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Polarized Positron Sources[1]

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The collisions of both polarized electron and positron high energy beams is very attractive for fine physics beyond the standard model. Some schemes discussed what is able to prepare such a statement for future linear collider.

0. Introduction[3].

The importance of a polarization for high energy physics was discussed a lot of times by many authors. Even in simple treatment one can see the gain of factor 2 in output of an ordinary and routine reactions if both of the beams are polarized. This is due to the circumstance that high energy statement is a polarized one, so each particle of the beam looks only for an appropriate polarized one from incoming beam. Or with the other words, each particle can see only half of the particles from incoming beam in case on nonpolarized second beam. In [2] the importance of polarization for seeking new bosons beyond the Standard Model carefully discussed. The output made there is that at the energy range $\sqrt{s} \cong 500GeV$ for the settings the Z' boson mass, the polarized beams gives the luminosity gain by ~ 5 times, or with unpolarized beams the total energy need to be 2-3 times higher. This is impressive figure.

Requirements, arising from typical linear collider design, one can find, for example, in [3,4]. The general output of the requirements is that the power carried by the beams is of the order of few Megawatts. So the efficiency of the particle generation is one among important components of the any project. Some projects from the very beginning include the possibility to collide polarized particles, both electrons and positrons or the electrons only.

1. Polarized statements of electron and gamma radiation.

For description the electron polarization the usual convention is in defining the polarization vector $\vec{P}$ (spin vector) in the rest frame of the positron or electron (see for example [5]). The fields are defined in the laboratory system. The equation of the spin motion can be represented as following [6]

$$\frac{d\vec{P}}{dt}=\frac{2\mu mc^2+2\mu'(E-mc^2)}{\hbar E}(\vec{P}\times\vec{H})+\frac{2\mu' E}{\hbar(E+mc^2)}(\vec{\beta}\vec{H})(\vec{\beta}\times\vec{P})+\frac{2\mu mc^2+2\mu' E}{\hbar(E+mc^2)}(\vec{P}(\vec{E}\times\vec{\beta}))$$

where $\vec{\beta}=\vec{v}/c$, c is a speed of light, $\vec{E}$, $\vec{H}$ are correspondly the electric and magnetic fields in the laboratory frame, E is the particle energy, t is the time in lab frame, $\mu=-\frac{e\hbar}{2mc}=-9.3\cdot10^{-21}\,erg\cdot Gauss$, $\mu'\cong\mu\frac{\alpha}{2\pi}=-1.1\cdot10^{-23}\,erg\cdot Gauss$, $\alpha=e^2/\hbar c=1/137$ is the fine structure constant, $\hbar$ is the Plank constant.

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[3] We shall see, that the methods of polarized *positron* generation, discussed below, can be applied to polarized *electron* generation with the same success. The question here only in the cost of the method applied, compared with the other one, used an appropriate photocathod, illuminated with polarized laser light [1].

Polarization of the photon defined by the matrix of polarization, where the components are the products of different components of the electrical field vector $E_i E_j^*$, $i,j = 1,2$.

Spin handling and management for the linear collider complex was considered in [7]. The manipulation with the spin is based on the anomalous magnetic moment μ', what yields the rotation the vector of spin with respect to the vector of momenta. The angular frequency, same as the frequency of the vector momenta particle has at the energy 440.6 MeV.

Depolarization in the interaction region was a subject of considerations from the very beginning [8 a,b]. Due to huge magnetic field of incoming beam the vector of spin rotates at the angle $\varphi \equiv 2\pi \cdot E[GeV]/0.4406$ with respect to the vector of momenta. The very first estimations as well as the last one [9] shows that this effect yield a lost of a few percent of polarization and need to be taken into account.

2. Conversion of polarized gammas obtained from the wiggler.

Basically the idea [10] of polarized particles generation is rather simple. It is based on very well described components. The content of this idea is to irradiate the thin target with circulary polarized photons of sufficient energy and to collect the positrons at the top of its energy, Fig.1. Due to specific properties of interaction of the photons with the matter, the positrons at the high (or lower) energy top spectra has a longitudinal polarization. The source of radiation could be a primary one (helical wiggler of undulator) or the secondary one (back scattered photons). The target also could be treated as real material or the photon one. If the source is not polarized, then the output positrons or electrons are also not polarized.

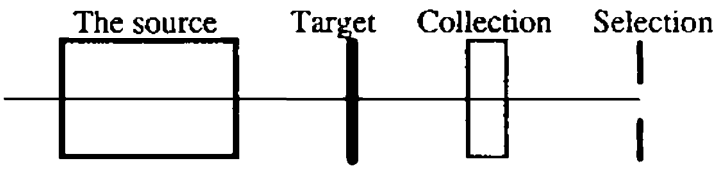

Fig. 1. The basis of conversion system for polarized particle production.

2.a. This idea with real helical wiggler was under consideration in [11-14]. In [15] one can find the latest results on this subject. On Fig. 2 there is represented more detailed view of the scheme with a wiggler.

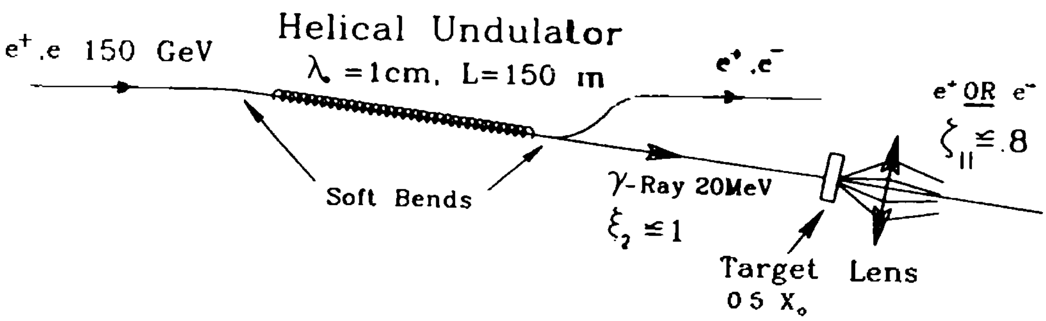

Fig. 2. The helical wiggler based scheme.

Let us make more detailed description of the elements of this scheme.

Interaction of the photons with the nuclei was described in many tutorials. We will use here [16,a,b] as a reference. The differential cross-section of the pair production by the photon has a rather complex dependence near the maximal possible positron (or electron) energy. Some additional difficulties connected with the screening the Coulomb field of the nuclei by the electrons. The screening becomes important when the minimal wavelength, connected with the momentum q_{min}, transferred to the nuclei, becomes bigger, than the size of the nuclei, i.e. $\hbar / q_{min} \geq a_0 Z^{-1/3} \equiv \hbar^2 / e^2 m \cdot 1/Z^{1/3}$, where Z is the atomic number of the conversion target and it was substituted the Bohr radius value $a_0 = \hbar^2 / e^2 m$. This gives $q_{min} \leq mc\alpha Z^{1/3}$. Since $q_{min} \approx p_+ + p_- - \hbar\omega / c \approx mc\dfrac{mc^2\hbar\omega}{2E_+E_-}$, where E_+ is the positron total energy, $E_- = E_\gamma - E_+$ is the electron total energy, $E_\gamma = \hbar\omega$ is the energy of the incoming photon, that yields $\dfrac{mc^2\hbar\omega}{2E_+E} \leq \alpha Z^{1/3}$, or $\chi = \dfrac{mc^2\hbar\omega}{2\alpha Z^{1/3}E_+E_-} \leq 1$. The parameter χ describing the screening. Thus as we are interesting the situation, when $E_+ \approx E_\gamma \approx 20 MeV$, $Z \approx 80$, $\alpha Z^{1/3} \equiv 0.03$, so $\chi \approx 32mc^2 / E_- >> 1$, i.e. no screening. We will represent here an analytical expression what is valid in Born approximation [16a]

$$\frac{d\sigma(E_\gamma,E_+)}{d(E_+/E_\gamma)} = 4\alpha Z^2 r_0^2 G(E_+,E_-) = \alpha Z^2 r_0^2 \frac{p_+p_-}{E_\gamma^2}\left\{-\frac{4}{3} - 2E_+E_-\frac{p_+^2+p_-^2}{p_+^2p_-^2} + m^2c^4\left(\frac{E_+l_-}{p_-^3} + \frac{E_-l_+}{p_+^3} - \frac{l_+l_-}{p_+p_-}\right) + \right.$$

$$\left. + L\left[\frac{E_\gamma^2(E_+^2E_-^2 + p_+^2p_-^2)}{p_+^3p_-^3} - \frac{8}{3}\frac{E_+E_-}{p_+p_-} - \frac{m^2c^4E_\gamma}{2p_+p_-}\left(\frac{E_+E_- - p_-^2}{p_-^3}l_- + \frac{E_+E_- - p_+^2}{p_+^3}l_+ + \frac{2E_\gamma E_+E_-}{p_+^2p_-^2}\right)\right]\right\},$$

where $\alpha = e^2/\hbar c = 1/137$ is a fine structure constant, $r_0 = e^2/mc^2$ is the electron classical radius, $l_\pm = ln\dfrac{E_\pm + p_\pm}{mc^2}$, $L = ln\dfrac{E_+E_- + p_+p_- + m^2c^4}{mc^2E_\gamma}$ and the relation between the energy and momentum is the following $p_\pm^2 = E_\pm^2 - m^2c^4$ (c included in p, definition of [16a]). This cross-section dependence has a view, represented on the Fig. 3A. It is clear, that the spectral density goes to zero, when E_+, or E_- goes to the maximum possible (or the lowest) value. When the $E_\gamma, E_+, E_- >> 2mc^2$ the angles of electron and positron with respect to direction of the photon incident have the order $\theta_\pm \approx mc^2 / E_\pm$ and formula looks like

$$\frac{d\sigma(E_\gamma,E_+)}{d(E_+/E_\gamma)} \cong 4\alpha Z^2 r_0^2 \ln(183/Z^{1/3})\hat{G}(E_+/E_\gamma) \equiv \frac{A}{N_0X_0}\hat{G}(E_+/E_\gamma),$$

where A is its atomic weight, $N_0 \equiv 6.022 \cdot 10^{23}$ is the Avogadro number, radiation length X_0 is defined by

$$X_0^{-1} \cong 4r_0^2\alpha\frac{N_0}{A}Z^2 ln(\frac{183}{Z^{1/3}})[cm^2/gramm],$$

function $G(x)$ has a rather simple form in this case

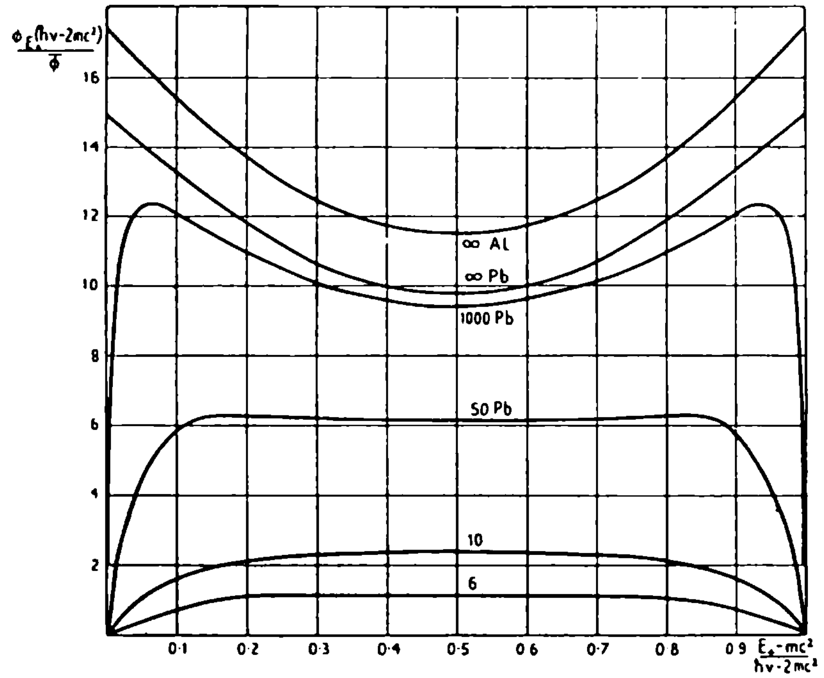

Fig. 3A. The differential cross-section of the pair production $\frac{E_\gamma - 2mc^2}{\alpha Z^2 r_0^2}\frac{d\sigma(E_\gamma, E_+)}{dE_+}$ as the function of the positron partition energy $y = \frac{E_+ - mc^2}{E_\gamma - 2mc^2}$ [16a]. The numbers at the top of each curve indicates the energy of incoming quanta in units mc^2. The curves for $E_\gamma = 6, 10\,mc^2$ are valid for any element.

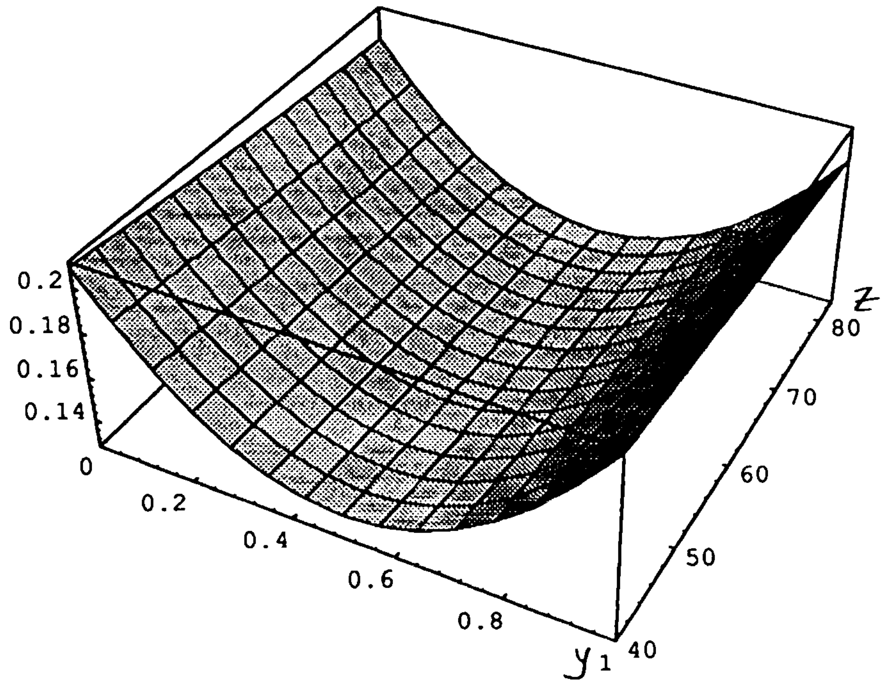

Fig. 3B. The differential cross-section of the pair production $\frac{E_\gamma}{4\alpha Z^2 r_0^2}\frac{d\sigma(E_\gamma, E_+)}{dE_+}$ as the function of the positron partition energy $y = \frac{E_+}{E_\gamma}$ when $E_\gamma, E_+, E_- >> 2mc^2$

$$\tilde{G}(x) = x^2 + (1-x)^2 + \frac{2}{3}x(1-x) - \frac{x(1-x)}{9\ln(183Z^{-1/3})}.$$

This function is represented on Fig. 3B. There is no dependence of the incoming photon energy in this function, the ratio only. This is sequence of the assumption $E_\gamma, E_+, E_- >> 2mc^2$ or when the energy of each particle far from the limit arising from the energy conservation law.

The values at the boundary condition, when $E_+, E_- \approx E_\gamma$, the function $G \to 0$. For analytical calculations, in more realistic case of intermediate energies of gammas this function can be approximated as having a root dependence. For example, for $E_\gamma \approx 50mc^2$, Z = 82 this function can be represented [12] as

$$G(x) \approx \begin{cases} 4.75\sqrt{x}, & 0 \le x < 0.11 \\ 1.55, & 0.11 < x < 0.89 \\ 4.75\sqrt{1-x}, & 0.89 < x \le 1 \end{cases},$$

One can see, that the number of the particles here is lower than for intermediate partial energies. As the method requires to collect the particles near the top of the energy, this circumstance reduces the efficiency of positron generation. In any case the higher energy is desirable from the point of conversion efficiency. For example, the increasing the energy of incoming photos from 5 to 25 MeV yields increasing the efficiency about 6 times.

The mostly important property of the pair generation by the polarized photon, is the transferring the polarization from the gamma to the positron and electron created. Basically this is a sequence of the conservation low for the momentum. The polarization phenomena in positron production is carefully investigated in [17]. The longitudinal polarization of the particle created is a function of its energy, E_+, E_- and the polarization ξ_2 of the incoming gamma

$$\vec{\varsigma} = \xi_2 \cdot \left[f(E_+, E_-) \cdot \vec{n}_\parallel + g(E_+, E_-) \cdot \vec{n}_\perp \right] = \vec{\varsigma}_\parallel + \vec{\varsigma}_\perp ,$$

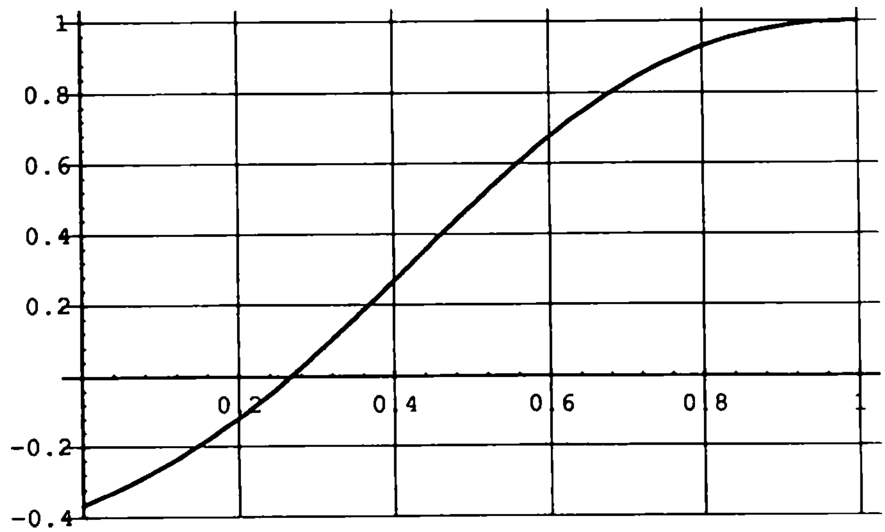

Fig. 4. The longitudinal polarization of the positron created as a function of its fractional energy [17].

where $\vec{n}_\parallel$ is directed along the initial direction of the gamma radiation and $\vec{n}_\perp$ is rectangular to it. An analytical expression for f has a form

$$f = E_\gamma \frac{E_+\psi_1 - E_-(\psi_1 - 2\psi_2/3)}{(E_+^2 + E_-^2)\psi_1 + 2E_+E_-/3} = \frac{x\psi_1 - (1-x)(\psi_1 - 2\psi_2/3)}{(x^2 + (1-x)^2)\psi_1 + 2x(1-x)/3},$$

where $\psi_1 = ln183Z^{-1/3} - F(\alpha Z)$, $F(\sigma) = \sigma^2 \sum_{n=1}^{\infty} \frac{1}{n(n^2+\sigma^2)}$, $\psi_2 = \psi_1 - \frac{1}{6}$, $x = E_+/E_\gamma$.
The view of this function is represented on Fig. 4. The function f is weakly dependent of Z.

As the polarization ξ_2 defines the longitudinal polarization of the particles generated, it is clear that the source of gammas must generate them with highest possible value ξ_2 with the necessary amount.

Let us make preliminary estimation of the efficiency of the gamma conversion. Integrating the formula for spectral distribution, one can obtain the cross-section per one atom

$$\sigma_{tot} \cong \int_0^1 \frac{A}{N_0 X_0} G(x) dx \cong \frac{7}{9} \frac{A}{N_0 X_0}.$$

If t is the thickness of the target, then the number of the atoms N in the volume having a height t and a cross-section $1cm^2$, will be $N = N_0 \frac{g[g/cm^3]\cdot 1cm^2 \cdot t[cm]}{A[g]}$, where g is the specific weight of the target material. So the number of the positrons what will be created at the exit of the target will be $N_+ \cong N_\gamma \sigma_{tot} N \cong \frac{7}{9} N_\gamma \frac{gt}{X_0} = \frac{7}{9} N_\gamma \tau$, where $\tau = \frac{gt}{X_0}$ is the target length, measured as a fraction of the radiation length. We will be interesting in $\tau \le 0.5$ and taking into account that about 1/5 positrons only carrying the necessary level of polarization, we can finally estimate the conversion efficiency of the photons as $N_+/N_\gamma \approx 0.077$, or 7.7%. This estimation looks very close to that obtained from numerical calculation (se lower). We supposed also, that the phase volume of the positrons created, corresponds mostly to multiscattering in a target, and the particles could be accepted by appropriate collecting system. The conditions for acceptance is a subject of special considerations made in section **2.c**.

For obtaining the formula, describing the spectrum of the positrons created, we can write

$$\frac{d^2N_+}{dE_+ d\tau} = \frac{1}{\sigma_{tot}} \iint \frac{d\sigma(E_\gamma, E_+)}{dE_+} \frac{d^2N_\gamma}{dE_\gamma dS} dE_\gamma dS,$$

where $\frac{d^2N_\gamma}{dE_\gamma dS} = \frac{d^2N_\gamma}{dE_\gamma R^2 do}$ is the spectral density of the photon source, illuminating the target, $dS = R^2 do$, do is the solid angle, R is the distance from the source to the target.

The photon source what is planned to use here -- is the wiggler. Formulas for wiggler radiation are represented in the next section.

For obtaining the spectrum of the positrons at *the output* of the conversion target, we need to take into account the fluctuations of the energy losses in the target at the distance from the point, where the positron was generated to the output surface of the target. We suppose that the target has a thickness, measured as a fraction of the radiation length, $\delta = gd/X_0$, where d is a geometrical thickness (measured from the front surface of the target). The probability WdE_+ that the positron, created by the photon at the depth τ with initial energy E_+, will have the energy in the interval from E_+^{out} to $E_+^{out}+dE_+^{out}$ at the output of the target, is described by the formula [48]

$$W(E_+,E_+^{out},\delta-\tau)dE_+^{out}=\frac{dE_+^{out}}{E_+}\cdot\left(ln\frac{E_+}{E_+^{out}}\right)^{(\delta-\tau)/ln2-1}\Bigg/\Gamma\left(\frac{\delta-\tau}{ln2}\right),$$

where $\Gamma(x)=\int\limits_0^\infty t^{x-1}e^{-t}dt$ is the Gamma function. So, the number of the positrons, generated by the photon flux, having spectral-angular density $d^2N_\gamma/dE_\gamma dS$, and with the initial energy in the interval from E_+ to E_++dE_+ and leaving the converter at the energy interval from E_+^{out} to $E_+^{out}+dE_+^{out}$ is [11,12]

$$\frac{d^2N_+}{dE_+dE_+^{out}}=\int\frac{d^2N_+}{dE_+d\tau}\cdot exp(-\frac{7}{9}\tau)\cdot W(E_+,E_+^{out},\delta-\tau)d\tau\ ,$$

where the factor $exp(-\frac{7}{9}\tau)$ reflects the photon flux attenuation by the target.

Finally, the energy spectrum at the output of the target becomes

$$\frac{d^2N_+}{dE_+^{out}}=\int\frac{d^2N_+}{dE_+dE_+^{out}}dE_+=\frac{1}{\sigma_{tot}}\int\frac{d\sigma(E_\gamma,E_+)}{dE_+}\frac{d^2N_\gamma}{dE_\gamma dS}\cdot exp(-\frac{7}{9}\tau)W(E_+,E_+^{out},\delta-\tau)d\tau dE_+dE_\gamma dS$$

Temporary we leave this formula until the end of the next section, where the detailed properties of the photon source are investigated.

The main requirements for the photon beam is the monochomaticity and sufficient flux, because even simple estimations made, indicates the necessity for about 15 initial photos for the one positron to be generated. We will see that the undulator radiation satisfy this requirement.

Types of the undulators. In general case, the wiggler generates the axis field type as the following

$$\vec{H}_\perp(z)=\vec{e}_xH_{xm}Cos\frac{2\pi z}{\lambda_u}+\vec{e}_yH_{ym}Sin\frac{2\pi z}{\lambda_u},$$

where x, y are the transverse coordinates, z is the longitudinal one, λ_u is the period of the wiggler, H_{xm},H_{ym} are the magnetic field amplitudes in corresponding directions. The transverse motion is characterized [19] by relative velocities

$$\vec{\beta}(t')=\{\beta_{xm}Cos\Omega t',-\beta_{ym}Sin\Omega t',\overline{\beta}-(\delta\beta_z)_m Cos2\Omega t'\}$$

and the radius vector

$$\vec{r}(t')=\{x_mSin\Omega t',y_mCos\Omega t',\overline{\beta}ct'-(\delta z_m)Sin2\Omega t'\},$$

where

$\Omega=2\pi\overline{\beta}c/\lambda_u$, $\beta_{xm}=H_{ym}/H_c$, $\beta_{ym}=H_{xm}/H_c$, $(\delta\beta_z)_m=(\beta_{xm}^2-\beta_{ym}^2)/4$, $x_m=c\beta_{xm}/\Omega$, $y_m=c\beta_{ym}/\Omega$, $\delta z_m=c(\delta\beta_z)_m/2\Omega$, $\overline{\beta}=\beta(1-\beta_m^2/4)$, $\beta_m=(\beta_{xm}^2+\beta_{ym}^2)^{1/2}$, $H_c=2\pi mc^2/e\lambda_u\cong10700[G\cdot cm]/\lambda_u[cm]$, $t'=t-R(t')/c$ is the time in the moment of radiation, R is an actual distance between the particle and the point of observation, c is speed of light. These expressions give the possibility to calculate the electromagnetic field, radiated by the particle

$$\vec{E}(t) = \frac{e(\vec{n}\times((\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}))}{cR(1-\vec{n}\vec{\beta})}\Big|_{t'=t-R(t')/c},$$

where $\vec{n}$ is the unit vector in the direction of observation and all values are taken in the moment of radiation. Spectral angular distribution of the energy emitted by the particle on the area dS is determined by expression

$$\frac{\partial^2\varepsilon}{\partial\omega\partial S} = c|\vec{E}_\omega|^2 .$$

where $\vec{E}_\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty}\vec{E}(t)exp(i\omega t)dt$ is a Fourier image of the electric field as a function of the time of observation t.

Helical undulator or wiggler. In the case $H_{xm} = H_{ym} = H$, we obtain circulary polarized radiation. The common parameter, characterized the radiation in this case is

$$K = \beta_\perp\gamma = eH_\perp\lambda_u/2\pi mc^2 \cong 93.4\cdot H_\perp[Tesla]\lambda_u[m].$$

This is so called deflection parameter or the undulatority factor.

For ultrarelativistic particle the frequency of radiation ω is a function of the angle of observation and the K value

$$\omega_n = \frac{n\Omega}{1-\vec{\beta}\vec{n}} \cong \frac{2n\Omega\gamma^2}{1+K^2+\gamma^2\vartheta^2} = \frac{\omega_{n max}}{1+\frac{\gamma^2\vartheta^2}{1+K^2}},$$

where n is the number of the harmonic, ϑ is the angle of observation calculated from the forward direction, $\omega_{n max} = 2n\Omega\gamma^2/(1+K^2)$ is the frequency of each harmonic, corresponding to the radiation in the forward direction, i.e. $\vartheta = 0$. We will use a parameter $s = \omega_n/\omega_{n max}$ [11,12] (what is the fraction of the energy with respect to its maximal possible value), so the angle of observation and the energy of the photon are connected by the relation

$$\frac{\omega_n}{\omega_{n max}} = s = \frac{1}{1+\frac{\gamma^2\vartheta^2}{1+K^2}}, \text{ or } \gamma\vartheta = \sqrt{(1+K^2)(1-s)/s}.$$

Notice here that this angle does not depend of the harmonic number.

The solid angle can be expressed as $do = 2\pi\cdot Sin\vartheta\cdot d\vartheta \cong \pi d\vartheta^2 = -\frac{\pi(1+K^2)}{s^2\gamma^2}ds$. So the spectral distribution of the photons emitted and its degree of circular polarization can be expressed as follows [11,12,19]

$$\frac{dN_{\gamma n}}{ds} = \frac{\pi(1+K^2)}{s^2\gamma^2}\frac{dN_{\gamma n}}{do} = 4\pi\alpha nM\frac{K^2}{1+K^2}F_n(K,s),$$

$$\xi_{2n} = \frac{\sqrt{1+K^2}}{K}\frac{2s-1}{\sqrt{s(1-s)}}\frac{J_n(n\kappa)J_n'(n\kappa)}{F_n(K,s)},$$

where $\kappa = 2K\sqrt{s(1-s)/(1+K^2)}$, $F_n(K,s) = J_n'^2(n\kappa) + \frac{1+K^2}{4K^2}\frac{(2s-1)^2}{s(1-s)}J_n^2(n\kappa)$, J_n and J_n' is the Bessel function and its derivative, M is the number of the wiggler periods, $\alpha = e^2/hc = 1/137$ is a fine structure constant.

In dipole approximation, $K \le 1$, using expansion of the Bessel function, we came to expressions [11]

$$F_n(K,s) = \frac{(nK)^{2(n-1)}}{2(n-1)!(n-1)!}\left(\frac{s(1-s)}{1+K^2}\right)^{n-1}\left(1-2s+2s^2-\frac{2n}{n+1}\cdot\frac{K^2}{1+K^2}s(1-s)[1+n(1-2s+2s^2)]\right)$$

$$\xi_{2n}(K,s) = \frac{(2s-1)\left(1-\frac{2n^2+1}{n+1}\cdot\frac{K^2}{1+K^2}s(1-s)\right)}{\left(1-2s+2s^2-\frac{2n}{n+1}\cdot\frac{K^2}{1+K^2}s(1-s)[1+n(1-2s+2s^2)]\right)}.$$

On Fig. 5 there is represented the polarization as a function of K, s.

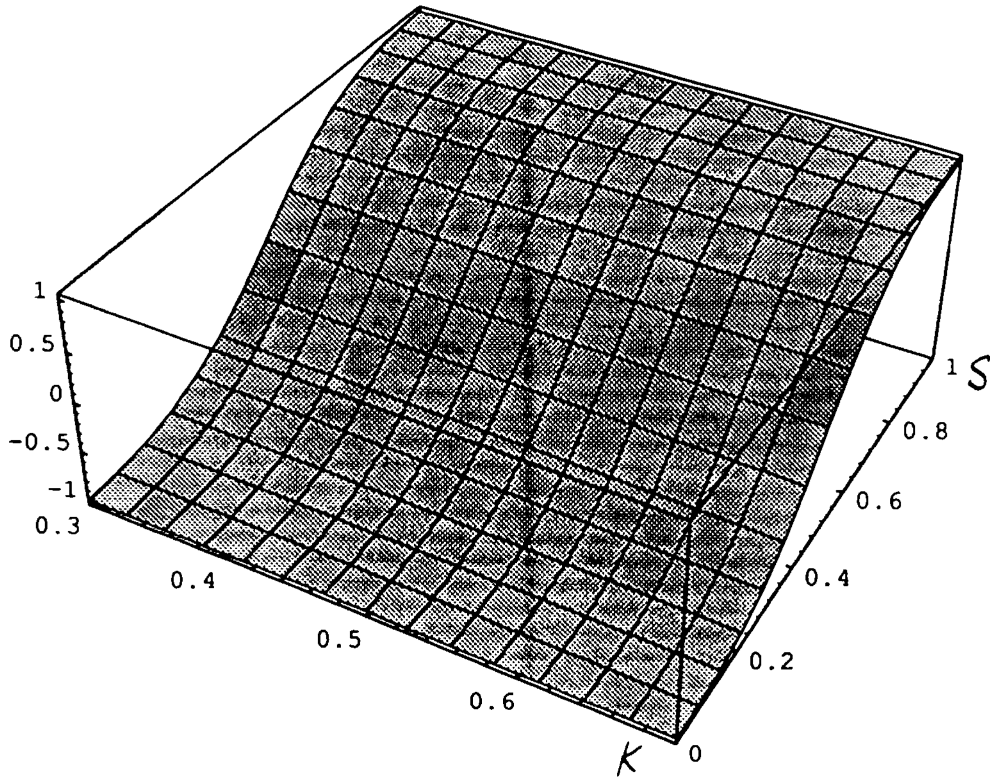

Fig. 5A. The polarization of the radiation emitted as a function of the K and $s = \omega_n/\omega_{n\,max}$ for the first harmonic.

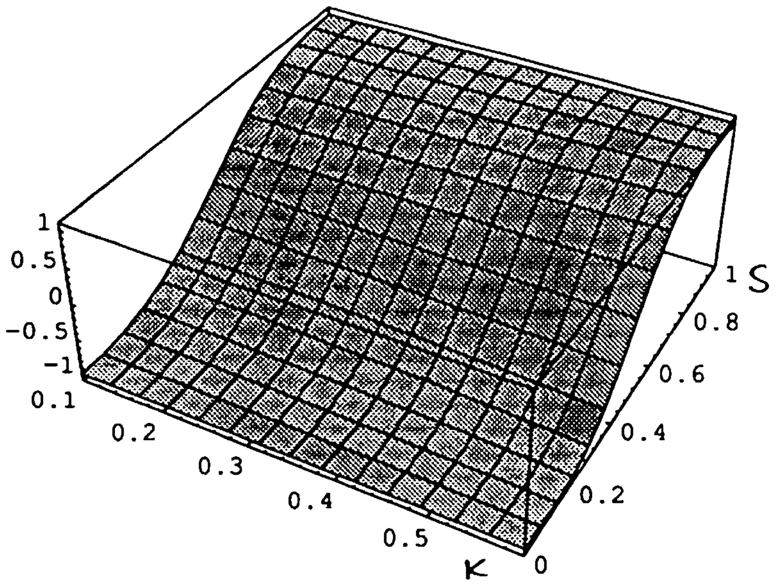

Fig. 5B. The polarization of the radiation emitted as a function of the K and $s = \omega_n / \omega_{n\,max}$ for the second harmonic.

For harmonics $n = 1,2$ in approximation $K^2 << 1$ it follows from here

$$F_1(K,s) \equiv \frac{1}{2}(1-2s+2s^2), \quad F_2(K,s) \equiv 2s(1-s)(1-s+2s^2)K^2, \quad \xi_{21} = \xi_{22} = \frac{2s-1}{1-2s+2s^2}$$

These expressions as a function of the angle can be represented as

$$F_1(\vartheta) = \frac{1+\gamma^2\vartheta^2}{2(1+\gamma^2\vartheta^2)^2}, \quad F_2(\vartheta) = 2(K\gamma\vartheta)^2 \frac{1+\gamma^4\vartheta^4}{(1+\gamma^2\vartheta^2)^4}, \quad \xi_{21} = \xi_{22} = \frac{1-\gamma^4\vartheta^4}{1+\gamma^4\vartheta^4}.$$

One can see from expression for angular dependence of polarization, that the polarization becomes linear ($\xi_{21} = \xi_{22} \cong 0$), when the angle of observation $\vartheta \cong 1/\gamma$.

Let us compare this angle with the maximal angular spread of the particles in the beam. The last is given by expression $\vartheta_m \cong \sqrt{\gamma\varepsilon / \gamma\beta_w}$, where $\gamma\varepsilon$ is a normalized emittance, β_w is an envelope function value in the wiggler. As the envelope function is of the order of the wiggler length $\beta_w \approx 100\, m$, then for $\gamma\varepsilon = 10^{-4}\, cm \cdot rad$, $\gamma \approx 4 \cdot 10^5$ (200 *GeV*), $1/\gamma \cong 2.5 \cdot 10^{-6}$, one can estimate $\vartheta_m \cong \sqrt{10^{-4}/4 \cdot 10^9} \approx 1.6 \cdot 10^{-7}$, so $\gamma\vartheta_m \cong 0.06$. Thus there is no input to the photon flux on the target due to the angular spread in the beam. The beam dimensions in the wiggler will be of the order $r_\perp \cong \sqrt{\gamma\varepsilon\beta_w / \gamma} \approx \sqrt{10^{-4} \cdot 10^4 / 4 \cdot 10^5} \approx 1.6 \cdot 10^{-3} cm$. The 10σ criteria gives $10 \cdot r_\perp \approx 0.016 cm$ or 0.16 *mm*, what gives the idea about possible aperture of the wiggler and also an influence of the field inhomogeneties across the aperture.

We will be interesting in the number of the photons emitted by the particle on the n-th harmonic in the range of relative frequency from $s = 1$ (corresponding the straight forward direction), to the threshold value $s = s_t$. This threshold value defined by the maximal possible angle of incoming radiation, selected by the diaphragm $\gamma\vartheta_t = \sqrt{(1+K^2)(1-s_t)/s_t}$. The number of the photons radiated in the range discussed is

$$N_{\gamma n}(K,s_t) = \int_{s_t}^{1} \frac{dN_{\gamma}}{ds} ds = 4\pi\alpha n M \frac{1+K^2}{K^2} \int_{s_t}^{1} F_n(K,s_t) = 4\pi\alpha n M \frac{1+K^2}{K^2} \Phi_n(K,s_t),$$

In approximation $\kappa = 2K\sqrt{s(1-s)/(1+K^2)} \leq 1$ ($K \leq 1$ or/and $\gamma\vartheta \leq 1$) for harmonics with the numbers $n = 1,2$ one can obtain [11]

$$\Phi_1(K,s) = \frac{1}{6}(1-s_t)(2-s_t+2s_t^2) - \frac{K^2}{2(1+K^2)}(1-s_t)^2\left(\frac{4}{15}+\frac{8}{15}s_t-\frac{1}{5}s_t^2+\frac{2}{5}s_t^3\right)$$

$$\Phi_2(K,s) = \frac{K^2}{10(1+K^2)}(1-s_t)^2\left[(1+2s_t-2s_t^2+4s_t^3) - \frac{20K^2(1-s_t)}{21(1+K^2)}\left(\frac{2}{15}+\frac{2}{5}s_t+\frac{4}{5}s_t^2-s_t^3+2s_t^4\right)\right].$$

The number of the photons on the first and second harmonic as a function of K, s_t is represented on Fig. 6A, Fig. 6C and Fig. 6B.

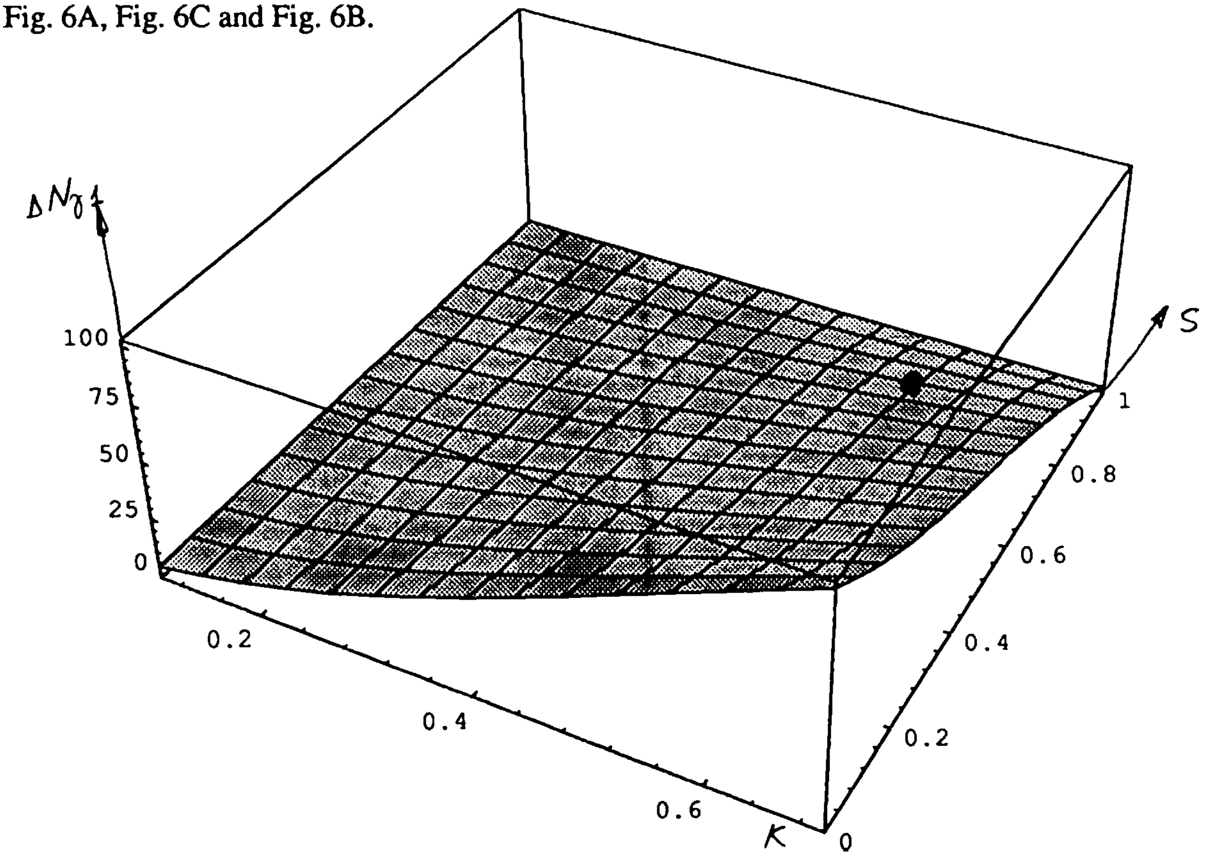

Fig. 6A. The number of the quants radiated on the first harmonic as a function of K and $s = \omega_n / \omega_{n\,max}$ in all possible energy range.

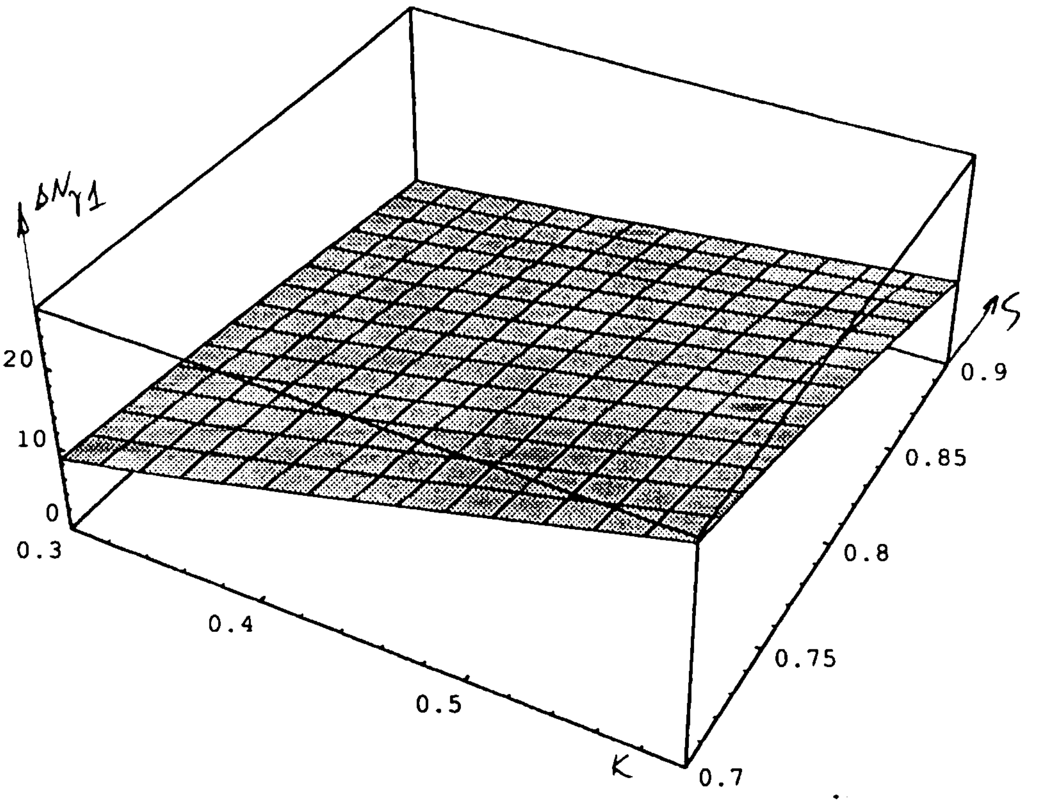

Fig. 6B. The number of the quants radiated on the first harmonic as a function of K and $s = \omega_{x} / \omega_{x\,max}$ in the interval what may be interested for practical conversion system.

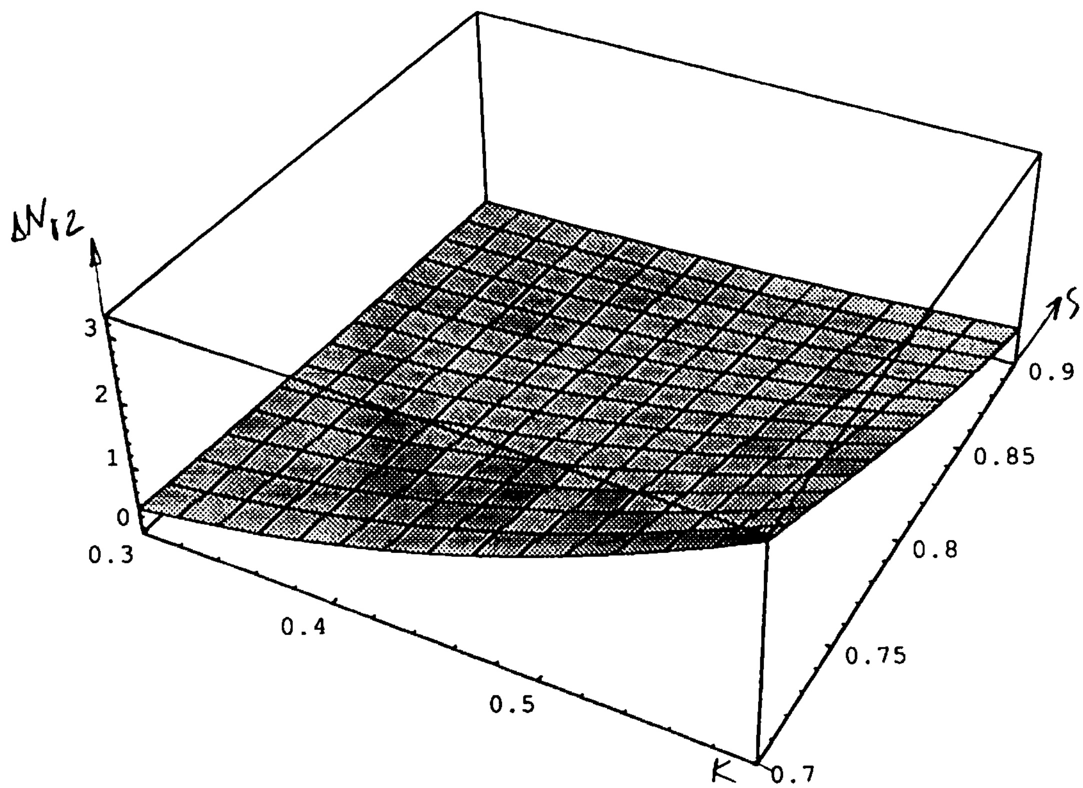

Fig. 6C. The same as above for the second harmonic.

For preparing the gamma flux of maximal possible polarization, the angular separation is necessary. That formally gives the same threshold parameter in description of mean level polarization of the flux.

An averaged value of circulary polarization of the photons concentrated in the solid angle between 0 and $\gamma\vartheta_t = \sqrt{(1+K^2)(1-s_t)/s_t}$ can be evaluated as

$$< \xi_{2n} > = \frac{\int_{s_t}^{1} \xi_{2n}(s) \frac{dN_n}{ds} ds}{\int_{s_t}^{1} \frac{dN_n}{ds} ds} = \frac{\int_{s_t}^{1} \xi_{2n}(s) \frac{dN_n}{ds} ds}{N_n}.$$

Substitute here the expressions for ξ_{2n}, one can obtain in approximation $K^2 \leq 1$

$$< \xi_{21} > = \frac{3s_t}{2 - s_t + 2s_t}, \quad < \xi_{22} > = \frac{5s_t}{1 + 2s_t - 2s_t^2 + 4s_t^3}.$$

These functions are represented on Fig. 7.

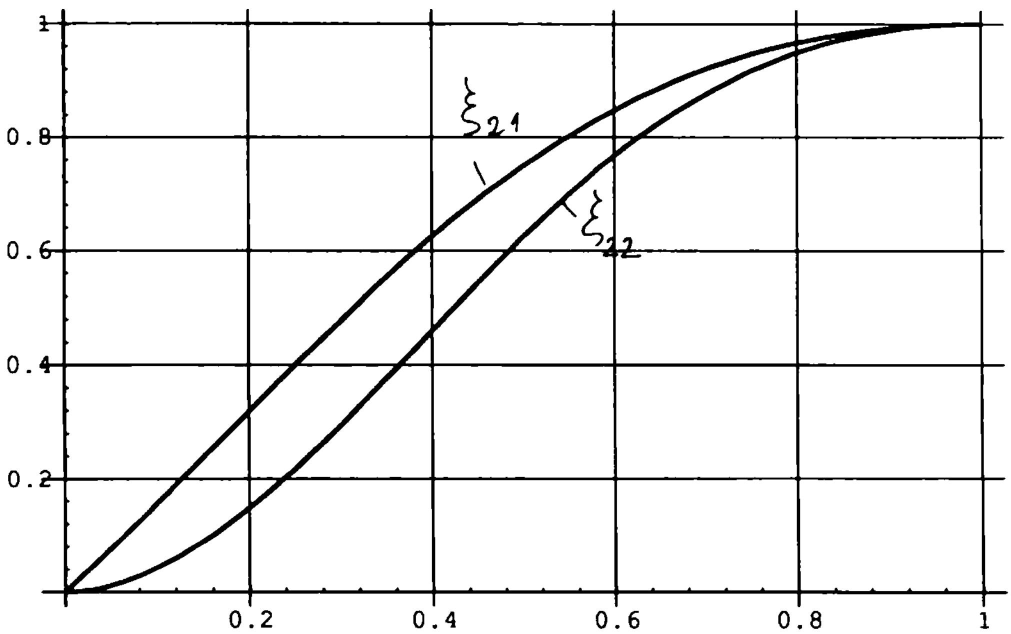

Fig. 7. The averaged polarization of the photon flux as the function of the energy interval of selection. $s_t = 0$ corresponds to the absence of any selection, $s_t = 1$ corresponds to the straight forward direction. $s_t = 0.8$ corresponds to selection in 20% of the energy interval down from the maximal possible energy of the quanta for corresponding harmonics

If selection system collects only in the energy interval 20% of maximal possible energy down from the maximum, i.e. $s_t \cong 0.8$, then $< \xi_{21} > \cong 0.96$, $< \xi_{22} > \cong 0.95$. For $s_t \cong 0.7$ (30% interval) $< \xi_{21} > \cong 0.92$, $< \xi_{22} > \cong 0.89$. These figures indicates that the level of polarization is rather high.

The corresponding maximal values of the angles for selection (minimal value is zero for the forward direction) are

$$\vartheta(s_t = 0.7) = \sqrt{(1+K^2)(1-s)/s} \cong \frac{0.65\sqrt{1+K^2}}{\gamma} \text{ and } \vartheta(s_t = 0.8) \cong \frac{0.5\sqrt{1+K^2}}{\gamma}.$$

If the distance L between the end of helical wiggler and the target is $L \approx 200\,m$, $\gamma = 4\cdot 10^5$ (200 *GeV*), $1/\gamma \cong 2.5\cdot 10^{-6}$, $K^2 \approx 0.25$, $s_t \cong 0.8$, then the corresponding radius of the diaphragm at the face of target need to be $r_D \cong L\cdot\vartheta_t \cong 2.8\cdot 10^{-2}\,cm$, what gives the diaphragm diameter 0.56 mm.

So, in the *first approximation*, the level of polarization of the positrons created, can be estimated by averaging the function $f(E_+,E_-)$, describing the longitudinal polarization of the positron

$$<\vec{\zeta}_1> \cong <\xi_2>\cdot<f(E_+,E_-)>\cdot\vec{n}_1 .$$

For $E_+,\ E_- > E_{+max}/2$, where $E_{+max} = E_{+max}(s) = s\hbar\omega_{nmax} - 2mc^2$, the function $f(E_+,E_-)$ can be approximated

$$f(E_+,E_{+max}) \cong 1-2\left(\frac{E_{+max}-E_+}{E_{+max}}\right)^2 = 1-2\left(\frac{s\hbar\omega_{nmax}-2mc^2-E_+}{s\hbar\omega_{nmax}-2mc^2}\right)^2 = 1-2(1-x)^2,$$

where $x = \frac{E_+}{E_{+max}}$. By averaging this expression one can obtain

$$<f(E_+,E_{+max})> = \frac{\int_\Delta^1 [1-2(1-x)^2]dx}{\int_\Delta^1 dx} = 1-\frac{2}{3}(1-\Delta)^2,$$

where $\Delta = \frac{E_{+cap}}{E_{+max}}$, E_{+cap} is the minimal energy of the positron, captured by the focusing system after the target. For $E_{+cap} \cong 0.5E_{+max}$ (the positrons in the energy interval 50% down to the maximal possible energy) $<f(E_+,E_{+max})> = 1-\frac{2}{3}(1-0.5)^2 = 0.83$, so

$$\left|<\vec{\zeta}_1>\right| \cong <\xi_2>\cdot<f(E_+,E_-)> \cong 0.96\cdot 0.83 = 0.8 .$$

i.e. rather high level of polarization. In next approximation we need to take into account that there are few of particles with maximum energy according to the $G(E_+,E_\gamma)$ dependence

$$\left|<\vec{\zeta}_1>\right| \cong \frac{\int_{E_{+cap}}^{E_{+max}} \xi_2(E_\gamma)f(E_+,E_-)\frac{d\sigma(E_\gamma,E_+)}{dE_+}dE_+}{\int \frac{d\sigma(E_\gamma,E_+)}{dE_+}dE_+},$$

where N_+ is the number of positrons in the energy interval from the maximal possible $E_{+max} = sE_{\gamma max} - 2mc^2$ to E_{+min}. Notice here that the energy distribution must be taken in the moment of pair production without recalculation with the probability W. The changing of the ς_l, when the particle goes from the point of creation τ to the output surface of the target is described by the length of depolarization $l_{dep} \cong 3X_0$ [17], so $\varsigma_{l out} = \varsigma_l \cdot \exp(-\frac{\delta - \tau}{3X_0})$, and the final expression has a form, what includes the spectral properties of the photon flux

$$|<\zeta_l>| \cong \frac{1}{\delta} \frac{\iiint \int_{E+cap}^{\delta} \xi_2(E_\gamma) f(E_+, E_-) \frac{d\sigma(E_\gamma, E_+)}{dE_+} \frac{d^2 N_\gamma}{dE_\gamma dS} \exp(-\frac{7}{9}\tau) \exp(-\frac{\delta - \tau}{3X_0}) dE_+ d\tau dE_\gamma dS}{\int \frac{d\sigma(E_\gamma, E_+)}{dE_+} \frac{d^2 N_\gamma}{dE_\gamma dS} \exp(-\frac{7}{9}\tau) dE_+ d\tau dE_\gamma dS}$$

We will see, that the thickness of the target is of the order of $\delta \le X_0 / 2$, so the factor of radiativ depolarization in the target after creation in less than $\exp(-\frac{1}{2 \cdot 2 \cdot 3}) \cong 1 - \frac{1}{12} \cong 0.917$, where additional factor 1/2 reflects the mean path length of the individual positron in the target. Numerical calculations shows that the mean path length even less than 1/2 reflecting the total tendency that the particles created at the out side of the target have more probability to come out of the target.

The spectral angular distribution of the gammas from undulator has a form [19]

$$\frac{d^2 N_\gamma}{dE_\gamma dS} = \sum_n \frac{d^2 N_{\gamma n}}{dE_\gamma dS} = \sum_n \frac{1}{E_{\gamma n}} \frac{d^2 \varepsilon_n}{dE_\gamma dS} = \sum_n \frac{1}{E_{\gamma n} R^2} \frac{d^2 \varepsilon_n}{dE_\gamma do} = \sum_n \frac{1}{E_{\gamma n} R^2(\gamma\vartheta)} \frac{M}{E_{\gamma 1}} \frac{\partial \varepsilon_n}{\partial o} Sinc^2 \sigma_n,$$

where $Sinc(x) = Sin(x)/x$, $\sigma_n = \pi n M \frac{(\omega - \omega_n)}{\omega_n}$, M is the number of periods in the undulator. When $M \gg 1$, $Sinc^2 \sigma_n \cong \frac{E_{\gamma 1}}{M} \delta(\omega - \omega_n(\gamma\vartheta))$, so

$$\frac{d^2 N_{\gamma n}}{dE_\gamma dS} = \frac{1}{E_{\gamma n}} \frac{d^2 \varepsilon_n}{dE_\gamma dS} = \frac{1}{\hbar \omega_n(\gamma\vartheta)} \frac{1}{R^2(\gamma\vartheta)} \frac{\partial \varepsilon_n}{\partial o} \delta(\omega - \omega_n(\gamma\vartheta)).$$

The last expression reflects the fact that the angular and spectral distributions of the radiation are connected due to the fact that the energy of the photon emitted is a function of the angle. The same distribution must be substituted into the formula for spectral density of the positrons

$$\frac{d^2 N_+}{dE_+^{out}} = \frac{1}{\sigma_{tot}} \int \frac{d\sigma(E_\gamma, E_+)}{dE_+} \frac{d^2 N_\gamma}{dE_\gamma dS} \cdot \exp(-\frac{7}{9}\tau) W(E_+, E_+^{out}, \delta - \tau) d\tau dE_+ dE_\gamma dS$$

Exactly speaking, the formulas represented above are valid is the case when the distance L from the end of the wiggler to the target is much bigger, than the length of the wiggler itself

$L_u = M\lambda_u \ll L$. Otherwise we need to average the flux density falling onto fixed point over different angles of coming radiation arising from the different positions of radiated electrons along the undulator. The geometry of averaging is represented on the following Fig. 8.

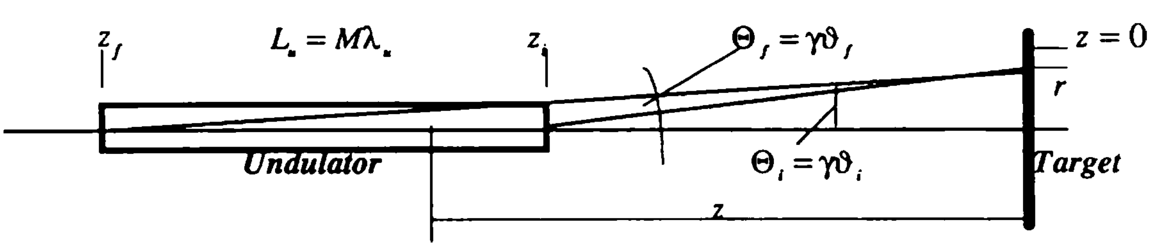

Fig. 8. The geometry of irradiation of the target, when the distance from the end of the undulator is comparable with the length of the undulator itself. $z = 0$ corresponds to position of the target, z_i corresponds to beginning of the undulator, z_f corresponds to it's end.

So the spectral density of the energy falling onto the converter 's area dS becomes [11,12]

$$\frac{d^2N_{\gamma n}}{dE_\gamma dS} \to \frac{1}{L_u}\int_{z_i}^{z_f}\frac{d^2N_{\gamma n}}{dE_\gamma dS}dz = \frac{1}{M\lambda_u}\int_{z_i}^{z_f}\frac{1}{R^2(\vartheta)}\frac{\partial N_n}{\partial o}\delta(\omega-\omega_n(\vartheta))dz = \frac{1}{M\lambda_u r}\int_{\vartheta_i}^{\vartheta_f}\frac{\partial N_n}{\partial o}\delta(\vartheta-\vartheta(\omega_n))\frac{\partial\vartheta}{\partial\omega}d\vartheta\,,$$

where it was used that the angle ϑ is connected with the transverse coordinate r of the point on the target surface (see Fig. 8) by the relation $\vartheta = -\tan^{-1}(\frac{r}{z}) \cong -\frac{r}{z}$. In the same approximation it was used, that $R(\vartheta) \cong z$, $\frac{dz}{z^2} = -\frac{d\vartheta}{r}$. Substituted also $\delta(\omega-\omega_n(\vartheta)) = \delta(\vartheta-\vartheta(\omega_n))\cdot\frac{\partial\vartheta}{\partial\omega}$. Using the relations $\frac{\partial N_n}{\partial o} = 4\alpha nM\frac{s^2\gamma^2K^2}{(1+K^2)^2}F_n(K,s)$ and expressing the frequency with the parameter s used above one can finally evaluate the number of photons irradiating the target on n-th harmonic [11,12]

$$N_n = 2\pi\iint\frac{dN_n}{dSdE_\gamma}dSdE_\gamma = \frac{4\pi\alpha\gamma K^2}{\lambda_u(1+K^2)^{3/2}}\int_0^{r_m} rq_n(K,r_m)dr = \frac{4\pi\alpha\gamma K^2}{\lambda_u(1+K^2)^{3/2}}Q_n(K,r_m)\,.$$

where $q_n(K,s) = \frac{n}{r}\int_{s_i}^{s_f}\sqrt{\frac{s}{1-s}}F_n(K,s)ds$, $s_i = \cfrac{1}{1+\cfrac{\gamma^2(r/z_i)^2}{1+K^2}}$, $s_f = \cfrac{1}{1+\cfrac{\gamma^2(r/z_f)^2}{1+K^2}}$ and r_m is the radius of the target (the radius of the diaphragm installed before the target). For the first harmonic

$$N_{\gamma 1} = \frac{4\pi\alpha\gamma K^2}{\lambda_u(1+K^2)^{3/2}}\left[\frac{1}{2}\frac{\gamma r_m^2}{\sqrt{1+K^2}}\left(\frac{1}{z_f}-\frac{1}{z_i}\right)-\frac{5\gamma^3r_m^4}{24(1+K^2)^{3/2}}\left(1+\frac{4}{5}\frac{K^2}{1+K^2}\right)\left(\frac{1}{z_f^3}-\frac{1}{z_i^3}\right)\right]$$

For the number of the positrons created in the energy interval $\Delta E_{+cap} = E_{\gamma n}^{max} - 2mc^2 - E_{+cap}$ by the undulator radiation on the n-th harmonic one can obtain

$$\Delta N_{+n}(E_{+}^{out}, E_{+}^{max}) \cong \frac{\alpha K^2 \delta}{c\gamma \log(183 Z^{-1/3})} \Gamma_{*},$$

where

$$\Gamma_{*} = \int_{0}^{r_1} dr \int_{s_t}^{s_1} \frac{F_n ds}{\sqrt{s(1-s)}} \int_{E_{-}}^{E_{+}} G(E_{+}, E_{+}^{max}) \hat{Y}(E_{+}, E_{+}^{out}) dE_{+},$$

$$\hat{Y} = \frac{1}{\delta} \int_{E_{+}^{-}}^{E_{+}} dE_{+}^{out} \int_{0}^{\delta} I(E_{+}, E_{+}^{out}) d\tau = \frac{1}{\delta} \int_{E_{+}^{-}}^{E_{+}} dE_{+}^{out} \int_{0}^{\delta} \exp(-\frac{7}{9}\tau) W(E_{+}, E_{+}^{out}, \delta - \tau) d\tau,$$

and function $\hat{Y}$ defines the share of the positrons produced with the energy E_{+}, that have the out energy in the interval $\{E_{+}, E_{+}^{out}\}$. One can evaluate [11]

$$\hat{Y}\left(\frac{E_{+}}{E_{+}^{out}}\right) \cong \frac{\ln 2}{\delta \ln \Delta}(1 - \Delta^{\delta/\ln 2}),$$

where $\Delta \cong \frac{E_{+} - E_{+}^{out}}{E_{+}}$. For thin target $I(E_{+}, E_{+}^{out}) \cong \delta(E_{+} - E_{+}^{out})$ and hence $\hat{Y} \approx 1$. Finally

$$\Delta N_{+n}(E_{+}^{out}, E_{+}^{max}) \cong \frac{\alpha K^2 E_{+}^{max} \delta Q_n}{c\gamma n \log(183 Z^{-1/3})} \int_{\zeta_{-}}^{1} G(\zeta) d\zeta,$$

where $\zeta_{cap} = \frac{E_{+}^{cap} - 2mc^2}{s_t E_{\gamma}^{max} - 2mc^2}$. For n=1, $r_{-} = \kappa r^{*} = \kappa z_f \frac{\sqrt{1+K^2}}{\gamma}$ the evaluation is the following

$$\Delta N_{+1} \cong 3 \cdot 10^{-2} \kappa^2 M \delta \frac{K^2}{1+K^2} \frac{z_f}{z_i} (1 - \zeta_{cap})$$

For $\kappa = 1/2$, $M = 10^4$, $\delta = 0.2$, $K = 1$, $z_f = M\lambda_u = 2z_i$, $\zeta_{cap} = 0.7$ $\Delta N_{+1} \cong 5$.

The formulas represented above gives to anyone a possibility to estimate the number of the photons, its average polarization and the number of positrons and its average polarization.

Codes for calculation the efficiency of the photons interaction with media. We interrupt here for discussion about existing numerical codes what are able to do this. First of all there are the codes used for modeling the high energy physics phenomena, for example EGS type codes [18 a-b]. The output file of UNIMOD2 (an analog of EGS) is used by the code CONVER [18c] for rather fast calculations with the targets having a different size and form. This output file, obtained on the big computer, having typically 2 MB of memory and describing the individual history of about 6000-10000 incoming photons (depending of the accuracy required), can be preloaded in a Personal Computer. For example, the 486DX-2 66 MHz notebook computer requires about 5 minutes (including the input) to obtain efficiency as a function of the thickness of the target. In [13] described some further modifications, including the codes, which uses the output files from CONVER for the further analysis such as energy distributions, space and angular distribution, distribution of the path lengths, polarization in the target and so on. Al these codes also working on

PC. Below the results, obtained with these codes, are described [13]. The main output of these considerations that the efficiency of the particle production could be made around 6% for each initial photon. The mean polarization can reach 70% total.

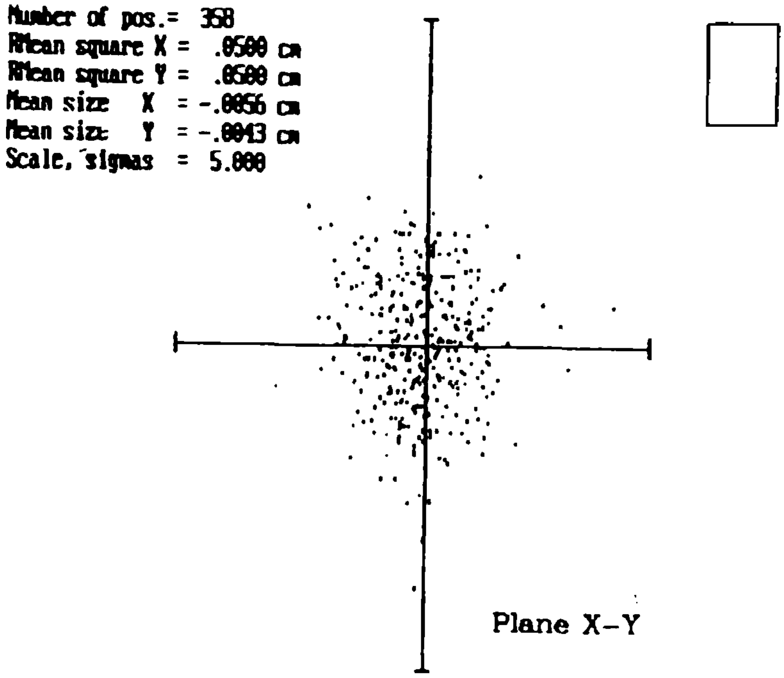

Fig.9 The transverse space distribution of the positrons at the output surface of the target.

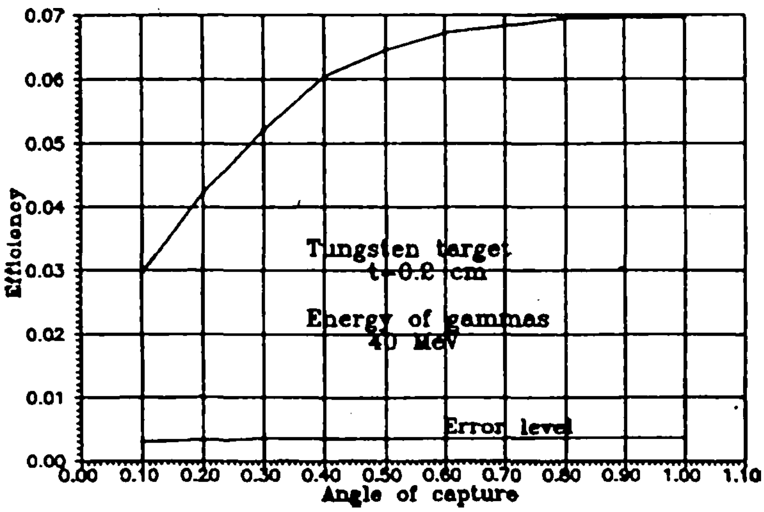

Fig.10. Efficiency of the pair production as a function of the captured angle.

Some special considerations was made to estimate the energy deposition in the material of the target. It was found that this value is around 250 Mev/gram at the end of the target. The thickness of the target was about 0.2 cm. This yield the temperature gain of the order 116 deg for the beam with 10^{10} positrons in the bunch.

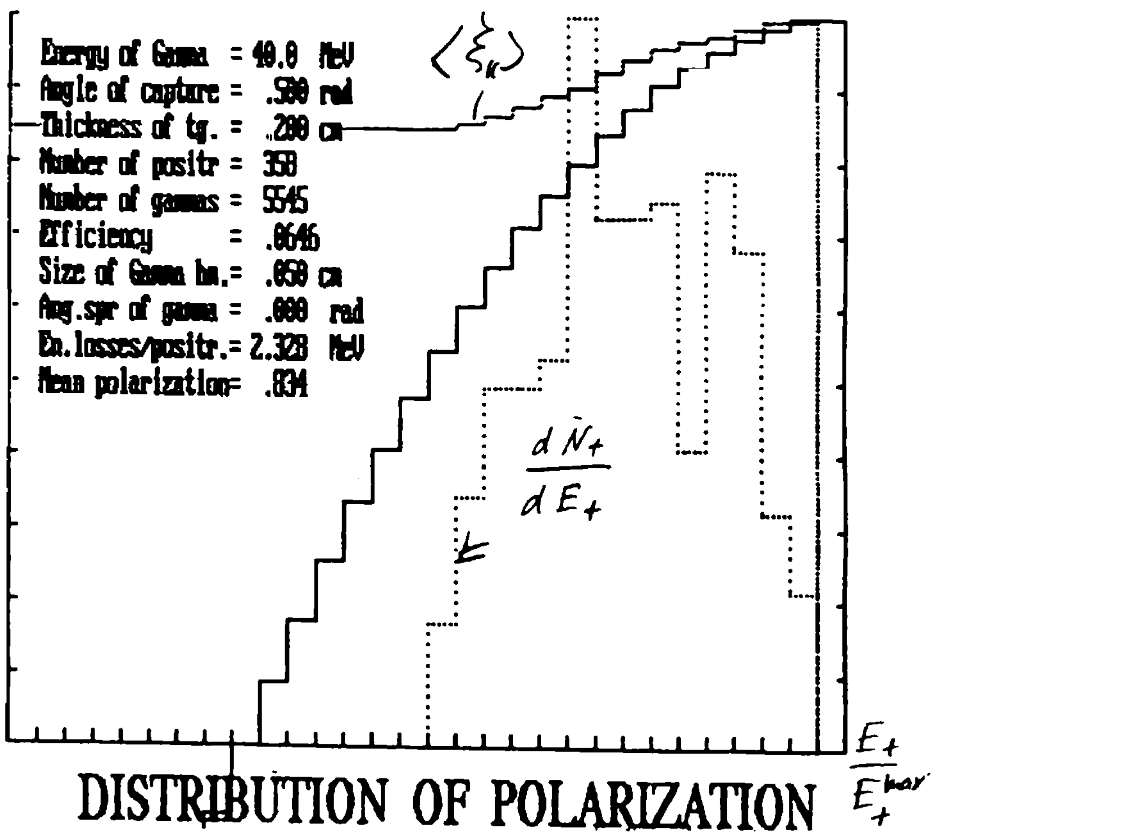

Fig. 11. The energy distribution and polarization. The energy distribution is shown at the moment of positron creation.

The technical proposition for helical field generation was made in [20]. This is a bifilar helix with currents opposed. There are some computer codes for the helical field design [21]. Basically it the same as the codes for calculation of two dimensional fields, but with substitution of coordinate dependence

$$\xi = x + iy \rightarrow \xi e^{-i\varphi} = \xi \cdot exp(-i2\pi \frac{z}{\lambda_*}),$$

what is, basically, the twist with the wiggler period.

The progress in design of short period wigglers with high field one can find in [22-24]. In [24] the results of calculations and testing the models with the period 0.7 and 1 cm are represented.
The photography of the tested superconducting undulator is represented on Fig.12. One interesting moment what can be noticed here is that this undulator was supplied with the captured flux. That was made with the help of superconducting transformer. The current in one of 22 turn coil was around 200 A. The impulse undulator (0.7 cm period) has a current around 10 kA with the pulse duration about 50 μsec. the voltage was 1.19 kV.

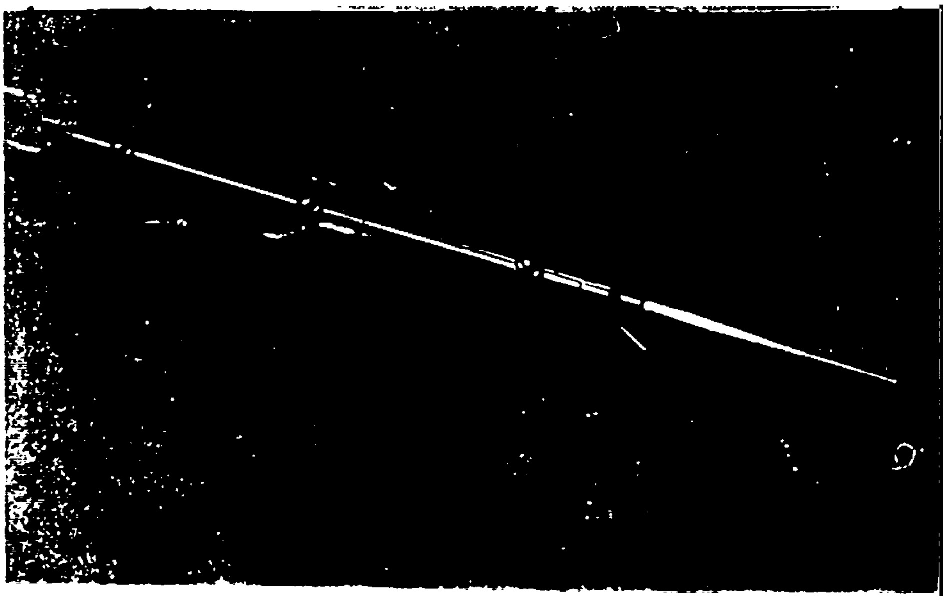

Fig. 12. The 30 cm long superconducting undulator with period 10 mm and the axis field ~ 5 kG.

The wall illumination was considered in [25]. The resistive wall instability if the beam, moving in the vacuum chamber of the wiggler considered in [26]. This looks as weak requirement.

One interesting possibility is connected with further utilization of the gamma -beam, passed through a thin converter. The attenuation coefficient $k \approx \exp(-7/9\tau)$ for $\tau \approx 0.5$ is around 0.68, so in principle the second target can be used as well [13]. The combining schemes is based on the possibility to stack the bunches with slightly different energy in the longitudinal space. On Fig. 13 there is represented one of these schemes. Here the gamma beam from the wiggler is coming from the

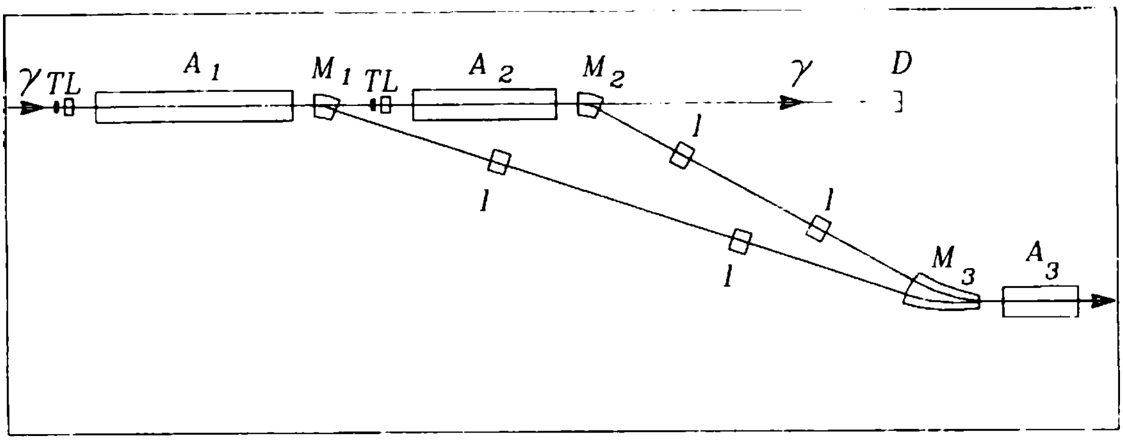

Fig. 13. The combining scheme.

left side and illuminates the target T . The focusing lens L collects the particles and adjusts for further optics. An acceleration section A_1 gives the energy E_1 for the secondary beam. This beam of positrons is bend with the help of magnet M_1. The magnets M_1 and a part of the magnet M_3 with the quadrupoles l make an achromatic parallel shift of the first beam [27]. The second acceleration section A_2 gives to the beam, collected from the second target, the lower energy E_2 . The magnet M_2 with the part of the magnet M_3 and the lenses, adjusted for parallel shift of the beam with the energy E_2. The difference in the path lengths of these two lines in an integer of the wavelength and a half of the section A_3. This section eliminates the energy difference. D is the gamma beam dump.

2.b. A plane wiggler with a sqew dipole field. One interesting class of wigglers considered in [30-32]. This is so called the wigglers with elliptical polarization. This wigglers can be very effective, unfortunately the numerical analyses was not made yet. The basical internees arises from the possibility to have a big undulatority factor in one plane, while in the other plane the undulatority factor is around 1. this can reduce the length of the wiggler.

Micropole wigglers described in [35]. There is no visible applications for its utilization in polarized particle production.

2.c. Collection of the particles with the help of flux concentrator and the lithium lens one can find in [33-34]. The first selection system described [22] used a lithium lens and a diaphragm as energy separator : the particles with the lower energy was overfocused.

3. Pair production in the high external electromagnetic field.

3.a. The laser flash mostly.

Historical review. The idea of direct pair generation in vacuum comes from [36], where the critical field strength E_{cr} was estimated in a pure electrostatic field. Namely $eE_{cr}\lambda_c \approx 2mc^2$, where $\lambda_c = \hbar / mc = 3.86 \cdot 10^{-11}\ cm$ is the Compton wavelength of electron. The last means, that the work, made by electric field on the distance of the Compton wavelength, is equal to the doubled rest energy of an electron. The numerical value $E_{cr} \cong m^2c^3 / e\hbar \cong 1.3 \cdot 10^{16}\ V / cm$. In [37] the proposal made, how to use the *focused* laser beam instead of the static field. In [38] the *multi-photon*, or *coherent* pair production in an alternating field was described. The next step was made in [39], where discussed the method of pair generation by electron, accelerated by intense circulary polarized laser light in plasma. The electrons generated the pairs as a result of interaction with the nuclei. The final approach to the problem of the pair production with help of intense laser flash was made in [40]. Let us discuss it more carefully.

The method proposed in [40] has some common moments with the production of $\gamma\gamma$ collisions by means of Compton back scattering of the incoming light from FEL by the relativistic electrons [41]. For $\gamma\gamma$ collisions the pair production is a background process, what yields a restriction in the photon energy.

Generally, the high energy beam (the beam after collision can be used also) are compressed in a small size and collides with an appropriate photon beam obtained from a high power laser of any type. In a strong field on the first stage the high energy photons are created. On the second stage these photons are interact with the same electromagnetic field of the laser flash and converts into electron-positron pairs. Some specific moments connected with strong quantum regime and multiphoton interaction was taken into account.

The main criteria what indicates the coherent regime of interaction is the deflection angle $\theta_D \approx \Delta p / p$ compared with the angle of radiation $\approx 1/\gamma$. From equation of motion one can obtain $\Delta p \approx 2eE / \omega$, where the influence of the magnetic field as well as the electric field was taken into account. So $\theta_D \approx \Delta p / p \cong \dfrac{2eE}{\omega mc\gamma}$. If $\dfrac{2eE}{\omega mc} >> 1$ then the deflection angle is much bigger than the divergence of radiation, one can describe it as a multiphoton absorption (synchrotron radiation). In a case when the angle of deflection is less than $\approx 1/\gamma$, or $\frac{2eE}{\omega mc} = \frac{2eE\lambda}{mc^2} << 1$, then the one photon interaction becomes significant. The last relation also indicates that in this case the work of the electric field on the distance of the wavelength is negligible. Another important parameter what describes the interaction is

$$Y = \gamma \frac{2E}{B_c} = \gamma \frac{2E}{(m^2c^3)/e\hbar} = \gamma \frac{2eE\hbar}{m^2c^3} = \gamma \frac{2eE(\hbar / mc)}{mc^2} = \gamma \frac{2eE\lambda_C}{mc^2},$$

where $B_c = m^2c^3 / e\hbar \approx 4.4 \cdot 10^{13}\ G$ is the Schwinger field. The physical sense of the Y parameter is the increased by γ factor the work of electromagnetic field on the Compton wavelength λ_C of an electron. The multiphoton regime can be either classical, when $Y <<1$, or quantum, when $Y>>1$. For typical laser bust needed for the purpose of gamma-production [40], energy of the laser flash $J \cong 15\, Joules$, laser wavelength $\lambda \cong 350\, nm$, pulse length $\sigma_T \approx 0.5\, ps$ or 150 μm space duration, the focused beam with a radius $r_F \approx 2\ \mu m$ with a depth of focus 70 μm yields the electric field value $E \cong \left[\frac{120\pi J}{\sqrt{2\pi} c \sigma_T \pi r_F^2} \right]^{1/2} \cong 2.4 \cdot 10^{13}\ V/m$, and $B = E/c \cong 0.8 \cdot 10^9 G$. So, the parameter $\gamma\theta_D = \frac{2eE}{\omega mc} \approx 2.7$, Y ~20 and, hence, here we have multyphoton quantum regime. The number of the photons emitted by the initial electron per unit length can be evaluated [49]

$$\frac{dn_\gamma}{dz} \cong \frac{5}{2\sqrt{3}} \frac{\alpha Y}{\lambda_C \gamma} \frac{1}{\sqrt{1+Y^{2/3}}}.$$

The photons emitted are interact with the same electromagnetic field . The energy spectrum of the pair production is the following [40,43]

$$\frac{d^2 n_+}{dx dz} = \frac{1}{\sqrt{3}\pi} \frac{\alpha}{\lambda_C \gamma'} \left\{ \left[\frac{1-x}{x} + \frac{x}{1-x} \right] K_{2/3}(\xi) + \int_\xi^\infty K_{1/3}(z) dz \right\},$$

where $\xi = \frac{2}{3Y'} \frac{1}{x(1-x)}$, $Y' = \gamma' \frac{2E}{B_c}$, $\gamma' = E_\gamma / mc^2$, $x = E_+ / E_\gamma$. The number of the positrons can be evaluated

$$\frac{dn_+}{dz} = \frac{1}{\sqrt{3}\pi} \frac{\alpha}{\lambda_C \gamma'} \int_{x_1}^{x_2} \left\{ \left[\frac{1-x}{x} + \frac{x}{1-x} \right] K_{2/3}(\xi) + \int_\xi^\infty K_{1/3}(z) dz \right\} dx,$$

where parameters x_1, x_2 defined by possible gates of capture. The typical values of parameters required for the conversion efficiency, based on calculations, made in [40], $n_{e+} / n_{e-} \approx 4.5$ (full efficiency). This requires the energy of the laser flash around $J \cong 15\, Joules$, laser wavelength $\lambda \cong 350\, nm$, pulse length $\sigma_T \approx 0.5\, ps$ or 150 μm space duration. These figures gives the power required $W_{flash} \cong J / \sigma_T = 30 \cdot 10^{12}$ W or 30 TW. This is about 100 times more than the power required for $\gamma\gamma$ collisions. For electron energy $E_e \cong 250 GeV$, the necessary emittance for focusing electron beam to the transverse dimension $\gamma\varepsilon \cong 10^{-6} m \cdot rad$. In the energy interval

$\Delta E_+ / E_+ \cong \pm 2.5\%$ around 1.6 GeV, the efficiency is around 1.6 .The polarization could be obtained here by using polarized laser light. At the lower boundary of the spectrum the longitudinal polarization could achieve the level of circular polarization of incoming radiation. The other useful possibility in this method, is that for positrons created, the emittance is even less than the emittance of incoming electron beam by a factor 0.03, defining the small angle of created positrons $\approx 1/\gamma'$.

Let us estimate the possibility to obtain a laser flash of such a high energy. In [42] there made a consideration for two stage FEL arrangement for obtaining the power in a flash light around $W_{flash} \cong 3 \cdot 10^{11}$ W or 0.3 TW for $\gamma\gamma$ collisions. On the first stage a small power FEL is using as a master oscillator. On the second stage, a powerful FEL is feeding by a E_b=2 GeV electron beam with a peak current $I_b \approx$ 2.5 kA. So the peak power in the electron beam is around $W_{beam} \cong I_b \cdot E_b \approx 5$ TW. The efficiency about 6% was supposed. The charge what corresponds to the current of 2.5 kA can be estimated as $Q_b \cong I_b \cdot \sigma_T = 2.5 \cdot 10^3 \cdot 0.5 \cdot 10^{-12} \cong 1.25 \cdot 10^{-9}\ C = 1.25\ nC$.

If we consider a CLIC design as example [50], we can accept, that the drive beam of the energy around E_b=3 GeV (to feed the acceleration sections in case of CLIC) and the charge about $Q_b \approx 40$ nC (and even more) is available. So the total energy in the electron beam will be in this case around $E_b\ Q_b \approx 120$ Joules. So if one suppose that the efficiency of the energy transformation from the beam to the photon flux is around 10% that brings us to the necessary level of the FEL flash. The length of the wiggler must be around 50 meters, the wiggler period must be from ≈ 20 cm at the beginning of the wiggler and ≈ 10 cm at the end of the laser adjusting the resonant frequency $\lambda_w \cong 2\lambda\gamma^2/(1+K^2)$. The wiggler must be a helical one.

Instead the FEL, one can use a *solid state laser* as a master oscillator (the first stage of amplifier). This solid state laser must provide a peak power around 10 MW and there is no limitation to get it.

3.b. The pair production in the field of incoming beam in collision point. In the same line of investigations, the considerations made in [43]. This is generally the exact description of the pair production in strong electromagnetic field including the pair production through a virtual photons. The authors considered mostly the field of incoming beam. The value of such a field can reach the order of MG. So the Y parameter can reach the level of tens. However the polarization is not available due to absence of controlled polarized statements in this reaction.

4. Natural polarization in a damping ring. In [44,45] a self polarization due to synchrotron radiation is predicted. The time dependence of polarization could be described by the formula

$$P(t) = P^{\infty}(1-\exp(-\frac{t}{\tau_p})),$$

where the asymptotic level of polarization $P^{\infty} = \frac{8}{5\sqrt{3}} \cong 0.9238$ and the characteristic time of polarization is

$$\tau_p = \frac{8m|\rho|^3}{5\sqrt{3}r_0\hbar\gamma^5} \equiv \frac{8\cdot mc^2|\rho|^3e^2}{5\sqrt{3}\cdot e^2 r_0\hbar c^2\gamma^5} = \frac{8|\rho|^3\alpha}{5\sqrt{3}r_0^2\gamma^5 c}.$$

where ρ is a bending radius in the magnetic field. This time can be compared with the time of radiation damping

$$\tau_{rad} = \frac{3}{2}\frac{\rho^2}{r_0\gamma^3 c}.$$

So the ratio of these times is

$$\frac{\tau_p}{\tau_{rad}} = \frac{16|\rho|\alpha}{15\sqrt{3}r_0\gamma^2}.$$

Even simple radiation damping of the emittance is a problem due to high repetition rate required. One can see that the huge factor $|\rho|/r_0$ cannot be neutralized. So, self polarization is not useful for the purposes of preparing the beam.

5. Cleaning the beam in the damping ring by blowing out the positrons with unnecessary polarization with the polarized laser beam.

The proposal was made [46] to illuminate the beam in a damping ring by a laser light with appropriate polarization. Due to dependence of cross section of the polarization one can hope to kick out the positrons (or electrons) with unnecessary polarization.

So only half of positrons are rest. Not taking into account the time of the process, what depends of the intensity of the light, one can estimate that this is an extremely extensive way.

6. Radioactive decay.

The radioactive decay [47] is not able to provide the necessary amount of positrons, having appropriate brightness. Remember, the average flux is about $10^{13} \div 10^{14}$ positrons in a second.

7. Discussion, Conclusion.

In conclusion we can say that for future linear colliders the method of polarized particle production with the help of circular radiation from the undulator or wiggler looks attractive. The typical length of helical wiggler for production one polarized particle per one initial is about 100 meters. The degree of polarization could achieve 70%. There is no apparent limitation to applying this method.

The method of positron production using conversion of the high intensity laser beam comes to difficulty to find a souse of such powerful flash. The FEL scheme looks as the only possibility to do this.

Very attractive may be utilization of the wigglers with elliptical polarization. This requires more detailed calculations.

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