ISBN: 978-3-95450-227-1 ISSN: 2673-5490 doi:10.18429/JACoW-IPAC2022-WEPOTK063

A WIRELESS METHOD TO OBTAIN THE IMPEDANCE FROM SCATTERING PARAMETERS

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Abstract

The coaxial wire method is a common and appreciated choice to assess the beam coupling impedance of an accelerator element from scattering parameters. Nevertheless, the results obtained from wire measurements could be inaccurate due to the presence of the stretched conductive wire that artificially creates the conditions for the propagation of a Transverse ElectroMagnetic (TEM) mode.

The aim of this work is to establish a solid technique to obtain the beam coupling impedance from electromagnetic simulations, without modifications of the device under test. In this framework, we identified a new relation to get the resistive wall beam coupling impedance of a circular chamber directly from the scattering parameters and demonstrated that it reduces to the exact theoretical expression. Furthermore, a possible generalization of the method to arbitrary cross section geometries has been studied and validated with numerical simulations.

INTRODUCTION

The beam coupling impedance describes the electromagnetic interaction between the particle beam and the accelerating structure. Ideally, the beam coupling impedance of a device should be evaluated by exciting the device with the beam itself. However, in most cases, this solution is not possible, and one must resort to alternative methods to consider the effect of the beam.

A well-established technique is to simulate the beam by a current pulse flowing through a wire stretched along the beam axis, resulted in the development of the stretched Wire Method (WM) [1]. Nevertheless, the results obtained from wire measurements might not entirely represent the solution of our initial problem, because the presence of the stretched wire perturbs the EM boundary conditions. The most evident consequence of the presence of another conductive medium in the centre of the device under study is the artificial propagation of the TEM mode through the device, with zero cut-off frequency. The presence of a TEM mode among the solutions of the EM problem will have the undesired effect of causing additional losses. In this regard, the attention has been focused to possible approaches without modification of the Device Under Test (DUT). Wireless measurements have already been proposed in [2] and performed in [3] above the cut-off frequency of the device under test (DUT), where an approximated formula has been employed.

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An exact formula to obtain the longitudinal beam coupling impedance of the accelerator components is presented in this paper. The new formula, relating the longitudinal beam coupling impedance and the scattering parameters, has been analytically validated for a resistive circular chamber also below its cut-off frequency. Furthermore, a possible generalization to arbitrary chamber shapes above the cut-off frequency has been explored.

WIRELESS METHOD

The longitudinal beam coupling impedance is essentially related to the energy loss of the electromagnetic wave propagating in the structure and, therefore, is intrinsically linked to the transmission scattering parameter. Given these considerations and by analogy with the WM we looked for a Log-formula to express the beam coupling impedance by using the first propagating Transverse Magnetic (TM) mode in the DUT. The proposed relation to evaluate the impedance, without modifications of the DUT, has the following form:

$$Z = -K \cdot Z_{mode} \ln \frac{|S_{21DUT}|}{|S_{21REF}|}. \quad (1)$$

The Z_{mode} is the characteristic wave impedance of the TM propagating mode. The transmission scattering parameter S_{21DUT} refers to the 2-Port DUT, that is the chamber with finite electric conductive walls, while the S_{21REF} refers to the related reference structure, in this case, the chamber with Perfect Electric Conductive (PEC) walls. The term K is a possible constant to be determined in the analytical derivation.

Analytical Validation

The proposed formula has been analytically validated for the case of a circular resistive wall chamber of radius b, wall conductivity σ and length L, both below and above the cut-off frequency of the chamber. The longitudinal beam coupling impedance of the circular resistive chamber can be analytically calculated by using the following well-known equation [4]:

$$Z^{theory} = \frac{\zeta_s}{2\pi b} L, \quad (2)$$

where in the classical thick wall regime $\zeta_s = \zeta(1+j) = \sqrt{\frac{\omega\mu_0}{2\sigma}}(1+j)$, ω is the angular frequency and μ_0 the permeability of free space. In order to validate the proposed approach it has to be demonstrated that the formula of Eq. (1) reduces to the theoretical formula of Eq. (2). The analytical expression of the S_{21} of the resistive circular pipe

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is derived from the attenuation constant α by imposing the conservation of the energy. It turns out to be the following:

$$S_{21} = e^{-|\alpha| z}. \quad (3)$$

The attenuation constant α for a lossy circular pipe is obtained in [5] applying the Leontovich boundary condition and can be written as follows:

$$\alpha = Im\sqrt{k_0^2 - \frac{1}{b^2}\left[u_{nm} + \frac{j k_0^2 (\frac{u_{nm}}{b})^2}{\omega(\frac{u_{nm}}{b})^3(\frac{\mu_0}{\zeta_s} + \epsilon_0 \zeta_s)}\right]}. \quad (4)$$

The terms k_0, ϵ_0 are the wave number and the permittivity of free space and u_{nm} is the m^{th} zero of the Bessel function J_n.

Figure 1 shows the perfect agreement that has been reached between the simulated S_{21} for a given resistive chamber and, the analytically computed S_{21} computed with Eqs. (3), (4). Afterwards, Eqs. (3), (4) can be substituted in (1),

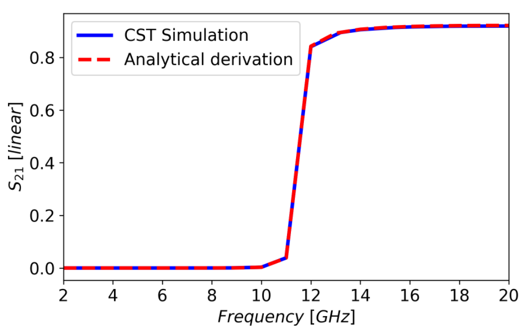

Figure 1: Comparison between the S_{21} from the analytical computation and simulation for a circular chamber with $b = 10\ mm$, $\sigma = 3000\ S/m$ and $L = 50\ mm$.

where:

$$Z_{mode} = Z_{TM} = \frac{\sqrt{k_0^2 - \frac{u^2}{b^2}}}{\omega \epsilon_0},$$

and the following expressions of the longitudinal impedance are obtained below and above the chamber cut-off frequency [6]:

$$Z_{below} = -K \cdot Z_{TM}(\sqrt{A} - \sqrt[4]{B^2 + C^2})L \quad (5)$$

$$Z_{above} = -K \cdot Z_{TM}\left(\sqrt[4]{B^2 + C^2} \cdot \frac{-C}{2B}\right)L, \quad (6)$$

where $A = k_0^2 - \frac{u^2}{b^2}$, $B = k_0^2 - \frac{u^2}{b^2} + 2\omega\epsilon_0\frac{\zeta}{b}$ and $C = 2\omega\epsilon_0\frac{\zeta}{b}$. Under the assumption that the structure can be treated as a planar geometry, which means that the radius of curvature is much greater than the skin depth ($b >> \delta = \sqrt{\frac{2}{\omega\mu_0\sigma}}$), it can be shown that, both below and above the chamber cut-off frequency, the longitudinal impedance of Eq. (1) reduces to Eq. (2), by placing the constant term K equal to $\frac{1}{2\pi}$ [6].

This proves the correctness of the relation between the scattering parameter S_{21} and the longitudinal beam coupling impedance proposed in Eq. (1) that becomes:

$$Z = -\frac{1}{2\pi} \cdot Z_{TM} \ln\frac{|S_{21DUT}|}{|S_{21REF}|} \quad (7)$$

Figure 2 displays the comparison between the impedance from the analytical derivation and the theoretical one.

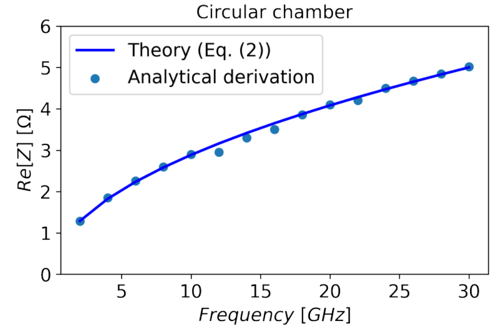

Figure 2: Comparison between the longitudinal impedance of Eqs. (5), (6) and Eq. (2). The agreement is not perfect around 11 GHz, the cut-off frequency of the chamber, due to the analytical approach which does not provide an estimation of the impedance at the cut-off frequency.

GENERALIZATION OF THE WIRELESS METHOD TO ARBITRARY CROSS SECTION GEOMETRIES

The generalization of the method to arbitrary shapes of the vacuum chambers has been explored. As a first step, the case of the rectangular chamber has been studied to investigate the potential of the method with non-axially symmetric structures. The result is that, above the cut-off frequency of the chamber, the impedance can be derived from the scattering parameter S_{21} with the following relation [6]:

$$Z = -\frac{G \cdot F}{2\pi} \cdot Z_{TM} \ln\frac{|S_{21DUT}|}{|S_{21REF}|}, \quad (8)$$

where F and G are geometrical factors. F is the longitudinal form factor for the rectangular chamber (see [7], [8]). Analytical expressions of F exist for the rectangular and elliptical chambers and could be computed for any geometry with simulations, since it is given by the ratio of the wake function of the chamber under test and the wake function of the reference circular chamber (see [8]):

$$F = \frac{w(z)^{DUT}}{w(z)^{CIR}}.$$

The theoretical expression of the beam coupling impedance (Eq. (2)) can be generalized to arbitrary cross section chambers using the form factor F. The G factor is related only to the maximum half-width a and half-height b of the cross section of the DUT as defined by the following expression [6]:

$$G = \frac{(b^2+a^2)a}{b^3+a^3}.$$

Its behaviour versus the aspect ratio of the chamber is displayed in Fig. 3.

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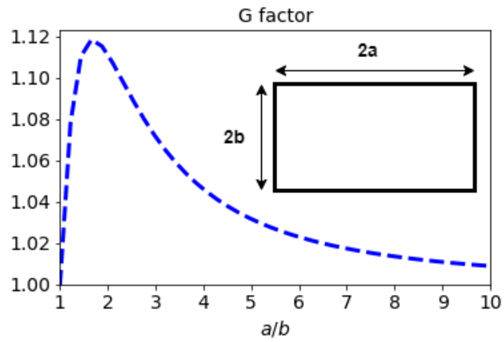

Figure 3: G versus $\frac{a}{b}$.

It is worth mentioning that Eq. (8) is a more general expression of Eq. (7). In fact, for circular chambers, F and G are equal to 1 and Eq. (8) would reduce exactly to Eq. (7). Consequently, the developed method has been tested in simulation for resistive wall chambers with elliptical and octagonal cross section. These simulations suggest that Eq. (8) is a general expression that could be applied to obtain the longitudinal beam coupling impedance of arbitrary cross section chambers.

SIMULATION RESULTS AND COMPARISON WITH THEORY

The simulation studies are carried out using a 3D electromagnetic tool, CST Studio Suite, which makes available several simulation solvers [9]. In this framework, the choice fell on the frequency domain solver because it is equipped with tetrahedral mesh cells, that allow a better discretization of the calculus domain, contrary to the time domain solver where only hexahedral mesh cells are available. Indeed, the aim of the proposed method is also to establish an accurate procedure to compute the impedance of curved and complex geometries. The DUT is excited using the Waveguide Ports which allow only the desired TM mode to be launched. The longitudinal impedance computed from frequency domain simulations by using Eq. (8), has been compared with the exact theoretical evaluation in Fig. 4. The results show an almost perfect agreement between the two curves suggesting that the proposed simulation approach, with the related formula, is a suitable and accurate method to compute the beam coupling impedance of arbitrary shaped chambers.

CONCLUSIONS AND OUTLOOK

We identified a logarithmic formula that relates the longitudinal beam coupling impedance and the transmission scattering parameter without modification of the DUT. The new formula has been analytically validated for a resistive circular chamber below and above the cut-off frequency.

The generalization of the method to arbitrary chamber shapes by means of appropriate factors, such as the form factor F and the geometrical factor G, has also been discussed and successfully benchmarked with simulations.

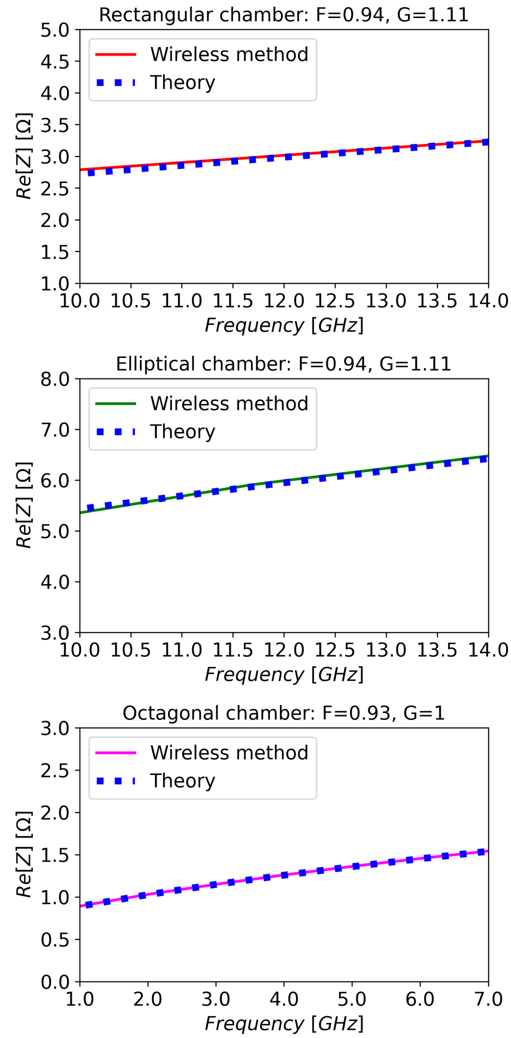

Figure 4: Comparison between the longitudinal impedance obtained from CST simulations (wireless method) and the theoretical impedance for various shapes of the chamber, above their cut-off frequencies. At the top: rectangular chamber with $a = 1.5\ mm$, $b = 10\ mm$, in the middle elliptical chamber with $a = 1.5\ mm$, $b = 10\ mm$, at the bottom octagonal chamber with $a = b = 50\ mm$. The imaginary part is not displayed since, in the frequency range analysed (thick wall regime), it is exactly equal to the real part (see Eq. (2)).

Moreover, since the new formula relates the longitudinal beam coupling impedance to the scattering parameter, which is also an output of the measurements, this very promising method could pave the way to develop a bench measurement technique, that will not require modifications of the DUT.

The key point of the development of a bench measurement technique is the engineering of an excitation able to emulate as much as possible the ideal excitation of the first TM mode provided by the Waveguide Port. This stage is currently under development as well as the possible extension of the method to resonant structures.

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