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MAD-X FOR FUTURE ACCELERATORS

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Abstract

The development of MAD-X was started more than 20 years ago, and it still remains the main tool for single particle dynamics for both optics design, error studies as well as for operational model-based software at CERN. In this article, we outline some of the recent development of MAD-X and plans for the future. In particular, we focus on development of the TWISS module used to calculate optics functions in MAD-X which is based on first and second order matrices. These have traditionally been calculated as an expansion around the ideal orbit. In this paper we describe explicitly how an expansion around the closed orbit can be employed instead, in order to get more precise results. We also describe the latest development of the beam-beam long range wire compensator in MAD-X, an element that has been implemented using the aforementioned approach. Finally, we also present the newly implemented estimate of the C^{-}.

INTRODUCTION

MAD-X is a code that has been used for more than 20 years to perform optics and beam dynamic studies for a range of accelerators [1]. MAD-X is currently still in the center of many optics, aperture and emittances studies as well as a crucial part for various operational tools [2]. MAD-X itself is written in a combination of C, Fortran and C++. A relatively recent extension of MAD-X in the form of a Python interface called Cpymad has been made available and can be installed via pip [3]. There is also an internal interface to Polymorphic Tracking Code (PTC) [4] in MAD-X, which enables the user to utilise the two different physics engines while using the same sequence structure. This is very beneficial for cross-checks and to access features such as higher order normal forms, currently available through PTC.

In this article, we focus on recent MAD-X developments in terms of physics and functionalities. All the new features for each release, *e.g.* the recently implemented exact drift and the possibility to specify the number of particles directly in a beam-beam element can be found on the website under *Releases* [1].

EXPANSION

In the TWISS module, the closed orbit is first calculated and afterwards the optics is calculated for that orbit. This makes it possible to first find the closed orbit using the element transfer map and then calculate the first and second order transfer matrices based on the derivatives of the map. This is different from the previous approach in MAD-X where the matrices were calculated around the zero-orbit [5]. The previous method works well for small deviations, which is normally the case for machines such as the LHC, but in accelerators such as the FCC-ee, which is designed to operate with large crossing angles and momentum deviation, this approximation does not always hold. In order to overcome this, a Python code based on SymPy [6] has been developed in which one can input the exact equations and then the derivatives are calculated. There is a check to verify that the resulting matrix is symplectic and afterwards a Fortran code is generated in MAD-X format that can directly be included in the MAD-X source code. In the following sections we will show how this has been used for the exact drift and for Long Range Beam-Beam (LRBB) wire compensator.

EXACT DRIFT

The drift, when approximated, takes a simple form where the final position simply depends on the transverse momenta. In MAD-X the approximate drift has the following form:

$$x_f = x_i + l\,p_x\left(1-\frac{p_t}{\beta_s}\right) \tag{1}$$

where l is the length of the drift, x_f is the final position, x_i is the position before the drift, p_x is the transverse normalised momenta, β_s is the relativistic factor for the reference particle and p_t is the energy deviation divided by the reference momenta P_s (see [5] for exact definitions). The drift when described exactly has interesting features such as introducing dependencies between planes. This can easily be seen by just observing the exact drift map:

$$x_f = x_i + l\frac{p_x}{\sqrt{\left(p_t^2+\frac{2p_t}{\beta_s}+1\right)-p_x^2-p_y^2}}. \tag{2}$$

In Fig. 1 the deviations between the MAD-X expanded and PTC exact drifts and the new exact drift in MAD-X TWISS and PTC are shown. We can observe that there is a deviation in both the orbit and the β-function for the old method, while the new one follows within numerical precision the results from PTC. In order to activate the exact drift, one simply adds the keyword EXACT when calling TWISS. Even though this method requires more calculations, no measurable slowdown was observed when using it for the LHC lattice.

A recent related development on the analytical side [7] described the evolution of the W function in a drift considering the chromatic map shown in Eq. (1).

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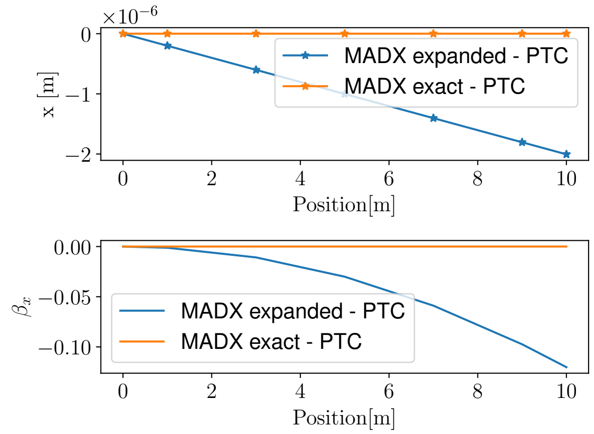

Figure 1: Deviations between the MAD-X TWISS and PTC at different locations along a drift space with initial conditions $p_x = 0.001, p_y = 0.02$ using the new exact option and the previous approximation method in MAD-X.

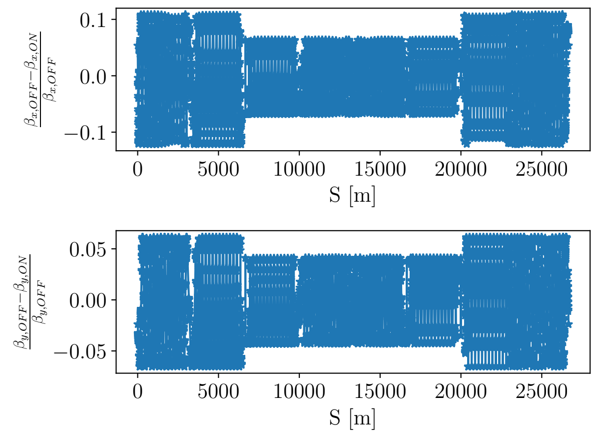

Figure 2: The introduced β-beat from a LRBB wire on the LHC optics when powered to 300 A at 6.8 TeV and at β^* of 30 cm for Beam 1.

WIRE

The wire element is planned to be used to compensate the effect of the Long Range Beam Beam (LRBB) effect in the LHC in Run 3 [8]. The wire used can be installed in combination with a collimator. In general overlapping elements are not allowed in MAD-X and therefore a special wire collimator has been implemented. This enables the user to install a wire-collimator in the sequence while keeping the functionality of both a wire and collimator internally. This means for example that when converted to SixTrack [9] it is converted to both a wire and a collimator. The physics model [10] of the wire is also the same as implemented in SixTrack [9] and the input format tries to follow the same conventions when possible.

The implementation in the TWISS module has been done using the method described in the expansion section meaning that the derivatives are calculated around the closed orbit. It is also convenient to have the possibility to perform simulations where the wire does not impact the closed orbit. This is the default setting, but in order to have an impact on the closed orbit from the LRBB one simply changes the option BBORBIT to TRUE.

In Fig. 2 the induced β-beat from the wires in the LHC when powered to 300 A at 6.8 TeV and at β^* of 30 cm is shown.

COUPLING

The way to correct the global transverse coupling in MAD-X has traditionally been to match the tunes as close as possible to each other and try to adjust the skew quadrupoles to minimize the distance between the tunes. This quantity is refereed to as $|C^-|$ or ΔQ_{min}. In Fig. 3 this quantity is indicated with the arrow and it is the minimum distance between the red lines which are the eigentunes. The black lines show the tunes in case there is no transverse coupling. Even

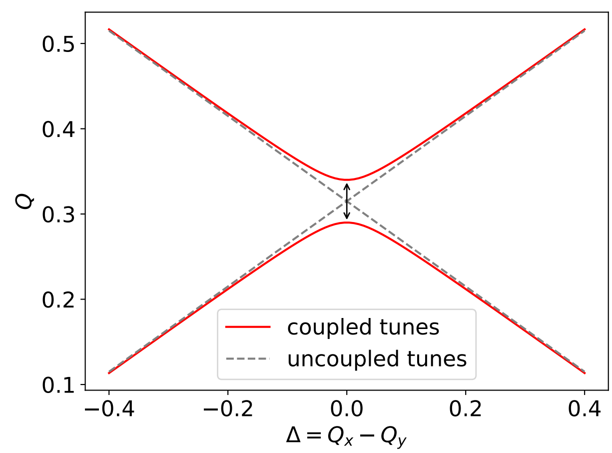

Figure 3: Coupled and uncoupled tunes versus uncoupled tune split. The arrow denotes the ΔQ_{min}.

though this method works well, it is desirable to have an estimate of the ΔQ_{min} without having to change quadrupole strengths performing a full matching. This estimate can then be used for coupling minimization or simply as a quality check that the coupling is below a certain level. Already an estimate of the ΔQ_{min} based on the skew quadrupolar strength was derived by Guignard [11]. Even though it is possible to get the exact quadrupolar strength in MAD-X, it is not a convenient way since skew fields might also derive from higher order fields through feed-down. Instead, it is preferable to have an estimate that is only based on previously calculated values. Several of such estimates do exist, since they are also very useful to estimate the ΔQ_{min} from measurements. In the "Thin Element Program For Optics and Tracking" (TEAPOT) [12] manual an expression is given based on the one-turn-map that later was translated into Resonance Driving Terms (RDTs) formalism [13]. In

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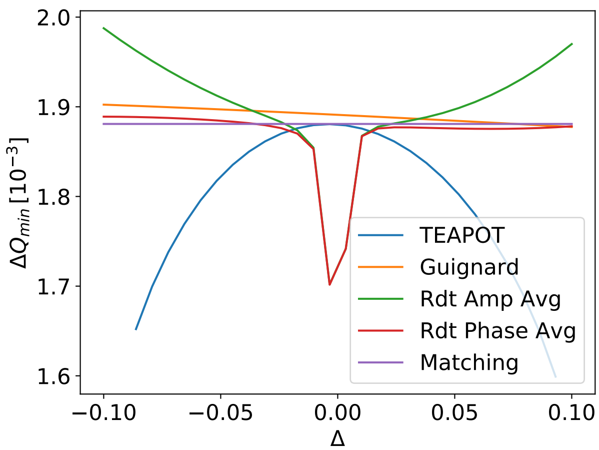

Figure 4: ΔQ_{min} plotted against fractional tune split for estimates described in the article with $Q_x = 0.3 + \Delta/2$ and $Q_y = 0.3 - \Delta/2$.

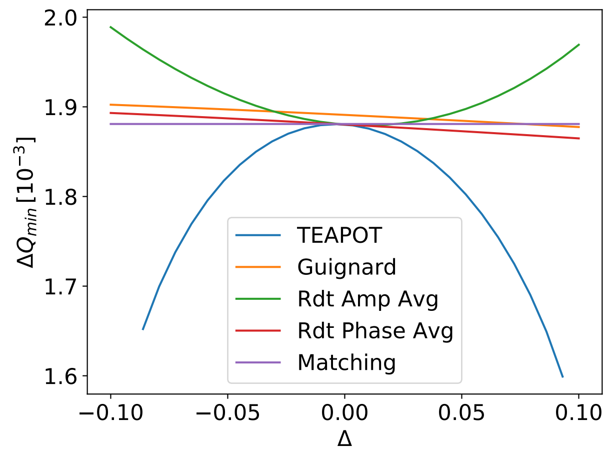

Figure 5: ΔQ_{min} plotted against fractional tune split for estimates described in the article with $Q_x = 0.3 + \Delta/2$ and $Q_y = 0.3 - \Delta/2$. Including substitutions specified in Eq. (6) for RDT Amp Avg and RDT Phase Avg.

Eq. (3) we give the expression in the RDT form as:

$$\Delta Q_{\mathrm{min}}^{TEAPOT} = \frac{|\tan(\pi\Delta')|}{\pi} \frac{4\sqrt{|f_{1001}|^2 - |f_{1010}|^2}}{1 + 4\left(|f_{1001}|^2 - |f_{1010}|^2\right)} . \quad (3)$$

where Δ' is the difference of the horizontal and vertical eigentunes (Q1 and Q2 in MAD-X), f_{1001} is the difference RDT and f_{1010} is the sum RDT. In this formalism the ΔQ_{min} was also dependent on f_{1010} which is not the case in the approximations we are about to see. In [14] the following equation was proposed:

$$\Delta Q_{\mathrm{min}}^{rdt\,amp\,avg} = \frac{4|\Delta|}{N} \sum_{w}^{N} \left|f_{1001}^{w}\right| , \quad (4)$$

where Δ is the difference in the uncoupled tunes and the sum goes over the observation points.

A conceptually similar equation was derived in [15, 16] with the difference that it takes a phase average of the f_{1001} rather than an average of the amplitude of the f_{1001}. The equation is written as:

$$\Delta Q_{\mathrm{min}}^{rdt\,phase\,avg} = \left|\frac{4\Delta}{2\pi R} \oint \mathrm{d}s f_{1001} e^{-i(\phi_x - \phi_y) + i\frac{s\Delta}{R}}\right| , \quad (5)$$

where R is the radius of the machine, s the position, ϕ_x and ϕ_y are the horizontal and vertical phase advance respectively. Equation (5) yields better results compared to Eq. (4) when evaluated further away from the $|Q_x - Q_y|$ resonance as seen in Fig. 4.

However, both Eq. (4) and Eq. (5) don't correctly predict the behaviour when evaluated close to the ΔQ_{min}, as seen in Fig. 4. This behaviour is not observed when using the equation from TEAPOT.

One can observe that by making the following substitutions:

$$f_{1001} \mapsto \frac{f_{1001}}{1 + 4|f_{1001}|^2} , \; \Delta \mapsto \Delta' . \quad (6)$$

in Eq. (4) and make the following approximation for Eq. (3): $\Delta' \ll 1$ and $|f_{1010}| \ll |f_{1001}|$ the two equations become identical which is shown in detail in [17]. The same substitution can be made to Eq. (5), which then results in the Eq. (7) as implemented in MAD-X and ouput in the TWISS table starting from release 5.08.01 and onward. The equation used is also very similar to what is derived in [16]:

$$\Delta Q_{\mathrm{min}}^{rdt\,phase\,avg} = \left|\frac{4\Delta'}{2\pi R} \oint \frac{\mathrm{d}s f_{1001}}{1 + 4|f_{1001}|^2} e^{-i(\phi_x - \phi_y) + i\frac{s\Delta'}{R}}\right| . \quad (7)$$

Even though the equation works very well in most cases (as shown in Fig. 5) there are still some limitations where the equation is less reliable [17]. This is in particular when $|f_{1010}|$ becomes comparable to $|f_{1001}|$. When this happens, a warning is emitted by MAD-X to notify the user that the estimate might not be accurate. However, in the cases where it has been used to minimize the coupling it has shown a significant reduction in time to compute a correction compared to the matching method.

CONCLUSION AND OUTLOOK

MAD-X continues to be one of the most used tool for accelerator simulations with an active user community requesting new features and reporting bugs. Planned features for coming releases include enabling the spin from PTC as well as implementing exact rotations in a similar manner as what was done for the exact drift.

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