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IMPACT OF LONGITUDINAL GRADIENT DIPOLES ON STORAGE RING PERFORMANCE*

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Abstract

Innovative new magnets with longitudinally varying dipole field are being produced for installation in a few modern light-source storage rings. We investigate some of the associated beam-dynamics issues, in particular the photon spectrum and quantum fluctuation associated with such magnets, and we study whether the resulting equilibrium emittance may deviate from the value expected in the long-magnet limit.

INTRODUCTION

Dipole magnets with longitudinally varying bending field are proposed to minimise the emittance, or cost, of linear collider damping rings [1] and storage-ring light sources [2–6]. A "trapezium" magnet was designed that could provide a factor ~7 emittance reduction for the CLIC Damping Rings [7, 8]. A similar magnet is being considered for reducing the horizontal emittance of the ELETTRA 2.0 light source by almost a factor of two [9].

The approach of tailoring the dipole field with longitudinal position is based on the standard expression for the horizontal equilibrium emittance (assuming separate function magnets and no linear coupling)

$$\varepsilon_x = C_q \gamma^2 I_5 / I_2 \ , \tag{1}$$

with $C_q = (55/(32\sqrt{3})\hbar c/(m_e c^2)$ (m_e electron mass, c speed of light, $\hbar$ the reduced Planck constant), the radiation integrals [10] $I_2 = \int 1/\rho(s)^2 ds$ and $I_5 = \int \mathcal{H}_x(s)/\rho(s)^3 \ ds$, where the integrals are evaluated around the storage ring, $\rho(s)$ designates the local bending radius, and $H_x(s) = \beta_x D_x'^2 + 2\alpha_x D_x D_x' + \gamma_x D_x^2$ the value at location s of the "curly-H" function defined by Sands [11]. The integral I_2 is related to the energy loss per turn, and the integral I_5 describes the quantum excitation. The dipole field is varied longitudinally to minimise the ratio I_5/I_2 and, hence, the emittance.

Specifically, when choosing the same cell length and betatron phase advances as in a conventional lattice and assuming a practical maximum field, the variational bend "TME" lattice was shown to reduce the storage ring emittance by a factor of 2–3, without a significant change in the dynamic aperture [1]. Reference [1] also pointed out that, alternatively, one could use variational bends to obtain the same emittance as with constant bends, while reducing the number of cells by 25% to 30%, which might lead to a larger dynamic aperture, a larger momentum compaction, and shorter circumference, as was achieved for the CLIC Damping Rings [8].

CLASSICAL QUANTUM EXCITATION

The radiation integral I_5 in Eq. (1) stems from the classical quantum excitation due to photon emission in a constant magnetic field [11]

$$\Delta\varepsilon_x = \int \dot{N}_{\rm ph} \left\langle u^2 \right\rangle \mathcal{H}_x \ ds \ . \tag{2}$$

The "long-magnet" photon spectrum for a single electron passing through a magnet with constant field B_0 and length $2L$ is given by

$$\frac{dN_{\rm ph}}{d\nu} = \frac{4 r_e e}{9\hbar} \frac{B_0 2L}{\nu} S\left(\frac{\nu}{\nu_c}\right) \ , \tag{3}$$

with

$$S\left(\frac{\nu}{\nu_c}\right) = \frac{9\sqrt{3}}{8\pi} \frac{\nu}{\nu_c} \int_{\nu/\nu_c}^{\infty} K_{5/3}(x') dx' \ , \tag{4}$$

$\nu_c = (3/2) c\gamma^3 B_0 e/p$, and r_e the classical electron radius.

The mean-square photon energy is $\left\langle u^2 \right\rangle = (11/27)\epsilon_{\rm cr}^2$ with critical photon energy $\epsilon_{\rm cr} = (3/2)\hbar c\gamma^3/\rho$, and $\dot{N}_{\rm ph} = (15\sqrt{3}/8) P_\gamma/\epsilon_{\rm cr}$, with $P_\gamma = cC_\gamma/(2\pi)E^4/\rho^2$, E the beam energy, $C_\gamma = (4\pi/3) r_e/(m_e c^2)^3$ and $1/\rho = B_0 e/p$. Note that $S(u)$ is normalized such that $\int_0^\infty S(u) du = 1$ and $\int_0^\infty (1/u) S(u) du = 45/(8\sqrt{3})$.

LONGITUDINAL GRADIENT DIPOLES

For a magnet with varying dipole field $B_y(s)$, from Eq. (3) the magnetic field variation can be taken into account as

$$\frac{d^2 N_{\rm ph}}{d\nu \ ds}(s) = \frac{4 r_e e}{9\hbar} \frac{B_y(s)}{\nu} S\left(\frac{\nu}{\nu_c(s)}\right) \ , \tag{5}$$

where now $\nu_c(s) = (3/2) c\gamma^3 B_y(s) \ e/p$. Integrating over the magnet length $2L$ we obtain the classical photon spectrum for this dipole

$$\frac{dN_{\rm ph}}{d\nu} = \int_{-L}^{L} \frac{d^2 N_{\rm ph}}{d\nu \ ds}(s) ds \ . \tag{6}$$

However, there may be a limitation to this approach. For a dipole magnet with non-constant field, we can define an effective "local" magnet length as

$$l_{\rm eff}(s) \approx B_y(s) / \left| dB_y(s')/ds' \right|_{s'=s} \ , \tag{7}$$

while the emission "source length" of synchrotron radiation is of order $l_{\rm source} \sim \pm\rho/\gamma$, where $\rho(s) = p/(eB_y)$. In the

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case that the effective local magnet length is comparable to, or shorter than, the source length, i.e., $l_{\rm eff} \lesssim l_{\rm source}$, or

$$\frac{1}{B_y^2(s)} \left| \frac{dB_y(s')}{ds'} \right|_{s'=s} > \frac{e}{m_e c} , \quad (8)$$

the radiation spectrum extends to higher frequencies, and Eq. (1) with (3) or (6) ceases to apply [12, 13]. In the case $l_{\rm eff} < l_{\rm source}$, the spectrum becomes [12]

$$\frac{dN_{\rm ph}}{d\nu} \approx \frac{r_e e^2 c}{2\pi m_e \hbar \nu} \int_1^\infty \frac{y^2 - 2y + 2}{y^4} \left| \tilde{B}\left(\frac{\nu}{2\gamma^2} y\right) \right|^2 dy , \quad (9)$$

with $\tilde{B}(\nu) = \int_{-\infty}^{\infty} B(t) e^{-i2\pi\nu t}\, dt$.

We note that the condition (8) only depends on the magnetic field profile and is independent of beam energy. The condition includes the particle mass, and, therefore, it is much more easily met for hadron beams.

LORENTZIAN PROFILE

As a first example, we follow Ref. [12] and consider a Lorentzian field of length $2L$ (full-width at half maximum)

$$B_y(s) = \frac{B_0}{1 + s^2/L^2} , \quad (10)$$

with magnet peak field B_0.

From Ref. [12], the photon spectrum is

$$\frac{dN_{\rm ph}}{d\nu} = \frac{\pi r_e e^2 c^2 L^2 B_0^2}{2 m_e c^2 \hbar c \nu} \left[\frac{2}{3} e^{-x} \left(1 + x + x^2/2\right) + x(1 + x - x^2/3) \mathrm{Ei}(-x) \right] , \quad (11)$$

with E_b the beam particle energy, $x = 4\nu/\nu_1$, $\nu_1 = 2\gamma^2 c/(\pi L)$, and Ei the exponential integral

$$\mathrm{Ei}(x) = \int_{-\infty}^{x} \frac{e^{x'}}{x'}\, dx' . \quad (12)$$

GAUSSIAN PROFILE

We take a second example from Ref. [12], this time a Gaussian field of length $\sim 2L$ (full-width at half maximum equal to $1.66L$)

$$B_y(s) = B_0 e^{-z^2/L^2} , \quad (13)$$

with magnet peak field B_0.

In this case the photon spectrum is [12]

$$\frac{dN_{\rm ph}}{d\nu} = \frac{r_e e^2 c^2 L^2 B_0^2}{2 m_e c^2 \hbar c \nu} \left[\frac{1}{3} e^{-x^2} \left(1 + 4x^2\right) + x\sqrt{\pi}(1 + 4x^2/3)\mathrm{erfc}(x) - x^2 \mathrm{Ei}(-x^2) \right] , \quad (14)$$

where E_b the beam particle energy, $x = \sqrt{2}\nu/\nu_1$, $\nu_1 = 2\gamma^2 c/(\pi L)$, and

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-x'^2}\, dx' . \quad (15)$$

TRAPEZIUM PROFILE

For the actual gradient dipole of ELETTRA 2.0 [9], the bending radius is linearly increasing, or decreasing, as

$$\rho(s) = \rho_1 + (L_1 - |s|)(\rho_1 - \rho_2)/L_2 , \quad (16)$$

from $|s| = L_2 + L_1$ to L_1 around a short central section of constant field, with a length $2L_1$ of 5.5 cm, surrounded by two gradient sections of length $L_2 = 37.2$ cm each. At a beam energy of 2.4 GeV, the corresponding field varies as $\sim 1/s$ from $B_2 = 0.62$ T to $B_1 = 2.32$ T, with $B_{1,2} = (B\rho)/\rho_{1,2}$ and $(B\rho)$ the magnetic rigidity.

DISCUSSION

Inserting (10) for the Lorentzian, inequality (8) becomes

$$|s| > eB_0 L^2/(2m_e c) \approx 292 \text{ m } B_0[\mathrm{T}] L[\mathrm{m}]^2 . \quad (17)$$

For the Gaussian profile (13) we find the condition

$$LB_0 e^{-s^2/L^2} < m_e c^2/(ec) \approx 0.0017 \text{ T m} , \quad (18)$$

while for the trapezium profile (16) we obtain

$$B_1 B_2 L_2/(B_1 - B_2) < m_e c^2/(ec) \approx 0.0017 \text{ T m} , \quad (19)$$

which is not fulfilled for the case of the ELETTRA magnet.

SPECTRA

As an illustration, we consider an electron beam of 3 GeV passing through a longitudinal-gradient dipole with a peak field of 1 T. Figure 1 compares spectra for a dipole half length of $L = 1$ cm, with either Lorentzian or Gaussian field profile. We note that the spectrum from the Lorentzian resembles the one from the Gaussian magnet. However, the results obtained from the long magnet calculation and those from the short magnet calculation are quite different, with the short-magnet spectral density significantly higher at low frequencies, and rolling off earlier. Figure 2 shows the results for a ten times shorter magnet, $L = 1$ mm. Even for this short a magnet length the magnet spectra (11) and (14) fall off earlier than the long-magnet spectrum (6). While the long-magnet spectrum [in blue] is essentially unchanged (only shifted downward due to the smaller number of photons emitted over the reduced magnet length), the short-magnet spectrum now extends to higher frequencies than in Fig. 1. Reducing the magnet length further to $L = 0.1$ mm, the spectral ranges of (11) and (14) start to extend in frequency beyond the "classical" spectrum (6), as is shown in Fig. 3. The numerically computed photon spectra for the trapezium ELETTRA magnet with a peak field of 2.32 T and an electron beam energy of 2.4 GeV are displayed in Fig. 4.

QUANTUM EXCITATION

The quantum excitation of Eq. (2) depends on the mean energy squared times the number of emitted photons,

$$\langle u^2 \rangle N_{\rm ph} = h^2 \int_0^\infty \left(\nu^2 dN_{\rm ph}/d\nu\right) d\nu . \quad (20)$$

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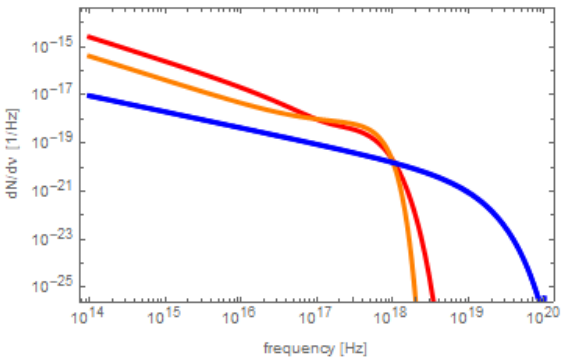

Figure 1: Photon spectra for a Lorentzian according to (6) [blue] and (11) [red], and for a Gaussian field profile (14) [orange], with $L = 1$ cm, at a beam energy of 3 GeV.

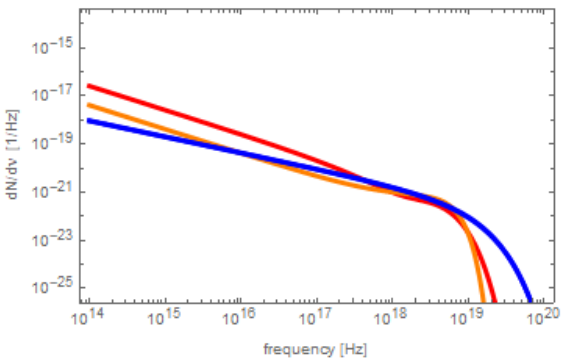

Figure 2: Photon spectra for a Lorentzian according to (6) [blue] and (11) [red], and for a Gaussian field profile (14) [orange], with $L = 1$ mm, at a beam energy of 3 GeV.

A few example results are compiled in Table 1. For the short magnet spectra (11) and (14) the quantum excitation (20) is independent of L, whereas for (6) it is proportional to the length of the magnet $2L$. This can also be directly seen by inspecting the spectral formula. The quantum excitation terms are approximately equal for $L \approx 14$ mm.

Table 1: Quantum excitation term (20), $\langle u^2 \rangle N_{\mathrm{ph}}$, in units of $(\mathrm{eV})^2$, according to the long-magnet expression (6) for a Lorentzian, and the short-magnet formulae (11) and (14), for a dipole with Lorentzian or Gaussian profile of peak field $B_0 = 1$ T and of varying length L, and considering a 3 GeV electron beam. The long-magnet values for the Gaussian are lower than for the Lorentzian by a factor 3.7/4.0.

L	"long"	Lorentzian	Gaussian
0.1 mm	4.0×10^3	6.2×10^5	9.0×10^5
1 mm	4.0×10^4	6.2×10^5	9.0×10^5
10 mm	4.0×10^5	6.2×10^5	9.0×10^5
100 mm	4.0×10^6	6.2×10^5	9.0×10^5

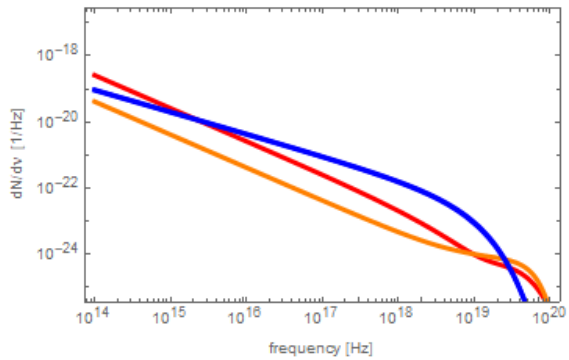

Figure 3: Photon spectra for a Lorentzian according to (6) [blue] and (11) [red], and for a Gaussian field profile (14) [orange], with $L = 0.1$ mm, at a beam energy of 3 GeV.

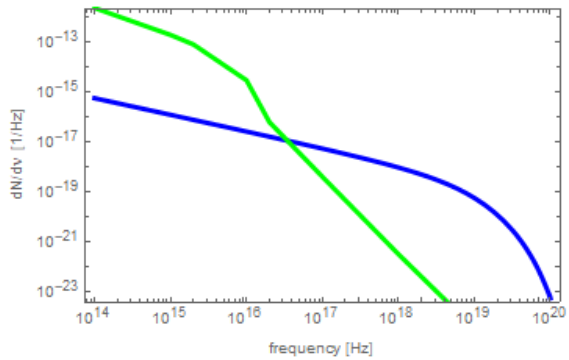

Figure 4: Photon spectra according to (6) [blue] and (9) [green] for the 0.8 m dipole with trapezium profile, at a beam energy of 2.4 GeV.

CONCLUSIONS

From Eqs. (17) and (18) for the profiles (10) or (13), and from our numerical examples, we infer that, in the case of an electron beam, and for peak magnetic fields of order 1 T, if the length over which the dipole field significantly changes is of order a cm, or shorter, the standard expression for quantum excitation ceases to be valid, and the latter can become much larger than naively expected. This effect may need to be considered when optimizing magnet field profiles for future extreme electron storage rings.

For protons, replacing m_e by the proton mass m_p in (8), the quantum excitation terms may already be enhanced when the field changes over a length scale of metres. Therefore, this effect needs to be taken into account when calculating equilibrium emittances for future highest-energy hadron storage rings, such as the 100 TeV collider FCC-hh [14, 15].

In this article, we have computed and compared spectra for the two limiting cases of constant-field "long" and varying-field "short" magnets, respectively. We note that more accurate emission spectra for arbitrary magnetic field shape, valid also in the transition region between "short" and "long" magnets, could be obtained numerically by starting from the retarded Liénard-Wiechert potentials [16, 17]; see e.g. Ref. [18].

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