

# OPTIMIZATION OF A 600 MHz TWO-CELL SLOTTED WAVEGUIDE ELLIPTICAL CAVITY FOR FCC-ee

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## Abstract

The radio-frequency (RF) system of the future circular lepton collider (FCC-ee) must cope with different machine parameters ranging from Ampere-class operation required for the Z-peak working point to the high-gradient operation for the  $t\bar{t}$  threshold. The Superconducting Slotted Waveguide Elliptical cavity (SWELL) concept was recently proposed as an alternative to the challenging RF baseline design of the FCC-ee. In this paper, random optimization methods are used to minimize the peak surface magnetic field and the maximum longitudinal impedance of the higher order modes (HOM) of a two-cell 600 MHz SWELL cavity. In the next step, the waveguide slots are optimized to first have a smooth transition from the cavity to the slots to avoid large peak surface fields and second to achieve high transmission at dipole mode frequencies and low transmission at fundamental mode frequency while keeping the design compact.

## INTRODUCTION

The strong wakefield effects in the radio frequency (RF) cavities of the Future Circular lepton Collider (FCC-ee) at the Z and W working points are one of the many problems facing the RF design of FCC-ee [1, 2]. A Superconducting Slotted Waveguide Elliptical cavity (SWELL) with two cells and four slots at 600 MHz is proposed in [3] as a possible efficient alternative for the baseline design of FCC-ee described in [4]. The SWELL geometry consists of four quadrants separated by narrow waveguide (WG) slots that provide strong damping of transversal higher-order modes (HOM). The SWELL cavity has several advantages compared to the standard elliptical cavities such as: heavy transversal mode damping via WG slots, easier access to the cavity surface for niobium coating, no welded joints at high magnetic field zones of the cavity, and a robust shape against microphonics. The SWELL design has very weak interaction with transverse-magnetic (TM) monopole modes, i.e., the fundamental mode (FM) and the monopole HOMs which can have a large longitudinal impedance. This paper aims to optimize the design of the SWELL cavity to untrap the longitudinal HOMs (thus eliminating the need for additional HOM couplers), minimize the peak surface magnetic field, and have strong damping of the dipole modes via WG slots.

## ELLIPTICAL CAVITY OPTIMIZATION

The elliptical cavity forms the basis of the SWELL cavity and must be designed first. Parametrization of a two-cell elliptical cavity is shown in Fig. 1. The two cells are assumed to be identical and the inner and outer half-cells can

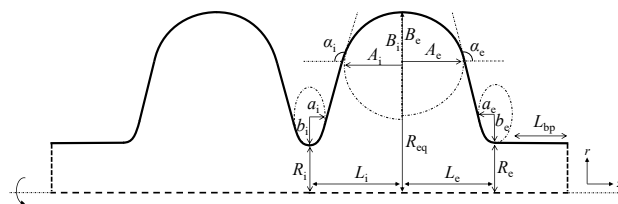


Figure 1: Parametrization of a two-cell elliptical cavity. In the simulations  $L_{bp}$  is assumed to be  $4L_i$ .

have different shapes. Since the WG slots of the SWELL do not damp the longitudinal HOMs, we try to minimize the peak of the real part of the longitudinal impedance of the HOMs ( $\Re(Z_{\parallel, f > f_{FM}})$ ) when designing the elliptical cavity. Low  $\max(\Re(Z_{\parallel, f > f_{FM}}))$  normally degrades the normalized magnetic field on the cavity surface ( $B_{pk}/E_{acc}$ ). Maintaining a low surface electromagnetic (EM) field is another critical design parameter, as surface fields are expected to enhance in the next step when WG slots are connected to the cavity. Thus, the following problem is formulated for the optimization of the elliptical cavity:

$$\begin{aligned} & \text{minimize}_{R_i, A_i, B_i, a_i, b_i, R_e, A_e, B_e, a_e, b_e} \left( B_{pk}/E_{acc}, \max(\Re(Z_{\parallel, f > f_{FM}})) \right) \\ & \text{subject to} \quad f_{FM} = 600.00 \text{ MHz}, \quad 130^\circ \geq \alpha_i \& \alpha_e \geq 90^\circ. \end{aligned} \quad (1)$$

The FM frequency ( $f_{FM}$ ) is set fixed and wall angles are maintained in an acceptable range as constraints. Ten geometric parameters can be varied in the optimization problem.  $L_i$  and  $L_e$  are fixed at a quarter of the wavelength of the FM, and  $R_{eq}$  is used to tune  $f_{FM}$ . The geometric constraints in the optimization problem is given in Table 1.

Table 1: Lower Bound (LB) and Upper Bound (UB) of the Optimization Parameters

	$(R_i)$	$(R_e)$	$(A_i, A_e, B_i, B_e)$	$(a_i, a_e, b_i, b_e)$
LB [mm]	85	100	50	20
UB [mm]	105	105	110	80

To find a Pareto front between the two objective functions of Eq. (1), we used the genetic algorithm (GA) method as implemented in MATLAB's global optimization toolbox [5]. For each individual (candidate solution) of GA, a 1D optimization problem on  $R_{eq}$  was first solved to tune  $f_{FM}$  to 600 MHz, then an eigenvalue problem was solved with SLANS [6] to calculate  $f_{FM}$  and  $B_{pk}/E_{acc}$ , and a wakefield simulation was done with ABCI [7] to obtain  $\max(\Re(Z_{\parallel, f > f_{FM}}))$ . The resulting Pareto front is shown in Fig. 2. In the next step, Monte-Carlo (MC) method was applied in the neighborhood of the selected geometry from the GA method (shown by

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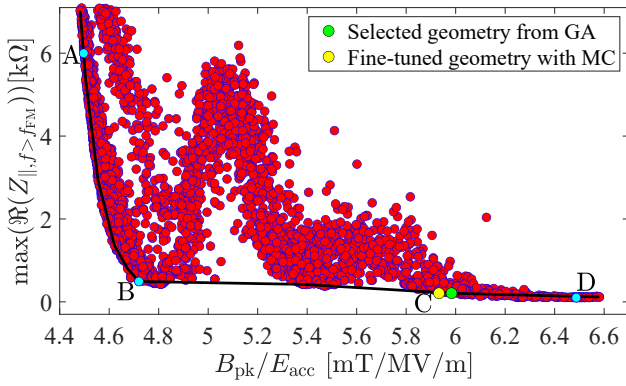
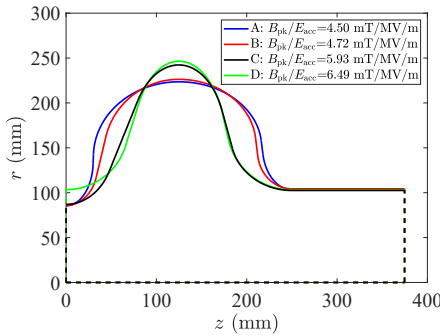
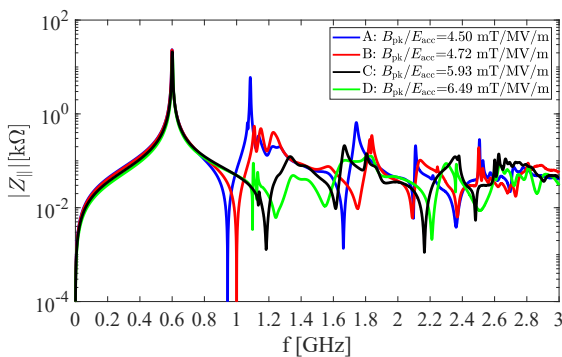


Figure 2: The two objective functions of Eq. (1) plotted for all individuals in the GA method (10000 samples). Point C was selected for the SWELL design.

green marker in Fig. 2) to minimize  $B_{pk}/E_{acc}$  without deteriorating  $\max(\Re(Z_{||,f>f_{FM}}))$ . This could slightly improve the result as shown by the yellow marker in Fig. 2. The shapes of four cavities on the Pareto front and their respective longitudinal impedances are shown in Fig. 3. A small volume around the equator ellipse of the cavity helps to untrap the longitudinal HOMs at the cost of a higher  $B_{pk}/E_{acc}$ . The geometric data and some performance parameters of the selected optimized cavity (point C in Fig. 2) are shown in Table 2.



(a) Cavity shapes

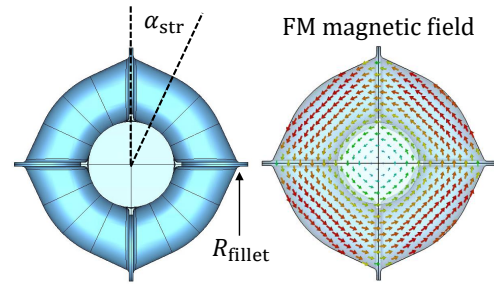


(b) Longitudinal impedance derived from 100 m wavelength

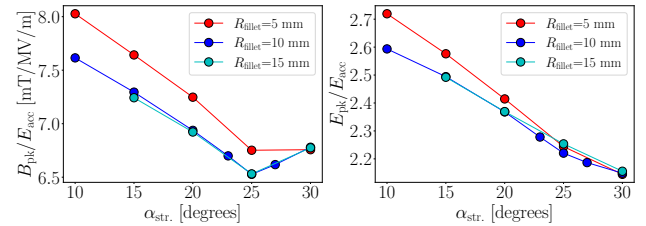
Figure 3: The shapes and the longitudinal impedances of the four points highlighted in Fig. 2.

## CAVITY TO SLOT CONNECTION

In the next step, four narrow WG slots with a width of 10 mm must be placed perpendicular to the cavity surface. The slot WG increases the peak surface EM field at the slot entrance. The use of large fillet radii at the connection point is not sufficient to alleviate surface field enhancement. Therefore, the azimuthal symmetry of the cavity is broken by adding a straight profile on the cavity surface right before the cavity-slot connection point, as shown in Fig. 4(a). This method helps to reduce the FM field near the slot entrance. The two important parameters affecting the EM surface fields are the angle of the straight profile ( $\alpha_{str}$ ) and the fillet radius at the intersection ( $R_{fillet}$ ).



(a) Parametrization of the cavity to slot-entrance connection



(b) Normalized surface magnetic field (c) Normalized surface electric field

Figure 4: Parameter study of cavity-slot connection to minimize peak surface fields.

A parameter study was performed to find an acceptable value for  $\alpha_{str}$  and  $R_{fillet}$  as shown in Fig. 4(b). For each point,

Table 2: Geometric Gata and Some Figures of Merit of the Optimized Elliptical Cavity.

$R_i$ [mm]	$A_i$ [mm]	$B_i$ [mm]	$a_i$ [mm]	$b_i$ [mm]
87.0	54.3	87.4	53.5	67.0
$R_e$ [mm]	$A_e$ [mm]	$B_e$ [mm]	$a_e$ [mm]	$b_e$ [mm]
102.5	52.4	86.2	67.6	61.4
$R_{eq}$ [mm]	$L_i = L_e$ [mm]	$\alpha_i \& \alpha_e$ [°]	$k_{  }$ [V/pC]	$f_{FM}$ [MHz]
242.313	124.91	111.9&105.2	0.335	600.00
$R/Q$ [Ω]	$G$ [Ω]	$B_{pk}/E_{acc}$ [mT/MV/m]	$E_{pk}/E_{acc}$ [-]	$G.R/Q$ [Ω <sup>2</sup> ]
167.1	210.5	5.93	1.99	$3.52 \times 10^4$

$R_{eq}$  of the original cavity is changed to bring  $f_{FM}$  to 600 MHz. Large  $\alpha_{str}$  decreases the peak surface fields, but above  $25^\circ$  an increase in  $B_{pk}/E_{acc}$  is observed because at this point the location of the maximum magnetic field jumps from the slot-entrance to the straight-to-circular profile junction. Finally, a value of  $\alpha_{str} = 25^\circ$  and  $R_{fillet} = 10$  mm is selected, and the optimum  $R_{eq}$  for frequency tuning is 239.457 mm. The increase in the  $B_{pk}/E_{acc}$  and  $E_{pk}/E_{acc}$ , compared to the initial elliptical cavity, is around 10% and 12%, respectively.

## WG SLOT DESIGN

The SWELL cavity is made up of four tightly clamped quadrants each precisely machined into a copper block and coated with a layer of superconducting material. Two coaxial couplers are added to each WG slot for HOM extraction, as shown in Fig. 5. Each coaxial coupler is terminated with a standard feedthrough. There is a very weak leakage of the FM field into the WG slot that is aggravated by a small misalignment of the quadrants. To prevent FM energy extraction by the coaxial couplers, a lambda half-coaxial rejecter with a T-shape of about 250 mm length was first used in [3]. This length could be reduced by a factor of about two by adding a capacitance gap at the end of the rejecter, as shown in Fig. 5.

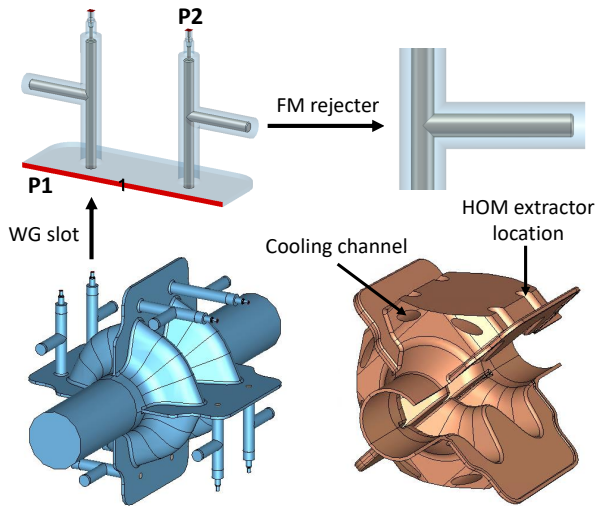


Figure 5: A full picture of the SWELL cavity with two coaxial couplers per slot (top-left) each equipped with a FM rejecter (top-right).

The geometry of the WG slot is optimized before it is connected to the SWELL cavity. For this reason, the slot entrance and the feedthroughs are terminated by WG ports, and the transmissions between the ports are calculated. The first objective is to have a notch at the FM frequency. This can be achieved by changing the dimensions of the rejecter, i.e. the total length and the gap. Compared to the lambda-half rejecter, the FM notch of the rejecter with gap has a wider bandwidth. Additionally, the lambda-half rejecter has a notch at harmonics of the FM frequency while with this design the second notch occurs at higher frequencies as shown in Fig. 6. The second objective is to maximize transmission

at frequencies where we expect high transversal impedance (typically in the first dipole passband). The dimensions of the WG slot are thus varied to maximize transmission at these frequencies (see Fig. 6). The external quality factor of the mode with the highest transversal impedance is  $\sim 154$ . The longitudinal and transversal impedance of the SWELL cavity calculated by the wakefield solver of the CST Studio Suite® [8] is shown in Fig. 7.

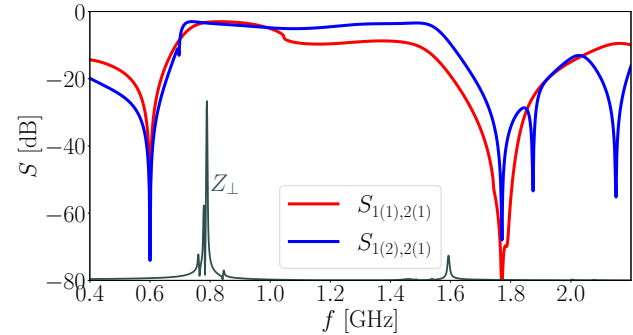


Figure 6: Scattering curves of the WG slot between port 1 and port 2 of Fig. 5 for the first two WG modes.  $Z_{\perp}$  shows the transversal impedance of the elliptical cavity.

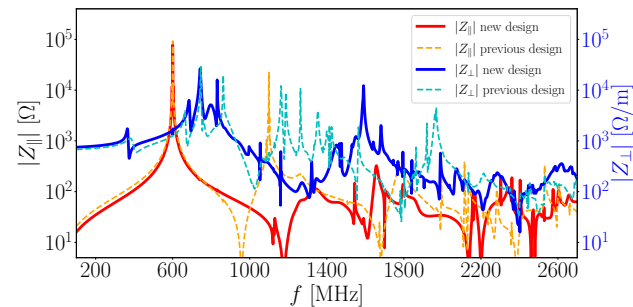


Figure 7: Longitudinal and transversal impedance of the SWELL cavity compared with the previous design [3] (dashed lines). Impedances are obtained from wakefield analysis with 200 m wavelength and a beam offset of 5 mm from the cavity center.

## CONCLUSION

The SWELL cavity, despite its many advantages, does not damp the longitudinal modes of the cavity and has high surface EM fields compared to an equivalent standard elliptical cavity. In this paper, these two problems were addressed using GA and MC methods by designing a SWELL cavity that has no trapped longitudinal mode, while minimizing the surface magnetic field. The WG slots and the coaxial extractors were also optimized to have high damping of dipole modes and a rejection at the FM frequency. The SWELL design is a work in progress, and other results of this work such as multipacting analysis, frequency tuning mechanism, module design, etc. will be presented in a dedicated paper.

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