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SYNCHROTRONS FOR HADRON THERAPY
PART I*

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Abstract

The treatment of cancer with accelerator beams has a long history with linacs, cyclotrons and now synchrotrons being exploited for this purpose. Treatment techniques can be broadly divided into the use of spread-out beams and scanned 'pencil' beams. The Bragg-peak behaviour of hadrons makes them ideal candidates for the latter. The combination of precisely focused 'pencil' beams with controllable penetration (Bragg peak) and high, radio-biological efficiency (light ions) opens the way to treating the more awkward tumours that are radio-resistant, complex in shape and lodged against critical organs. To accelerate light ions (probably carbon) with pulse-to-pulse energy variation, a synchrotron is the natural choice. The beam scanning system is controlled via an on-line measurement of the particle flux entering the patient and, for this reason, the beam spill must be extended in time (seconds) by a slow-extraction scheme. The quality of the dose intensity profile ultimately depends on the uniformity of the beam spill. This is the greatest challenge for the synchrotron, since slow-extraction schemes are notoriously sensitive. This paper reviews the extraction techniques, describes methods for smoothing the beam spill and outlines the implications for the extraction line and beam delivery system.

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1 INTRODUCTION

The treatment of cancer with accelerator beams has a long history dating from the 1930s and the invention of the cyclotron. Linacs, cyclotrons and more recently synchrotrons are all being exploited for this purpose. Treatment techniques can be broadly divided into the use of spread-out beams and scanned 'pencil' beams. The Bragg-peak behaviour of hadrons makes them ideal candidates for the latter. The combination of precisely focused 'pencil' beams with controllable penetration (Bragg peak) and high, radio-biological efficiency (light ions) opens the way to treating the more awkward tumours that are radio-resistant, complex in shape and lodged against critical organs. When scanning, the tumour is treated in a series of slices at decreasing ranges. Once a slice has been 'painted' by the magnetically-steered 'pencil' beam, the energy is lowered to reduce the depth of the Bragg peak and 'painting' is repeated on the next slice. Light ions have an additional advantage compared to protons because they are less affected by multiple scattering in the patient's body and hence small spot sizes can be produced more accurately.

To accelerate light ions (probably carbon) with pulse-to-pulse energy variation a synchrotron is the natural choice and to meet the prescribed dose uniformity (ideally ±2.5%) the synchrotron is required to operate with a slow-extraction scheme [1]. The third-integer resonance is preferred for this task and extends the spill time up to a few seconds, which facilitates the on-line measurement of the particle flux entering the patient and the 'painting'. Ideally, machine operation would be 'ramp and hold' with a minimum cycle rate of about 0.5 Hz. This allows synchronisation with breathing if required. Some 60 spills of maximum 4×10^8 carbon ions each, or 60 spills of maximum 10^{10} protons each would comprise a single treatment (known as a 'fraction') with a minimum treatment time of 2 min. The beam spot should be variable between 4 and 10 mm full width at half height and have a positional stability and precision of a few tenths of a millimetre [2]. For carbon ions the synchrotron would be ~75 m in circumference and would produce ions of up to ~400 MeV/u, whereas for protons these figures would be ~45 m and ~250 MeV. The quality of the dose intensity profile ultimately depends on the uniformity of the beam spill. This is the greatest challenge for the synchrotron, since slow-extraction schemes are notoriously sensitive. This paper reviews the extraction techniques and their relative merits, describes methods for smoothing the beam spill and outlines the implications for the extraction line and beam delivery system.

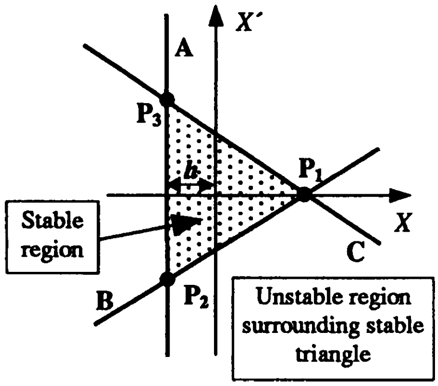
2 BASIC THEORY

2.1 Kobayashi hamiltonian

The suggested technique for designing a third-integer, resonant extraction system is to use the simplified Kobayashi Hamiltonian, H that describes the particle motion perturbed by the resonance sextupole in the plane of extraction (usually horizontal) [3,4]. This gives a good physical insight that can then be refined with tracking programs [5,6].

$$H = \frac{\varepsilon}{2}\left(X^2 + X'^2\right) + \frac{S}{4}\left(3XX'^2 - X^3\right) \tag{1}$$

where, (X, X') are the normalised co-ordinates of a particle, the normalised sextupole strength is $S = \frac{1}{2}\beta^{3/2}_{x,\text{sextupole}} \frac{\ell_{\text{sextupole}}}{|B_0\rho|}\left(\frac{\mathrm{d}^2 B_z}{\mathrm{d}x^2}\right)_0$, $\varepsilon = 6\pi\delta Q$, δQ is the distance from the resonance $Q = n \pm 1/3$ and the other symbols have their usual meanings. Figure 1 summarises the phase-space geometry derived from (1) at the position of the resonance sextupole for mono-energetic particles whose equilibrium orbits are centred on the origin.



Separatrices:	Fixed points:
A $X = -\frac{2}{3}\frac{\varepsilon}{S}$	P_1 $(\frac{4}{3}\frac{\varepsilon}{S}, 0)$
B $-\frac{\sqrt{3}}{2}X' + \frac{1}{2}X = \frac{2}{3}\frac{\varepsilon}{S}$	P_2 $(-\frac{2}{3}\frac{\varepsilon}{S}, -\frac{2\varepsilon}{\sqrt{3}S})$
C $\frac{\sqrt{3}}{2}X' + \frac{1}{2}X = \frac{2}{3}\frac{\varepsilon}{S}$	P_3 $(-\frac{2}{3}\frac{\varepsilon}{S}, \frac{2\varepsilon}{\sqrt{3}S})$
$h = \frac{2}{3}\frac{\varepsilon}{S} = \frac{4\pi}{S}\delta Q$	

Figure 1: Phase-space geometry at the resonance sextupole

The sense of rotation in the stable region is clockwise when the particles are above resonance and the whole diagram will rotate clockwise if the observer moves downstream. The resonance sextupole is best put in a dispersion-free region otherwise it affects the chromaticity. The equation of a separatrix anywhere in the machine, including the effect of dispersion, is

$$\left(X - D_\mathrm{n}\frac{\Delta p}{p}\right)\cos\alpha + \left(X' - D'_\mathrm{n}\frac{\Delta p}{p}\right)\sin\alpha = h, \tag{2}$$

where $(D_\mathrm{n}, D'_\mathrm{n})$ is the normalised dispersion function and α is the angle measured from the X-axis in the anticlockwise direction to the perpendicular, h, to the extraction separatrix. A particle that becomes unstable moves in growing steps along the separatrices until it reaches the electrostatic septum and is extracted. The spiral step, ΔX_{ES}, at the electrostatic septum (shown in Figure 5) is given by,

$$\Delta X_{\mathrm{ES}} = \left\{\frac{2}{3}\frac{\varepsilon^2}{S} - \frac{3S}{4}\left(\left[X_{\mathrm{ES}} - D_\mathrm{n}\frac{\Delta p}{p}\right]^2 + \left[X'_{\mathrm{ES}} - D'\mathrm{n}\frac{\Delta p}{p}\right]^2\right)\right\}\left|\cos(\alpha - \pi/2)\right|. \tag{3}$$

2.2 Hardt condition

When the beam enters the resonance, there will be a range of momentum and emittance values that will define the boundary between the stable and unstable regions (see Figure 2). The large amplitude/emittance particles will enter the resonance first, while still some distance from the resonant tune, and there will be a continuous range of stable-triangle sizes down to zero for the zero-amplitude/emittance particles that must be exactly on resonance to be unstable. In general, the dispersion function will spread out this range of phase-space triangles and separatrices so that particles of different momenta will reach the electrostatic septum with different angles. The condition to set the optics such that all the extraction separatrices are aligned to give minimum losses on the electrostatic septum is known as the Hardt condition [7,8],

$$D_n \cos\alpha + D_n' \sin\alpha = -\frac{4\pi}{S} Q' \tag{4}$$

where $Q' = \mathrm{d}Q/(\mathrm{d}p/p)$. Effectively the Hardt condition moves the stable triangles along the direction of the dispersion vector by adjusting the chromaticity. For this reason it is best to separate chromaticity and resonance sextupoles and to place the resonance sextupole in a dispersion-free region although this is not absolutely necessary. Figure 2 describes a general scenario, but the Hardt condition is only effective if the resonance is stationary and the beam is moved, for example, by a betatron core or by stochastic rf noise.

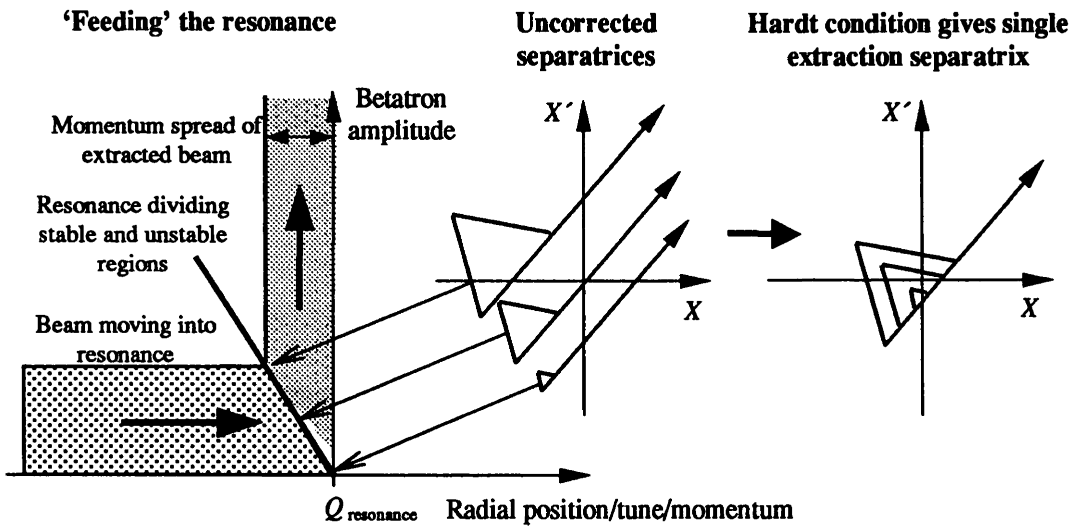


Figure 2: Description of the Hardt condition at the electrostatic septum.

Medical machines work below transition and ideally the chromaticity should be negative to ensure the transverse stability of the 'waiting' beam. Extraction is best made to the outside of the ring and this choice limits the possibilities for applying the Hardt condition to positions where the electrostatic septum has a positive dispersion with a negative slope as in the second half of a dispersion bump, for example.

There is also the trivial mathematical solution of (4) given by $D_n = D_n' = 0$ and $Q' = 0$, which corresponds to an important practical configuration known as the rf knockout slow extraction that will be described in Section 3.

2.3 *Tune ripple*

The tune of the resonance is an absolute number, whereas the tune of the beam is affected to some degree by almost all power converters in the synchrotron, vibrations and eddy currents. Thus, the tune separation between the beam and the resonance (and hence the area of the stable triangle) is subjected to a perpetual jitter. The flux of particles, ϕ, entering the resonance under these conditions, but with a single ripple component of the form $Q_r = \delta Q\cos(\omega t)$, will be,

$$\phi = \lambda\left(\frac{dQ_0}{dt}+\frac{dQ_r}{dt}\right) = \lambda\left[\frac{dQ_0}{dt}+\omega\delta Q\cos(\omega t)\right] \tag{5}$$

where $\lambda = dN/dQ$ is the particle line density in the 'waiting' beam and dQ_0/dt is the quasi-constant rate of tune change imposed by the extraction system to bring the beam into resonance and form the spill. To give some feeling for practical values, consider the critical point at which the ripple error equals the main term in (5) and the particle flux becomes 100% chopped,

$$\frac{dQ_0}{dt} \le \omega\delta Q\,. \tag{6}$$

Inspection of (6) shows that for normal values of dQ_0/dt, the main term will be quickly dominated by the error term $\omega\delta Q$ as ω enters the kHz range. As an example, consider a small synchrotron working with the parameters: resonant tune $Q_{res} = 1.666$, a spill time $T_{spill} = 1$ s, a tune shift needed to 'consume' the beam of $\Delta Q = 0.01$ and a relation between the power converter current ripple and the tune ripple of $\Delta Q/Q = \Delta I/I$. Equation (6) then shows that the spill will be 100% chopped at and above the current ripples quoted in Table 1 over the frequency range 50 to 10 kHz. The upper frequency limit is set by the sampling frequency of the on-line dosimetry system.

Frequency [Hz]	$(\delta Q/Q_{res}) = \Delta I/I$
50	2×10^{-5}
100	10^{-5}
300	3.2×10^{-6}
1000	10^{-6}
10000	10^{-7}

Table 1: Conditions for 100% modulation of the spill.
[$Q_{res} = 1.666$, $T_{spill} = 1$ s, $dQ_0/dt = 0.01$, $\Delta Q/Q = \Delta I/I$.]

It is likely that several ripple frequencies will be present in the spill and that Table 1 is therefore an under-estimate of the situation. This points to the need for extremely tight power converter specifications and the need to find ways of desensitising the machine to tune ripple. To this end, it is worthwhile checking the eddy current smoothing in the magnets and vacuum chamber. The disruptive effects of eddy currents during the ramp must be limited, but it is counter productive to over-design with ultra-thin-walled chambers of high resistivity. If a quadrupole is used to drive the extraction, it is important to ensure that this unit will filter out kHz frequencies and not pass them on to the beam. Eddy currents and the intrinsic smoothing that will be discussed in Section 2.4 are useful because they both improve in efficiency as the frequency increases and enters the kHz region. It is also advantageous to choose an extraction scheme that does not require changes in the optics of the machine during the spill, since the current from a ramping power converter has a structure at the DAC* frequency. A more active defence is to increase the velocity of the particles (dQ_0/dt) as they enter the resonance. This can be done, without affecting the spill time, by creating a local region close to the resonance where the particle density is low, but the velocity is high, so that the flux can be kept constant. This is known as 'front-end' acceleration. The empty channelling rf bucket described in Section 4.2 is one way of achieving this. Feedback systems will be needed to shape the spill and to compensate ripple, but their performance is adversely affected by the resonance transit time described in Section 2.4.

2.4 Transit time and intrinsic smoothing

The transit time, or storage time, in the resonance is the principal problem for feedback systems, but it also has the mitigating effect of smoothing the spill at high frequencies. By first studying the behaviour of a single particle and then the 'strip' of particles sitting along the edge of the stable triangle in phase space and finally the 'band' of particles that comprise all the 'strips' of different momenta that become unstable at any one time, it is possible to build up simulations of the micro-time profile of the spill under the influence of tune ripple and to compare the different extraction techniques [9,10].

Consider first a step function in the tune of a monoenergetic beam towards the resonance. This sharp movement will lead to a burst of particles entering the resonance and, some time later, appearing in the spill. The step function input appears in the output as a narrow peak followed by a flat plateau (see Figure 3). The particles are shared about equally between the peak and the plateau. The time needed for the leading peak to appear (transit time, T_0) depends on the area of the stable triangle. The length of the plateau is also approximately equal to this time. In brief, the tune movement maroons a narrow 'strip' of particles around the stable triangle in the unstable region. Since the particles move slowly when they are close to the stable fixed points, there is a higher density in these regions and it is these particles that, after the many turns needed to escape from the fixed point, arrive quickly and virtually all together in the spill to form the leading peak. The remaining particles (about 50%) have to migrate along the side of the stable triangle and then step slowly past the fixed point. These particles are stretched out in time and form the plateau.

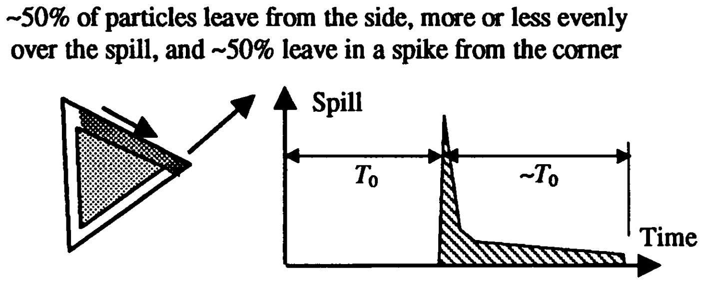


Figure 3: A 'strip' spill with the characteristic leading peak and plateau.

* Digital to Analogue Converter (DAC).

By mixing different initial amplitudes (momenta) the spikes are blurred out. This is best done by moving a wide beam in momentum space into the resonance as shown in Figure 2 for the Hardt Condition. There is then a continuum of narrow 'strip' spills spread out in betatron amplitude and momentum. The result is called a 'band' spill and it contains an intrinsic smoothing of the leading spikes (see Figure 4).

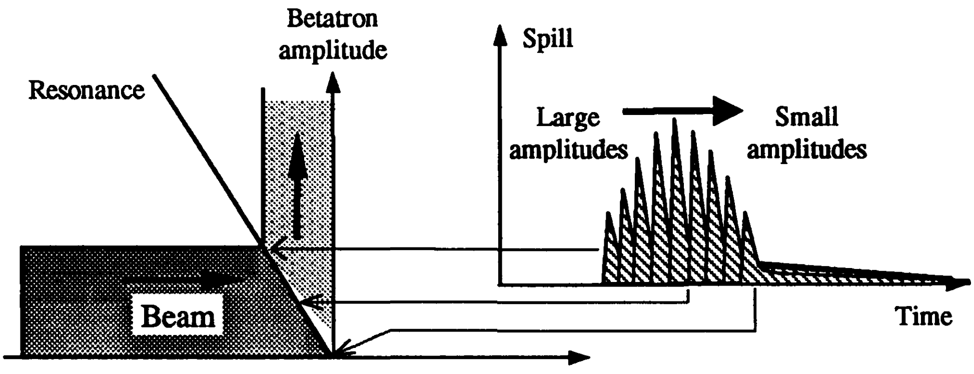


Figure 4: Intrinsic smoothing of the spill.

2.5 Extraction emittance

The emittance in the plane perpendicular to the extraction will be essentially equal to that of the circulating beam. However, it will suffer adiabatic damping in the same way as the circulating beam at the different extraction energies. Only small deviations from this situation will exist due to coupling in the resonance sextupole, especially if the beam is off-axis vertically.

In the plane of extraction, the situation is different. Firstly, the separatrices are determined by the geometry of the resonance and do not suffer adiabatic damping. Secondly, the extraction emittance is small compared to that of the circulating beam. This can be estimated by Liouville's theorem by comparing the phase-space volume (transverse × longitudinal emittances) in the ring to that in the spill. The momentum spreads will be of the same order, but the extracted transverse emittance will suffer a strong reduction by the ratio of the revolution time to the spill time ($\sim 10^6$). Coupling from the vertical emittance in the sextupoles will increase this value and dispersion effects, closed-orbit changes and ripple, will hide it, but the true extracted emittance will still be very small. An upper limit on the maximum emittance that can be extracted is derivable from the Hamiltonian by considering the whole beam as being exactly on resonance and tracing its phase-space area as it leaves the machine along the separatrices. For typical parameters, this leads to a maximum of a few percent of the initial beam emittance. To get exact figures for a specific case, it is necessary to track particles through the resonance. Figure 5 shows the overall view of such a simulation in phase space and Figure 6 shows in detail the extracted beam segment.

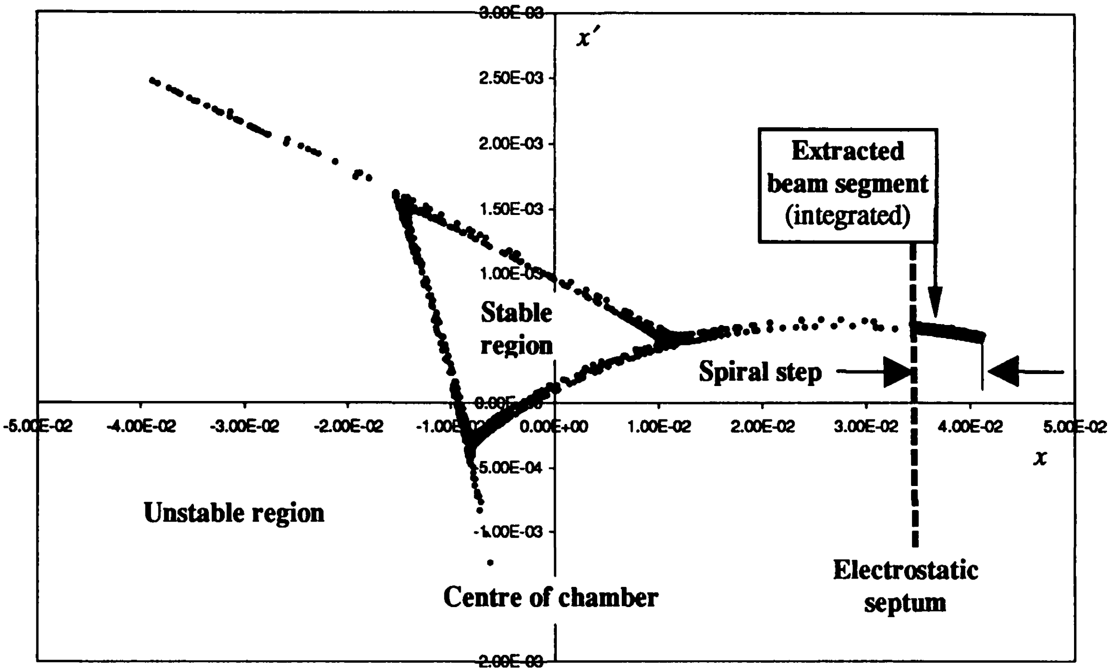


Figure 5: Simulation of the extraction phase space at the electrostatic septum.

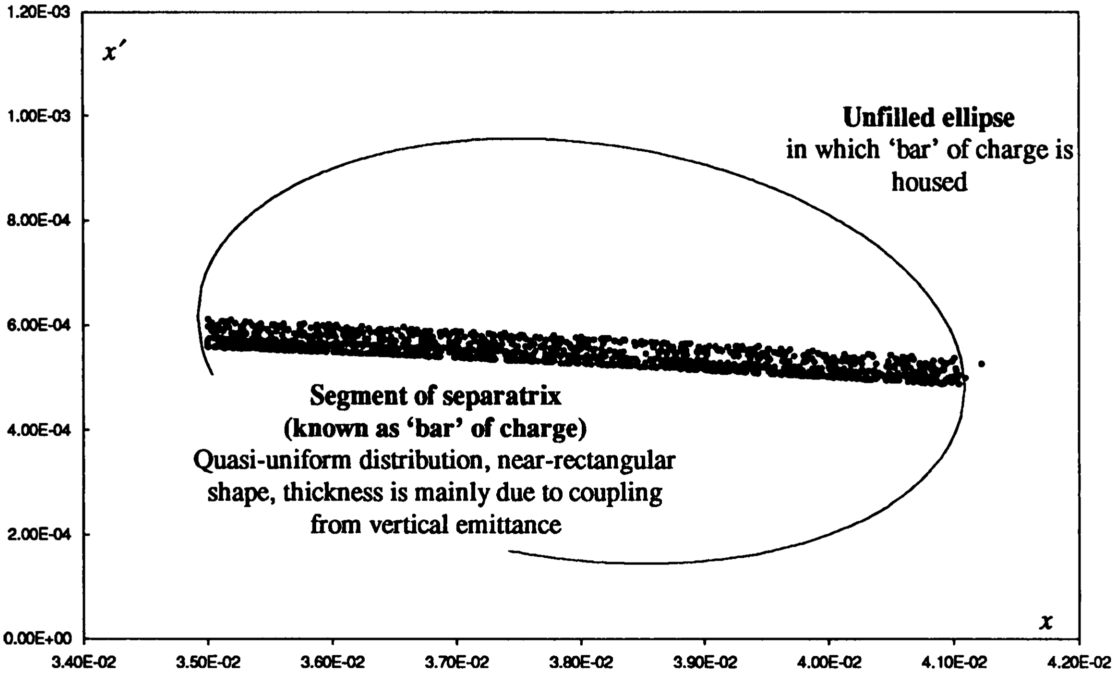


Figure 6: Detail of the extracted beam 'footprint' in horizontal phase space.

In Figure 6, the width of the trace is mainly due to the coupling from the vertical emittance. The final special feature of the extracted beam is the narrow 'rectangular' shape of the phase-space footprint with an almost constant particle density. This is known as the 'bar' of charge and is best treated as a diameter of an unfilled emittance ellipse as shown in Figure 6. The strong asymmetry in the extracted beam emittances, the adiabatic damping in only one plane and the rectangular phase-space 'footprint' have to be specially treated in the extraction line when matching to the gantry and when designing the scanning system.

3 RESONANT EXTRACTION

Resonant extraction can be classified by the techniques used to bring the beam into resonance (Figure 7). This is an important classification, since the properties of the extracted beam are strongly affected by this choice. The three main methods are:

- *Quadrupole-driven extraction (classic).* Selects large through to small betatron amplitudes by moving the resonance. The advantage of this technique is its simplicity. The disadvantages are the need to change machine parameters during the extraction, an unavoidable movement of the beam at the septum and changes in the spiral step as the betatron amplitude of the particles entering resonance is reduced. The resonance tuning quadrupole should be designed with thick laminations to exploit eddy current smoothing in order to reduce the possibility of passing high-frequency noise onto the spill.

- *Acceleration-driven extraction.* Selects particles with a fixed betatron amplitude-momentum relation by moving the beam into a stationary resonance. The disadvantage is the need for additional equipment to accelerate the beam. This can be a stochastic noise system or a betatron core. The advantages are that the transverse optics of the ring can be kept rigorously constant throughout the extraction, there is no movement of the extraction separatrices, the Hardt Condition can be applied, intrinsic smoothing is active and 'front-end' acceleration can be applied.

- *RF knockout extraction* [11]. Selects constant betatron amplitude by blowing the beam up onto a stationary resonance. The disadvantages are the need for additional equipment to excite the beam and the high ripple sensitivity. The advantages are that the transverse optics of the ring can be kept constant throughout the extraction, the zero-chromaticity Hardt condition exists and the spill can be conveniently switched on and off rapidly.

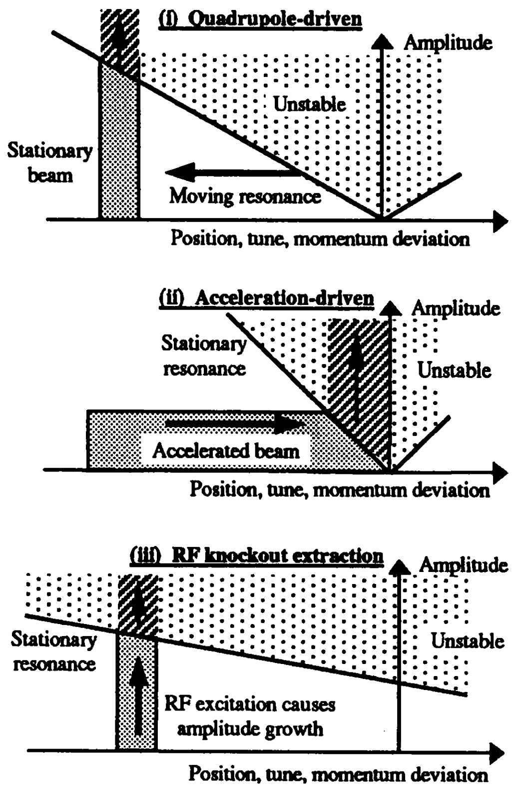


Figure 7: Slow-extraction schemes.

4 CONTROLLING RIPPLE

Providing there are no constraints from existing equipment, or other limitations, then configuring the synchrotron for an acceleration-driven extraction offers the smoothest spill. The preferred beam configuration [12] in this case is a wide momentum spread, as shown in Figure 7(ii), with approximately equal transverse emittances that are adapted in the vertical plane to achieve the required spot sizes at the patient with convenient optics in the transfer line and gantry. The chromaticities should be negative to ensure stability of the 'waiting' beam (assuming operation below transition). The Hardt condition should be applied to minimise the losses on the extraction septum. There is a limited choice of possible configurations of which one is shown in Figure 8.

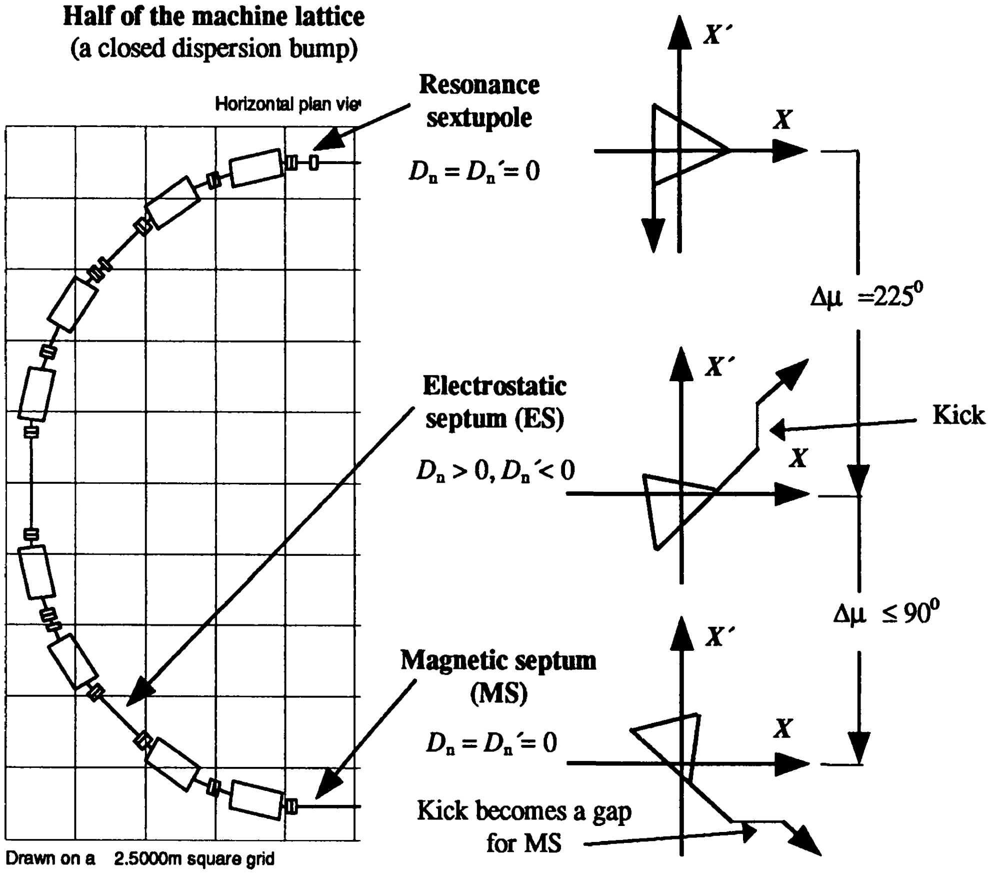


Figure 8: Possible layout of the extraction elements.

In Figure 8, the arc forms half of the machine and is a closed dispersion bump. The 'waiting' beam is kept in the inner half of the chamber. The resonance is placed on the central orbit to give the separatrices a balanced aperture for their growth and the extracted beam leaves from the outside of the ring (see Figure 9). The ring is made from two closed dispersion bumps joined by dispersion-free straight sections, which are used for the rf cavity and the resonance sextupole. Although these last two points are not essential, they give the advantages of independent chromaticity and resonance control and suppression of coupling between the transverse and longitudinal phase spaces in the cavity. In order to satisfy the Hardt Condition, the electrostatic septum is placed in the second half of one of the dispersion bumps were $D_n > 0$ and $D_n' < 0$.

Finally the magnetic septum is placed at the entry to the next dispersion-free region where the closest approximation to an achromatic transfer between the septa can be obtained. It is possible to make a completely achromatic transfer between septa by placing both units in the same drift space, for example. However, this inevitably leads to high field strengths in the septa due to the small phase advance and, generally, this type of solution would be used for protons (low rigidity) and notions.

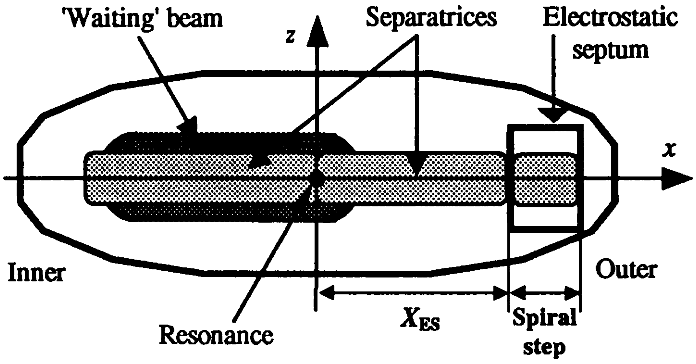


Figure 9: Schematic layout of the machine aperture.
[Typically X_{ES} = 35 mm, Spiral step = 10 mm]

4.1 Betatron core

The acceleration-driven extraction has the great advantage that it maintains all transverse optical parameters (and hence the power converters) in the machine constant. In this situation, a good method for accelerating a beam of small emittance and large momentum spread into the resonance is a betatron core [13]. This also ensures that all betatron amplitudes in the beam are 'fed' into the resonance concurrently in order to enhance the intrinsic smoothing. The 'stepping' from the DAC of the betatron core would be a source of ripple, but, since this is just one unit, the extra care and cost of providing a high precision DAC and some smoothing would be feasible. In any case, the core, being a high-inductance device, is smooth in its operation.

4.2 Channelling rf bucket

The acceleration-driven extraction is the only technique that allows the 'front-end' acceleration described in Section 2.3 for reducing the effect of tune ripple. There are basically two methods for creating a region of high tune speed near the resonance. The first uses stochastic noise [14] and the second uses an empty channelling rf bucket [15,16]. Stochastic noise requires considerable power when used with a short extraction time (~1 s), therefore the latter method is preferred. Furthermore, no new equipment is needed since the main rf cavity can be used. The action of the cavity is based on a technique known as phase-displacement acceleration [17]. All particles in the beam are accelerated by the betatron core and, at the same time, the rf cavity is set so that it would decelerate particles by the same amount if they were trapped inside the bucket. The beam, however, is outside the rf bucket and the influence of the cavity is only felt as the particles approach the empty buckets in longitudinal phase space. The particles are then compressed into a narrower and narrower region of phase space and have to move rapidly around the buckets, which remain empty (see Figure 10). This can be visualised by thinking of a river flowing past the piers of a bridge. The narrower the space allowed between the piers, the greater the river's velocity.

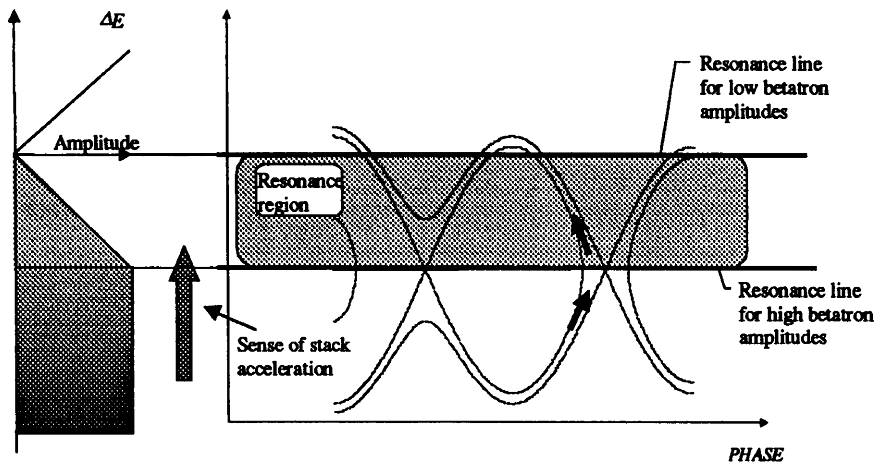


Figure 10: RF channelling with empty buckets.

5 CONCLUSIONS

The demonstration of voxel scanning at PSI [18] and raster scanning at GSI [19] has opened the way towards a new generation of high-precision synchrotrons for cancer therapy. To benefit fully from the new scanning techniques, it will be necessary to achieve smooth spills with a slow-extraction scheme and to design precise optics for beam delivery systems. Both problems depend on a detailed understanding of the extraction mechanism and the extracted beam. From the work done in the Proton-Ion Medical Machine Study at CERN, it has been possible to compare the different extraction techniques for sensitivity to tune ripple and suitability for 'pencil' beam scanning. The choice of acceleration-driven extraction has been integrated into an overall design strategy with several features for obtaining a smooth spill with minimum loss on the electrostatic septum The study of the extracted beam has also revealed the detailed structure of the phase-space footprint and the transfer characteristics of the resonance. This has prepared the way for a specialised optics for the transfer lines, gantry and delivery system.

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