect you want to observe is covered up by something else. Typically it was thought that wide-angle deuterons produced in a (p,d) reaction would be due to neutrons with large transverse momenta, so that one could find something out about the momentum distribution at high momenta. However, it is now evident that most, if not all, of these wide-angle deuterons are due to incident protons which scatter first in the nucleus and then pick up a low momentum neutron on the way out.

I have already remarked on the possibility that if a proton interacts with a nucleon close to, and interacting strongly with, another nucleon, then one would expect both nucleons to be ejected from the nucleus. Thus observation of, say, a (p,3p) reaction might tell us about close correlations. However, a (p,3p) reaction can also be due to two successive collisions in the same nucleus. And we have already seen, in talking about the ^{6}Li(p,pd) reaction that a two-nucleon cluster can be present in the nucleus and be ejected by a proton without it being a particularly close correlation. To separate these, and possibly other processes, will need much detailed study of the reaction mechanisms. It will also, I would think, need detailed investigation of the energy and angular correlations of all three protons, which would be difficult to do. All that is prospectively available is about four hundred events in bubble chamber investigations of the $^{12}C(p,3p)$ reaction at Oxford and at Orsay.

Another possibility may be the investigation of wide-angle elastic or inelastic scattering. If this were due to a single hard collision in the nucleus this would mean that the struck nucleon in the nucleus had a high momentum either before or after the collision, so one might hope in this way to find something about the high momentum part of the momentum distribution. However, wide-angle scattering can also be due to a number of softer collisions. In this latter case one might expect the spectrum of states produced in inelastic scattering to be much the same as for scattering at forward angles : the collective excitations of the ground state. As these collective excitations are an aspect of the shell-model structure of the nucleus one might think that they would not be picked out so particularly when a single hard collision takes place, a collision with a nucleon with a high momentum which is thus representative of a part of the nucleus where the shell model fails. Thus one might expect such hard collisions to be indicated by excitation of comparatively all the nuclear states available, so that at wide angles one would hope to see, in the spectrum of states produced in inelastic scattering, an increase in the relative production of the general mass of excited states as compared with the production of the collective states. There is some indication of this in the 100 MeV $^{12}C(p,p')$ work of Strauch and Titus at Harvard and we are investigating it further.

Such work does present the possibility of investigating current theories of nuclear matter. Such theories seem to do well in predicting bulk properties such as binding energies and densities but it would be fun to test them further. The situation is perhaps analogous to the state of affairs in the theory of electrons in metals. Up to a few years ago this theory did a good job with bulk properties but now it has been tested much further by measurements of the details of the Fermi surface. To do a similar job for nuclear matter seems to me a very interesting problem. The only trouble is that I do not know how to do it and the only prospect seems to be a lot of work. It will - I am sure - require a very detailed understanding of the reaction mechanisms involved.

My ideas in this field have been influenced by discussions with many people, in particular D.M. Brink, G.E. Brown, K.J. Foley, N. Marty, D. Newton, P. Radvanyi, M. Riou, B. Rose, D.J. Rowe, G.L. Salmon, G.R. Satchler, R.E. Segel, D.H. Wilkinson and W.S.C. Williams. I greatfully acknowledge the ideas I have gained from them.

SESSION VI

FRAGMENTATION INDUCED BY HIGH ENERGY PARTICLES

Speaker :

N.A. PERFILOV

FRAGMENTATION INDUCED BY HIGH ENERGY PARTICLES

N.A. Perfilov (presented by O.V. Lozhkin)
V.G. Khlopin Radium Institute, Leningrad.

Among the products of nuclear reactions emitted in interactions of protons having energies in the range of hundreds of MeV or of other particles with corresponding energies, there can be observed particles with big charges and masses. These particles are called fragments and the process of their emission is called fragmentation.

The study of the fragmentation process is of considerable interest for understanding the inner couplings in certain states of nuclear matter and nuclear structure. Up to date there have already been a large number of investigations dealing with the problem. The present situation of the problem of fragmentation was described in a review [1] and in the book "Nuclear Reactions Induced by High Energy Particles" [2] published by the Academy of Science of the U.S.S.R. in 1962. The book has an extensive list of references. This report contains only a brief recapitulation of the main facts and a discussion of the possible mechanism of fragmentation.

I. BASIC EXPERIMENTAL RESULTS

1. The dependence of the yields of fragments on the incident proton energy and the mass number of target nuclei

Let us consider interactions of protons with relatively heavy nuclei when the observed multicharged particles are not residual nuclei. The emission cross-section of fragments depends markedly on the energy

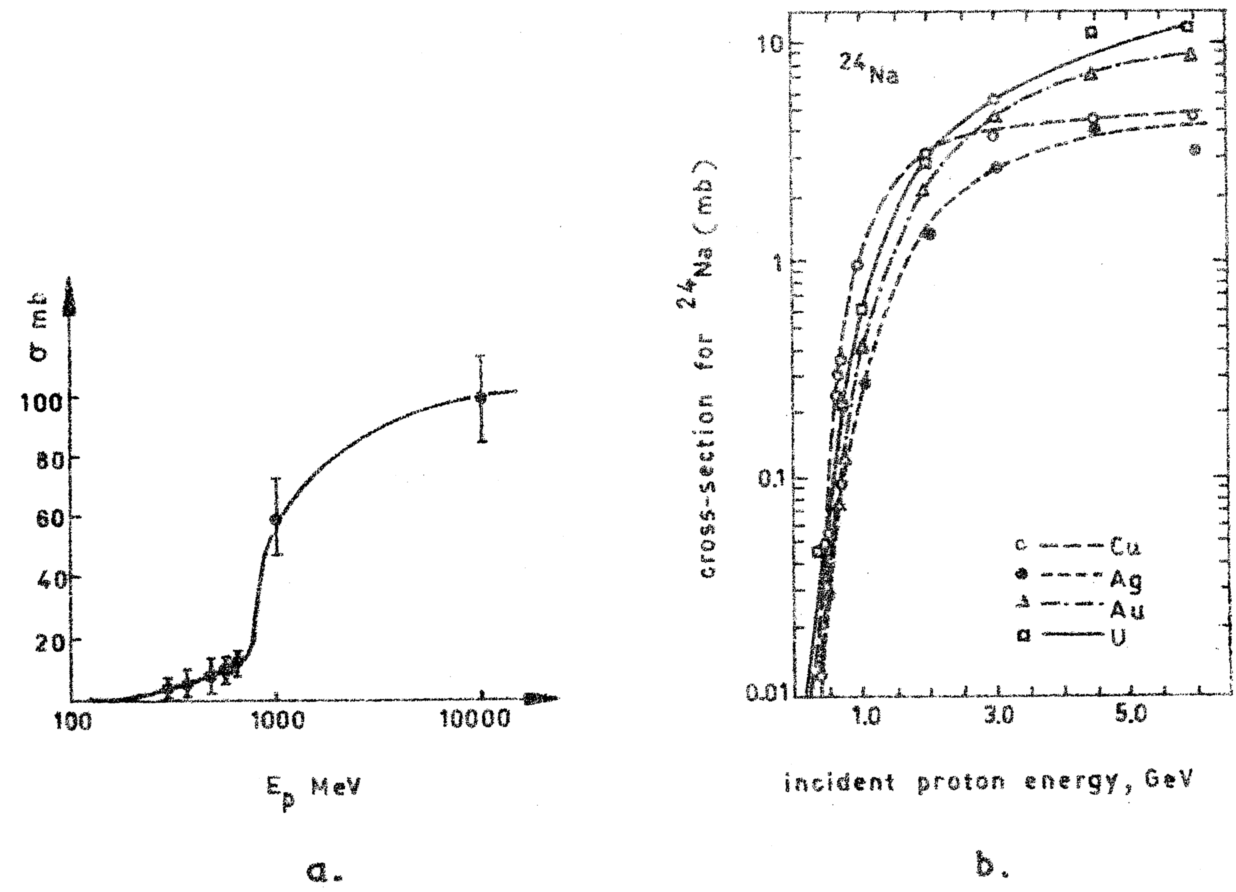

Fig. 1 Fragment cross-section vs proton energy for :
a) $Z \geq 4$; b) ^{24}Na

of incident protons and increases with their energy. The steepest growth is observed in the energy region from several hundred MeV up to 1 GeV. The curves showing this growth are given in Fig. 1 : curve a) represents measurements in emulsions for fragments with $Z \geq 4$ and curve b) shows radiochemical measurements of yields of ^{24}Na Refs. 4)-12).

The growth of the function $\sigma(E)$ is related to the increase in the probability of large energy transfers to target nuclei due to the increase of the incident proton energy 3),13)-16). The latter fact can easily be seen in studies of the relationship between the probability of fragment emission and the number of prongs in a star in disintegrations of Ag and Br nuclei in nuclear emulsions (Fig. 2a). The greater the number of the prongs (this number characterizes the energy transferred to

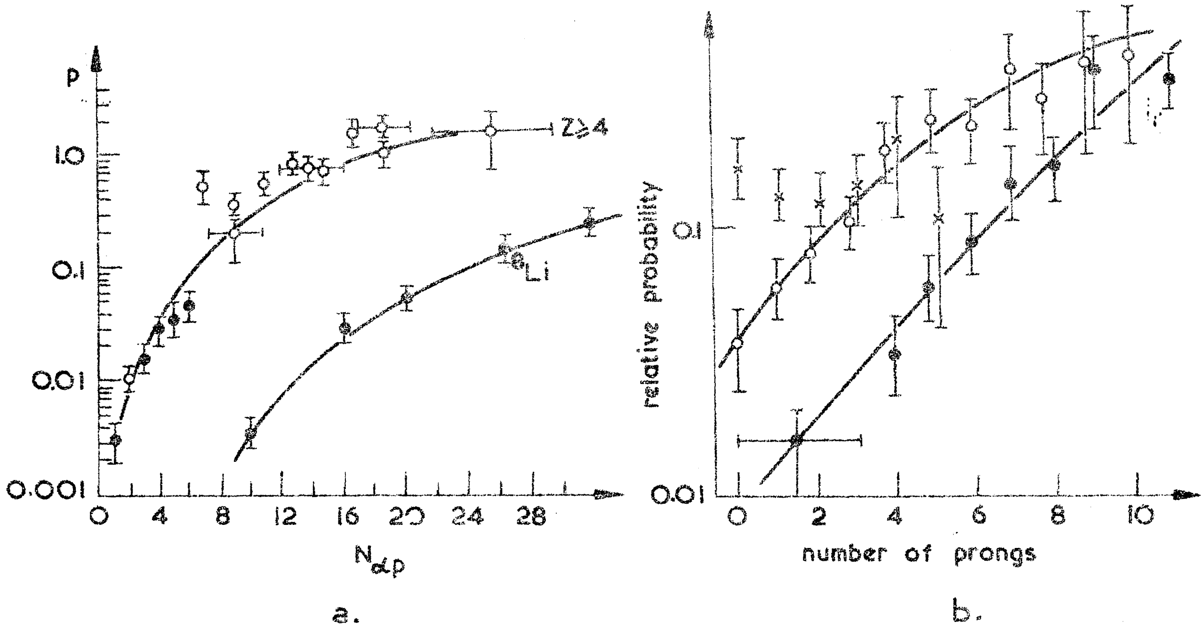

Fig. 2 Fragment probability vs :

a) number of prongs; b) number of "black" prongs (•), "grey" prongs (o); and "thin" prongs (×).

the nucleus), the greater is the probability of fragment emission *). When the energy transferred to the nucleus becomes of the order of the total binding energy of a nucleus, fragment emission becomes almost certain.

The yield of multicharged particles in disintegrations increases not only with the growth of energy transferred to a nucleus which is defined by the number of black prongs, but also with the increase of the number of cascade particles [3),15),17)]. At the same time the yield does not depend on the number of shower particles [3),13),18)] (Fig. 2b).

The same relationship holds for energetic fragments $(E>E_{Coulomb})$. Not all results agree on this point. For example, Nakagawa, Tamai,

*) The number of prongs here contains protons with energies up to 30-40 MeV and α particles of all possible energies.

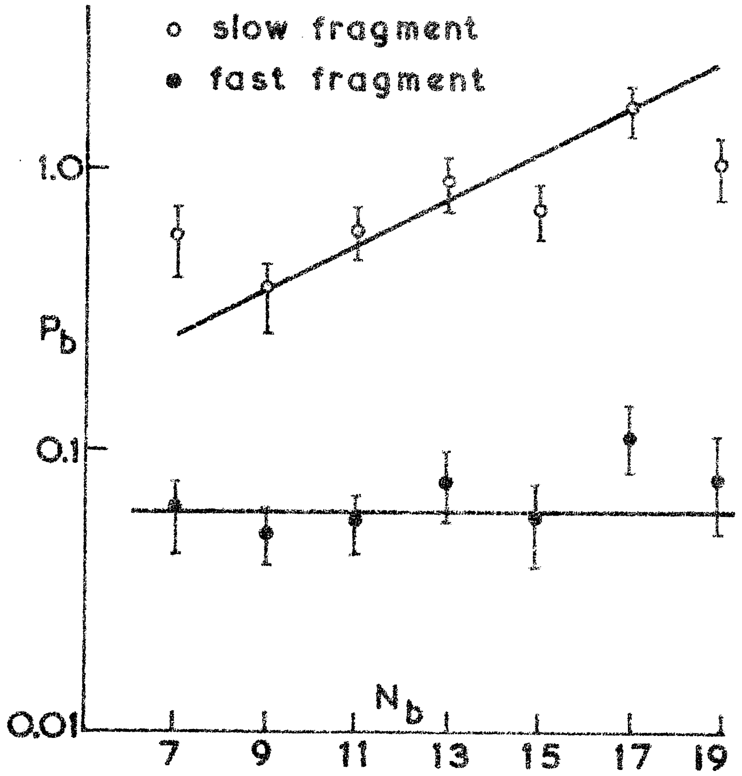

Fig. 3 Fragment production vs number of black prongs.

Namoto[18] studying the production of ^8Li fragments, in disintegrations induced by 6.2 GeV protons, found that the probability of emission of energetic fragments (E>60 MeV) does not depend on the number of black prongs in a sta (Fig. 3).

When the energy of the incident proton exceeds 1 GeV the disintegrations with 2 or more fragments appear to have an appreciable yield. The cross-section of disintegrations of Ag and Br nuclei with 2 or more fragments with $Z \geq 4$ is only 0.5 mb (that is only 4% of the total fragmentation cross-section) at 660 MeV incident proton energy and is about 16 mb (that is, 16% of the total fragmentation cross-section) when the incident proton energy is equal to 9 GeV.

The relationship between the fragmentation cross-section and the mass number of target nuclei happens to be complex. Measurements in nuclear emulsions at 660 MeV proton energy showed that the fragmentation cross-section is proportional to the geometric cross-section of the nucleus[19),20)]. But the investigation of the yields of radioactive isotopes from different target nuclei at various energies do not give similar results (see Fig. 4). Experimental data, obtained by the two methods, can be reconciled, on the assumption that the ratio between the measured radioactive isotope and other isotopes, including the stable isotopes of a given element, vary with the mass number of target nuclei. Such a conclusion, if it is true is very important because investigations in this field can give certain possibilities of studying structural peculiarities of nucle...

All these facts concerned fragmentation induced by protons. There is no reason to suppose that the results would change for other bombarding particles and this is confirmed by experiments with high energy neutrons and π mesons Refs. 13), 21)-25).

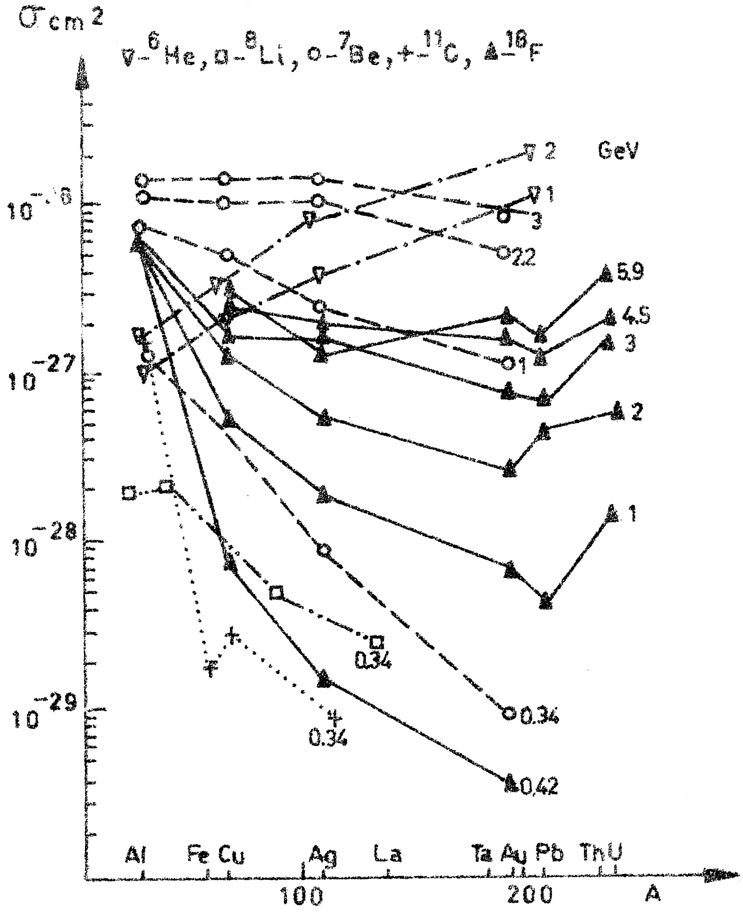

Fig. 4 Fragment production vs mass number.

2. Charge and mass distribution of fragments

Experimental data obtained with a nuclear emulsion method, Refs. 3),15),17),26),27),31),47), show an exponential decrease in the probability of fragment emission from Ag and Br nuclei with increasing Z for fragments with Z>3. The charge distribution does not change essentially in a wide range of energy transfer to nuclei (Fig. 5). On the basis of experimental data obtained from nuclear emulsions one can take the mass distribution of fragments to be similar to their charge distribution assuming that Z~M. But investigations of yields of separate radioactive isotopes do not give such numbers when A>13, and the yields are appreciably dependent, each in its own way, on the mass number of target nuclei and on the incident proton energy (Fig. 6). For example,

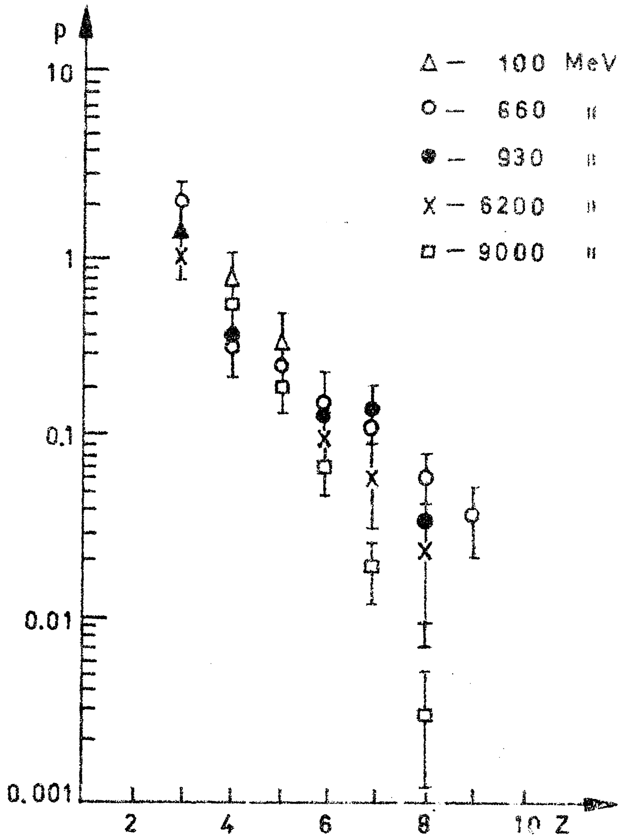

Fig. 5 Charge distribution of fragments.

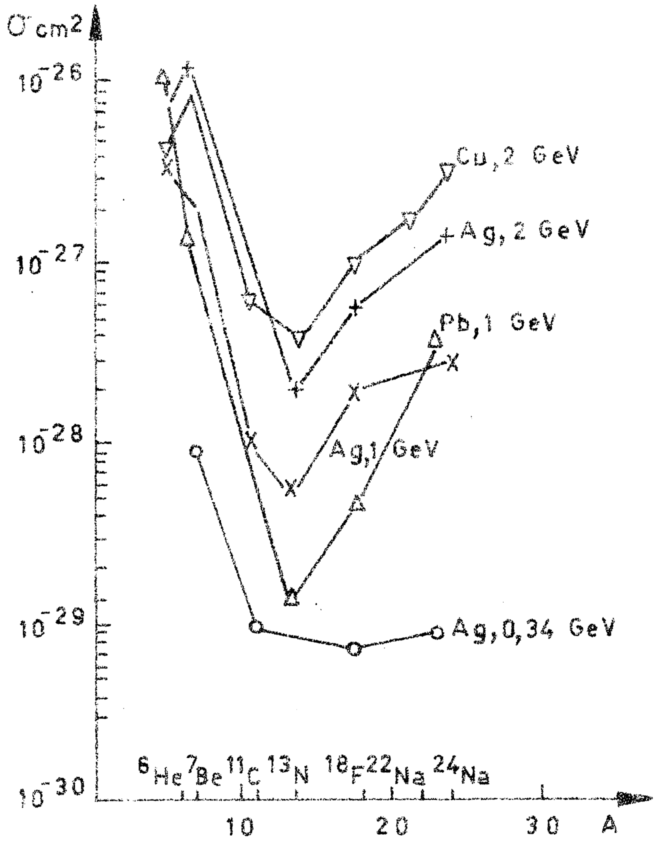

Fig. 6 Mass distribution of fragments.

the $^{24}Na/^{18}F$ yield ratio appears to be larger than unity for different targets and various bombarding energies although $Z_{Na}>Z_{F}$. Analyzing the data obtained from radiochemical measurements, it is necessary to take into consideration the following facts. a) In irradiations of targets with small atomic numbers some isotopes may happen to be residual nuclei from different nuclear reactions. This fact can change the form of the dependence on A towards an increase in the yield of fragments with bigger mass numbers. b) It seems that the ratio of the yield of a given isotope to the yield of all the other isotopes of the element essentially depends on the mass number. In fact, Table 1 shows that the yield data obtained by two different methods can differ by a factor of ten or more.

The investigation of the ratio between the yields of stable and of radioactive isotopes for all fragments with $Z \geq 4$ leads to the conclusion that about 90% of all fragments are not β emitters. Moreover, the presence of such isotopes as ^{8}Li provides grounds for supposing that fragments are often emitted with low excitation energies (otherwise the decay of ^{8}Li to $^{7}Li + {}^{1}_{0}n$ would have taken place).

Nuclear emulsion method Ag, Br nuclei			Radiochemical method Ag nuclei		
Z fragm.	E proton MeV	cross-section $10^{-29} cm^2$	isotope	E proton MeV	cross-section $10^{-29} cm^2$
4	350 660	100 350	${}^{7}_{4}Be$	335 1000	10 250
6	350 460	50 100	${}^{11}_{6}C$	340 480	1.0 ~ 3.0
9	350 460 660	12 20 40	${}^{18}_{9}F$	340 420 1000	1.0 1.6 20
11	350 460 660	8 12 24	${}^{24}_{11}Na$	340 480 1 000	1.0 ~ 3.0 30

Table 1

The available experimental data on the yields are not complete enough to say how the ratio of the yield of separate radioactive isotopes to the yields of other isotopes of the element changes with variation of excitation energy, or even to say whether it changes or not.

3. Energy and angular distributions of fragments

The energy distribution of fragments is characterized by the following features at all energies of incident protons : a) the most probable energy is usually defined by the magnitude of the nominal

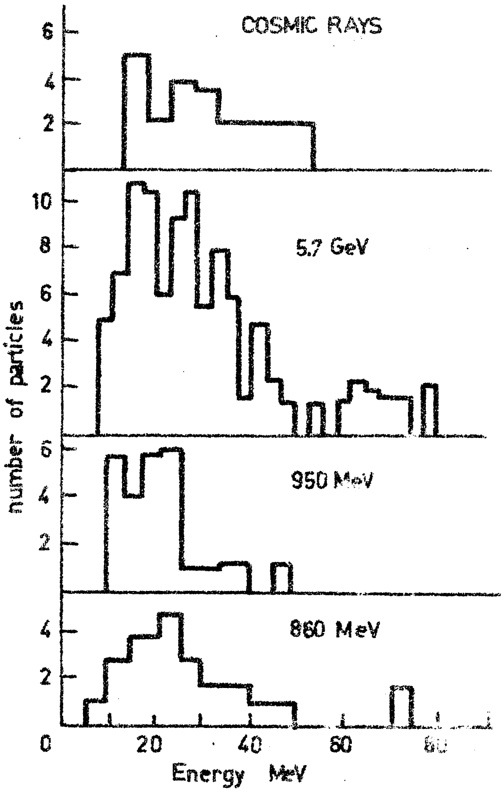

Fig. 7 Energy distributions of ^{8}Li.

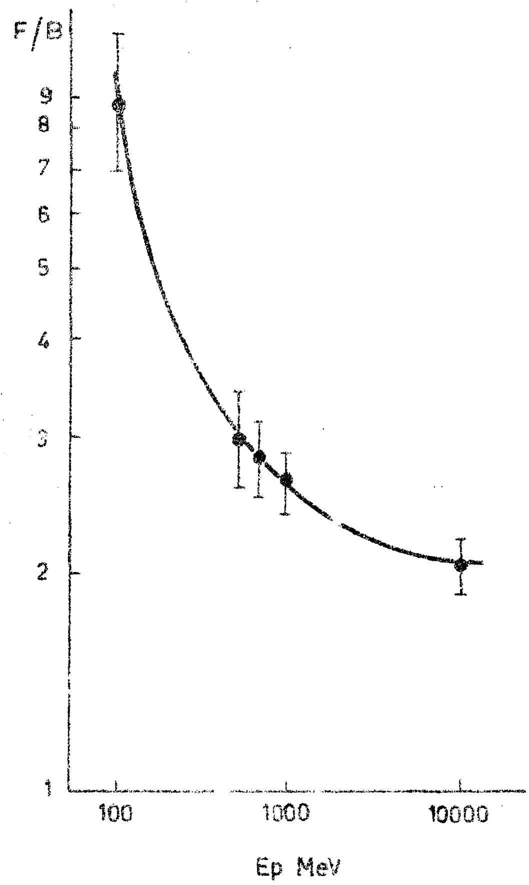

Fig. 8 Ratio of F/B vs incident proton energy.

Coulomb barrier of the residual nucleus. b) the increase in incident proton energy mainly causes an increase in the proportion of particles with $E>E_{Coulomb}$ in the energy distribution, without any change of the most probable energy (Fig. 7), Refs. 17),28),29),30).

Fragments are characterized by the anisotropy of their angular distribution which changes with variations of energy both of the primary protons and of the fragments themselves.

Figure 8 shows the dependence of the forward-to-backward ratio (F/B) with respect to the direction of the primary beam for with $Z\geq4$ upon the energy of incident protons. It can be seen that the (F/B) decreases with the growth of the average value of energy transferred to the nucleus.

When separate angular distributions for fragments with different energies were obtained it was found that the anisotropy increases with the rise in fragment energy. For example the (F/B) ratio for fragments with energy exceeding 8 MeV per nucleon is about 10 when the energy of the incident protons is 6.2 GeV, (Ref. [13]).

II. DISCUSSION OF POSSIBLE MECHANISMS FOR THE FRAGMENTATION PROCESS

Up to now there is no coherent explanation of the mechanism of fragment emission, despite considerable investigation of the problem. The fragment emission can be connected either with the development of a cascade in a nucleus (i.e., with processes which take place during the passing of the fast particle through the nucleus) or with de-excitation of the excited residual nucleus to its ground state by means of an evaporation process, during which the division of a nucleus into two parts can take place with a probability depending on a position of the nucleus in the periodic table of elements. It follows that on the basis of nuclear processes known to us there are three possible mechanisms of fragment emission.

1. Highly asymmetric nuclear fission

At the beginning of the investigation of the problem it seemed that the most natural supposition was that of the emission of fragments in a process of highly asymmetric fission [5)-8), 32)-34)]. There were some theoretical reasons in favour of this point of view. Fujimoto and Yamaguchi [35] showed that when the temperature of the nucleus is of the order of the nucleon binding energy the fission width becomes comparable with the neutron width. There were very poor experimental data on nuclear fission at high excitation energies.

The experimental facts obtained since that time lead one to believe that the fission mechanism of fragment emission is very unlikely.

a) The kinetic energy of fission fragments is determined by their Coulomb interaction at any excitation energy. The kinetic energy of a considerable portion of the emitted multicharged particles exceeds the energy of Coulomb interaction.

b) The yield of nuclear fission falls rapidly with the increase of mass asymmetry of fission. The yield of fragmentation, on the contrary, increases when the mass of the fragment decreases.

c) Nuclear fission into more than two fragments is a very rare event. But multiple production of multicharged particles is an important feature of fragmentation and is a rather frequent event.

d) Fission fragments have in most cases the same n/p ratio as the original nucleus. Multicharged particles emitted in nuclear disintegrations are in most cases stable isotopes and the corresponding residual nuclei have originally an excess of neutrons.

e) The angular distribution of fission fragments has but a small anisotropy with respect to the primary beam direction and it is much less pronounced than that of multicharged particles which are emitted predominantly into the forward hemisphere.

f) The cross-section of nuclear fission increases rapidly with the growth of the atomic number of the target (approximately by the factor of 10^3 from Ho to U). The cross-section of fragmentation grows comparatively slowly with the increase of atomic number of the target (by a factor of 2 at the most in the same region of target masses).

The facts stated above compel one to think of some other possible mechanisms of fragment emission.

2. Evaporation of fragments by the excited nucleus after the cascade dies out

In experiments on fragmentation with the nuclear emulsion technique the cases of ^{8}Li emission can be identified most readily. This is so, because after stopping in the emulsion the ^{8}Li nucleus, through β decay, goes into one of the excited states of ^{8}Be which decays into two α particles. The track of the ^{8}Li fragment has a T shape (a so-called hammer track) and can therefore be identified easily and reliably. The proportion of T shape events from ^{8}B and ^{9}Be transitions to ^{8}Be (at corresponding excitation energies ^{9}Be emits a neutron) is small (about 3% of ^{8}Li events).

Several authors [15),18),31),36)-42)] have carefully, and with good statistics, studied the emission of ^{8}Li fragments from various nuclei (though mainly from Ag and Br) and compared their results with predictions of the theory of charged particle evaporation using the known theoretical expressions for energy and angular distributions [43),2),31)].

Experimental data on angular and energy distributions of light multicharged particles like ^{8}Li can be satisfactorily described with evaporation theory formulae using suitably chosen parameters, such as the nuclear temperature T, the Coulomb barrier V, the relative velocity of evaporating nuclei v· but the chosen values of these parameters are physically unreasonable. For example, in the paper by Skjeggestad and Sörensen [31)] the energy spectrum of ^{8}Li fragments emitted from a silver target irradiated with cosmic rays fits evaporation expressions (see Fig. 9a) when T= 11.5 MeV, V= 6 MeV and v=0.016 c. At such a temperature the excitation energy is equal to $E_{ex} = \frac{A}{12.4} \cdot T^2 \approx$ 1000 MeV and the total binding energy is only about 800 MeV. The value V=6 MeV is too small even if the decrease of the barrier at high excitations is taken into consideration.

Deka and collaborators [39)] investigated the ^{8}Li fragment emission from 4.6 GeV/c π meson interactions with Ag and Br nuclei. The energy distribution of ^{8}Li fragments for 884 events is given in

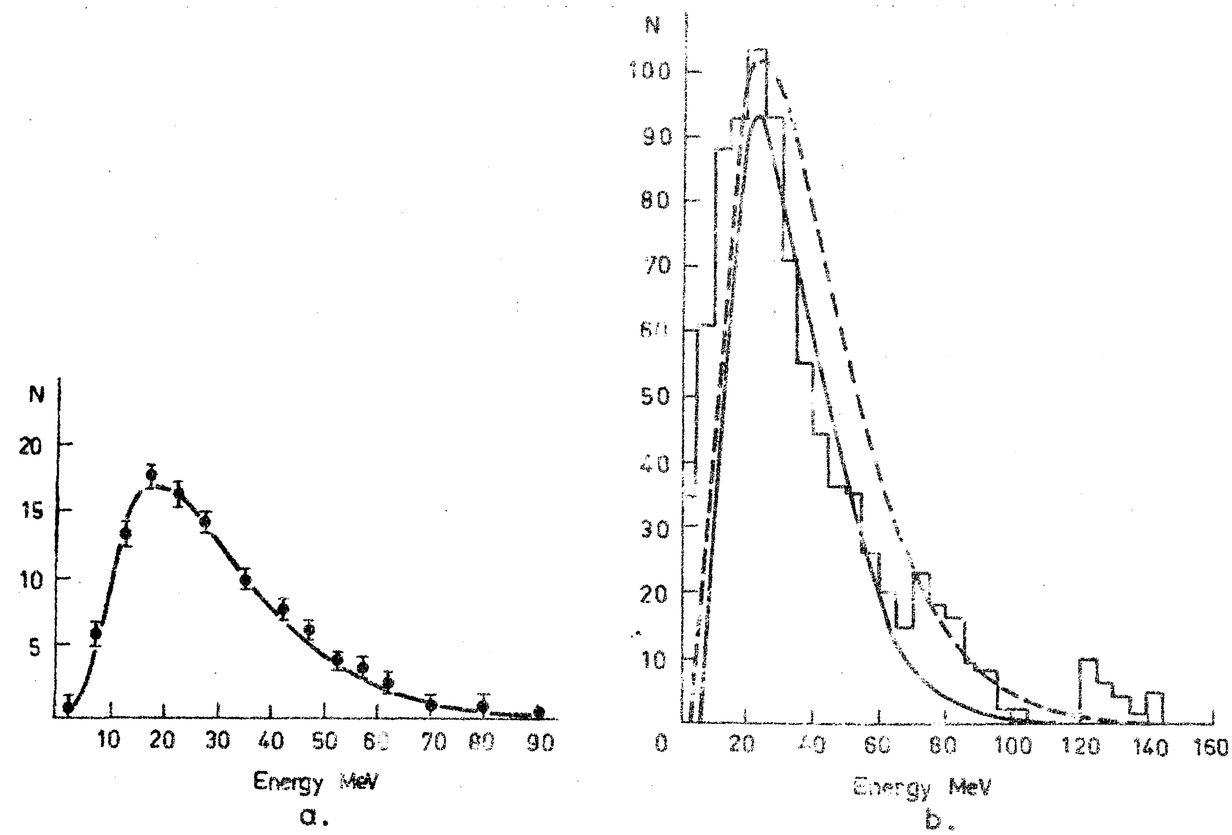

Fig. 9 Energy distributions of ^{8}Li. Solid curves represent evaporation theory corrected for nuclear motion.

Fig. 9b. The energy spectrum for fragments with energies 20 MeV<E<60 MeV is described almost satisfactorily when the parameters have the following values : T=9 MeV; V=13.5 MeV; v=0.02 c. But a considerable fraction of fragments with energies exceeding 60 MeV is not included in the distribution. To describe them in terms of the evaporation model one needs the following values of parameters : T=14 MeV; V=9 MeV and v=0.02 c. Such a magnitude of temperature is obviously far beyond the limits within which a nucleus can exist as a system.

Table 2 gives the set of values for the parameters T, V and v gathered from several papers [31),38),40),42)] on the observation of ^{8}Li fragments for various proton energies. It is only in the two papers of Cüer et al. [38),40)] that for spectra of ^{8}Li from Ag under the effect of 17 GeV/c pions and 24 GeV/c protons a value of 8 MeV was obtained for T and hence $E_{ex} < E_{binding}$.

Incident particles	Refs.	Nuclear temperature (T) MeV	(V) MeV	Nuclear velocity (v_o)
Cosmic rays	31)	11.5	6	0.016
Protons 9 GeV	41)	10	5	0.015
Protons 9 GeV	42)	15	-2 ± 1	0.012
Protons 24 GeV	42)	15	-2 ± 1	0.015
π mesons 17 $\frac{GeV}{c}$	40)	8	8	0.015
Nucleons 24.8 $\frac{GeV}{c}$	38)	8	5	0.02

Table 2

When we want to describe by an evaporation formula the whole energy spectrum of ^{8}Li with its long tail and its low energy part, we must use a large value of T, and a small value of V. This was shown in Ref. 42).

Figure 10 shows energy distributions of ^{8}Li fragments from various targets irradiated with 2.2 GeV protons in comparison with spectra predicted by evaporation theory 36). In the case of Ag it is possible to speak about the coincidence of some parts of the energy spectrum with evaporation theory, but in the cases of Cu and Au, experimental and theoretical spectra do not coincide. Similar results

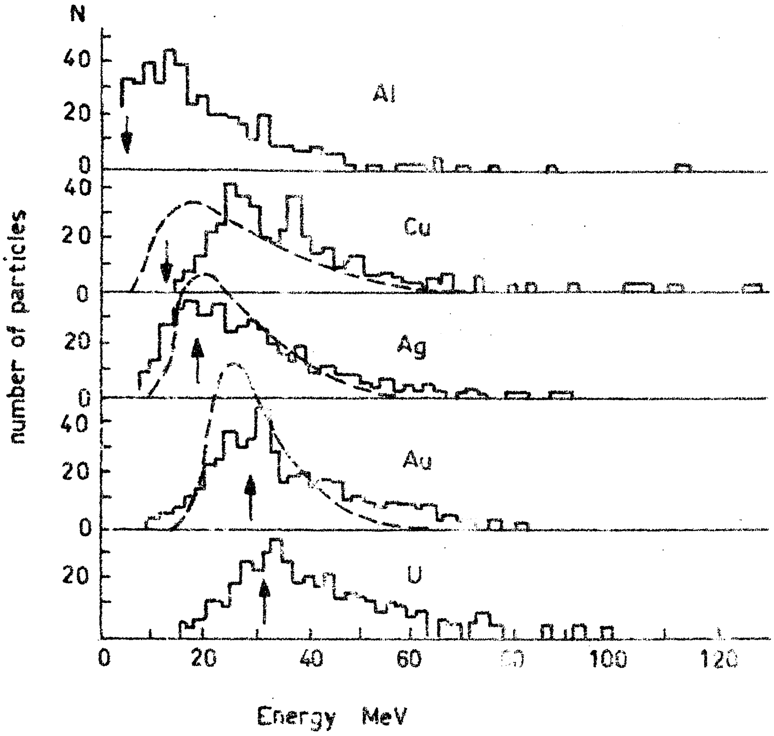

Fig. 10 Energy distributions of ^{8}Li.

showing the discrepancy between the energy spectra given by the evaporation theory in its present state and the experimental data were obtained by other authors [37),38),40)-42)] for ^{8}Li fragment emission at other energies of bombarding protons.

Assuming that fragments are evaporated from nuclei Hudis and Miller [44)] calculated the cross-sections of ^{7}Be emission from Cu, Ag and Au nuclei for incident proton energies 460 MeV, 940 MeV and 1840 MeV.

Similar calculations were later performed by Dostrovsky, Fraenkel, Rabinowitz and Hudis [45),46)] for ^{6}He, ^{8}Li, ^{7}Be, ^{12}N isotopes evaporated from Zn, Cu, In, Pb, U at incident proton energies 0.84 and 1.84 GeV. The cross-sections were calculated for three different expressions for the radius of interaction in the hope of achieving a satisfactory agreement with experimental data

1) $R = 1.5(A_1^{1/3}+A_2^{1/3})\cdot 10^{-13}$ cm;

2) $R = \left[1.5(A_1^{1/3}+A_2^{1/3})-1.2\right]\cdot 10^{-13}$ cm;

3) $R = \left[1.1(A_1^{1/3}+A_2^{1/3})+2\right]\cdot 10^{-13}$ cm.

These authors assume that R in the form 2) takes into consideration the presence of diffused boundaries in nuclei and their distortions at the points of interactions with high energy protons. The form 3)

target	light nucleide	940 MeV				1840 MeV				2.9 GeV
		σ experim.	σ calculated			σ experim.	σ calculated			σ experim.
			1	2	3		1	2	3	
Cu	^{6}He	2 ± 1	3.29	1.83	3.56	4 ± 2	6.73	4.10	9.01	
	^{8}Li					3	3.97	2.26	4.72	
	^{7}Be	4.4 ± 1.1	6.06	2.80	3.66	11.7 ± 2.9	13.52	7.56	6.45	
Zn	^{13}N	0.13	0.085	0.028	0.029	0.33	0.157	0.079	0.056	0.51
Ag	^{6}He	4 ± 2	6.66	3.68	6.51	7 ± 4	14.78	7.65	13.18	
	^{8}Li					4	7.25	3.61	5.75	
	^{7}Be	25 ± 0.6	6.69	3.02	4.11	11.3 ± 2.8	16.75	7.38	8.75	
In	^{13}N	0.056	0.074	0.020	0.025	0.19	0.116	0.041	0.044	0.27
Au	^{8}Li					9	20.10	8.45	10.75	
	^{7}Be	1.3 ± 0.3	4.78	1.31	2.07	5.9 ± 1.5	16.22	6.12	6.50	
Pb	^{6}He	10 ± 5	13.35	6.11	9.55	21 ± 11	38.30	18.95	29.60	
	^{13}N	0.011	0.042	0.008	0.007	0.11	0.151	0.035	0.028	0.16
U	^{13}N	0.025	0.163	0.030	0.023	0.075	0.550	0.111	0.094	0.125

Table 3

Experimental and calculated cross-sections (in mb) for the formation of ^{6}He, ^{8}Li, ^{7}Be and ^{13}N from various targets corresponds to the experimental value of the radius taken from electron scattering measurements with the addition of a constant term $2.0\cdot10^{-13}$ cm.

The results obtained with all three expressions for the interaction radius compared with experimental data are given in Table 3.

It can be seen that application of expressions 2) and 3) improves the agreement between experimental and calculated values of cross-sections for ^{6}He, ^{8}Li and ^{7}Be isotopes for all targets. But for ^{13}N, although the agreement improved in case of the U target, it remains unsatisfactory for other targets. It is possible that a more complex expression for R would give a better agreement with experiment. For Zn targets the discrepancy between experimental and theoretical values can be explained easily because ^{13}N fragments are also produced in part as residual nuclei, when the evaporation mechanism of fragmentation is assumed.

Dostrovsky, Fraenkel and Hudis [46] note that the cross-section of multicharged particle emission is very sensitive to changes of the level density parameter a. Their calculations show that the variation of a from a=A/10 to a=A/12.5 is equivalent to the variation of R from the value given by 2) to the value given by 3). It demonstrates that the lack of more accurate knowledge of the level density parameter, of its dependence on A and possibly on E prevents a more accurate determination of the interaction radius.

The calculated values of cross-sections for different isotopes and the comparison of energy spectra cannot be considered to be a proof of the evaporation mechanism of emission of all fragments, in the present state of the theory of particle evaporation by nuclei.

Attempts were made recently to improve the evaporation theory by taking into account the probability of the pick-up of particles with certain quantum characteristics from the nuclear surface by the evaporated particle. The simplest case for such a consideration is when an evaporated nucleon picks up another nucleon and forms a deuteron. Such a process can possibly be extended to the case when, for example, two α particles form a ^8Be nucleus, or to some other combinations, not necessarily binary, leading to the formation of complex nuclei observed in experiments.

Kikuchi [49] on the basis of the general theory of the (p,d) reaction was the first to study the pick-up process for evaporated particles. He obtained an expression for the probability of deuteron emission in the pick-up process. For highly excited nuclei the probability of such an indirect evaporation exceeds the probability of direct evaporation of the deuteron as a whole.

Ismailov and P'yanov [48] studied the additional yield of tritium from the indirect evaporation process for the case when the particles composing tritium are evaporated directly and then form a tritium nucleus near the surface of evaporating nuclei. Obviously, the following processes can take place :

A) direct evaporation of a deuteron and a neutron and their joining to form tritium;

B) direct evaporation of two neutrons and one proton and the consequent joining together. Table 4 gives the ratio of indirect evaporation widths to the widths of direct evaporation.

T MeV	$\frac{\Gamma_A}{\Gamma_n}$	$\frac{\Gamma_B}{\Gamma_n}$	$\frac{\Gamma_A+\Gamma_B}{\Gamma_n}$
0	0	0	0
1	1.28	0.004	1.28
2	1.35	0.12	1.47
3	1.36	0.37	1.78
4	1.35	0.65	2.00
5	1.32	0.91	2.23
6	1.28	1.49	2.77

Table 4

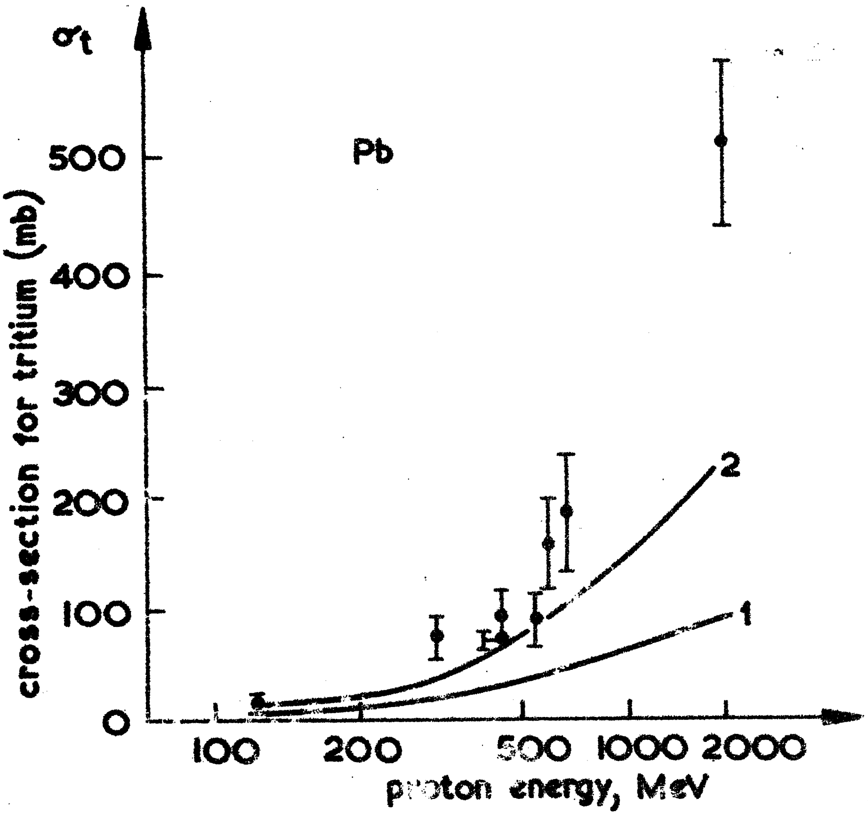

Fig. 11 Tritium yield vs proton energy. Curves 1 and 2 calculated from direct and indirect evaporation.

The data in Table 4 show that the importance of the indirect process grows with increasing T.

Figure 11 gives a comparison between the calculated and experimental data on tritium production from lead induced by protons of various energies [50),51)].

It can be seen that taking into account indirect evaporation improves the agreement with experiment in the range of energy of incident particles up to 500-700 MeV. It seems that at higher energies the role of the direct process during the cascade stage increases in the case of tritium production.

Energy spectra of tritium produced in indirect evaporation processes of types A and B differ from energy spectra of direct evaporation. The energy distribution of tritium produced in these three processes is described by

$$P_n(E)\, dE \sim \frac{1}{T}\left(\frac{E-V_{eff}}{T}\right)^{\frac{3n-1}{2}} \exp\left[-\frac{n+1}{2}\left(\frac{E-V_{eff}}{T}\right)\right] dE$$

Parameter n=1 corresponds to direct evaporation;
n=2 corresponds to indirect evaporation of type A;
n=3 corresponds to indirect evaporation of type B.

The maxima of the energy spectra will be at the points $T+V_{eff}$, $\frac{5}{3}T+V_{eff}$ and $2T+V_{eff}$ and the total spectrum as a spectrum of superposition will be wider than the spectrum in the case of direct evaporation only. Similar calculations of evaporation of more complex particles than tritium could possibly improve the agreement between experimental and theoretical data.

3. Emission of fragments during the development of the nuclear cascade

From the outset we have to reject the hypothesis of fragment emission as the result of the direct interaction of an incident proton of several hundred MeV with nuclear clusters. Many experimental results contradict this hypothesis :

a) the independence of the most probable energy of a fragment on the energy of the incident particles;

b) the small cross-section of high energy proton scattering to large angles [56];

c) the disagreement between expected fragment energy as a function of angle calculated assuming the mechanism of elastic collision, and the experimentally observed [57] dependence (Fig. 12);

d) the emission of several fragments in one interaction of a fast proton with the nucleus.

Instead, the variation of the angular anisotropy with the proton bombarding energy and the increase of the single and multiple fragmentation cross-section with the growth of the number of cascade particles make it natural to connect fragment emission with the development of the nuclear cascade. There can be several mechanisms for fragment emission during the cascade process. Let us consider two of these possibilities.

A) The experimental data do not contradict the hypothesis of fragment emission as the result of the collision of cascade particles with instantaneously forming groups of nucleon clusters, during the

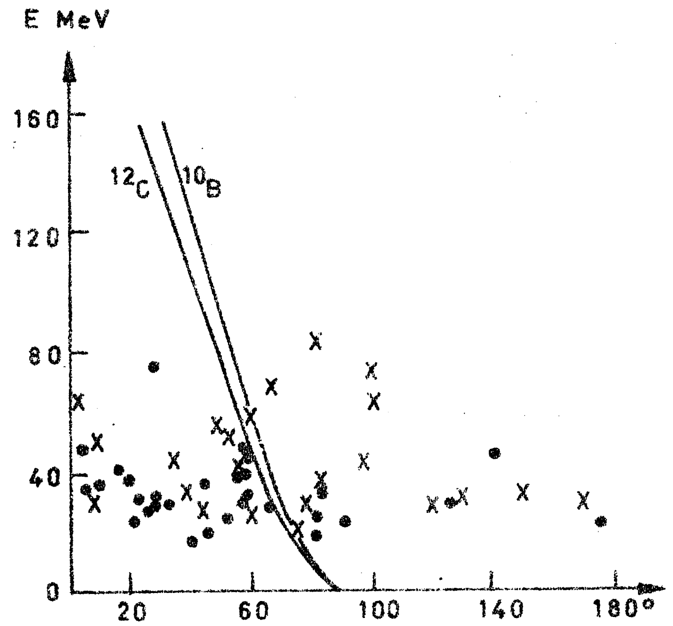

Fig. 12 Fragment energy vs emission angle. Solid curve assumes emission by elastic collision at 660 MeV. Z=5 (•); Z=6 (×).

de[illegible]ment of the cascade. As was mentioned before [52),53)], cascade α particles directed predominantly to the forward hemisphere with energies considerably exceeding the potential barrier of the residual nuclei should also be taken into account in studies of the fragmentation process. Possibly there is no difference between α particles and more complex particles, such as lithium and nuclei with higher Z. This proposition was based on the similarity of excitation functions and of angular distributions for cascade α particles and fragments with $Z \geq 4$ emitted by Ag and Br nuclei bombarded by protons with energies up to 660 MeV (Fig. 13).

Ostroumov and Filov [54)] calculated the cross-section for the "knock out" of α particles with energies of more than 30 MeV from the surface layer of nuclei by nucleons with given energy. They had taken into account the movement of α particles inside the nucleus and evaluated the probability of the existence of α particles inside nuclei. Their results are given in Fig. 14 and Fig. 15. The curves 1,2,3 and 4 represent kinetic energies W of 0,5,10 and 20 MeV of the alpha particle in the nucleus.

The curves for $\sigma_\alpha(E)$ show that 50-70 MeV protons happen to be most effective in knocking out α particles from nuclei and a further increase of proton energy leads to a decrease of the cross-section of the process. The calculated results can be set into agreement with the experimental $\sigma_\alpha(E)$ curve if one assumes that the secondary

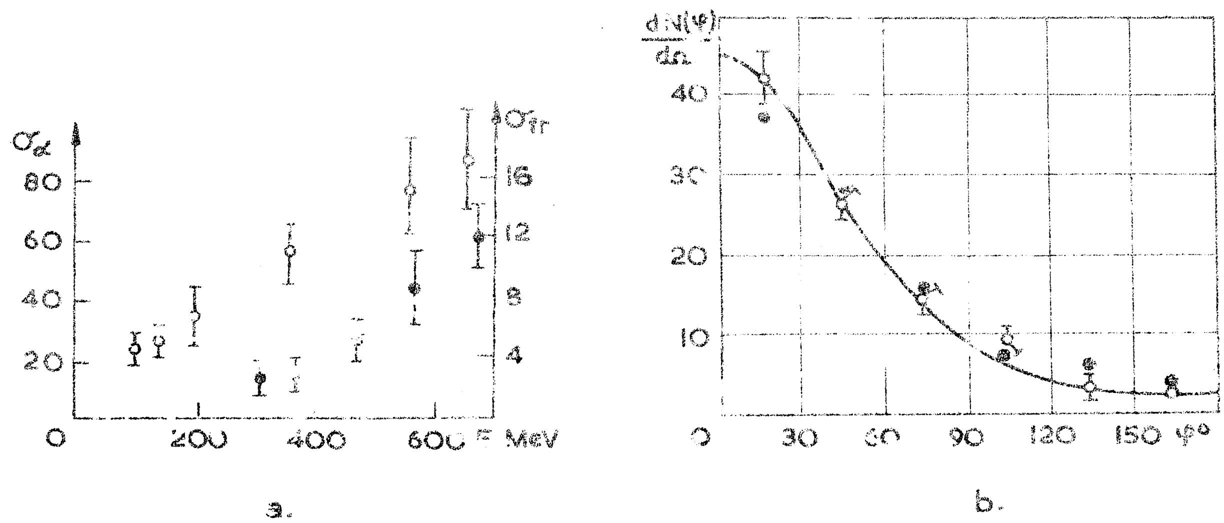

Fig. 13 a) Energy and b) angular distribution of cascade α particles (o) and fragments of Z ≥ 4 (•).

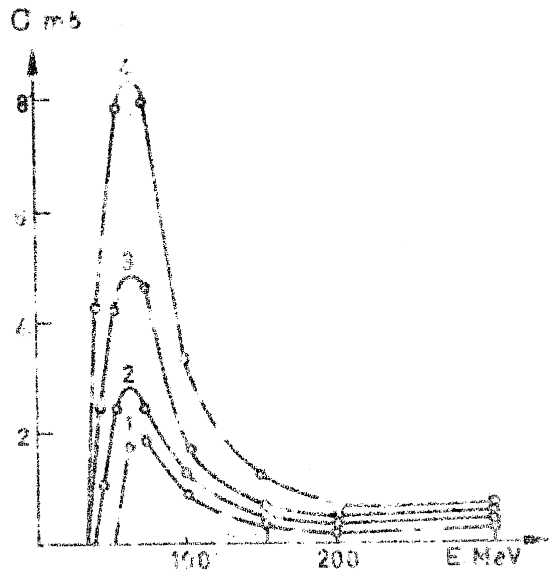

Fig. 14 Calculated cross-section for knock-out α particles vs incident energy for various values of W.

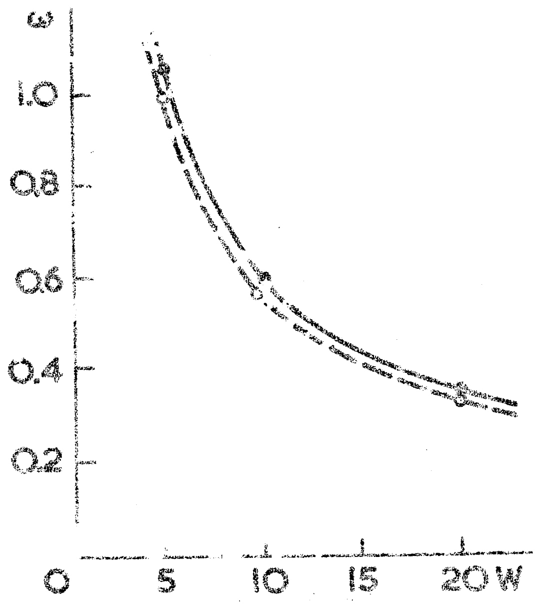

Fig. 15 Probability of nuclear α particles vs its kinetic energy in the nucleus.

cascade nucleons (not the primary high energy protons) are responsible for the process of the knocking out of α particles. The number of cascade nucleons grows continuously with the increase of incident proton energy. The probability of the existence of α particle type clusters inside nuclei comes out to be a decreasing function of the energy of α particle motion inside the nucleus.

The agreement between the characteristics of cascade α particles and fragments and the cross-section calculated on the basis of the probability of the existence of α particle clusters inside nuclei, makes it possible to assume that the multicharged particle emission is due to some extent to quasi-elastic collisions of cascade nucleons or their compounds with corresponding instantaneous nuclear groups.

Our group has made some experiments [55] on fragments emitted by Ag and Br nuclei bombarded with 100 MeV protons. The relationship between the fragment energy E_f and the angle ϑ between the direction of emission and the direction of the primary beam is given in Fig. 16. It can be seen that several points (about 20% of all events) are on the curve calculated for the case of elastic collisions of 100 MeV protons but that the majority of the points are below the curve. It can be interpreted as an indication that in this case it is also necessary to connect the majority of events with protons with energies less than 100 MeV. The calculation did not take into account the movement of clusters inside nuclei which can cause fluctuations of values of the $E_f(\vartheta)$ curve.

B) To explain the fragmentation process, Denisov, Kosareva and Čerenkov [57] proposed a cascade mechanism according to which the separation of fragments from nuclei takes place during the cascade process, as a result of the breaking of internuclear connections. The authors of the calculation assume the following. In the Fermi gas model used in the calculation the shape of a nucleus was assumed to be uniform and spherical. For zero temperature and negligible interaction between nucleons, one gets uniform density and spherical symmetry in this model. In reality it may be that nucleons are not uniformly distributed in the nucleus owing to nuclear interactions and that nuclear matter consists of continuously forming and dissociating clusters of nucleons. If the lifetime of any group of nucleons in a nucleus is set roughly equal to the ratio of the nuclear radius to the velocity of the nucleons, then it turns out that the duration of the nuclear cascade is of the same order or even less than the lifetime of a nucleon group for an energy of the incident particle of about 1 GeV. Therefore the nuclear cascade takes place in a certain space structure which the nucleus has at the moment the fast particle enters. Thus, it is assumed that locally correlated groups of nucleons, connected by only a small number of connections to the rest of the nucleons, exist inside nuclei. In the process of the nuclear cascade the nucleons acting as connections can be knocked out and a group of nucleons will be separated from the residual nucleus. If this group is not trapped back by the nucleus then emission of a fragment takes place.

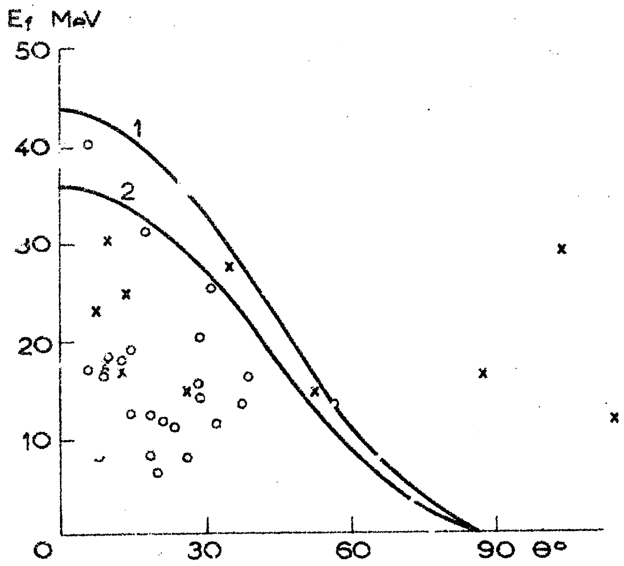

Fig. 16 Fragment energy vs emission angle. Curves 1 and 2 represent elastic collisions for mass 7 and 9. Experimental points · Li (o) and Be (x).

A detailed calculation of the cascade mechanism of fragment emission can be carried out using the model of the nucleon cascade in its first part (separation of a fragment) and a liquid drop model in its second part (oscillations of the deformed nucleus with recapture of the emitted fragment). The authors of the paper in question used a Monte-Carlo method only to obtain the volume distribution of knocked-out nucleons in the nucleus. The other part of the problem concerning separation of a fragment was treated as a problem of random choice. The energy of incident particles was chosen to be 660 Mev.

Figures 17a,b,c,d show the comparison of calculations with experimental results. The energy distribution of ^{9}Be and ^{7}Li fragments was compared with data obtained by Nakagawa et al.[15] (6.2 GeV protons, Ag and Br nuclei). Figure 17a) shows that calculations explain satisfactorily the energy distribution of fragments with energies about the value of the Coulomb barrier but it cannot explain the emission of fragments with $E \gg E_{Coulomb}$. Figure 17b) gives the comparison of calculated and experimental (Ref. [47]) angular distributions. There is a satisfactory agreement except in the region of small angles where the calculation gives smaller values. Denisov et al. [57] note that this disagreement is due to the fact that their model does not explain the emission of energetic fragments (with $E \gg E_{Coulomb}$) which have the most anisotropic angular distribution and which were included in the experimental results.

Good agreement exists between the charge distributions of fragments *) (Fig. 17c) and also between experimental and calculated excitation functions for different proton energies (Fig. 17d).

Denisov et al., using the suggested cascade model, calculated the dependence of fragment emission on the number of prongs in a star assuming different numbers of bonds (N) between a fragment

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*) Curve 1 was calculated assuming that different fragments are present in nuclei with equal probability. Curve 2 was calculated assuming that this probability is inversely proportional to Z.

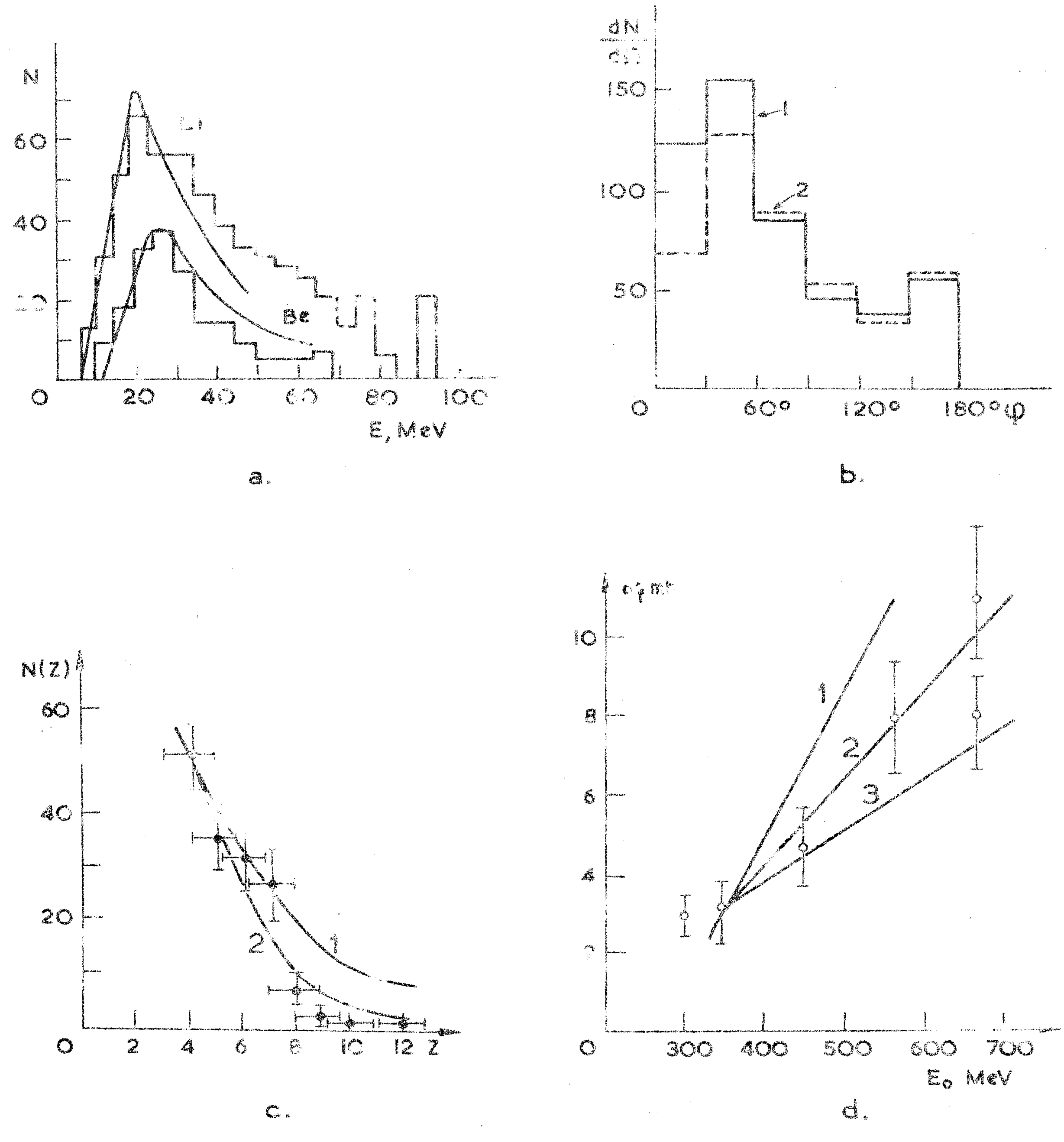

Fig. 17 a) Energy distributions of Li and Be b) angular distribution of fragments with Z≥4 c) charge distribution of fragments d) fragment production cross-section vs incident energy. Curves 1,2 and 3 refer to the number of bonds between fragment and nucleus.

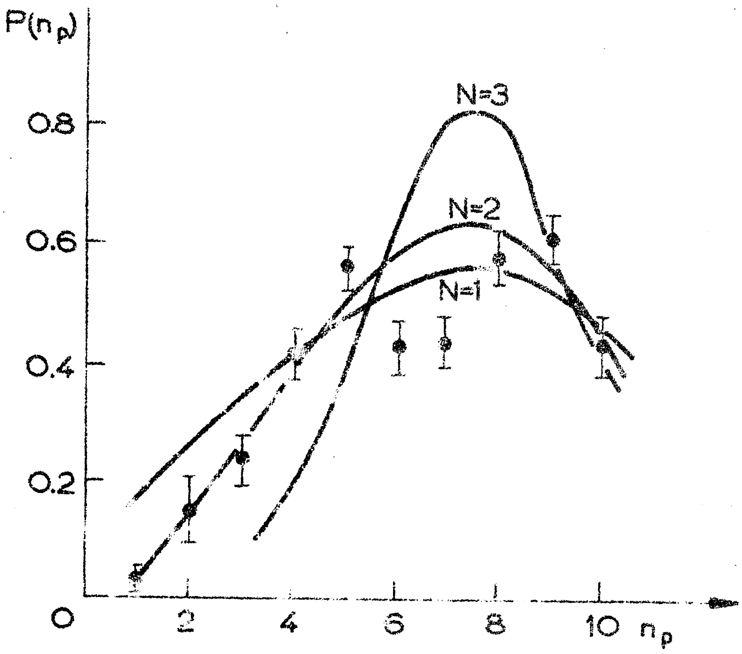

Fig. 18 Emission probability vs number of prongs.

and a nucleus. Figure 18 presents this dependence for N=1,2,3. The results are compared with experimental data [19] at 660 MeV incident proton energy. The agreement is better for N=2. Denisov et al. also give the calculated values of $\sigma_f(E)$ up to 660 MeV proton energy which is in satisfactory agreement with experiment and with the dependence of the cross-section for two-fragment emission on the energy of incident protons. This dependence is also in agreement with experimental data at 0.66 GeV and at several GeV proton energies when N=2 and the number of fragments simultaneously existing in a nucleus is five.

Thus the described model gives many features characteristic of the fragmentation process and this coincidence can hardly be considered fortuitous. The model is especially interesting because some information about the spacial structure within nuclei can be derived from comparison of experimental and calculated results.

It is necessary, however, to explain the mechanism of high energy fragment emission. It is not clear whether it is possible to find this explanation on the basis of a model in which the energy distribution of the fragments depends on the instantaneous distribution of internal nuclear motion and on the Coulomb interaction of a fragment with the residual nucleus.

III. FRAGMENTATION OF LIGHT NUCLEI

Investigations of nuclear reactions on light nuclei induced by high energy protons with analysis of all products of nuclear reactions must undoubtedly give valuable information about the mechanism of fragmentation, particularly about the part of the process which is not connected with evaporation. Theoretical considerations probably should also be based on interactions of fast particles with light nuclei because structural data for them are defined better than for heavy nuclei. Let us limit ourselves and consider only several investigations of fragmentation on light nuclei from which it is possible to draw some conclusions about the mechanism of fragmentation.

To explain the discrepancy between the experimental results and those calculated with the help of a Monte-Carlo method for the angular distributions of protons from interactions of 340 MeV incident protons with light nuclei of emulsion, Cüer, Combe and Sammam [58)-60)] supposed the existence of instantaneous substructures, mainly α particle groups, in light nuclei. In Ref. [60)] Cüer and Samman gave the results of their investigation of 106 events of the reaction of

$$p + {}^{12}C \rightarrow p + 3\alpha .$$

The forward to backward ratio for α particles turned out to be equal to 2. To explain the experimental data the authors developed the concept of a two-stage mechanism of reactions by assuming

1) collision of the primary proton with an α particle cluster and the knock-out of the α particle;
2) isotropic decay of the residual nucleus ${}^{8}Be$ into two α particles.

Analysis of angular and energy distributions of emitted particles confirmed the existence of two stages in the reaction. Thus, for the majority of stars it was established that the angular and energy

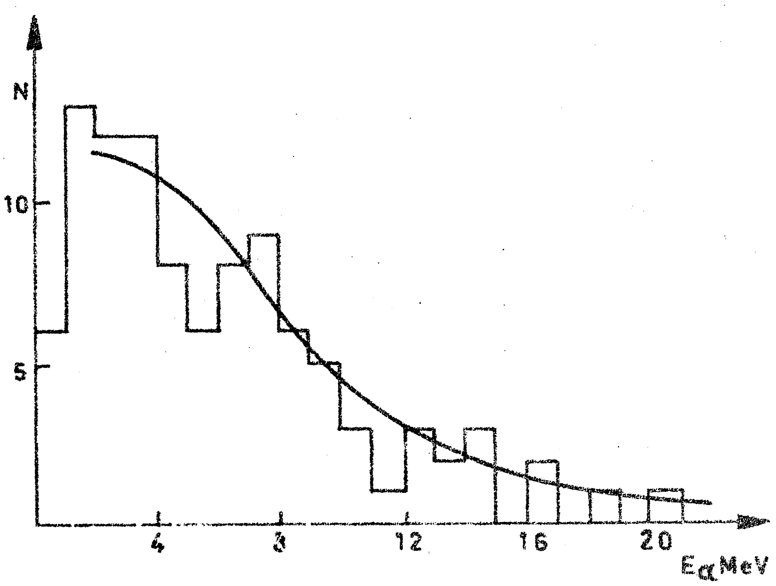

Fig. 19 Energy distribution of alphas from ^{12}C.

distributions of the proton and α particle emitted into the forward hemisphere correspond to a collision of a proton with an α particle. Certain fluctuations of angles and energies could be due to the internal motion of the α particle in the nucleus. On the basis of collision kinematics, Cüer and Samman calculated the energy distribution of α particles inside the nucleus (Fig. 19). The value obtained for the average energy is 5.8 ± 0.5 MeV. This is in good agreement with that calculated by Herzenberg [64] who used an α particle model of ^{12}C nucleus. The cross-section for the reaction in question was found to be 33 and 22 mb for 180 and 340 MeV incident protons, respectively.

Fedotov [65] studied disintegrations of carbon nuclei induced by 660 MeV protons. He charged emulsions with diamond dust and was able to identify carbon disintegrations very reliably. He analyzed 505 events. Experimental spectra (angular and energy distributions) were compared with Monte-Carlo calculations which took into account :

1) the presence of α clusters inside nuclei;

2) the density variation of nuclear matter defined by the expression

$$\rho(r) = \rho_o \left(1+\alpha \frac{r^2}{a_o^2}\right) \exp\left(-\frac{r^2}{a_o^2}\right) \quad ; \quad \text{and}$$

3) the production, absorption and scattering of π mesons inside the nucleus.

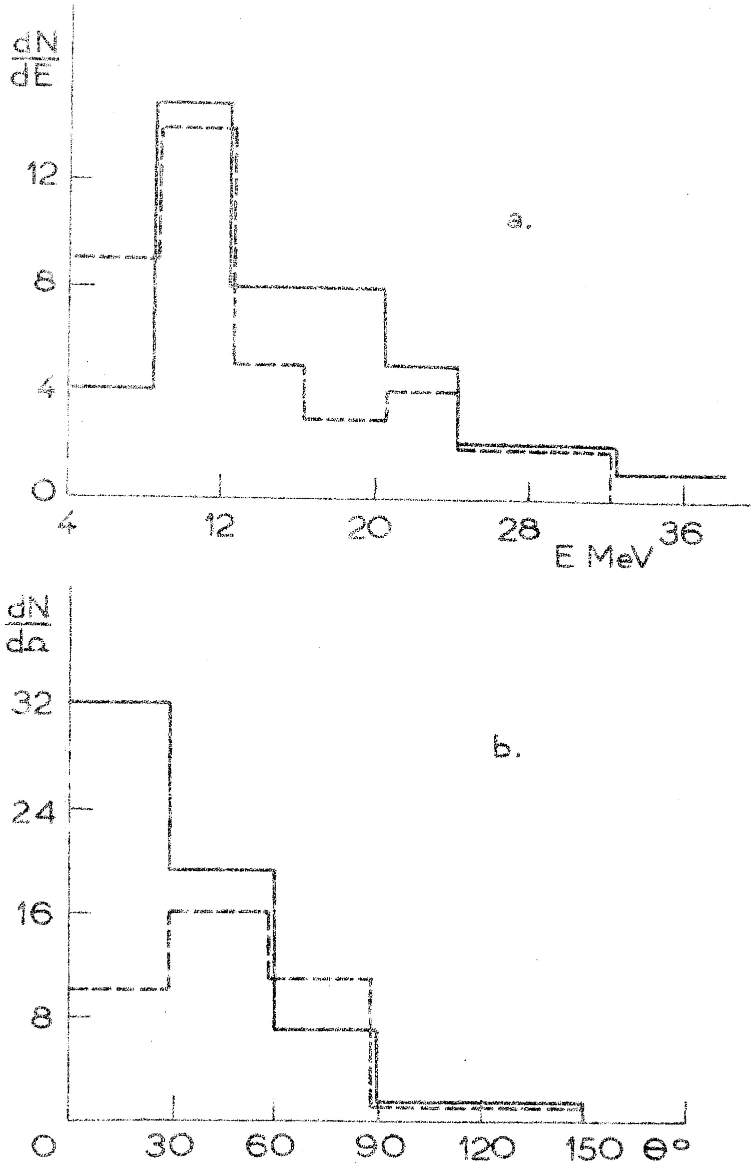

Fig. 20 Cascade alphas from ^{12}C. a) energy b) angular distribution. Experimental line solid, calculated one dotted.

Experimental data and calculations are in good agreement for cascade protons, but there is poor agreement for α particles. The calculated energy distribution does not give particles with energies of more than 30 MeV, while the experimental energy distribution extends up to 70 MeV. This fact is seen also in the angular distribution : there is no agreement between experimental results and calculations in the 0-30° region, where most energetic α particles are emitted (Fig. 20a,b). Fedotov [65] thinks that those α particles emitted at small angles to the primary protons possibly originate in some process other than direct knock-out. Similar calculations of the interaction of 300 MeV protons with carbon nuclei were carried out by E. Abate et al. [75].

Ostroumov and Yakovlev [67] investigated Z>3 fragment emission from carbon nuclei induced by 660 MeV protons. A thin polystyrol film

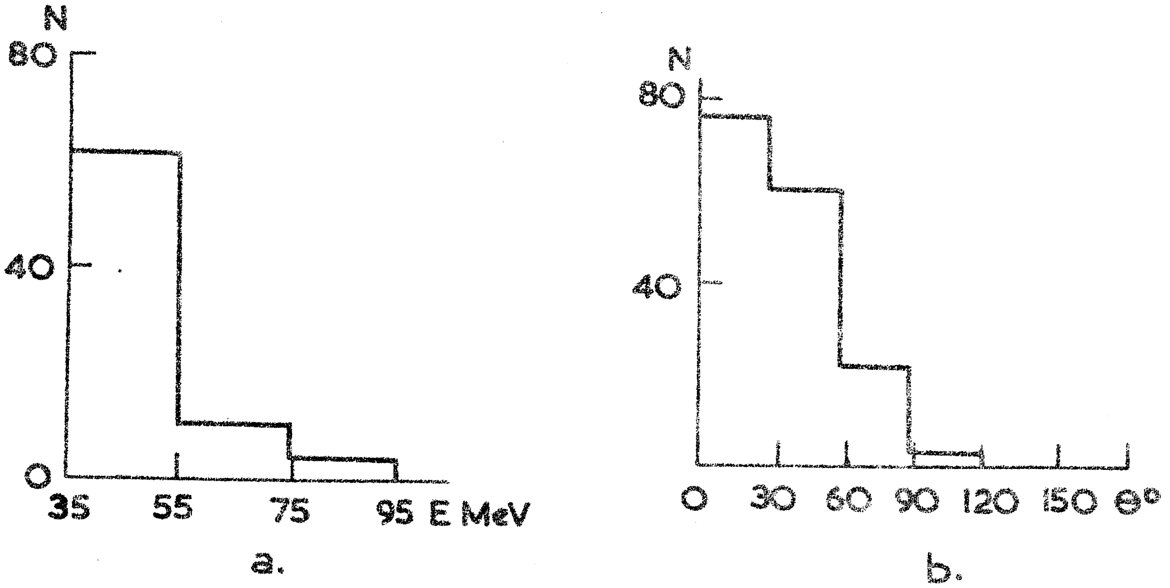

Fig. 21 a) Energy and b) angular distribution of $Z > 3$ fragments at 660 MeV.

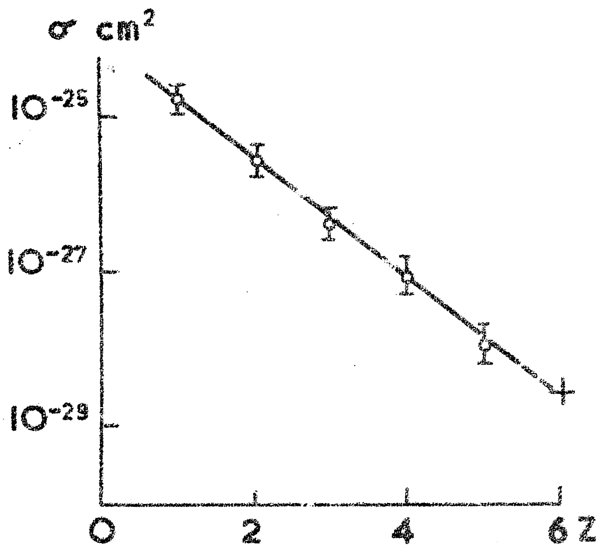

Fig. 22 Charge distribution of cascade particles at 660 MeV.

was placed above the emulsion during the irradiation. 204 tracks of multicharged particles originating in the film were selected for measurements from the total number of 302. Tracks of α particles, ^{8}Li and ^{5}B from these emulsions and tracks of ^{14}N from similar emulsions, were used for charge calibration. The charge was determined from the integral track width for particles with ranges more than 40 micron. Fig. 21a,b shows the energy and angular distributions of fragments and Fig. 22 presents the charge distribution. Points for $Z=2$ and 3 were taken from [63].

The angular distribution shows that a very small number of particles are emitted into the backward hemisphere and the charge distribution is almost the same as in the case of Ag and Br targets. The point for Z=6 was found by means of extrapolation for comparison with scattering of 660 MeV protons on carbon nuclei [56]. No fragments with Z=6 were found in this experiment. The total cross-section of emission of fragments with ranges more than 20 micron was found to be 4 mb.

An investigation of disintegrations with fragments from C,N,O nuclei (light nuclei of emulsion) was carried out [68] at 100, 200 and 300 MeV proton energy. It was found that : 1) all the fragments are emitted into the forward hemisphere and with a decrease of incident proton energy the distribution becomes more peaked forward; 2) the most probable range is the same for 100 and 200 MeV incident proton energy within experimental errors. It is significant that the energy of incident protons in these experiments was below the threshold of meson production so that here the mechanism of fragment emission could not be connected with production and reabsorption of mesons [69)-71)]. Emission of fragments only in the forward direction suggests a knock-on mechanism for fragmentation.

Interesting results, suggesting some mechanism other than the knock-on process were obtained by Varfolomeev [72] from the disintegrations of light nuclei induced by slow π^- mesons. To study the reaction

$$A + \pi^- \rightarrow B^* + {}^1_0n$$

after absorption of a slow π^- meson by a nucleus Varfolomeev irradiated plates loaded with BeF_2 by slowed-down π^- mesons. The expected reaction was :

$$\begin{array}{l} {}^9Be + \pi^- \rightarrow {}^8Li + {}^1_0n \\ \qquad\qquad\quad \downarrow \\ \qquad\qquad {}^8Be^* + 2\,\alpha . \end{array}$$

12 such events were observed and the photomicrograph of one of them is presented in Fig. 23.

From measurements of the energy of ^{8}Li and the two α particles and from the law of conservation of momentum, the energy of neutron was found to be 108 MeV and the total energy of the reaction, taking into consideration the binding energy of a neutron, was 140.6 MeV (in good agreement with the rest mass of π^- meson, 139.6 MeV). As the author notes, the ^{8}Li nucleus was produced either in its ground state or with excitation not exceeding 2 MeV because otherwise ^{8}Li would decay with the emission of a neutron. Hence the fast neutron receives its energy in the process of an interaction in which all the nucleons of the residual ^{8}Li nucleus take part. If, for example, the absorption of mesons by heavy nuclei could proceed so that all the energy was distributed between a light fragment and a residual nucleus, this would have been a suitable explanation of emission of fragments with energies much greater than the Coulomb barrier. It is well known that the hypothesis about fragment emission in the process connected with reabsorption of mesons [69)-71)] has been discussed in literature.

Varfolomeev [73)] has also observed (Fig. 24) the decay of an oxygen nucleus into three particles : ^{8}Li, ^{7}Be and $^{1}_{0}n$. This fact indicates the possibility of the correlation of large groups of nucleons inside a nucleus which interact as a whole .

Concluding this brief review of results of investigations of nuclear reactions with emission of multicharged particles, it is possible to note the following : the total sum of available experimental data on heavy and light nuclei cannot be explained on the basis of any one of the known mechanisms but can be explained by a combination of them. This is not surprising and is due to the fact that the collision of high energy particle with nuclei is a complex phenomenon and is accompanied by the development of a cascade in a "cold" nucleus (during a "nuclear

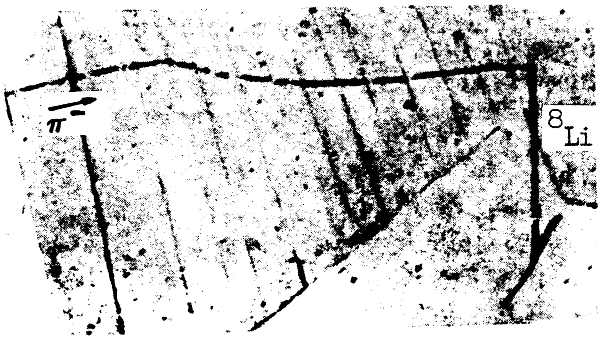

Fig. 23 Microphotograph : $^{9}Be+\pi^{-} \rightarrow {}^{8}Li+{}^{1}_{0}n.$

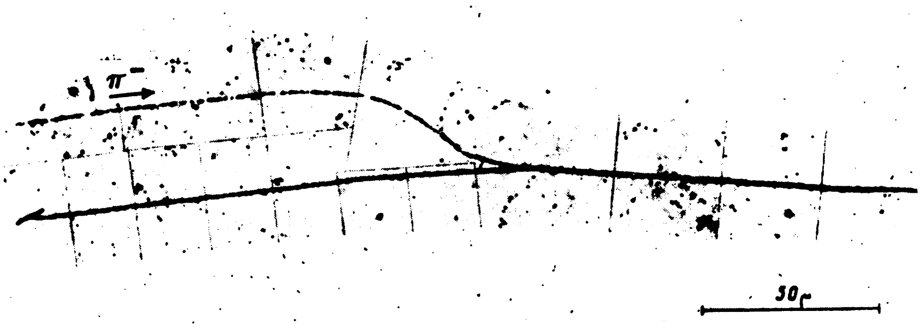

Fig. 24 Microphotograph : $^{16}O+\pi^{-} \rightarrow {}^{8}Li+{}^{7}Be+{}^{1}_{0}n.$

time") and with the production of a highly excited residual nucleus. Further behaviour of such an "overheated" nuclear matter substantially depends on the value of the excitation energy and on the mass of the residual nucleus (i.e., on its position in the periodic table). Further theoretical investigations are necessary to comprehend the behaviour of nuclear matter in highly excited state. I have already mentioned one of the attempts to improve the existing evaporation theory by taking into account interaction between evaporating particles which under certain conditions leads to their association in final states.

The majority of facts concerning the high energy fraction of fragments corresponds to the emission of fragments connected with the development of a cascade in the nuclear interior where some correlated groups of nucleons which are weakly bound to the rest of the nucleons are present. It is possible that these groups are situated in the layer close the nuclear surface.

If fragments emitted during the cascade process are structural elements of a cold nucleus, then the investigation of fragmentation would be a source of information on the structure of the original nuclei. The independence of the charge distribution of fragments on the energy of bombarding particles confirms this supposition. If the foregoing corresponds to the real nature of the phenomenon (i.e., the observed mass and energy spectra are due to at least two processes - evaporation from a highly excited nucleus and to the development of a cascade in a cold nucleus) then investigation of fragmentation will be a source of information about the structure of nuclei both in ground and highly excited states. It is necessary to note that all the experimental results available up to this moment are only a first approach to the true picture. It is necessary to develop new methods which will give better experimental possibilities to analyze all the products of nuclear reactions at various energies of bombarding particles for different targets.

During the recent years there have appeared new possibilities of theoretical analysis of high energy particle interactions with nuclei (so far only for light nuclei) on the basis of dispersion relations [76]. The future will show what this method can contribute to the understanding of nuclear reactions with the emission of multicharged particles.

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SESSION VII

MUONS AND NUCLEAR STRUCTURE

Speaker :

J.C. SENS

MUONS AND NUCLEAR STRUCTURE

J.C. Sens
CERN, Geneva.

I. PROPERTIES OF THE MUON

The muon has been discovered 28 years ago. At first it was thought to be the particle required by the Yukawa field. Since its interactions have been shown to be electromagnetic and weak only, the muon has obtained special interest in two respects : being, to all appearance, merely a heavier version of the electron, its "raison d'être" is not clear; and secondly, due to its small Compton wavelength (~1.9 fermi) compared to that of the electron (386 fermi), it forms a useful tool to explore the shape of nuclei, somewhat like the shape of a hand is mapped out by pouring sand over it, rather than marbles or apples.

The muon has the following properties : its spin is $\frac{1}{2}$, as follows from the splitting into two lines of the 3d-2p and 2p-1s muonic X rays in heavy elements [1], and from the precession frequency of muonium in an external magnetic field [2]. The magnetic moment is 1.001162 ± 0.000005 muon Bohr magnetons, from the measurement of g-2 [3]. The positive muon mass is 206.768 ± 0.003 electron masses, from combined electron/proton and muon/proton precession ratios and the g-2 experiments on electron and muon [3]. The negative muon mass is $206.76^{+0.02}_{-0.03}$ from K edge absorption in muonic X rays [4]. From the equality of the masses of μ^+ and μ^-, found by different methods, one can conclude that the muon charge is 1.00000 ± 0.00005 electron charges [3]. The electric dipole moment is $\leq (0.6 \pm 1.1) \times 10^{-17} \times e$ cm e.s.u. [5] from data on the precession of the spin around the electric field set up in its rest frame when the muon moves through a magnetic field [6].

I wrote these numbers down to show where we stand in accuracy in the study of the muon as an elementary particle. From this data one concludes that the muon interacts only electromagnetically or weakly with its environment. In particular, from the agreement between the measured deviation of g from 2 and the predicted electromagnetic corrections (the "weak" corrections are too small at this level of accuracy), one concludes that the coupling constant of the muon to the nucleon field, $G^2/4\pi$, is less than 10^{-3}. One can thus safely use the muon as a probe for nuclear structure, without being troubled by strong effects.

Nuclear structure effects show up in three kinds of experiments : scattering of muons by nuclei, muonic X rays and capture by nuclei. We shall deal with the latter two topics; the scattering of muons is discussed in the electron scattering session. No attempt is made to summarize all the work done; instead, we shall single out some of the problems that are likely to be pursued further in the future.

II. MUONIC X RAYS

For a point nucleus, the energy levels of the muon are given by the Dirac formula :

$$E(n,j) = - mc^2 \frac{(\alpha Z)^2}{2n^2}\left[1 + \frac{(\alpha Z)^2}{n^2}\left(\frac{n}{j+1/2} - \frac{3}{4}\right)\right] . \qquad (1)$$

Taking the Pb nucleus as our example we have, in round numbers, $E(1s)=21$, $E(2s)=E(2p_{1/2})=5.4$, $E(2p_{3/2})=4.8$, $E(p_{3/2})-E(p_{1/2})=0.55$, $E(2p_{3/2})-E(1s)=16.2$ MeV.

The finite size of the nucleus is taken into account by computing

$$\Delta E = \langle\varphi| V-V_p |\varphi\rangle . \qquad (2)$$

Now you can calculate this expression with various degrees of refinement. For example, you can take for φ the ground state Schroedinger wave function corresponding to $V_p = -\frac{Ze^2}{r}$ and thus consider the whole finite size as a perturbation. In that case one gets

$$\Delta E = \frac{2\pi Z\alpha^2}{3} |\varphi(0)|^2 \langle r^2 \rangle \quad . \tag{3}$$

This shows, quite generally, that the ground state shift goes up like Z^4 and is proportional to the mean square radius of the nucleus. The ratio of ΔE to the binding energy is proportional to $\langle r^2 \rangle / a_B^2$ (a_B = Bohr radius). Since $\langle r^2 \rangle$ increases roughly like $A^{2/3} \simeq (Z/2)^{2/3}$ while a_B decreases, the effect of nuclear size increases with Z. If (3) were exact, $\ell \neq 0$ states would not shift, since their wave functions have nodes at the origin. With Dirac wave functions a finite but very small shift would be obtained also for p states (in fact, all $j=\frac{1}{2}$ (be it $\ell = 0$ or 1) Dirac functions are singular at the origin). The conclusion is thus that what is measured in a 2p-1s transition is the nuclear mean square radius.

Further refinement is obtained by solving the Dirac equation for an extended charge distribution. Cooper and Henley [7] reduce the equation to a Schroedinger equation with a relativistic correction term and a spin-orbit coupling term. This is justified since the muon-nucleus system is essentially non-relativistic. In treating the relativistic term as a perturbation they obtain results accurate to better than 1%. Fitch and Rainwater [8] separate out the radial part, and obtain the two radial equations :

$$\begin{aligned} \frac{dG}{dr} &= -\frac{k}{r} G + (E+m-V)F \\ \frac{dF}{dr} &= \frac{k}{r} F - (E-m-V)G \end{aligned} \tag{4}$$

where $F/r=f$ and $G/r=g$ are the small and large components. Eqs. (4) are then solved by series expansion. In both calculations the charge distribution is assumed uniform, so that the potential is given by

$$V = -\frac{Ze^2}{R}\left(\frac{3}{2} - \frac{1}{2}\frac{r^2}{R^2}\right) \quad r \leq R \tag{5a}$$

$$V = -\frac{Ze^2}{r} \qquad\qquad r \geq R \quad . \tag{5b}$$

Both calculations agree to better than 1%. The result for Pb is $E(1s)=10.11$ MeV, $E(p_{3/2}-1s_{1/2})=5.48$ MeV, showing the very pronounced effect of the finite nuclear size.

Deviations from uniformity will decrease the binding energy and (since the effect on the p state is negligible) reduce the transition energy. This follows simply from the fact that the electric field at a given distance $r<R$ is proportional to the total charge enclosed, and is thus less for distributions which have tails. In the work of Pustovalov and Krechko [9], the Schroedinger equation is solved for a uniform distribution. Non-uniform models are then calculated in perturbation theory, using the "uniform" solutions as zero order approximation. Figure 1 (taken from Ref. [9]) illustrates the sensitivity to deviations from a uniform charge distribution for the case of a Fermi type distribution for average values of electron scattering parameters.

All of the above-mentioned calculations are based on perturbation theory, i.e., formula (2). Relativistic corrections and deviations from uniformity in the charge distribution are both about 1-2% and to draw conclusions about the shape the relativistic corrections must therefore be carefully estimated, at least for large Z.

No such difficulty arises in an exact numerical solution of the Dirac equation (4) such as has been given by Ford and Wills [10]. In the work of Ford and Wills a non-uniform distribution is assumed from the outset. It assumes that the form

$$\rho(x) = \frac{Z}{4\pi r_1^3 N_o} \begin{cases} 1 - 0.5e^{-n(1-x)} & x < 1 \\ 0.5e^{-n(x-1)} & x > 1 \end{cases} \tag{6}$$

(with $x=r/r_1$ and $N_o=1/3+2/n^2+e^{-n}/n^3$) is valid for all 34 nuclei investigated. Let us look in slightly more detail at the results here, since they are at present the best values to compare experimental data with. The potential following from (6) is

$$V(r) = \frac{Ze^2}{r_1} J(x) \tag{7}$$

with

$$J(x) = \frac{1}{N_o}\left[\frac{1}{n^2} + \frac{1}{2} - \frac{1}{6}x^2 + \frac{e^{-n}}{n^2}\left(\frac{1-e^{nx}}{nx} + \frac{1}{2}e^{nx}\right)\right] \quad \text{for} \quad x < 1$$

and

$$J(x) = \frac{1}{x} - \frac{1}{N_o}\left[e^{-n(x-1)}\left(\frac{1}{x} + \frac{n}{2}\right)\Big/n^3\right] \quad \text{for} \quad x > 1 .$$

Evidently, for $n \to \infty$ the surface thickness becomes smaller, the distribution becomes more uniform and (7)→(5). The parameters r_1 and n are taken from electron scattering data, and range from $r_1/A^{1/3}=0.83$ to 1.11 and from $n=2.25$ to $n=10.00$ for Be and Pb respectively. The charge densities and the ground state muon densities are plotted in Fig. 2 for C($n=3.00$, $r_1=1.009A^{1/3}$), Zn($n=5.50$, $r_1=1.07A^{1/3}$) and Pb($n=10.00$, $r_1=1.11A^{1/3}$). The eigenvalues are accurate to 5 significant figures. A 10% increase in the radial parameter r_1 decreases the $E(p_{3/2}-1s)$ difference by 0.14% for C and by 8.19% for Pb. A 10% increase in surface thickness decreases the energy difference by 0.09% in C and 1.09% in Pb.

At this level of accuracy the transition energies in heavy elements become sensitive to properties of the nucleus other than $\langle r^2 \rangle$.

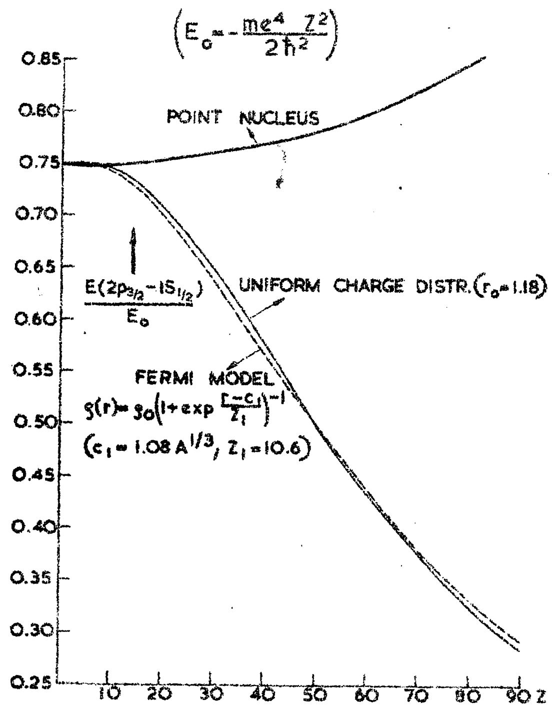

Fig. 1

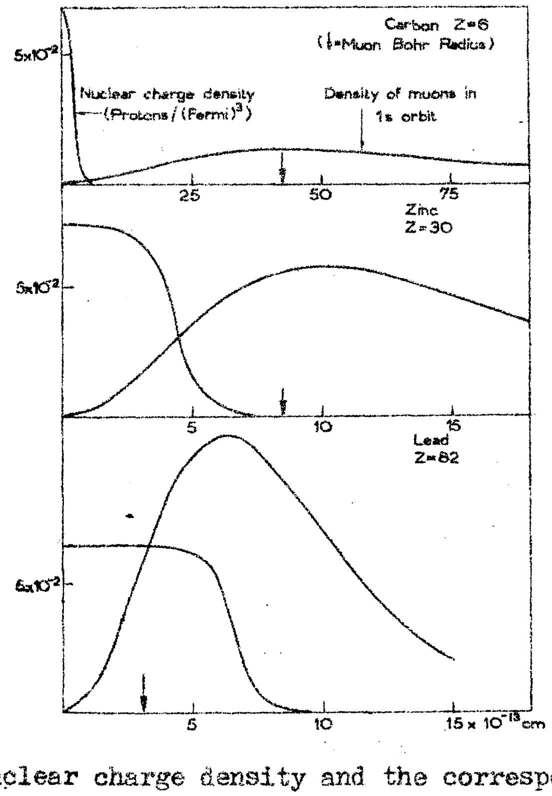

Fig. 2 The nuclear charge density and the corresponding ground state muon density distribution for carbon, zinc and lead. The arrows indicate the Bohr radius for a point nucleus. From Ref. 10).

Extra energy shifts are produced by the vacuum polarization correction, the nuclear quadrupole moment, and the excitation of specific nuclear states.

In muonic atoms the vacuum polarization is the only radiative correction : the radiation reaction which makes up the bulk of the Lamb shift in electronic atoms varies as $1/m^2$ and is thus negligible for muons. In the case of the phosphorus 3d-2p transition the experimentally observed transition energy is 88017^{+15}_{-10} eV [4]. The calculated contribution of the vacuum polarization is 331.42 eV [11]. Since at present we know the muon mass from independent experiments, we can use the X ray experiment (which was designed to measure this mass) to check the vacuum polarization. In this way we find agreement with theory to ~4% *). The finite size of the nucleus alters the correction by only 3.2±1 eV : the disturbance of the virtual electron states around the <u>extended</u> nuclear charge results in a slightly larger potential outside the nucleus (as compared to a point nucleus) and a slightly reduced potential inside; in the surface area, where the muon density is high, the effects nearly cancel.

We now discuss the quadrupole effects. Let us note first that the hyperfine splitting due to nuclear spin is very much smaller than the effect of a quadrupole moment since

$$\frac{(e^2 Q_N)/r^3}{(\mu\ \mu_N)/r^3} \approx 200 \qquad (8)$$

for muonic atoms; this in contrast with electronic orbits, where the ratio (8)≈1. The hyperfine splitting due to nuclear spin is of the order of electron Volts for medium Z nuclei and is thus undetectable with existing techniques. This circumstance facilitates the detection of quadrupole effects : whereas in the case of electronic spectra, quadrupole effects appear as <u>departures</u> from the interval rule for

*) The measured Lamb shift agrees with the calculated vacuum polarization to ~2%.

hyperfine splitting, in the muonic spectra they are responsible for the entire effect. In order to calculate the splitting, for instance, in the case of a uniform charge distribution, a term

$$V_\varphi(r) = -\frac{1}{2} Q\left(\frac{3}{2}\cos^2\vartheta - \frac{1}{2}\right)\frac{Ze^2}{r^3} \tag{9a}$$

must be added to (5a) and a term

$$V_\varphi^1(r) = -\frac{1}{2} Q\left(\frac{3}{2}\cos^2\vartheta - \frac{1}{2}\right)\frac{Ze^2 r^2}{R^5} \tag{9b}$$

to (5b), as pointed out in the classical paper of Wheeler [12]. A numerical solution of the Dirac equation for this potential has not yet been given. It is to be expected, however, that the splitting is smaller than in the case of a point nucleus, due to the decrease of the quadrupole field inside the nucleus. If we define the boundary of the non-spherical nucleus by

$$R = R_o\left(1 + \frac{2}{3}\epsilon\ P_2\ (\cos\vartheta)\right) \tag{10}$$

with $\epsilon = \frac{a-b}{R}$ (a,b, symmetry axis, azimuthal axis) then it is easy to show that

$$Q \sim \int \rho\ (3z^2 - r^2)\ r^2 dr \sim \epsilon < r^2 > \quad . \tag{11}$$

Since $<r^2>$ is known from nearby nuclei with no quadrupole moment, we thus obtain a measure for the deviation from a purely radial distribution of charge. As is well known, most quadrupole moments found so far are not in agreement with shell model predictions, but much larger. A large amount of configuration mixing is required to find agreement.

We have assumed so far that the nucleus remains in the ground state during the cascade process. If this is not so the equation for the muon-nucleus system

$$\left(H_N + H_\mu + \mathcal{H}\right)\Psi = W\Psi \tag{12}$$

is solved by perturbation theory on the joint Hamiltonian. We separate the wave function in the familiar way

$$\Psi^{(nm)} = \psi^{(n)}\varphi^{(m)} = \psi_o^{(n)}\varphi_o^{(m)} + \psi_1^{(n)}\varphi_1^{(m)} \tag{13}$$

where $\psi_o^{(n)}\varphi_o^{(m)}$ is the solution of the unperturbed system with energy $W_o=E_o+\epsilon_o$ (E_o=energy of the nucleus, ϵ_o=energy of the muon). By expanding

$$\psi_1^{(n)}\varphi_1^{(m)} = \sum_{k,\ell} c_{nmk\ell}\, \psi_o^{(k)}\, \varphi_o^{(\ell)}$$

and inserting it into (12) we get

$$\Psi^{(nm)} = \psi_o^{(n)}\varphi_o^{(m)} + \sum_{k,\ell} \frac{\langle k,\ell|\mathcal{H}|n,m\rangle}{(E_o^{(k)} + \epsilon_o^{(\ell)}) - (E_o^{(n)} + \epsilon_o^{(m)})}\, \psi_o^{(k)}\varphi_o^{(\ell)} \, . \tag{14}$$

From (14) we see that the conditions for nuclear excitation are two :

1) one must have deformation, even if the static quadrupole moment is zero;

2) there must be resonance between a muonic transition and a nuclear transition

$$\epsilon_o^{(m)} - \epsilon_o^{(\ell)} \approx E_o^{(k)} - E_o^{(n)} \, .$$

To first order the energy changes by

$$\Delta W_1 = \langle n,m|\mathcal{H}|n,m\rangle \tag{15}$$

resulting in the static quadrupole moment (zero for even-even nuclei) and in second order

$$\Delta W_2 = \sum \frac{|\langle k,\ell|\mathcal{H}|n,m\rangle|^2}{(E_o^{(k)}+\epsilon_o^{(\ell)}) - (E_o^{(n)}+\epsilon_o^{(m)})} \quad . \tag{16}$$

For a quadrupole interaction

$$\mathcal{H} \sim \frac{r_N^2}{r_\mu^3} Y_{20}(\vartheta_{N\mu}) \rightarrow \sim \frac{r_N^2}{r_\mu^3} \sum_m Y_{2m}(\vartheta_N) Y^*_{2m}(\vartheta_\mu) \quad . \tag{17}$$

The matrix elements in (14), (15) and (16) thus break up into a nuclear part and a muon part. The effect is largest for the 1s state. It increases as Z^4R. For p states the level splitting is much smaller; the ratio of the polarization effect to the 2p-1s transition energy varies thus as Z^2R. The effects are therefore to be found at the high Z values. Examples, calculated by Jacobsohn and Wilets [13] are $_{74}W$ $\overline{p}$ state fine structure 142 keV, excited state at 115 keV (average for $^{180}_{74}W$ and $^{186}_{74}W$), $Q=7\times10^{-24}$ cm^{2}7, $^{238}_{92}U$ (234 keV, 44 keV, 10×10^{-24} cm^2), $^{181}_{73}Ta$ (151 keV, 137 keV, 7×10^{-24}cm^2) and $^{230}_{90}Th$ (242 keV, 50 keV, 12.6×10^{-24} cm^2). In Fig. 3 (taken from G.E. Temmer, Rev. Mod. Phys. 30, 498 (1958)) the quadrupole pattern for ^{230}Th is shown in detail. The 2p-1s energy is ~7 MeV. The spectrum is spread out over several hundred keV. The zero point indicates the centre of gravity of the lines. The dashed lines represent the fine structure (F.S.) splitting for an undeformed nucleus. Solid lines show the pattern typical for prolate (top) and oblate (bottom) quadrupole deformations. Arrows indicate transitions which leave the nucleus in the 2^+ rotational state (50% probability). ^{230}Th has no static quadrupole moment. The fine structure intensity rule does not hold if quadrupole effects are present.

I would like now to turn to the experimental observations. The "Chang radiation" was first observed in 1949. Fitch and Rainwater published extensive data in 1953 and Stearns and Stearns in 1957. In 1962 three sets of data appeared by Johnson, Hincks and Anderson at the University of Chicago, by Brix, Engfer, Hegel, Quitmann, Backenstoss, Goebel and Stadler at CERN, and by Frati and Rainwater at Columbia University, all with greatly improved accuracy [1),14)]. In Table 1 these data are listed, along with the Ford-Wills predictions, calculated as discussed above; in these predictions the older scattering data are used (Hofstadter, Ref. [15)]). In Fig. 4 the percentage deviations from the Ford-Wills predictions are plotted. More recent data on electron scattering indicate that better fits are obtained by a distribution with somewhat larger density and smaller radius. We have plotted also in Fig. 4 the results of Bloore, Varshni and Pearson [16)], who have recently calculated the

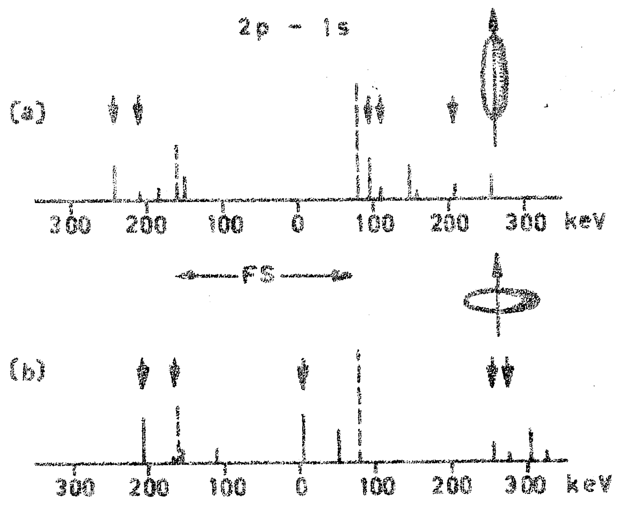

Fig. 3

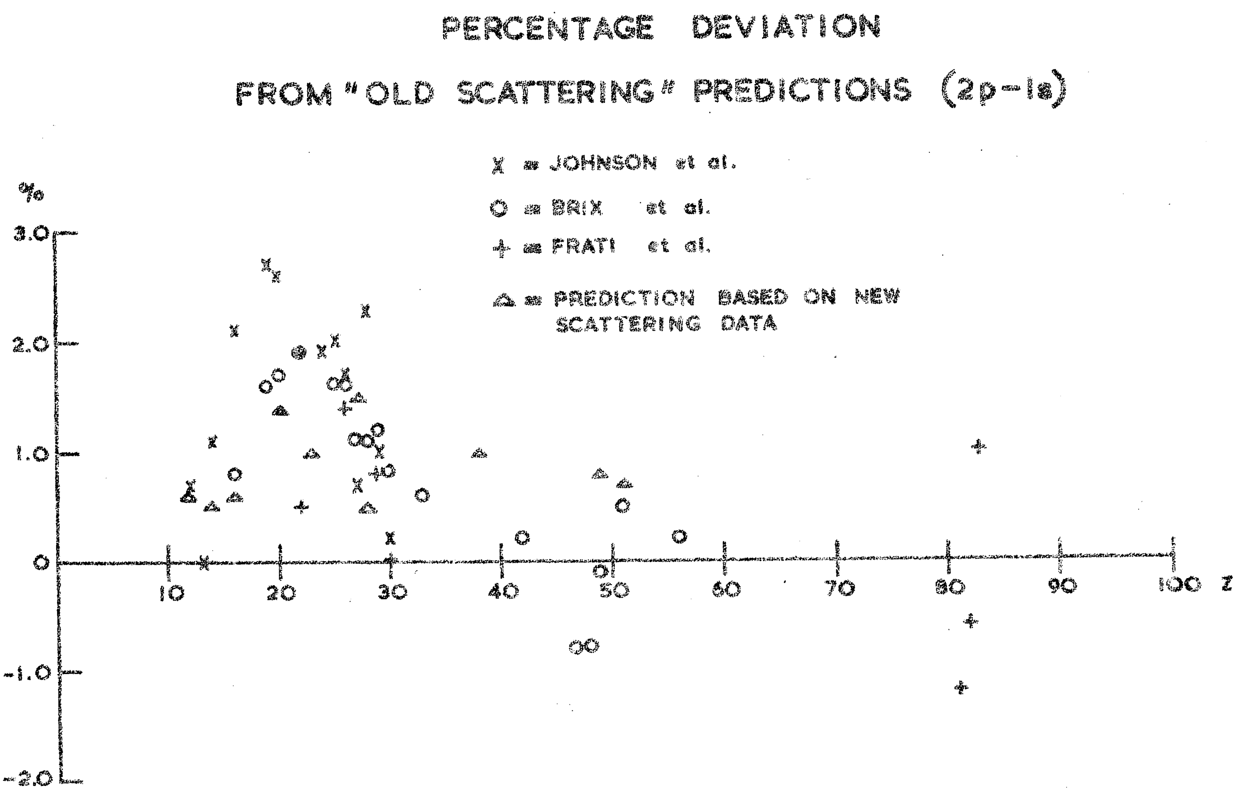

Fig. 4

Element	Z	ΔE_{Exp} 2p – 1s (Experiment)	Ref.	ΔE_{Th} 2p – 1s (Theory)	$\frac{\Delta E_{Exp} - \Delta E_{Th}}{\Delta E_{Th}}$ (%)
Mg	12	296.3 ± 2	(a)	294.3	0.7
Al	13	344.2 ± 9.1	(a)	344.3	0.0
Si	14	401.1 ± 1.8	(a)	396.6	1.1
S	16	522.7 ± 1.7	(a)	512 *)	2.1
		516 ± 4	(b)		0.8
K	19	721.5 ± 5.7	(a)	702.5	2.7
		714 ± 3	(b)		1.6
Ca	20	791.5 ± 2.0	(a)	771.4*)	2.6
		784 ± 3	(b)		1.7
Ti	22	937.0 ± 4.4	(a)		1.9
		937 ± 7	(b)	919.8	1.9
		924.7 ± 2.5	(c)		0.5
Cr	24	1094.4 ± 4.3	(a)	1075.8	1.9
Mn	25	1178.1 ± 4.3	(a)	1155.0	2.0
		1174	(b)		1.6
Fe	26	1257.6 ± 4.3	(a)		1.7
		1258 ± 6	(b)	1238.1*)	1.6
		1255.5 ± 2.4	(c)		1.4
Co	27	1332.8 ± 4.3	(a)	1323.0	0.7
		1337 ± 5	(b)		1.1
Ni	28	1442.3 ± 6.2	(a)	1409.9*)	2.3
		1426	(b)		1.1
Cu	29	1511.0 ± 5.5	(a)		1.0
		1515	(b)	1496.5	1.2
		1508.2 ± 4.0	(c)		0.8
Zn	30	1589.9 ± 6.2	(a)		0.2
		1600	(b)	1586.5	0.8
		1586.9 ± 4.5	(c)		0.0
As	33	1867 ± 7	(b)	1855 *)	0.6
Mo	42	2712 ± 5	(b)	2707.8	0.2
Rh	45	2977	(b)		
Pd	46	3068	(b)		
Ag	47	3163	(b)	3189.2	– 0.8
Cd	48	3254	(b)	3281 *)	– 0.8
In	49	3360	(b)	3365 *)	– 0.1
Sn	50	3454	(b)		
Sb	51	3546	(b)	3528.3	0.5
Ba	56	3985 ± 30	(b)	3975	0.2
La	57	4079	(b)		
Tl	81	5930 ± 11*)	(c)	6001	– 1.2
Pb	82	5990 ± 11**)	(c)	6025	– 0.6
Bi	83	6053 ± 9**)	(c)	5995	1.0

Table 1

Recently measured muonic X rays. a) C.Johnson et al., Ref. 15); b) P. Brix et al., Ref. 15); c) W. Frati and J. Rainwater, Ref. 1). The theoretical values are taken from Ref. 10). The symbol *) indicates that these values were interpolated from Ref. 10). All energies are measured from the centre of gravity of the p doublet, except for **), where the $2p_{3/2} - 1s_{1/2}$ energy is tabulated.

change that this narrower distribution produces in the Ford-Wills predictions. One sees that the bulk of the discrepancy which existed so far between the scattering data and the X ray data is taken care of by this correction. The remaining deviations are of the same order as the experimental errors.

Frati and Rainwater have succeeded in observing the fine splitting $2p_{3/2}-2p_{1/2}$ in both the 2p-1s and the 3d-2p transitions in Tl, Pb and Bi. The energy differences are in excellent agreement with the values computed from electron scattering data, but the intensity ratio deviates strongly from the value expected from the intensity rule in the 2p-1s transition for Tl and to a lesser extent in the 2p-1s and the 3d-2p transition for Bi. In Pb the intensity ratio is normal (see Table 2).

	Z	$\frac{I(2p_{1/2}-1s)}{I(2p_{3/2}-1s)}$	$\frac{I(3d-2p_{1/2})}{I(3d-2p_{3/2})}$
Tl	81	0.97 ± 0.09	0.53 ± 0.03
Pb	82	0.49 ± 0.07	0.51 ± 0.02
Bi	83	0.75 ± 0.05	0.59 ± 0.02

Table 2

Intensity ratios in 2p-1s and 3d-2p transitions in Tl, Pb and Bi. Experimental data from Ref. 1). The theoretical value is 0.5 in all cases.

To explain this interesting result the authors suggest that for Tl we have here a case of muon induced nuclear excitation. Indeed, the $2p_{3/2}-2p_{1/2}$ difference is 188 keV while the nucleus has its first excited state $d_{3/2}$ at 205 keV above the ground state $s_{1/2}$. As a result the $p_{3/2}$ wave function is no longer pure, but contains an admixture of $p_{1/2}$. Applying (14)

$$\Psi\left(\text{"}\tfrac{3}{2}\text{"}\right) \Rightarrow \psi^N(s_{1/2})\varphi^\mu(p_{3/2}) + a\psi^N(d_{3/2})\varphi^\mu(p_{1/2})$$

$$\Psi\left(\text{"}\tfrac{1}{2}\text{"}\right) \Rightarrow \psi^N(s_{1/2})\varphi^\mu(p_{1/2})$$

and inserting a weight factor 1/2 for the "1/2" state we get for the ratio of intensities $(p_{1/2}\to 1s)/(p_{3/2}\to 1s)$

$$\frac{1/2 + a^2/(1+a^2)}{1/(1+a^2)} = \frac{1}{2}(1+3a^2)$$

and this is experimentally found to be 0.97. Hence $a^2=0.31$. A consequence of this mixing is that one expects an energy shift, which is, using (16)

$$\Delta W = a^2\left[\left(E^N(d_{3/2}) - E^N(s_{1/2})\right) - \left(\epsilon(p_{3/2} - \epsilon(p_{1/2})\right)\right] = 5.3 \text{ keV} .$$

The interpretation of the Tl data in terms of nuclear excitation is confirmed by the fact that in the 3d-2p transition the intensity ratio is normal : the nucleus does not go to its first excited state until the muon has reached the 2p state.

In the case of Bi the interpretation is less clear. The ratio $(3d\text{-}2p_{1/2})/(3d\text{-}2p_{3/2})=0.59\pm0.02$ instead of 0.50. Frati and Rainwater give the following explanation

$$\mu^- + {}^{209}\text{Bi} \rightarrow {}^{209}\text{Pb}^* + \nu$$

$${}^{209}\text{Pb}^* \rightarrow {}^{208}\text{Pb}^*(3^-) + n$$

$${}^{208}\text{Pb}^*(3^-) \rightarrow {}^{208}\text{Pb}(0^+) + \gamma \ (2.615 \text{ MeV}) .$$

This γ ray has been seen very clearly, both by the Columbia and the CERN groups. It happens to fall just between the two 3d-2p lines, and although it is delayed by the lifetime for disappearance of the muon by capture or decay, a sizeable fraction of events occurs so close to "zero time" that they cannot be resolved. The nuclear γ ray affects the weaker of the two X rays most; it thus tends to <u>increase</u> the ratio (1/2)/(3/2), as observed.

The ratio $(2p_{1/2}\to 1s)/(2p_{3/2}\to 1s)$ in Bi has been measured by Frati and Rainwater, who find 0.75 ± 0.05; Brix et al., on the other hand, find no significant deviation from the intensity rule [17]. No anomaly is expected since the $p_{3/2}-p_{1/2}$ energy difference is 189 keV, while the first excited state is at $\approx$ 910 keV; we are thus far away from the resonance condition.

In Pb no such coupling is observed. This is expected since the first excited state is high up (2.615 MeV) and has anyhow the wrong parity.

With present techniques the level splittings which accompany the above described intensity variations are too small to be observed. Quadrupole effects show up most clearly in 3d-2p transitions in heavy elements; a resolution of ~10 keV in 2-3 MeV is thus required. This could be achieved by solid state detectors, by pair spectrometers or β ray spectrometers employing thin converters. Such detectors are practical only if very high muon fluxes are available.

III. TOTAL AND PARTIAL CAPTURE RATES

After completion of the cascade process, the muon sits in the 1s orbit at a distance

$$a_B^\mu \approx \frac{250}{Z} \text{ fermi}$$

from the nucleus, until it decays or is captured. In the basic capture process almost all of the available energy (~106 MeV) is carried off by one neutrino, leaving ~5 MeV kinetic energy for the struck nucleon. In a complex nucleus some of the available energy is used for emission of nucleons and excitation of the residual nucleus. The mean excitation energy is 15-20 MeV.

Very generally speaking, the reasons for studying muon capture in connection with nuclear structure are the following :

1) the muon is very close to the nucleus for a long time and is thus sensitive to the nuclear shape;

2) in muon capture there are many more excited states available than in e.g., electron capture. Although energy levels, spins, and parities are best determined by low energy physics methods, the wave function can, in specific cases, more conveniently be studied by μ capture, since here one goes from the ground state of one nucleus to the ground state and excited states of its neighbour;

3) spontaneous μ^+ or μ^- disintegration of nuclei is energetically forbidden, while the induced process is rare. Capture is thus the only remaining process; this in contrast to the β interactions.

I shall try to answer first the following question : how does the capture rate depend on the properties of the nucleus ? It is unfortunately necessary to write down several long expressions : since you are familiar with them already, I have collected them without derivations in Table 3. I follow mainly the notation of a recent review by Tolhoek [18]. Note that if in addition to the approximations made in Table 3 the induced term g_p is neglected the rate follows very simply from

$$\mathcal{H}_{eff} \approx G_V \left[1.1_i - x\vec{\sigma} \cdot \vec{\sigma}_i \right]$$

where $x = -G_A/G_V$, 1 and $\vec{\sigma}$ are the non-relativistic vector and axial vector operators for the muon, 1_i, $\vec{\sigma}_i$ the same for the i^{th} nucleon.

GOLDBERGER – TREIMAN – WEINBERG :

REQUIREMENTS :

① LORENTZ – INVARIANCE
② CHARGE – INDEPENDENCE OF STRONG INT.
③ TIME REVERSAL INVARIANCE

LEADING TO M.E. FOR CAPTURE :

$$\sqrt{2}\,M.E. = (\bar{\nu}(1-\gamma_5)\, i\, \gamma_\lambda \gamma_5 \, \mu) < \bar{n} \,|\, \mathcal{J}^A_\lambda \,| p > + (\bar{\nu}(1-\gamma_5)\gamma_\lambda \, \mu) < \bar{n} \,|\, \mathcal{J}^V_\lambda | p > \qquad (1)$$

$$< \bar{n} \,|\, \mathcal{J}^A_\lambda \,| p > = g_A (\bar{u}_n \, i \, \gamma_\lambda \gamma_5 u_p) - \frac{g_P}{m} (\bar{u}_n \, q_\lambda \gamma_5 u_p) + \frac{g_T}{2M} (\bar{u}_n \, \sigma_{\lambda\rho} \, q_\rho \gamma_5 u_p)$$

$$< \bar{n} \,|\, \mathcal{J}^V_\lambda \,| p > = g_V (\bar{u}_n \, \gamma_\lambda \, u_p) - \frac{g_M}{2M} \, i \, (\bar{u}_r \, \sigma_{\lambda\rho} \, q_\rho u_p) + \frac{g_S}{m} \, i \, (\bar{u}_n \, q_\lambda u_p)$$

STATEMENTS ABOUT THE COUPLING CONSTANTS :

① IF G-INVARIANCE HOLDS: $g_S = g_T = 0$
② IF CVC HOLDS: $g_M = (\mu_p - \mu_n)\, g_V = 3.7\, g_V$ (2)
③ IF 1π-STATE CONTRIBUTES: $g_P = 8\, g_A$

USE (2) PLUS DIRAC EQ. ON (1) (PRIMAKOFF) :

$$H_{eff} = \frac{1}{2}\tau^+ (1 - \vec{\sigma}.\hat{\vartheta}) \sum_i \tau_i^- \Big[G_V 1.1_i + G_A \vec{\sigma}.\vec{\sigma}_i - G_P (\vec{\sigma}.\hat{\vartheta})(\vec{\sigma}_i \hat{\vartheta}) - g_V (\vec{\sigma}.\hat{\vartheta})\Big(\frac{\vec{\sigma}.\vec{p}_i}{M}\Big)$$

$$- g_A (\vec{\sigma}.\hat{\vartheta}) \Big(\frac{\vec{\sigma}_i . \vec{p}_i}{M}\Big) \Big] \, \delta(r - r_i)$$

CAPTURE RATE $\Lambda \div |< b | H_{eff} | a >|^2$:

$$\Lambda(a \to b) = \frac{\nu^2}{2\pi} \int \frac{d\hat{\vartheta}}{4\pi} \Big[G_V^2 |\textstyle\int 1|^2 + G_A^2 |\textstyle\int \vec{\sigma}|^2 + (G_P^2 - 2G_P \, G_A) |\hat{\vartheta}. \textstyle\int \vec{\sigma}|^2$$

$$- \frac{G_V g_V}{M} \big((\textstyle\int 1)^* (\textstyle\int \vec{p}) \hat{\vartheta} + cc \big) - \frac{G_A g_A - G_P g_A}{M} \big((\hat{\vartheta}. \textstyle\int \vec{\sigma})^* \textstyle\int \vec{p}. \vec{\sigma} + cc \big) - \frac{G_A g_V}{M} i \, \hat{\vartheta} (\textstyle\int \vec{\sigma} x \textstyle\int \vec{p} - cc) \Big]$$

APPROXIMATIONS :

① PUT TERMS IN $1/M \approx 0$
② PUT $|\hat{\vartheta}. \int \vec{\sigma}|^2 = \frac{1}{3} |\int \vec{\sigma}|^2 = |\int 1|^2$

THEN THE CAPTURE RATE SIMPLIFIES TO :

$$\Lambda(a \to b) = \frac{1}{2\pi} \Big[G_V^2 + 3 G_A^2 + (G_P^2 - 2 G_P \, G_A) \Big] M^2 \quad \text{WITH}$$

$$\boxed{M^2 = \nu^2 \int \frac{d\hat{\vartheta}}{4\pi} \, |< b \,|\, \Sigma_i \, \tau_i^- \, e^{-i \vec{\vartheta}.\vec{r}_i} \, \varphi_\mu(r) \,|\, a >|^2}$$

ALL NUCLEAR STRUCTURE EFFECTS ARE COLLECTED IN M^2

Table 3

For the triplet state : $\mathcal{H}\sim 1-x$; weight 3/4; for the singlet state : $\mathcal{H}\sim 1+3x$; weight 1/4. Hence :

$$3/4(1-x)^2 + 1/4(1+3x)^2 \rightarrow G_V^2 + 3G_A^2 \ .$$

From the last line in Table 3 we see that nuclear structure effects enter in three ways : first of all through the overlap of the muon wave function with the charge distribution of the nucleus. We can extract $\varphi_\mu(r)$ from the matrix element provided we replace it by its average over the charge distribution

$$|\ldots\ldots\varphi_\mu(r)\ldots\ldots|^2 = |\varphi_\mu|^2_{av}\,|\ldots\ldots|^2$$

with

$$|\varphi_\mu|^2_{av} = \int |\varphi_\mu^2(r)|\,\rho(r)d\tau \ .$$

For a point nucleus we would have

$$|\ldots\ldots\varphi_\mu(0)\ldots\ldots|^2 = \frac{Z^4}{\pi a_B^3}\,|\ldots\ldots|^2 \ .$$

Therefore we define formally

$$|\varphi_\mu|^2_{av} = \frac{Z^4_{eff}}{\pi a_B^3}$$

so that the effect of the finite nuclear size can conveniently be expressed by the ratio Z^4_{eff}/Z^4. Secondly, structure effects enter into ν^2. For capture by a free proton $\nu\cong m$; for capture by a proton in a state a going to a state b $\nu\simeq m_\mu-(E_b-E_a)$. If we sum the contributions over various states a and b, we have $\nu_{av}=m_\mu-\langle E_b-E_a\rangle_{av}$. The uncertainty in $\langle E_b-E_a\rangle_{av}$ turns out to be the largest single source of uncertainty in the total capture rate. Finally, structure effects enter into the remainder of the matrix element, which is essentially

(leaving out summation signs)

$$|\langle b| e^{-i\vec{\nu}\vec{r}_i} | a\rangle|^2 \ .$$

The matrix element can be evaluated either by summing over all final states b which are accessible within the limits of conservation of energy, and for each state putting in the right neutrino momentum, or else by applying the closure approximation, i.e., by writing

$$\langle a| e^{+i\vec{\nu}\vec{r}_i} | b\rangle^* \langle b| e^{-i\vec{\nu}\vec{r}_j} | a\rangle = \langle a| e^{i\vec{\nu}\vec{r}_{ij}} | a\rangle \qquad (18)$$

since

$$\sum_b | b\rangle \langle b| = 1 \ .$$

We thus sum over all final states, whether energetically accessible or not, and replace the energy dependent quantities such as ν^2 by average values. Such an approach can work only if the matrix element falls off fairly fast for higher-lying states b; but this is generally so, since most higher-lying states have larger angular momenta.

Now the capture process is a coherent process, i.e., if the amplitude for capture on nucleon i is b_i, then the total probability is

$$\sim (\Sigma b_i)^2 \sim \Sigma (b_i)^2 + \sum_{i\neq j} b_i b_j$$

and hence the rate will depend both on $\rho(\vec{x})$=the nucleon density distribution and on $\rho(\vec{x}_1\vec{x}_2)$ which we define as the joint probability for finding one nucleon at $\vec{x}_1$ and another at $\vec{x}_2$. We can express this differently by regarding (18) as the average of an operator over the initial state. Now from matrix calculus we have

$$\bar{\Omega} = \frac{\langle a|\Omega| a\rangle}{\langle a| a\rangle} = \mathrm{Tr}(\Omega\rho)$$

with

$$\rho = |a\rangle\langle a| .$$

The operator consists of a one-particle part (in (18) this is 1) and a two-particle part $(e^{i\vec{\nu}\vec{r}_{ij}})$; hence we have also two density functions $\rho(\vec{x})$ and $\rho(\vec{x}_1\vec{x}_2)$ as defined above. Now, if all nucleons were independent we would have $\rho(\vec{x}_1\vec{x}_2)=\rho(\vec{x}_1)\rho(\vec{x}_2)$, but if there is a <u>correlation</u> between the position of the nucleons, we have

$$\rho(\vec{x}_1\vec{x}_2) = \rho(\vec{x}_1)\rho(\vec{x}_2) \left\{1+f(\vec{x}_1-\vec{x}_2)\right\}$$

where $f(\vec{x}_1-\vec{x}_2)$ is the nuclear pair correlation function.

The form of $f(\vec{x}_1-\vec{x}_2)$ depends on a model. If we use a Fermi gas model for instance, i.e., nucleons described by an antisymmetrized plane wave in a sphere of radius equal to nuclear radius, the function f can be shown to be [19]

$$f(\vec{x}) = \left(\frac{3j_1(k_F x)}{k_F x}\right)^2 = \left[3\left(\frac{\sin k_F x}{k_F^3 x^3} - \frac{\cos k_F x}{k_F^2 x^2}\right)\right]^2 . \qquad (19)$$

The form of $f(\vec{x})$ is a direct consequence of the Pauli principle.

All told, the matrix element M^2 depends thus on the proton density distribution $\rho(\vec{x})$ (this brings in two parameters, the radial constant r or c, and the skin thickness, n or z), the pair correlation function (this brings in k_F=radius of Fermi sphere, if a Fermi gas model is used) and the average excitation energy.

Luyten, Rood and Tolhoek [20] have recently investigated the sensitivity of the <u>total</u> capture rate to the values of these parameters, for ^{16}O and ^{40}Ca. In round numbers their conclusions are as follows :

a 5% decrease in the radial parameter → 6% decrease in total rate;

a 5% increase in average neutrino momentum → 16.5% increase in total rate;

a 5% increase in the radius of the Fermi sphere → 2.5% decrease in total rate.

The dependence on the radial parameter need not worry us : from electron scattering and muonic X rays combined, the radial parameter is known to 1-2%. The dependence on neutrino momentum is very strong, but not surprising, since it enters twice in the matrix element $((e^{i\nu r} \sim 1 + i\nu r)^2)$ and twice in the phase space factor (ν^2). We thus expect an uncertainty of the order of 20%. This uncertainty is inherent to the closure approximation. The dependence on k_F is not very strong; it must be noted, however, that the choice of the statistical model [i.e., the functional form of $f(\vec{x}_1 - \vec{x}_2)$] as a whole brings in an uncertainty. Using shell model wave functions instead, (with closure and a harmonic oscillator well) Luyten et al. obtain a ≈12.5% change in the capture rate; for other potential wells the difference is bigger, going up to ≈25%. As long as this sensitivity to the model persists it seems difficult to draw unambiguous conclusions from the total capture rate. However precise the experiment, it will always lead to one value for, say, the radial parameter in a given model, but to another value of the same parameter by just changing to another model. Figure 5 illustrates this point. The experimental data are taken from Ref. [21] and [22].

One can take another approach, however, that is to calculate partial capture rates. If you do that for enough states and then add them up, you have the total rate also but it is of more interest to take the ratios of various partial rates. This procedure is evidently much less dependent on the choice of the model. I shall discuss two experiments which have been done recently along this line.

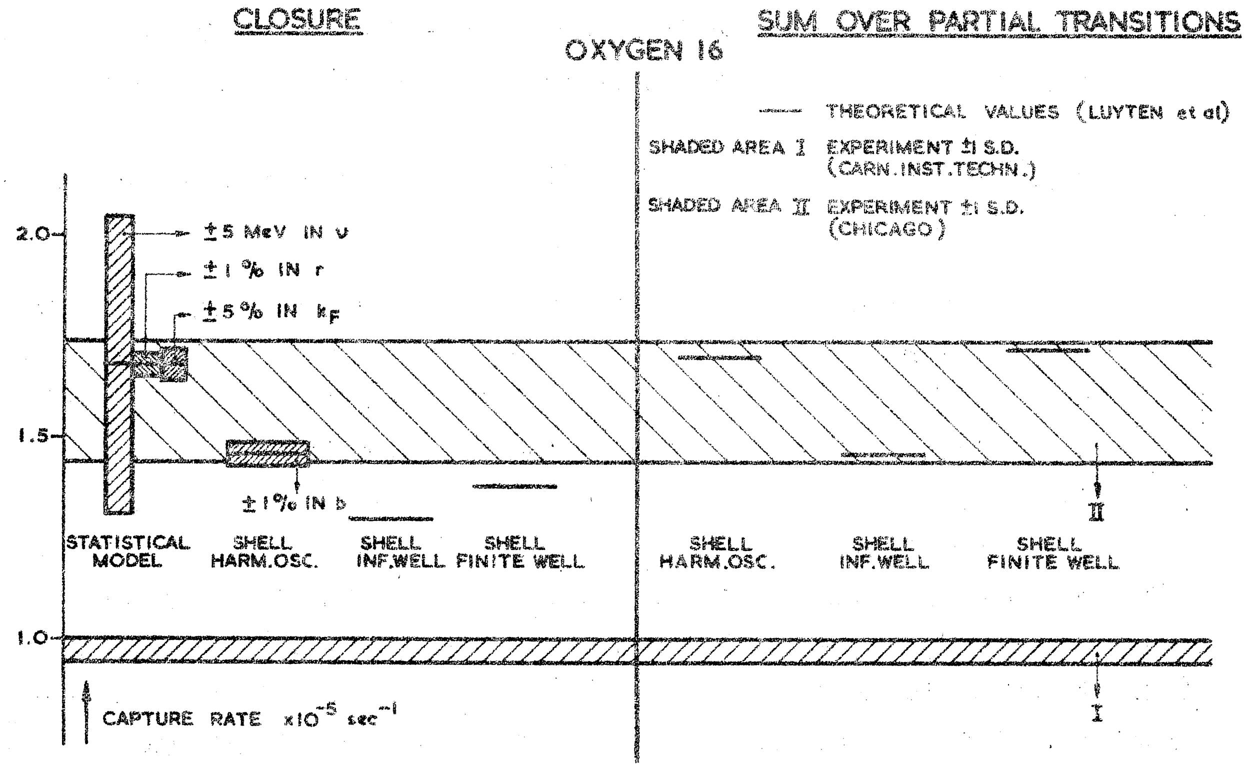

Fig. 5

The first is an experiment done by E. Maier [23] on capture in ^{12}C. As is well known the aim is to make a direct comparison of the coupling constant g_A for μ capture and β decay. This is done by measuring

$$\mu^- + {}^{12}C \rightarrow {}^{12}B \ \text{(gr. state)} + \nu$$
$${}^{12}B \ \text{(gr. state)} \rightarrow {}^{12}C + e + \nu$$

and taking the ratio. The difficulty is to know which fraction of the decays from the ground state got there directly and which fraction arrived via excited states. To find out, Maier has measured first the number of bound ^{12}B nuclei formed (by extrapolating the ^{12}B decay electron curve back to zero time) per decaying muon (by extrapolating the μ decay curve back to zero time). All three signals used : μ stop, μ decay electron and boron decay electron, required a pulse in a block of scintillator which served as the carbon target. One finds from these data that in 20% of the captures bound ^{12}B is formed ; in

80% of the captures one has $^{11}B+n$ etc. In the second part of the experiment a NaI crystal is added; a coincidence is required between a ^{12}B recoil (375 keV) pulse in the target and a de-excitation gamma ray in the NaI crystal. These recoil γ coincidences show a time distribution consistent with the muon disappearance rate in carbon. For each γ recoil coincidence the associated pulse height of either the γ or the recoil is recorded, provided the ^{12}B also decays back to ^{12}C. In the γ spectrum a peak is observed at 0.95 MeV corresponding to the first excited state of ^{12}B. It is concluded that of all the bound states formed, about 90% goes to the ground state, about $4.3\pm2\%$ to the first excited state, $6\pm4\%$ to the next (1.67 MeV) level, and $\approx 1.5\pm2\%$ to the 2.67 MeV level (see Fig. 6). The two last figures are upper limits. This leads to a capture rate into the ground state $\lambda_{ground}=(6.75^{+0.30}_{-0.75})\times$ $\times10^3 sec^{-1}$. This number is consistent with a V-A theory including CVC and $g_P/g_A=+8$. It also agrees with a V-A theory without CVC and without g_P as well as with V-A with g_P and without CVC; in short, it agrees with almost every assumption and this only goes to show that it is difficult to get out of a "total rate" experiment more than a consistency check. The interest of this experiment is that it shows perhaps a beginning of nuclear spectroscopy by means of muons. Once ratios such as the ones detected here are accurately measured, they could give information on the mixing of nuclear wave functions.

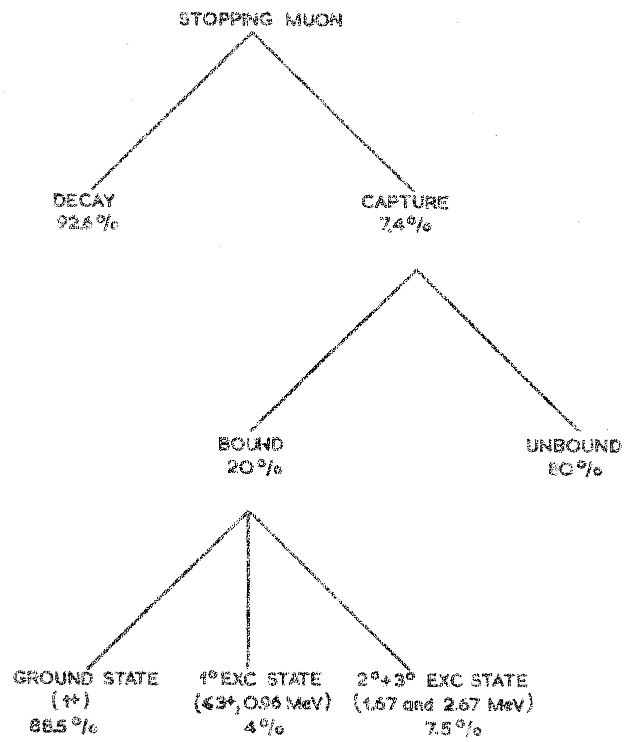

Fig. 6

The second experiment is the one recently reported by Cohen, Devons, Kamaris and Knipper [24] on partial capture of μ^- by ^{16}O. The ^{16}N nucleus formed is rightly famous as a test ground for the shell model : Elliott and Flowers [25] predicted the presence of 4 low-lying negative parity states of angular momentum 0,1,2,3 followed by a gap of several MeV. These levels have indeed been found by experiment. The order of the levels depends on the details of the interaction potential. Experimentally they are in the order (counting upwards) $2^-, 0^-, 3^-, 1^-$ in agreement with reasonable values of the parameters in the potential.

It has been pointed out by Blokhintsev and Shapiro [26] that this unusual nucleus can be used as a very sensitive detector for the induced pseudoscalar coupling constant g_P. Loosely speaking one can say that the capture process is governed by 4 diagrammes (Fig. 7). Now, in fact, what Blokhintsev and Shapiro suggest is to use the selection rules in complex nuclei to cancel as much as possible the contribution of the first two graphs thus enhancing the relative contribution of the "peripheral capture" diagram g_P (the sensitivity to g_M remains small). The capture by the 0^+ ^{16}O ground state with excitation of the 0^- state in ^{16}N is such a case : the "allowed" contribution is suppressed by the required change in parity. Let us consider the sum of the $0^+ \to 0^-$ and the $0^+ \to 1^-$ transitions. Both transitions only "go" by means of a change of orbital angular momentum of 1 unit, which is taken up by the neutrino. The sum of the 0^+0^- and 0^+1^- rates is given by

$$\Lambda_{0^- + 1^-} = \frac{\nu^2}{2\pi} \, |\varphi_\mu|^2_{av} \, I^2 \left[G_V^2 + 2G_A^2 + (G_P - G_A)^2 \right] \qquad (20)$$

where I is the radial matrix element. By computing

$$M^2_{0^- + 1^-} \equiv \nu^2 |\varphi_\mu|^2_{av} \, I^2$$

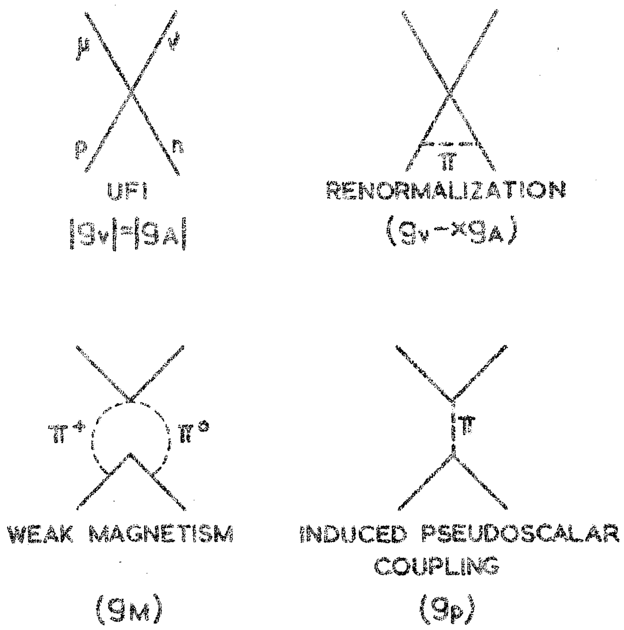

Fig. 7

and comparing this to the corresponding quantity M^2 of Table 3 for the total capture rate [20] we obtain

$$\frac{\Lambda_{0^-+1^-}}{\Lambda_{total}} = \frac{M^2_{0^-+1^-}}{M^2} \approx 2\% \quad . \qquad (21)$$

Now consider the 0^+-0^- transition alone. The vector part cannot contribute since the parity changes and thus $\Delta \ell$ must be $\neq 0$; since $\Delta S=0$ for the vector part we would have $\Delta J \neq 0$. Hence, the vector part contributes only to the 0^+-1^- transition. The spin flip part of the axial vector term (σ_+, σ_-) can also not contribute to the $0^+ \to 0^-$, because if we choose the z axis along $\hat{\nu}$ we have $\Delta m_\ell = 0$, while the spin flip operators σ_+ and σ_- give $\Delta m_s = \pm 1$. Thus $\Delta M \neq 0$, in contradiction with $\Delta J=0$. Only the term $(G_P - G_A)^2$ contributes to the $0^+ \to 0^-$ *). So we get for the ratio

$$\frac{(0^+0^-)}{(0^+1^-)} = \frac{(G_P - G_A)^2}{G_V^2 + 2G_A^2} \quad . \qquad (22)$$

*) The no-spin flip operator does not contribute to $0^+ \to (1^-, M = \pm 1)$, since $\Delta m_\ell = 0$ and $\Delta m_s = 0$ implies $\Delta M = 0$; nor does it contribute to $0^+ \to (1^-, M=0)$ as can be shown by explicit calculation. Both the no-spin flip axial part and the pseudoscalar term have the same matrix element.

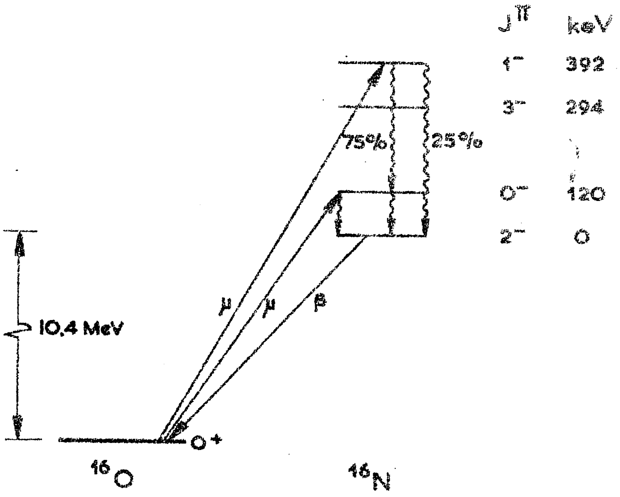

Fig. 8

All other factors cancel and we thus have a ratio which is quite sensitive to g_p and in addition is independent of factors such as the neutrino momentum and details of the wave functions. We shall see below that the interpretation of the measured ratio is unfortunately less unambiguous than presented here. Experimentally, the capture rates into 0^- and 1^- are identified by the γ decay of these states (see Fig. 8). The experiment of Cohen et al. consists of stopping muons in oxygen and measuring the β decay (7sec) of the ^{16}N ground state. This measures the combined rate of capture into all bound states (the neutron boil-off level is at 2.5 MeV; there are no levels between the 1^- state and 2.5 MeV). Subsequently the number of stops followed by 120 keV and the stops followed by 272 keV γ rays are detected. From these data the following result is deduced

$$\frac{(0^+0^-)}{(0^+1^-)} \approx 0.38 \ . \qquad (23)$$

Now in interpreting this result it must be noted that due to the absence of the allowed terms the nucleon velocity terms are likely to make an important contribution to the rate. In addition there are the admixtures, in particular capture from the $p_{3/2}$ shell to the $2s_{1/2}$, $1d_{5/2}$ or $1d_{3/2}$ neutron shells. If we use the admixture amplitudes as given by Elliott and Flowers for these configurations and including the nucleon velocity terms, we obtain instead of (22) *) :

$$\frac{(0^+0^-)}{(0^+1^-)} = \frac{0.84(G_P-G_A)^2-0.82g_A(G_P-G_A)+0.20g_A^2}{0.85G_V^2+1.40G_A^2+0.16g_VG_V+0.64g_VG_A+0.08g_V^2} . \qquad (24)$$

This leads to the following predictions for this ratio :

g_P/G_A	8	0	-8
uncorrected	0.12	0.32	0.59
corrected	0.50	0.91	1.44

The experiment would thus seem to agree with a value of g_P around $+12g_A$. This is a tentative conclusion, and both theory and experiment are clearly in need of further improvements.

The author is indebted to several physicists in this laboratory, in particular to Dr. T. Ericson for many interesting discussions.

*) It has been pointed out by Luyten et al. [20] that a term of the form $\hat{\nu} \cdot \int \vec{\sigma} \times \int \vec{p}$ must be added to the nucleon velocity terms as given in the original formulation by Fujii and Primakoff [27]. This changes the (0^+1^-) rate by -25% and has no effect on the (0^+0^-) rate. This term is also present in calculations by Duck [28] and by Delorme [29]. It has been included in Eq. (24). Eq. (24) was obtained in collaboration with T. Ericson and H.P.C. Rood.

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SESSION VIII

COMPLEX NUCLEI AS TOOLS IN ELEMENTARY PARTICLE PHYSICS

Speaker :

V. TELEGDI

SESSION IX

CONCLUDING REMARKS

Speaker :

R.E. PEIERLS

CONCLUDING REMARKS

R.E. Peierls
Department of Mathematical Physics, Birmingham.

Evidently I have taken on a very difficult assignment. I shall not be able to sum up all that we have learned during the week, or to collect all the ideas for new experiments and for their interpretation. One of the limitations I shall set myself is not to attempt to summarize Session VIII, which consisted mainly of Telegdi's paper. His topic was rather distinct from the rest, and I think we all felt after listening to this session, that we would have liked to spend more time on it. There was so much material in it and it was so concisely and clearly presented that any attempt to condense it further would be even more unsatisfactory than the rest of my summary.

In his opening of the conference, Weisskopf explained how the idea originated, and by now it need hardly be stressed that he was right in believing that such a discussion would be great fun, and that there was a lot to be learned from the cross-connections between nuclear structure and high-energy physics. I would not go with him quite as far as saying that the whole is just one branch of physics. After all, we have seen only too often that confusion arises both amongst physicists and administrators by saying nuclear physics when one means high-energy physics. In a sense, of course, the whole of physics is one field, but within it we have to have experts of different kinds. It is most important that they should on occasion talk to each other, and learn from each other, and this we have tried to do, and have enjoyed, all this week.

In sorting out my material, I intended originally to present the ideas not according to the techniques employed, but according to the information on nuclear structure which each method can provide. This, I thought, would be interesting, if only because it was presenting differently from the way it was done in the sessions. However, I soon found that I was unable to work it out that way, for reasons which, I think, are interesting.

One reason is that at the beginning of a new experiment one often does not know what exactly one is going to learn from it. New information usually takes the form of a disagreement between the results and our expectations. Our expectations were based on a number of assumptions and approximations, and the first result is only that one of these has to be modified. It often requires some quite sophisticated analysis before we know which one, and why, and only then it is clear what is the nature of the new information. If the experiment agrees with prediction, it confirms the accuracy of all the main ideas involved in the prediction, and we shall each, according to our taste, regard this as confirmation of the picture that we were least confident about.

Another reason (or perhaps another way of stating the same reason) is that most of the quantitative statements we have been discussing relate to specific models of nuclear structure. There are, of course, quantities which have a precise and objective meaning, such as binding energies, spins, transition rates, etc. These are best found by direct experiments, and form the raw material from which a discussion on nuclear structure begins. The questions which have figured in the sessions have mostly referred to such quantities as single-particle levels, momentum distributions, correlations, etc., and these acquire their meaning only through specific models, so that the experiment tests not only the particular quantities, but also the validity of the model. And very often new experiments suggest their own new questions and concepts.

I shall therefore have to follow the more pedestrian approach of discussing the material in the order in which it was presented. But first one more general point. If we tried to extract from this week's reports the new items of information about nuclear structure that we have acquired, the list might look rather disappointing; you might like to conclude that the initial optimism was not justified. But such a conclusion would be quite wrong. In the first place, much of the work has confirmed that current ideas about nuclei are valid to a good approximation. This is far from trivial, because some of these ideas had not been put to a precise test, or have, by the high-energy experiments been taken into new regions, and therefore been exposed to new kinds of test.

I should here remind you of what was probably the first important use of high-energy information for clarifying a problem in nuclear structure. When we found it difficult to understand the success of the nuclear shell model, in spite of the presence of strong short-range forces, the suggestion was made, I believe first by Teller and Johnston, that perhaps such forces did not exist in the nucleus, but that at high nucleon density the meson fields of all the nucleons acted together to give saturation, so that each nucleon was, in fact, moving in a reasonably uniform potential field. The strongest argument against such a view came from the general behaviour of high-energy reactions, in which one often sees processes in which a nucleon was hit while interacting closely with another. As we shall see, this type of reaction is not easy to analyse in detail, and it is still difficult to pick out one crucial experiment as proof of the existence of strong two-body forces in nuclear matter, but the impression of the whole evidence makes this rather convincing. We tend to forget this argument today, not only because we have since learned to understand the shell model in spite of strong two-body forces, but also because it simply confirmed what we had thought all along. Nevertheless it played a very important part in the development of our ideas.

Much of the value of the new work lies in confirming that our concepts are right, and that we may with confidence apply them to problems arising as ingredients in the work of the high-energy physicist.

In reviewing the major topics of this week's discussions I shall try to indicate for each method the kind of information that has come out or may be expected from it, and comment on the problems of analysis, i.e., of the steps necessary to get from the experiment to some useful nuclear structure information.

The first topic discussed related to strange particles. The first type of experiment was the study of hypernuclei. This, of course, provides information about hypernuclei, which in themselves form a very important and interesting aspect of nuclear structure. It provides information about hyperon-nucleon forces, which again are interesting in their own right, and also help to test our prejudices about nucleon-nucleon forces. Since, in a hypernucleus, the hyperon is inserted into an ordinary nucleus, the study also tests some ideas about ordinary nuclei, though so far nothing very sensitive seems yet to have emerged in that line.

A remark by de-Shalit seemed to open up a theoreticians paradise, in suggesting that it was legitimate to treat the interaction of the hyperon with the rest of the nucleus in low-order perturbation theory. Some more discussion would be required, however, before we are sure that this applies to a hypernucleus more accurately than to the problem of adding one more nucleon to an existing nucleus.

One point made about hypernuclei was that the present data are consistent with the assumption of there being only two-body forces and do not require us to postulate three-body forces. This is important, but we should be clear that we are here looking only at the possibility of attractive three-body forces. These might make a substantial difference to the comparison between the binding energies of light and heavy hyperfragments. Repulsive many-body forces would simply prevent too many particles getting too close together. Since this is already made difficult by the repulsive cores in the two-body forces, the effect is not very specific and to settle this question is just as hard as it has been to prove or disprove the existence of repulsive many-body forces between nucleons.

The field of hypernuclei has been widened and made more exciting by the discovery, reported to this conference, of the existence of double hypernuclei.

The next experiment discussed was the capture of K^- mesons from atomic orbits as a means of studying the nuclear surface. Wilkinson made the point that most of the K^- will be absorbed from nearly circular orbits in the outer surface region, and this seems a strong argument although still somewhat controversial. This experiment may therefore give information about the state of nuclear matter in the surface, in particular about the amount of clustering in the surface region. The analysis of this problem is far from trivial, because it is not easy to be sure what we ought to see in such processes if there was no clustering, or rather only the amount of clustering expected from the known two-body correlations, and even the question of the meaning of clustering came in for some discussion. This problem, as well as other reactions leading to the ejection of α particles and other clusters, still offers scope for more theoretical work, as well as for further experiments.

Processes in which K^- capture removes one nucleon from the nucleus can be used to study parentage, i.e., the proportion in which the ground state of the nucleus of A nucleons is made up from the various possible states of the nucleus $A-1$ plus one additional nucleon. This is similar to the study of knock-out and capture reactions. The analysis is involved if one wants quantitative results. Probably the best approach is to start from a model which predicts both the parentage coefficients and the dynamics of the process. If the predictions agree with experiment, this supports both parts of the model. In the event of disagreement it is not certain that one can still use the dynamical part of the model and calculate different parentage coefficients.

Wilkinson's report contains many other interesting suggestions of new reactions that might be looked at.

Session II discussed electron scattering. Elastic scattering by nuclei is by now a fairly old subject. It gives information about charge and current distributions. The analysis is direct, some minor corrections due to radiative effects and similar complications are well understood. In the case of the charge distribution all that need be said is that the accuracy is now such that it shows up differences between different shapes of the charge distribution, in other words, one is getting beyond the limits of the two-parameter representation of the nuclear charge distribution.

Much less is yet known about the current distribution, or the distribution of the magnetic moment in the nucleus. This is a very important problem, because here theory is in a rather helpless state. We have known for a long time that the charge and current in a nucleus are not just superpositions of those due to individual nucleons. This is evident from the existence of exchange forces. If, in the course of their interaction, a proton and a neutron exchange their charges, conservation of charge requires that a current must flow somewhere in the intervening space. Unfortunately this argument proves only the existence of "exchange currents" but is not adequate to determine these in detail. There exist various plausible or phenomenological hypotheses for exchange currents, but they lack in theoretical foundations. Ultimately the theoretical answer must come from a field theory of the nuclear forces, but this is in the future. It is perhaps conceivable that even now some dispersion relations could be used to link this problem with other data, such as nucleon form factors and scattering amplitudes. A bright idea on this would be very welcome. Meanwhile direct experimental evidence would be very interesting. Probably the elastic magnetic scattering is not very sensitive to such effects, and one would have to look at inelastic scattering.

Apart from this, inelastic scattering provides information about the matrix elements of the various multipole moments, and their distribution through the nucleus. The data are usually interpreted in terms of Born approximation, and there seems to be some doubts about its reliability. This is a rather trivial matter, because it is fairly easy to test the approximation by a more exact calculation. The experimental accuracy is limited by bremsstrahlung effects so for the present one cannot expect data of high accuracy.

These remarks relate to the inelastic scattering with excitation of a discrete nuclear level. In the continuous spectrum both measurements at given energy and sum-rule studies are interesting. The latter can be connected directly to the two-proton correlation, which is of fundamental importance. The experimental difficulties are the same as in the discrete case, and we are told that most of the effect is accounted for by the correlations due to the Pauli principle. One would therefore need an experiment of really high accuracy to show up the interesting dynamic correlations over and above the Pauli principle. The present data already show some indication of these.

Session III dealt with the use of pions. π capture is likely to give us some information about the momentum distribution of nucleons in the nucleus, particularly at the high-momentum end. This is one of the oldest arguments in this subject, and was used in the reasoning for the existence of two-body forces inside the nucleus, which I mentioned earlier. There are, however, still serious difficulties with a precise analysis, as was stressed in the discussion by Brown. The question is whether it is easier for the pion to find two nucleons which happen to be interacting closely and therefore have large and opposite momenta, or whether the same result can more easily be achieved by a two-stage or multi-stage process during which a virtual pion can transmit momentum between separated nucleons. This uncertainty is unfortunate, since the data would otherwise give direct and valuable information about a region which is hard to get at.

Elastic scattering of pions will give us information about the effective nucleus-pion optical potential. Why is this important ? Firstly because we have applied optical models successfully in other parts of nuclear physics. We think we understand them and we know the connection between the parameters of the optical potential at high energy and the two-body scattering amplitude, and what corrections to apply to this simple relation. It would increase our confidence in these arguments if we could apply them to the pion-nucleus problem and still found them right. Secondly a knowledge of the optical potential is necessary as a tool for describing accurately various other reactions of pions on nuclei which are important both for high-energy problems and for nuclear physics.

The connection between the observed scattering and the optical potential is, of course, straightforward, but the interpretation of the latter in terms of two-body scattering is still a matter of controversy. The problems are of great interest, and I am tempted to quote (though not quite in the spirit in which it was intended) Ericson's concluding remark : "the pion study of nuclei is here to stay". It is, but the theoretical problems associated with it are also here to stay for a while.

The next process to mention is pion charge exchange. This can enrich nuclear physics by producing and studying new nuclei, which are not accessible otherwise. Since this is a question of identifying the reaction product, and perhaps its energy levels, no difficult question of interpretation arises. This type of work opens up an exciting new field, particularly if the suggested double charge exchange mode (π^-, π^+) can be found.

In the same field, though by a very different experiment, there is pion production, which in the beautiful Frascati experiment of Argan and Piazzoli has enriched nuclear physics by the production of ^4H. We hope to learn much more about this object and others which may follow.

Sessions IV and V were concerned with nucleon studies. Elastic nucleon scattering is again related to the optical potential. This does not necessarily involve high-energy nucleons, but it turns out that the interpretation of the optical potential in terms of the nucleon-nucleon scattering amplitude, with various corrections, is much more direct at high energy. The purpose is again in part to test our understanding of what goes on in many-nucleon nucleus collisions and in part to determine what comes into them (as data needed for distorted wave calculations). For many of these a really deep understanding is required; for example, we must know whether in a highly excited state of the nucleus we should use a different optical potential from the usual and if so what it should be. This cannot be determined by direct experiment, therefore we must make sure that our theoretical understanding of the basis of the optical model is adequate.

In inelastic nucleon scattering we are essentially testing the nuclear transition matrix element. Before we can derive it from the data we need information about the distorted wave for the fast nucleon, and about the effective forces. This is therefore again one of the cases in which the experiment depends on a number of factors, and if we think we know some of them, the rest can be derived from the data. So one cannot be dogmatic about what information precisely follows from the experiment.

Inelastic scattering can be made more informative by observing also the polarization of the scattered nucleons. In the beautiful experiments of Mrs Marty this is used in the first place for labelling nuclear states where their quantum numbers are not known, since strong polarization effects appear only for some states and not others. (This is reminiscent of the old work on stripping, which is an early example of using high energy for nuclear structure studies, the high energy in that case being something like 8 MeV, higher than necessary to excite the states in question.)

Another refinement, as we heard from Clegg, is to use $(p,p\gamma)$ reactions, observing both the secondary proton and the γ ray in coincidence. Here the information involves several angles and energies, and the discussion necessarily becomes complicated, but the reward is impressively clear and detailed information, both about the interaction and about the nuclear wave functions. So far this method has been used mainly for cases in which the wave functions are reasonable well known already, and this gives a useful calibration of the method; no doubt this approach will be extended.

Next we come to knock-out reactions like $(p,2p)$ first studied by Tyrén and others. Here the energy balance tells us the energy used to make a "hole" in one of the occupied levels of the nucleus. At small energy transfer the new nucleus may be left in a discrete state, and an exact measurement of the energy must agree with one of the known states of the final nucleus. In that case the intensities with which the different states appear tell us something about their structure. For example, a state which in the shell model is described as being just the target nucleus with one proton removed from some level, is likely to be very strong. At higher energy transfer, where the spectrum is continuous, or where the excitation energies are no longer resolved, the same situation is reflected in the appearance of peaks in the energy distribution. The fact that these peaks exist is support for the shell model. Their position, and the variation of the intensity with the angles, can give information about the normally occupied shell-model states.

Similarly (p,d) reactions can give information about the filled neutron levels. Other knock-out reactions, such as (p,pd), (p,pt), (p,pα) can give information about clustering, though here again there is some controversy about interpretation.

We now turn to the subject of Session VI, fragmentation. So far this work looks like giving us information about the fragmentation process itself, which is an interesting, but rather complex phenomenon. It will clearly be some time before the mechanism is sufficiently well understood to give more general information about nuclei.

So far all that has been possible is to postulate a specific mechanism and see how far it is capable of accounting for the whole range of observed phenomena. It may yet be that no simple mechanism can do this, but that different mechanisms have to be invoked for the different ranges of the observed results. I am tempted to compare the problem with trying to study the result of throwing a stone through a glass window, when probably neither the evaporation model, nor the cascade model would work very well, but perhaps this comparison is unfair.

We might here remember that historically the first example of using high-energy particles to study a low-energy structure (or in other words using a sledge-hammer to crack a nut) was Rutherford's work on the scattering of α particles by atoms. We know that Rutherford was extremely surprised by the result, and we might therefore ask what he expected to see. I imagine he was looking for something not unlike the pictures of fragmentation which we have seen, and the fact that he still thought the experiment worth doing should give us some encouragement in continuing to look for an interpretation of fragmentation.

One of the simple and puzzling features of the process seems to be that in events in which there are two fragments emitted these come out predominantly in opposite directions, and if there are three they tend to form angles of 120° with each other. (I was not clear whether the three tend to be coplanar.) This striking effect is not predicted by any of the mechanisms discussed so far, and may be an important clue.

Work of this type may also provide statistical data about evaporation, from which one may deduce the specific heat of nuclear matter, which is a fundamental and interesting quantity.

Finally we come to Session VII with the fascinating results about the use of μ mesons. μ mesonic X rays can give us information about the nuclear charge distribution. The connection is very direct, and the first point to note is that the results are consistent with those from electron scattering, thus confirming our view that the only interaction involving muons is electromagnetic. The μ mesonic atoms involve only a fairly broad average of the nuclear charge distribution, and thus give less detailed information than electron scattering data, but it is interesting to note that the information which does come out is now very close to the accuracy with which electron scattering data are known.

It looks as if before long it may be possible to measure the fine structure of the levels of μ mesonic atoms. This would give information about nuclear magnetic dipole and electric quadrupole moments, and the latter would be particularly interesting because it would be independent of calculations of atomic or molecular fields, which are involved in all other methods of measuring quadrupole moments.

It has even been suggested by Telegdi that one day one may be able to determine octopole moments that way, but this seems definitely in the future.

As a future experiment inelastic scattering of muons looks very attractive, because of the absence of bremsstrahlung, which bedevils the electron experiments.

Muon capture still belongs to the class of experiments designed to test the fundamental interactions, and therefore I shall not comment here on the beautiful experiment of Devons, since it really belongs to the subject of Session VIII, on which I decided not to report. Perhaps one day the fundamental interactions will be so well known that we may use capture experiments to study nuclear properties, just as we now use electromagnetic interactions.

I have, in this summary, been able to mention only rather general points, and much interesting material, which related to points of detail, or to individual cases, had to be left out. Our general impression at the end of the conference is that we are learning and understanding many interesting and amusing things. We must not be surprised if we cannot summarize in a simple formula what we have learned and what exactly we have understood. If, before coming here, any of us needed convincing that it was a good idea to have a conference of this kind, their doubts will by now have been removed.

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