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A NEW CONCEPT FOR THE CONTROL OF A SLOW- EXTRACTED BEAM IN A LINE WITH ROTATIONAL OPTICS, PART II*

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Abstract

The current trend in hadrontherapy is towards high-precision, conformal scanning of tumours with a 'pencil' beam of light ions, or protons, delivered by a synchrotron using slow-extraction. The particular shape of the slow-extracted beam segment in phase space and the need to vary the beam size in a lattice with rotating optical elements create a special problem for the design of the extraction transfer line and gantry. The design concept presented in this report is based on telescope modules with integer-π phase advances in both transverse planes. The beam size in the plane of the extraction is controlled by altering the phase advance and hence the rotation of the extracted beam segment in phase space. The vertical beam size is controlled by stepping the vertical betatron amplitude function over a range of values and passing the changed beam size from 'hand-to-hand' through the telescope modules to the various treatment rooms. In the example given, a combined phase-shifter and 'stepper', at a point close to the synchrotron, controls both of these functions for all treatment rooms in the complex. The modular nature of the design makes it easy to extend the complex.

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1 INTRODUCTION

The current trend in hadrontheraphy is towards high-precision, conformal scanning of tumours with a variable-sized 'pencil' beam of light ions or protons. Since the production of light-ion beams and the energy flexibility needed for the active scanning are better provided by a synchrotron than a cyclotron, it has been necessary to study the implications of using slow extraction and to understand better the characteristics of the extracted beam [1,2]. The principal consequences of a third-integer, resonant slow extraction are:

- A strong asymmetry between the transverse beam emittances.
- The extracted segment of the separatrix appears as a narrow rectangle, or 'bar', of charge in the phase space of extraction (horizontal) as shown in Figure 1.
- The beam distribution in the orthogonal plane (vertical) is similar to that in the synchrotron.

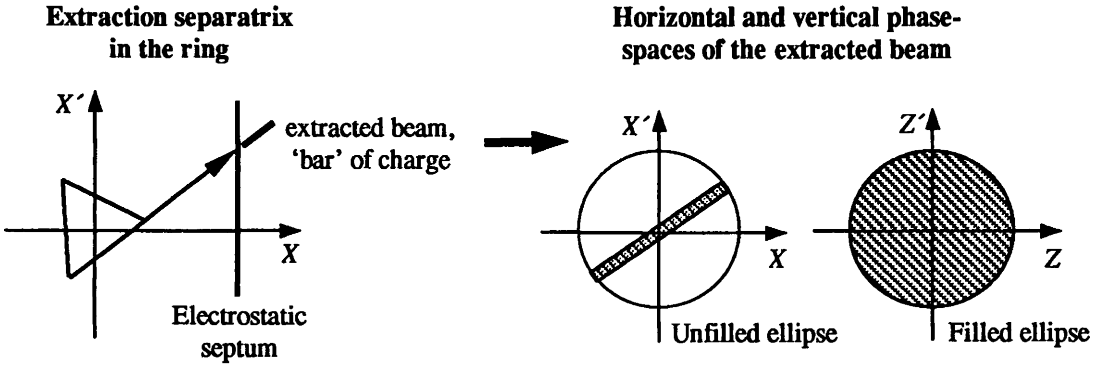


Figure 1: Normalised phase-space plots of the extraction and the slow-extracted beam at the electrostatic septum.

Due to higher-order effects the separatrices and the 'bar' of charge are slightly curved, but for all practical purposes the phase-space 'foot print' of the extracted beam segment is a narrow rectangle with an almost uniform particle density in the plane of extraction. The length of the rectangle for on-resonance particles is set by the extraction configuration and is constant whatever the energy level of the extraction. The lengths of the segments of particles that are slightly off-resonance (off-momentum) will differ somewhat from the on-resonance case. The thickness of the rectangle is small and changes according to the transverse coupling in the synchrotron that arises mainly from the resonance driving sextupole. This effect varies with adiabatic damping according to the energy level of the extraction and with the closed-orbit distortions at the sextupoles. The area (emittance) of the 'bar' of charge is typically of the order of one percent of the emittance circulating in the ring. However, strong coupling can increase this and dispersion and ripple effects can give an apparent emittance increase.

The usual Twiss representation of a beam is not suitable for the slow-extracted beam segment for the following reasons:

- Fitting an ellipse to such a narrow rectangle yields inconveniently large betatron amplitude functions.
- Particles with different momentum deviations have different spiral steps. This manifests itself as a strong dependence of the betatron amplitude function on momentum.
- The 'thickness' of the rectangle depends on the extraction energy (via adiabatic damping) and thus the value of the betatron amplitude function depends on the extraction energy.
- The 'thickness' of the rectangle depends also on closed-orbit distortions in the sextupoles and is thus subject to variations with time that cannot be conveniently included in the Twiss formalism.

These peculiarities make the use of standard optics programs difficult and yield exotic and complicated optical descriptions of the extraction line. It is simpler, and more meaningful, to consider the rectangle as a 'bar' across an unfilled phase-space ellipse as schematically shown in Figure 1. In this case, the ends of the 'bar' stay on the unfilled ellipse and rotate with the betatron phase advance. Making the unfilled ellipse much wider than the 'bar' adds two essential features:

- The optics becomes insensitive to changes in the 'thickness' of the extracted segment.
- The rotation of the 'bar' in the unfilled ellipse provides an independent way of controlling the horizontal beam size by adding a phase shifter at the entry to the line (see Section 2.1).

Finally, the asymmetry in the beam emittances complicates the matching to a rotating gantry. The matching has to be done such that the beam properties are transparent to the gantry rotation. For one class of gantries, the dispersion is zero at the entry to the gantry and only the lattice functions need to be matched and, in the second class, the dispersion function must also be rotated. For the former, there is the tight optical constraint of closing the dispersion bump within the gantry in order to obtain zero dispersion at the patient, whereas, for the latter, the dispersion bump is generated in the fixed line and closed in the gantry which facilitates the gantry design.

The method proposed for the matching is to use a 'rotator', which maps the betatron oscillations and the dispersion functions one-to-one from the fixed beam line to the rotating gantry [3,4].

2 BEAM SIZE CONTROL

In a dedicated hospital centre, there will be two or more gantries and one or more fixed beam lines. The medical specifications require spot sizes of 4-10 mm full width at half height with either protons between 60 and 250 MeV or carbon ions between 120 and 400 MeV/u with zero dispersion at the patient. The conventional approach is to control the beam sizes locally at the end of each beam line. Since only one beam line can be used at one time, it is possible to apply a different solution in which the beam sizes are all controlled from a common section of the line just after the exit of the synchrotron. In this solution, the strong asymmetry between the phasespaces of the slow-extracted beam is exploited to provide fully orthogonal controls of the horizontal and vertical beam sizes.

2.1 Horizontal beam size

The horizontal beam size at the patient will be given by the projection of the 'bar' onto the x-axis. By inserting a phase-shifter in the line, the 'bar' can be rotated, thus changing this projection without altering any other optical parameter of the line. In this way, the beam size can be varied at the patient from the minimum value (with the 'bar' upright) to the maximum value (with the 'bar' horizontal). This is shown schematically in Figure 2. For the efficient use of the phase shifter, the horizontal phase advances to the different treatment rooms should only differ by an integer number of π.

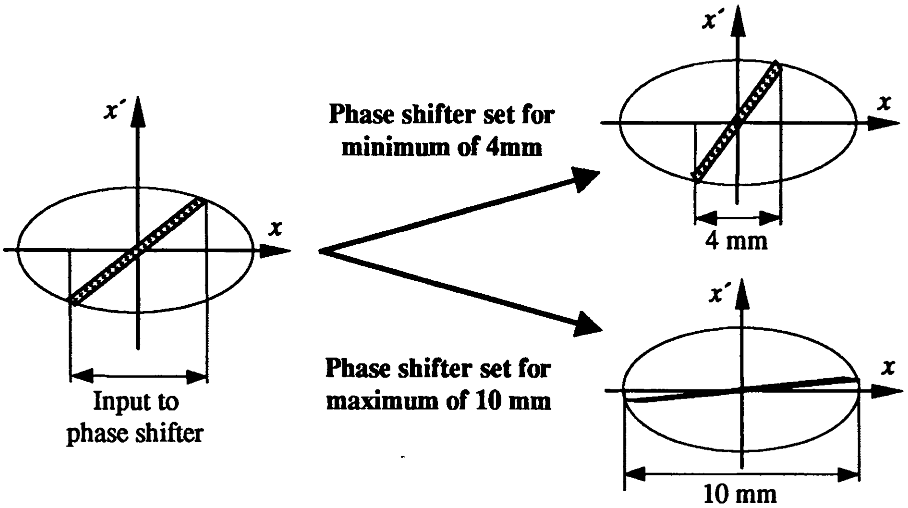


Figure 2: Horizontal beam size control by altering the horizontal phase advance with the phase shifter.

The maximum beam size at the patient (length of the 'bar') determines the product of the betatron amplitude function at the end of the line and the emittance of the unfilled ellipse. A criterion for deciding how much bigger the unfilled ellipse should be than the extracted segment, arises naturally from the minimum beam size that is needed at the patient. The width of the 'bar' must be no bigger than this dimension (and preferably smaller) when the 'bar' is upright in the ellipse. This sets a minimum height for the ellipse, since the phase-space area of the 'bar' is constant. Once this has been decided, the betatron amplitude function at the exit of the synchrotron is automatically fixed.

2.2 Vertical beam size

In the vertical plane, the phase-space ellipse of the beam is filled, the distribution is near-gaussian and a conventional Twiss description of the line is valid. Thus, the beam size can be controlled by adjusting the vertical betatron amplitude function at the patient. This is necessary over a wide range, since the vertical plane is additionally subject to adiabatic damping.

In the present design, a module, called a 'stepper' is introduced in the common section of the line. The 'stepper' changes the vertical betatron amplitude function according to the above requirements while leaving the horizontal betatron amplitude function and the horizontal phase advance unchanged. In order to transmit the action of the 'stepper' to each of the treatment rooms, the downstream lines consist of modules with integer-π phase advances and fixed magnification in the vertical plane (telescope modules).

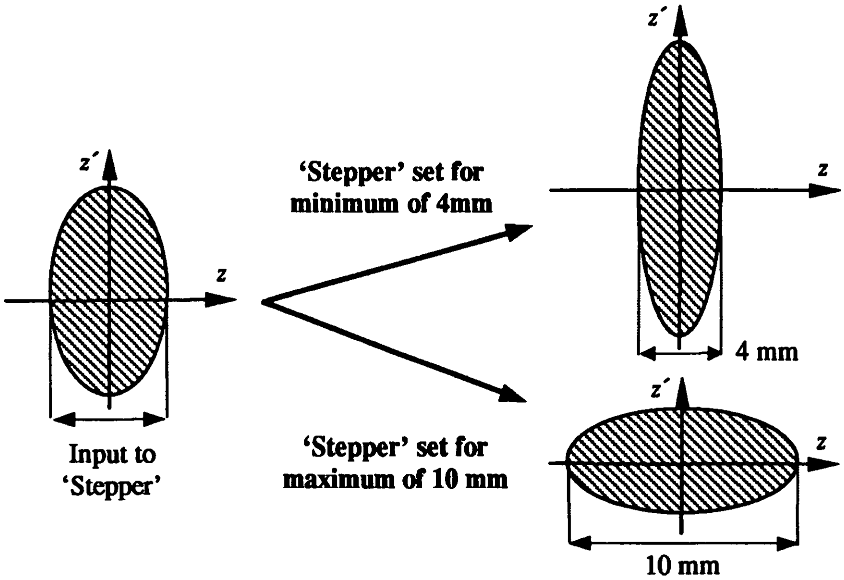


Figure 3: Vertical beam size control by adjusting the vertical betatron amplitude function with the 'stepper'.

3 BASIC DESIGN CONCEPTS

The principal steps in the design of the transfer lines can be summarised as:

- Exploiting the 'bar' of charge to create an independent control of the horizontal beam size by rotating the bar in an unfilled phase-space ellipse using a phase shifter at the common part of the lines.
- Controlling the vertical beam size by a 'stepper' that provides a variable magnification of the vertical betatron amplitude function while leaving the horizontal betatron amplitude function and the horizontal phase advance unchanged. The variable vertical betatron amplitude is 'handed' through the telescope modules all the way to the different treatment rooms.
- Placing the phase shifter and 'stepper' at the exit to the accelerator so that they can act for all treatment rooms in the complex.
- Using telescope modules (extension modules, switch modules and rotators) with integer-π phase advances in both planes that have one-to-one, or one-to-minus one, or fixed magnification properties.
- Matching the unequal emittances and non-zero dispersion functions to a rotating gantry by the use of a rotator.

The modular character of the extraction lines is shown schematically in Figure 4. The lay-out will be preceded by a matching module that makes the liaison from the synchrotron to the main extraction line. This module, which must be custom designed for a specific centre, is assumed to deliver zero dispersion and standard lattice functions to the modular line ($\beta_x = \beta_z = 3$ m and $\alpha_x = \alpha_z = 0$ for the present example). In the horizontal plane, the standard values are 'handed' from module to module throughout the line to the patient. In the vertical plane, the standard values are modified by the 'stepper' and then 'handed' through the modules to the patient. In this nomenclature, the planes between the modules are known as 'hand-over' planes.

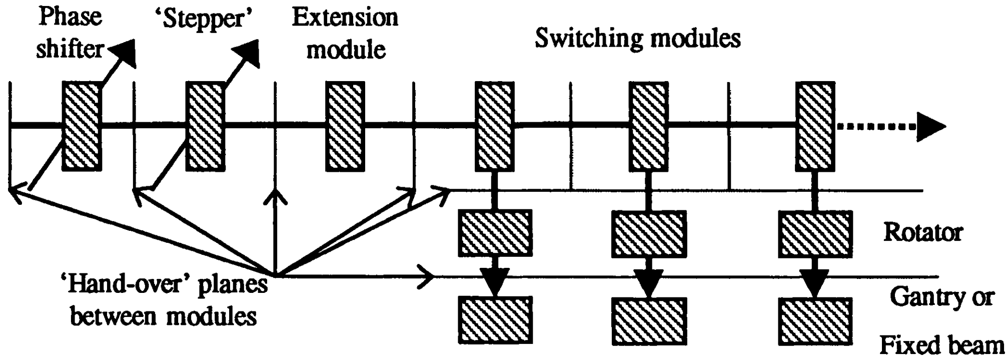


Figure 4: Schematic layout of extraction lines.

This general strategy has been adapted to a specific example for a cancer therapy facility*. In the following sections, the basic theory for the telescopes and the rotator is recalled and design examples of the various modules are given.

4 TELESCOPES

The basic matching scheme uses one-to-one and one-to-minus one modules with integer-π phase advances, but in certain cases there is a practical advantage in using the more general telescope module. Consider the transfer matrix,

$$\begin{pmatrix} C & S \\ C' & S' \end{pmatrix} = \begin{pmatrix} (\beta/\beta_0)^{1/2}(\cos\Delta\mu + \alpha_0 \sin\Delta\mu) & (\beta\beta_0)^{1/2}\sin\Delta\mu \\ -(\beta\beta_0)^{-1/2}[(\alpha-\alpha_0)\cos\Delta\mu + (1+\alpha\alpha_0)\sin\Delta\mu] & (\beta_0/\beta)^{1/2}(\cos\Delta\mu - \alpha\sin\Delta\mu) \end{pmatrix}, \quad (1)$$

where C and S are known as the principal trajectories and the other symbols have their usual meanings. If $\Delta\mu = n\pi$, then $\sin(\Delta\mu) = 0$ and $S = 0$. The transfer matrix is always independent of the choice of the initial Twiss parameters, therefore S must be zero for any initial α_0 and β_0. This shows the first result:

- A lattice with integer-π phase advance in one plane, has the same phase advance for any incoming lattice functions in that plane.

Now consider the transfer matrix for the Twiss parameters with $S = 0$:

$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix} = \begin{pmatrix} C^2 & -2CS & S^2 \\ CC' & CS'+SC' & -SS' \\ C'^2 & -2C'S' & S'^2 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{pmatrix} = \begin{pmatrix} C^2 & 0 & 0 \\ CC' & CS' & 0 \\ C'^2 & -2C'S' & S'^2 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{pmatrix}. \quad (2)$$

* The Proton-Ion Medical Machine Study (PIMMS) currently being hosted by CERN.

If the lattice is matched for $\alpha = \alpha_0$ in (2), then $C' = 0$ must be true since $CS' = 1$, being the determinant of the transfer matrix (1).

Thus it remains:

$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix} = \begin{pmatrix} C^2 & 0 & 0 \\ 0 & CS'=1 & 0 \\ 0 & 0 & S'^2 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{pmatrix} \tag{3}$$

and for any incoming set of lattice functions it follows that

$$\frac{\beta}{\beta_0} = C^2; \quad \alpha = \alpha_0 \quad \text{(constant}\,\beta\,\text{magnification)}\,. \tag{4}$$

Thus a telescope module provides a fixed magnification between the incoming and outgoing betatron amplitude functions while maintaining the alpha-functions equal. Normally, the extension and switching modules in Figure 4 would use $C^2 = 1$ and the action of the 'stepper' (see Section 6.2) would be transmitted directly to the patient. For practical reasons, it may be necessary to reduce beam sizes in certain regions in which case a demagnification followed by a magnification could be used. A single magnifying module can be used to move the range of the 'stepper'.

5 ROTATOR

Another key module in the extraction line is the rotator that transcribes the normal modes and dispersion functions from the fixed line into those of the rotating gantry. The ability of this module to also map the dispersion function to the gantry system opens the possibility of closing the gantry's dispersion bump in the fixed part of the line; a possibility that is not available for conventional gantries that use either the 'symmetric' or the 'round' beam method for matching [4]. The rotator is a quadrupole lattice with 2π and π phase advances in the two planes. The elements are mounted so that they can be rotated in proportion to the gantry angle. In the rotator lattice, the transfer matrix takes the simple form,

$$\begin{pmatrix} x \\ x' \\ z \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \\ z_0 \\ z'_0 \end{pmatrix}. \tag{5}$$

If the rotator is rotated by half of the gantry angle (2θ), then it maps the incoming beam into the co-ordinates of the gantry as shown by the matrix multiplication

$$\begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & \cos\theta & 0 & \sin\theta \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & -\sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & \cos\theta & 0 & \sin\theta \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & -\sin\theta & 0 & \cos\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{6}$$

where on the left-hand side the three matrices represent (right to left) the rotation by an angle θ before entering the rotator (to refer the beam to the proper axes of the rotator), the passage through the rotator and then a further rotation by θ before entering the gantry (to refer the beam to the proper axes of the gantry). The right-hand side shows the result, that the beam is mapped one-to-one or one-to-minus one from the fixed line to the gantry, making the rotation completely transparent to the optics. Note that since the rotator is dipole free, any dispersion function entering the lattice behaves, to first order, as a betatron oscillation. For this reason, the dispersion function is rotated in the same way and matched one-to-one from the fixed beam line to the rotating gantry.

The mathematics of the rotator is straightforward and exact, but the practical realisation needs a little more care. For example, three FODO cells with $2\pi/3$ and $\pi/3$ phase advances in the two planes would obey the mathematics, but at a rotation angle of $\theta = 90°$, corresponding to $2\theta = 180°$ rotation for the gantry, the F-quadrupoles become D-quadrupoles and vice versa and the beam sizes suffer wild fluctuations. The recipes for designing better-behaved lattices are given in [5].

6 MODULES

In the following sections, design examples for the various modules are presented. As mentioned above, the slow-extracted beam is described with an unfilled phase-space ellipse in the horizontal plane. This makes it possible to use the standard Twiss-formalism but gives an upper limit for the beam size. The effective beam size depends on the orientation of the 'bar' inside the ellipse. In order to see the true excursions of the beam, for a given setting of the phase shifter, it is necessary to track the four 'corners' of the 'bar' of charge. For designing the aperture, the unfilled ellipse is needed, since the beam will in principle move over the full ellipse area at different times, due to the action of the phase shifter ($0 < \Delta\mu < \pi/2$).

6.1 Phase Shifter

The horizontal beam size is controlled by varying the horizontal phase advance in the extraction line, so that the 'bar' of charge rotates in the unfilled ellipse. Figure 5 shows a dedicated insertion that varies the horizontal phase advance while keeping the horizontal and vertical Twiss functions at the exit constant, as well as the vertical phase advance (although this is not strictly necessary).

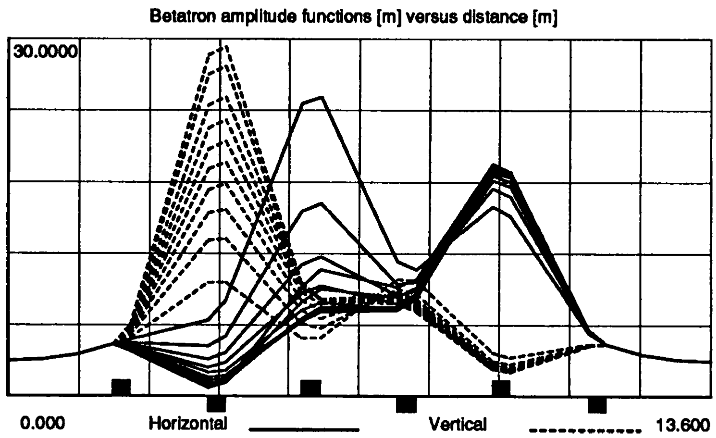


Figure 5: Optical functions in the phase shifter.
[$\Delta\mu_x$ = 3.9, 3.8 then in steps of 0.2 down to 2.2 rad,
$\Delta\mu_z$ = 2.9 radian, $\beta_x = \beta_z$ = 3 m and $\alpha_x = \alpha_z = 0$ at entry and exit.]

6.2 *'Stepper'*

For the control of the vertical beam size, a dedicated module, called the 'stepper', is used In the design example shown in Figure 6, β_z covers the range 1 to 17 m at the exit for an incoming value of 3 m with $\alpha_z = 0$ at entry and exit. In the horizontal plane, the module acts as a 1 to -1 structure with $\beta_x = 3$ m at entry and exit, $\alpha_x = 0$ and $\Delta\mu_x = \pi$. The vertical phase advance was kept constant for β_z varying from 1 to 11 m, but then, to facilitate the matching, it was allowed to vary which interrupts the continuity between the traces in Figure 6.

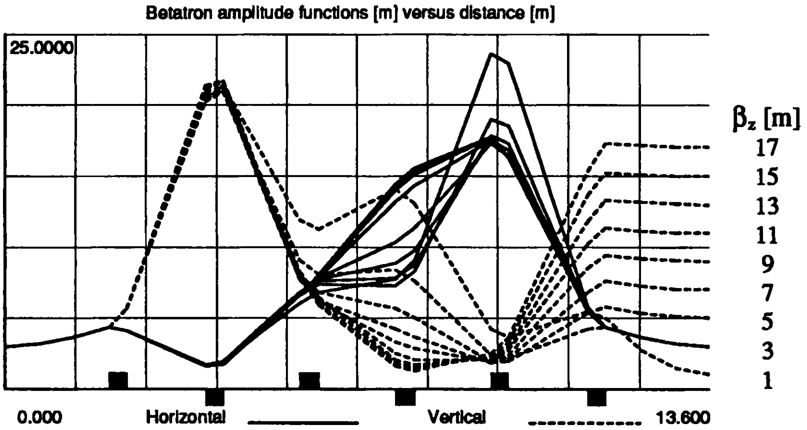


Figure 6: Optical functions in the 'stepper'.
[$\beta_z = 3$ m at the entry and steps from 1 to 17 m in steps of 2 m at the exit. $\Delta\mu_x = \pi$ radian, $\Delta\mu_z$ varies from 2.9 to 3.23 radian, $\beta_x = 3$ m and $\alpha_x = \alpha_z = 0$ at entry and exit.]

It should be noted that the important feature is the ratio in β_z provided by the 'stepper' and not the absolute values. These can, according to the vertical emittance in a specific machine design, be magnified in one of the downstream telescope modules to create whatever spot sizes are needed.

6.3 *Phase shifter - 'Stepper'*

The modules shown in Figures 5 and 6 are in fact identical and it is possible to combine their functions into a single unit. This is inconvenient inasmuch as a single module has to span over a two dimensional parameter space, which makes the operation more complicated and may reduce the global ranges, but it represents a considerable saving in space. Figure 7, shows the betatron amplitude functions in the combined phase shifter-'stepper' for four extreme cases in the parameter range.

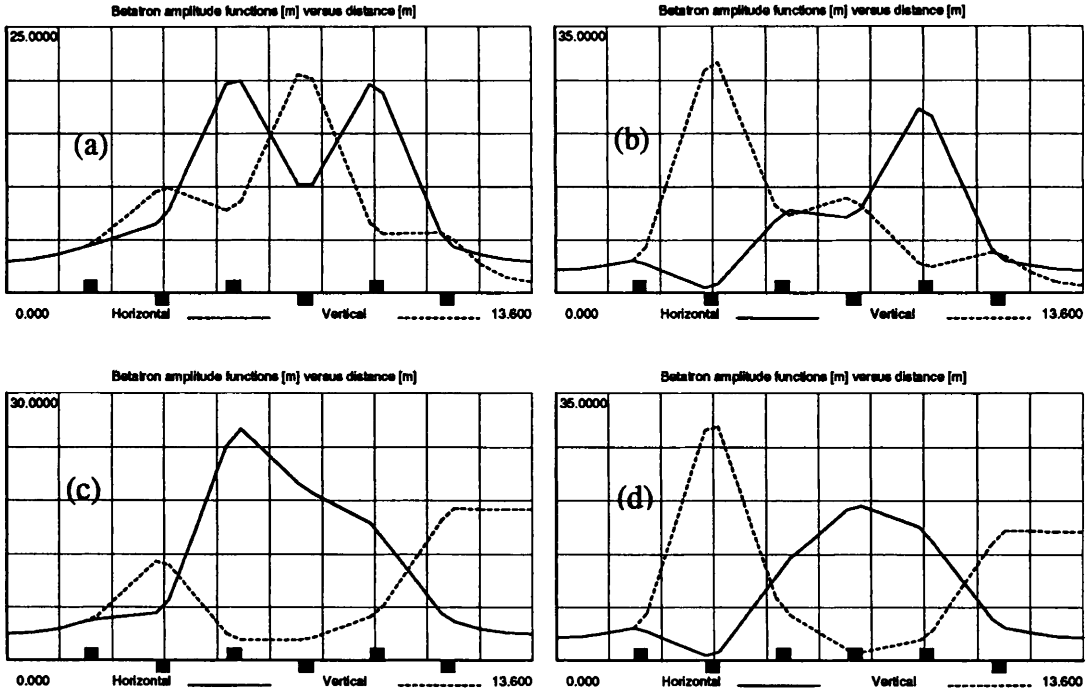
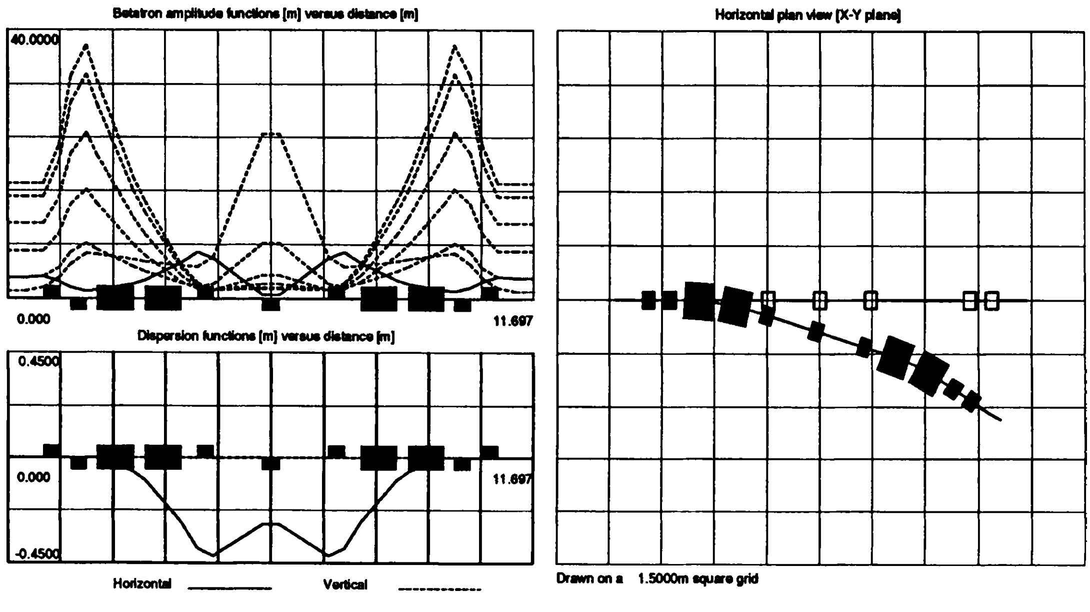


Figure 7: Extreme optical functions in the combined phase shifter-'stepper'.
[(a), (b) β_z = 1 m and $\Delta\mu_x$ =2.2 and 3.9 radian; (c), (d) β_z = 17 m and $\Delta\mu_x$ =2.2 and 3.9 radian]

6.4 Closed-dispersion bend and extension modules

When designing a centre, it is necessary to have a number of utility modules as indicated in Figure 4: a plane extension module, a switch with a closed dispersion bend and a switch with an open dispersion bend. A one-to-one module with a 2π phase advance in both planes is a very convenient structure in all of these cases. Figure 8(a) shows the lattice functions of a switching module with a closed dispersion bend and 8(b) shows the geometry. The lattice functions are shown with the range of 1 to 17 m in the vertical plane corresponding to the 'stepper' in Figure 6.



(a) (b)

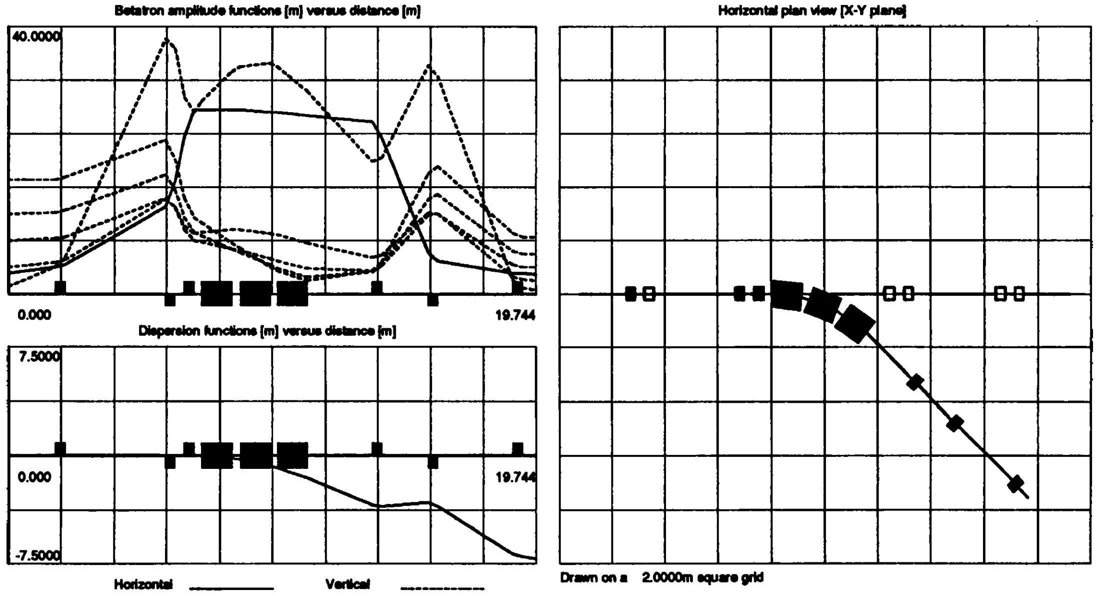
Figure 8: Switching section in the form of a one-to-one module.

This module would be used with a gantry that requires zero dispersion at its entry. In Figure 8(b), the outline of the straight extension module that continues the main extraction line is shown. The lattice functions in this module would be very similar to those in 8(a) since the underlying quadrupole lattice is identical.

6.5 Open-dispersion bend

Figure 9 shows a switching module which allows dispersion to pass to the gantry. The rotator module will match this dispersion to the gantry so that the bend can be used to close the dispersion bump from the gantry. This is particularly useful in exo-centric gantries. As an example and as anticipated in Section 6.2, a magnification factor of 0.5 in the vertical plane has been chosen for the switching module. Thus the module delivers a range of 0.5 to 8.5 m in vertical betatron amplitudes for the gantry. In the horizontal plane, the lattice is one-to one for $\beta_x = 3$ m $\alpha_x = 0$ at entrance and exit. The open-dispersion bend module has to create dispersion such that the dispersion bump closes inside the gantry. Therefore, in a practical case, the open dispersion bend and the gantry must be designed as an integral unit. Figure 9 (a) shows the lattice functions in the bend for the full range of the vertical betatron amplitude functions and Figure 9 (b) shows the geometry.

In Figure 9(b), the outline is shown of the extension module that would continue to the next gantry. The lattice functions in this module would be very similar to the those in 9(a) although an extra quadrupole has been added before the bend.



(a) (b)

Figure 9: Open-dispersion bend, vertical magnification 0.5.
[Initial optical functions, $\beta_z = 1, 4, 8, 12$ and 17 m]
[Exit optical functions, $\beta_z = 0.5, 2, 4, 6$ and 8.5 m]

6.6 Rotator

Figure 10 shows an example of a rotator module, when rotating the beam by 90° (physical angles: gantry 90°, rotator 45°). Note that the range of incoming beam sizes is transferred to the orthogonal plane when going through the rotator. For illustration the incoming dispersion is non-zero (as would be the case for an open-dispersion module) and also exchanges planes, like the beam sizes. Since the beam is coupled inside the module, the beam size is quoted rather than the Twiss-functions, however, at the exit, the distributions are uncoupled and the Twiss-formalism is again valid.

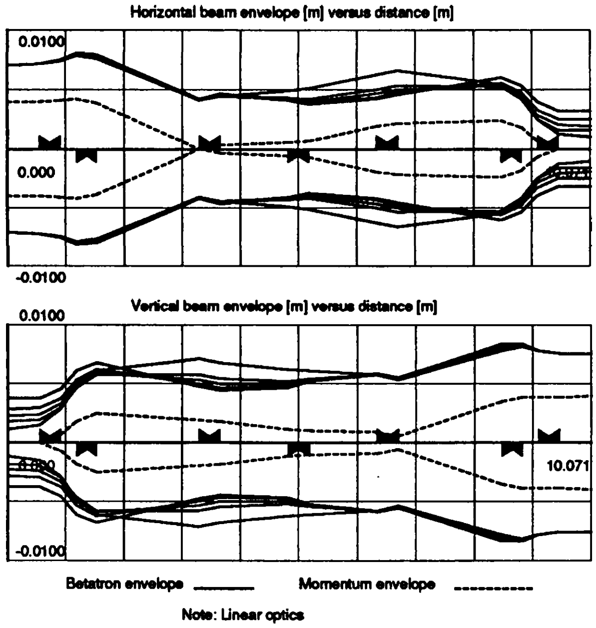


Figure 10: Beam sizes inside the rotator for beam rotation and gantry angle of 90°.

7 CONCLUSIONS

The extraction lines for a medical centre have been designed specifically to take into account the particular properties of a slow-extracted beam and to match these properties to the gantries with an exact optical system that is transparent to the gantry rotation. The design is modular in character and provides dedicated modules for each function. The control of the beam sizes has been integrated into the design in such a way that the dedicated modules can be positioned at the exit to the synchrotron so that all treatment rooms can be served by the same modules. The modular design makes it simple to extend the treatment complex. The similarity of the modules rationalises the number of different magnet types and reduces the need for spares. For bends a series of small dipoles is used rather than single large units, which also makes the provision and cost of spares less expensive. Matching to gantries with non-zero dispersion is included both for easing the gantry design and for exo-centric gantries where it is a requirement.

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