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NUMERICAL CALIBRATION OF THE BEAD-PULL SETUP FOR BEAM COUPLING IMPEDANCE EVALUATION

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Abstract

The bead-pull method is a commonly used electromagnetic field measurement technique exploited to tune a radiofrequency cavity to achieve design specifications. The frequency of a resonant cavity is perturbed by inserting a metallic or dielectric bead. For a given electromagnetic field, the amplitude of the perturbation depends only on the geometry of the perturbing object. Therefore, the calibration of the bead can be done in different resonant structures without loss of generality. In this paper a method to perform an accurate calibration of the bead with electromagnetic simulations is proposed. Compared to the common practice of measuring a reference cavity, the flexibility given by the simulation method to study different bead shapes and sizes could be advantageous to optimize the measurement setup. A calibrated bead-pull setup allows to quantify the electric field and, therefore, the shunt impedance of the resonant modes of the cavity. As experimental benchmark the beam coupling impedance measured with the calibrated bead-pull setup is compared with electromagnetic simulations.

INTRODUCTION

Beam coupling impedance measurements of a device are usually made by exploiting bench measurements techniques. A common and appreciated choice is to simulate the excitation due to a relativistic beam in the Device Under Test (DUT) by means of a conductive wire stretched along the axis of the structure, the so-called Wire Method (WM).

The WM is not recommended for cavity-like structures. The simple example of a pillbox with resistive walls can clarify the reason. Let us consider a cavity mode below the cut-off frequency of the attached beam pipe. In the real configuration of the structure (without wire) this mode can only get dissipated on the cavity wall. By introducing a conductive wire, the beam pipe is turned into a coaxial cable and its cut off frequency vanishes. The mode, which would be otherwise trapped in the cavity, will be able to lose power also through TEM propagation. Therefore, the quality factor measured with wire could be significantly lower than the actual quality factor of the mode (without wire) [1].

In this paper, the bead-pull method is explored as a method to measure the beam coupling impedance of structures, avoiding the issue of the conductive wire.

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BEAD-PULL METHOD

Bead-Pull Radio Frequency (RF) measurement systems consist of a small dielectric or metallic bead being pulled through a cavity while electric field measurements in the cavity are taken. Bead-pull measurements involve two types of perturbations:

1. Small material perturbation, like a small dielectric bead enters a large volume of cavity.
2. Small cavity volume change, like a small metallic bead enters a large volume of cavity.

The bead-pull method is widely used in the tuning of cavities to obtain the desired accelerating field. The method is based on the classical Slater perturbation theory which states that if any resonant cavity is perturbed by a small bead, its resonant frequency shifts from the original frequency. This frequency shift is proportional to the combination of the squared amplitudes of the electrical and magnetic fields at the location of the bead [2].

This relationship is given by the equation

$$\frac{\Delta\omega}{\omega_o} = \frac{\omega_p - \omega_o}{\omega_o} = k_{SLH}\frac{|H|^2}{U} - k_{SLE}\frac{|E|^2}{U} \qquad (1)$$

where ω_p and ω_0 are the perturbed and the original resonant angular frequencies respectively, k_{SLE} and k_{SLH} are the constants determined by the shape and material of the bead, U is the energy stored in the cavity while E and H are the electric and magnetic field amplitudes at the location of the perturbation, respectively.

Therefore, if the magnetic field or k_{SLH} is zero, the electric field is directly proportional to the change in resonant frequency. Hence, if the change in resonant frequency is known, the electric field can be determined by moving the bead along a line in the cavity. For calibrated beads (knowing the bead constants) and controlled bead speed in the traversal of the structures, the bead-pull method allows a full characterization of resonances.

NUMERICAL CALIBRATION OF THE BEAD

The bead-pull technique is a perturbative method. The dimension of the perturbing object must be chosen so that the field does not vary significantly over its largest linear dimension and at the same time introduces a disturbance large enough to be distinguishable from the measurement noise. Shaped beads are used to enhance perturbation and give directional selectivity among different field components.

Since the amplitude of the perturbation depends only on the shape, material and size of the bead, a resonant cavity

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of any geometry can be used to calibrate the method without loss of generality. In fact, using a circular pillbox cavity allows accurate studies due to its simplicity.

A resonant cavity is characterized by its resonant frequency f_0, the quality factor of the resonance Q_0 and its shunt impedance R.

To properly validate the results, it is necessary to model accurately the bead including its geometry.

Numerical Evaluation

The frequency shift of the resonance depends on the local electric and magnetic fields at the position of the perturbing object: the higher the field, the higher will be the frequency perturbation. An example of the frequency perturbation as a function of the longitudinal position for centered metallic beads in a circular pillbox cavity is displayed in Figure 1.

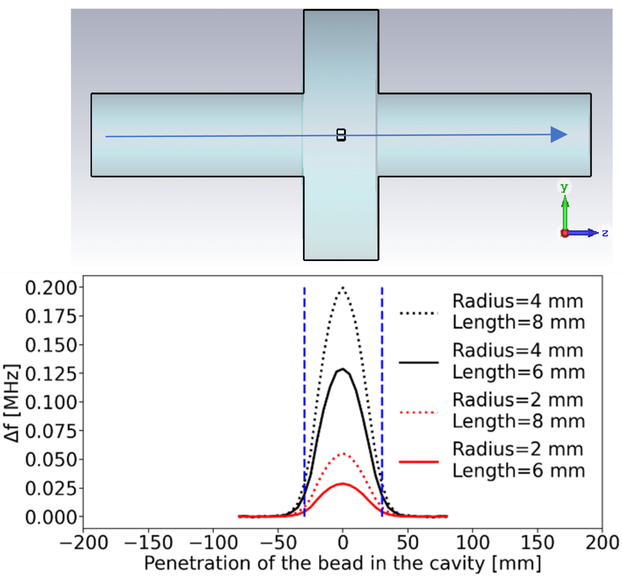

Figure 1: Top: longitudinal cross section of the circular pillbox cavity simulated with CST, in the center of the cavity a cylindrical bead is visible. Bottom: frequency perturbation of the TM_{01} mode for transversely centered beads having different length and radius as function of the longitudinal position.

Through CST simulations and some post-processing steps it is possible to derive the calibration constant (k_{SLE}). The mathematical equation that describes the system together with (1) is:

$$\frac{R}{Q_0} = \frac{V^2}{2\omega_0 U} \qquad (2)$$

Eq. (2) describes the well-known ratio of R/Q that depends on the square of the electrical potential V, the original resonant angular frequency of the cavity ω_0 and the stored energy U.

Rearrangement of equations (1) and (2) gives

$$\frac{V}{\sqrt{U}} = \int_0^L \sqrt{-\frac{\Delta\omega(z)}{k_{SLE}\omega_0}}\,dz \qquad (3)$$

$$\frac{V}{\sqrt{U}} = \sqrt{\frac{2\omega_0 R}{Q_0}} \qquad (4)$$

that combined provide:

$$k_{SLE} = \frac{Q_0}{2\omega_0 R}\left(\int_0^L \sqrt{-\frac{\Delta\omega(z)}{\omega_0}}\,dz\right)^2 \qquad (5)$$

where the parameters ω_0, Q_0, R are obtained from eigenmode simulations on the reference circular pillbox cavity.

To confirm the reliability of the method, the calibration of the beads was also studied for off-centered bead positions. In this case it must be underlined that the value of the shunt impedance has been calculated exactly where the bead is positioned. Figure 2, as an example, shows a comparison between the simulated longitudinal shunt impedance obtained from CST eigenmode simulations without bead and with the bead-pull simulation technique from the calibration constant of a centered bead.

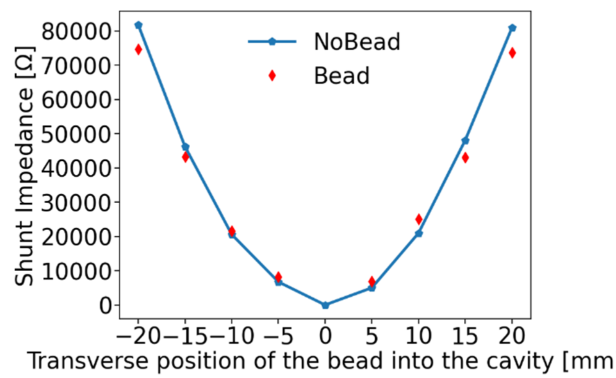

Figure 2: Comparison between the simulated longitudinal shunt impedance without bead and with the bead-pull numerical calibration method as function of the transverse position of the bead.

The good agreement obtained in Fig. 2 shows the potential of the method for an accurate evaluation of the transverse beam coupling impedance. In fact, the transverse impedance can be obtained from the variation of the longitudinal impedance by using Panofsky-Wenzel theorem [3].

EXPERIMENTAL VALIDATION

In order to assess the consistency of the calibration method, bead-pull measurements have been performed on a resonant pillbox like cavity with rectangular pipes and resonant frequency of f_0=2.4255GHz.

The measurement setup is composed by a pulley system, a dielectric wire, two spherical metallic beads (D_1=2.48mm, D_2= 3.05mm) whose movement inside the cavity is controlled by a stepper motor, antennas and a Vector Network Analyzer.

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As a first step, bead-pull measurements were performed for different positions of the measuring antenna inside the cavity. Knowing that the ratio between the phase variation respect the unperturbed case ($\Delta\Phi$) and the loaded Q (Q_L) is constant, it is possible to obtain the value of the frequency perturbation through the mathematical relation:

$$\frac{\Delta f}{f_0} = \frac{\tan\phi}{2Q_L} \approx \frac{\Delta\phi}{2Q_L} \quad (6)$$

The frequency shift shown in Fig. 3 highlight the robustness of the method that for a given bead independently of the value of the Q_L, i.e., penetration of the measuring antennas into the cavity, gives the same relative frequency perturbation.

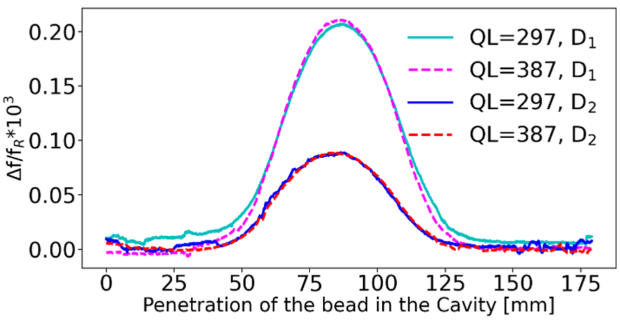

Figure 3: Relative variation of the resonant frequency as obtained from bead-pull measurements of $\Delta\Phi$ and using Eq. (6) for two different loaded Q values.

As a second step, CST eigenmode simulations of the DUT have been performed considering a deviation of 100 µm on the nominal value of radius of the beads. This has been represented with an error bar on the expected value of the frequency perturbation (see Fig. 4).

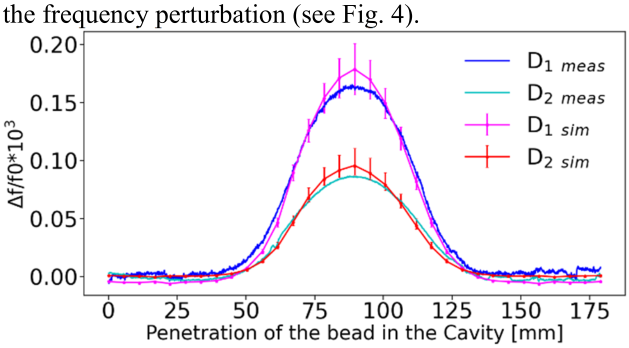

Figure 4: Comparison between measurements and simulations.

Measurements and simulations exhibit a reasonable agreement. As a final step to validate the virtual calibration method, the circular pillbox shown in Fig.1 has been adopted to obtain the calibration constant kSLE for the beads used in the experimental setup. Therefore, by using Eq. (5) to calculate R, the impedance of the DUT is obtained by means of the Resonator Model:

$$Z = \frac{R}{1+jQ_0\left(\frac{f}{f_0}-\frac{f_0}{f}\right)} \quad (7)$$

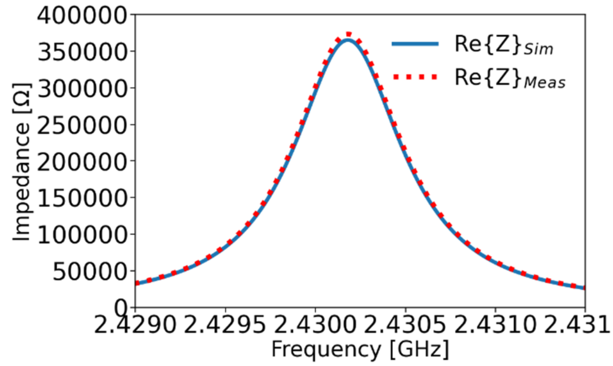

Figure 5: Real part of DUT's impedance. Measurements performed with the 3.05 mm spherical metallic bead.

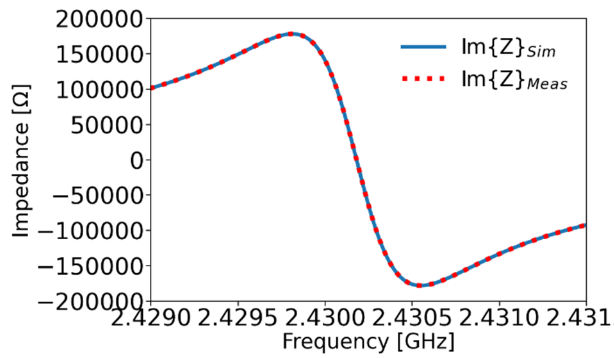

Figure 6: Imaginary part of DUT's impedance. Measurements performed with the 3.05 mm spherical metallic bead.

Figures 5 and 6 show the comparison between measurements and simulations respectively for the real and imaginary part of the impedance. The curves obtained with standard impedance simulations (without bead) are in very good agreement with the experimental measurements proving the high accuracy of the proposed calibration method for beam coupling impedance measurements.

CONCLUSION

The potential of the bead-pull method to perform beam coupling impedance measurements of cavity-like structures has been investigated. A simulation procedure has been established for an accurate calibration of the beads. The method has been experimentally validated for the derivation of the longitudinal beam coupling impedance and it has been shown to have potential also to measure the transverse beam coupling impedance.

REFERENCES

[1] C. Zannini, "Electromagnetic Simulation of CERN Accelerator Components and Experimental Applications", CERN-THESIS-2013-076, Mar. 2013.

[2] J. C. Slater, "Microwave Electronics", D. Van Nostrand Company, Inc., 1950.

[3] S. Heifets, A. Wagner, and B. Zotter, "Generalized Impedances and Wakes in Asymmetric Structures", no. SLAC/AP110, 1998.