

**EXPERIMENTAL STUDY OF CONTROLLED
LONGITUDINAL BLOW-UP**

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ABSTRACT

High frequency cavities with phase modulation are used at the CERN PS in order to increase longitudinal bunch emittance. The dependence of blow-up on such parameters as phase modulation amplitude and frequency, and the phase of the high frequency voltage relative to the fundamental, has been predicted by theory and is herein verified experimentally.

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1 Introduction

Since 1976, the CERN PS has used high-frequency cavities at ~ 200 MHz to blow up the longitudinal emittance of the beam bunched at ~ 10 MHz while preserving the quality of the distribution [1]. A phase-modulated RF voltage is developed in the HF cavities on a high harmonic of the beam revolution frequency. The parameters of the system have been optimized during many years of operation.

RF noise theory [2] has been applied to explain some experimental results. Other features of the technique were found later using computer simulations [3]. Balandin, Dyachkov and Shaposhnikova developed a resonance theory [4] to attempt to explain this controlled blow-up and good agreement is observed with the results of numerical simulations [3], [4], [5].

The purpose of the present work is to compare, for the first time, some experimental results with the resonance theory.

2 Review of the Theory

2.1 Basic Equations and Definitions

According to the definitions used in the theory [4], the total voltage, V , affecting the particle which crosses the gap of the main RF cavity at a phase ϕ is written:

$$V = V_0 \sin \phi + V_1 \sin(N\phi + \Phi(t)). \quad (1)$$

Here, V_0 is the main RF voltage at a frequency f_{RF} , while V_1 and Nf_{RF} are the voltage and frequency applied to the HF cavity. The phase $\Phi(t)$ is modulated according to:

$$\Phi(t) = \alpha \sin(\Omega t + \Theta) + \theta, \quad (2)$$

where α and Ω are the phase modulation amplitude and frequency, respectively, and Θ and θ are constants.

For a stationary bucket (synchronous phase, $\phi_s = 0$), the longitudinal motion of particles in the absence of the HF cavity is governed by the equations:

$$\dot{r} = 0, \quad \dot{\psi} = \omega_s(r), \quad (3)$$

where the variable r , which is defined by

$$r = \frac{1}{\sqrt{2}} \sqrt{1 - \cos \phi + \frac{\dot{\phi}^2}{2\omega_{s0}^2}},$$

is a constant of motion which varies from 0 to 1 inside the bucket and is related to the energy of the synchrotron oscillations. ψ is the phase of synchrotron oscillations whose frequency, $\omega_s(r)$, is such that $\omega_s(0) = \omega_{s0}$.

In the presence of the HF cavity, the equations of motion (3) can be transformed into the following approximate expressions [4], valid for $\varepsilon = V_1/V_0 \leq 0.3$ and $r < 2/3$:

$$\dot{r} = \varepsilon \frac{\omega_s(r)}{8Nr} \sum_{m=1}^{\infty} m k_0 (-1)^{m l_0} J_{m k_0}(2Nr) J_{m l_0}(\alpha) \times \quad (4)$$

$$\{[1 + (-1)^{m(k_0+l_0)}] \cos \theta \sin(m\psi_r) + [1 - (-1)^{m(k_0+l_0)}] \sin \theta \cos(m\psi_r)\},$$

$$\dot{\psi} = \omega_s(r) - \varepsilon \frac{\omega_s(r)}{2r} J_1(2Nr) J_0(\alpha) \cos \theta - \varepsilon \frac{\omega_s(r)}{4r} \sum_{m=1}^{\infty} (-1)^{m l_0} [J_{m k_0+1}(2Nr) - J_{m k_0-1}(2Nr)] \times$$

$$J_{m l_0}(\alpha) \{[1 + (-1)^{m(k_0+l_0)}] \cos \theta \cos(m\psi_r) - [1 - (-1)^{m(k_0+l_0)}] \sin \theta \sin(m\psi_r)\}. \quad (5)$$

Here, $\psi_r = k_0\psi - l_0\Omega t - l_0\Theta$ is the resonant phase and $J_k(x)$ is the Bessel function of order k .

This derivation is obtained by noting that the resonance condition

$$k\dot{\psi} - l\Omega = 0, \quad \text{with } k, l \text{ integers}, \quad (6)$$

must be satisfied for a systematic change of r to take place. At a given point, r_0 , only integer multiple resonances can exist

$$\frac{k}{l} = \frac{k_0}{l_0}, \quad \text{or} \quad k = m k_0, l = m l_0,$$

where k_0, l_0 are the smallest possible integers and where $m=1,2,\dots$. This is reflected by the sum over m in the expressions (4) and (5).

2.2 Consequences

The following are consequences of the above:

- It follows from expression (4) that, for $\theta = \pi/2$ and for odd k_0 and l_0 , resonances are suppressed. This prediction of the theory was checked experimentally and the results are discussed later.

- In the general case, expressions (4) and (5) are not very simple to analyse. In Table 1 the first finite term (i.e. $m = 1$ or $m = 2$) in the summation of (4) is given for the particular

Table 1: Dominant coefficients in expression (4) for \dot{r} .

l_0	k_0	k_0/l_0	θ	1-st term	value at $\alpha = 2.4$
1	2	2	0	$4J_2(\alpha)J_4(2Nr)$	$1.72J_4(2Nr)$
			$\pi/2$	$2J_1(\alpha)J_2(2Nr)$	$1.04J_2(2Nr)$
	3	3	0	$3J_1(\alpha)J_3(2Nr)$	$1.56J_3(2Nr)$
			$\pi/2$	0	0
	4	4	0	$8J_2(\alpha)J_8(2Nr)$	$3.44J_8(2Nr)$
			$\pi/2$	$4J_1(\alpha)J_4(2Nr)$	$2.08J_4(2Nr)$
	5	5	0	$5J_1(\alpha)J_5(2Nr)$	$2.6J_5(2Nr)$
			$\pi/2$	0	0
2	5	2.5	0	$10J_4(\alpha)J_{10}(2Nr)$	$0.64J_{10}(2Nr)$
			$\pi/2$	$5J_2(\alpha)J_5(2Nr)$	$2.15J_5(2Nr)$
	7	3.5	0	$14J_4(\alpha)J_{14}(2Nr)$	$0.9J_{14}(2Nr)$
			$\pi/2$	$7J_2(\alpha)J_7(2Nr)$	$3.0J_7(2Nr)$
	9	4.5	0	$18J_4(\alpha)J_{18}(2Nr)$	$1.2J_{18}(2Nr)$
			$\pi/2$	$9J_2(\alpha)J_9(2Nr)$	$3.9J_9(2Nr)$
	11	5.5	0	$22J_4(\alpha)J_{22}(2Nr)$	$1.4J_{22}(2Nr)$
			$\pi/2$	$11J_2(\alpha)J_{11}(2Nr)$	$4.7J_{11}(2Nr)$
3	7	2.33	0	$7J_3(\alpha)J_7(2Nr)$	$1.39J_7(2Nr)$
			$\pi/2$	0	0
	8	2.67	0	$16J_6(\alpha)J_{16}(2Nr)$	$0.06J_{16}(2Nr)$
			$\pi/2$	$8J_3(\alpha)J_8(2Nr)$	$1.6J_8(2Nr)$
	10	3.33	0	$20J_6(\alpha)J_{20}(2Nr)$	$0.07J_{20}(2Nr)$
			$\pi/2$	$10J_3(\alpha)J_{10}(2Nr)$	$2.0J_{10}(2Nr)$
	11	3.67	0	$11J_3(\alpha)J_{11}(2Nr)$	$2.2J_{11}(2Nr)$
			$\pi/2$	0	0
	13	4.33	0	$13J_3(\alpha)J_{13}(2Nr)$	$2.6J_{13}(2Nr)$
			$\pi/2$	0	0
	14	4.67	0	$28J_6(\alpha)J_{28}(2Nr)$	$0.1J_{28}(2Nr)$
			$\pi/2$	$14J_3(\alpha)J_{14}(2Nr)$	$2.8J_{14}(2Nr)$
16	5.33	0	$32J_6(\alpha)J_{32}(2Nr)$	$0.1J_{32}(2Nr)$	
		$\pi/2$	$16J_3(\alpha)J_{16}(2Nr)$	$3.2J_{16}(2Nr)$	

cases $l_0 = 1, 2, 3$, $k_0 = 2, 3 \dots 16$ and $\theta = 0, \pi/2$. The number of non-negligible terms in the summation of (4) as well as the significant values of the parameters k_0 and l_0 depend on the value of N and on the size of the bunch in phase space. Using a conventional approximation for the Bessel function of order mk_0 , one finds that the highest harmonic, m , of the modulation frequency at which particles inside the bunch are affected satisfies the condition

$$mk_0 + 0.8(mk_0)^{1/3} \leq 2Nr_b. \quad (7)$$

Here, r_b is the maximum value of r inside the bunch. For short bunches, r_b is equal to one quarter of the bunch length in radians: $r_b = \Delta\phi_b/4$.

- Each term in the summation of (4) is proportional to $J_{ml_0}(\alpha)$. Therefore, with an appropriate choice of the parameter α , those resonances with high l_0 as well as the subsequent terms (those with large m) can be made negligible.

- The modulation function of the form (2) produces a steady-state modulation of the synchrotron frequency within the bunch according to

$$\frac{\Delta\omega_s}{\omega_s} = -\varepsilon \frac{J_1(2Nr)}{2r} J_0(\alpha) \cos \theta. \quad (8)$$

This modulation is absent when θ is an odd multiple of $\pi/2$ and for zeros of the Bessel function of zero order (i.e. when $\alpha = 2.48, \dots$). The evolution of the synchrotron frequency inside the bunch, from its centre ($r = 0$) to its edge ($r = 1$), is illustrated in Fig.1 for three different values of α , with $\theta = 0$ and $\varepsilon = 0.1$.

3 Experimental Study

3.1 Experimental Set-up

Experiments were performed using the two sets of parameters listed in Table 2.

The measurement system is described in Appendix A. The basic hardware layout for the excitation of the HF cavities is sketched in Appendix B. As a measure of the degree of blow-up, we used two bunch length estimations, rough and fitted, as described in Appendix A.

Different bunch shapes have been observed in the course of the experiments. The relevant measurements are presented in section 3.5 and discussed with respect to theoretical expectations.

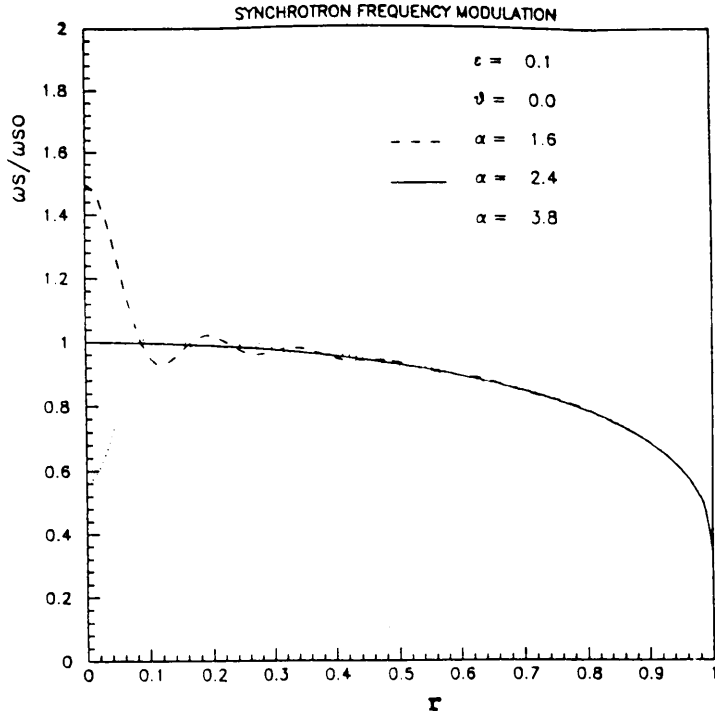


Figure 1: Normalized synchrotron frequency inside a bunch for $\alpha = 1.6, 2.48, 3.8$.

Table 2: Machine and beam parameters

	Parameter list 1	Parameter list 2
N	21+13/20	22
B (Gauss)	1692	1413
γ	3.918	3.318
β	0.9669	0.9535
f_{rev} (kHz)	461.34	454.96
f_{RF} (MHz), $h = 20$	9.227	9.102
V_0 (kV)	76-115	91
f_{so} (Hz)	757-920	1165
V_1 (kV)	17	17
$T_{\text{blow-up}}$ (ms)	60	60

3.2 PS Operational Blow-up at 3.5 GeV/c

The usual parameters during PS operation are given in parameter list 1 of Table 2. In addition,

$$V_0 = 76 \text{ kV, which gives } \omega_{s0}/(2\pi) = 0.757 \text{ kHz.}$$

The phase modulation of the HF cavities has

$$\Omega/(2\pi) = 3.0 \text{ kHz} \quad \text{and} \quad \alpha = \pi \text{ rad}$$

and the initial bunch length is

$$4\sigma_t = 32.8 \text{ ns.}$$

The initial and final bunches are shown in Fig.2(a), (b).

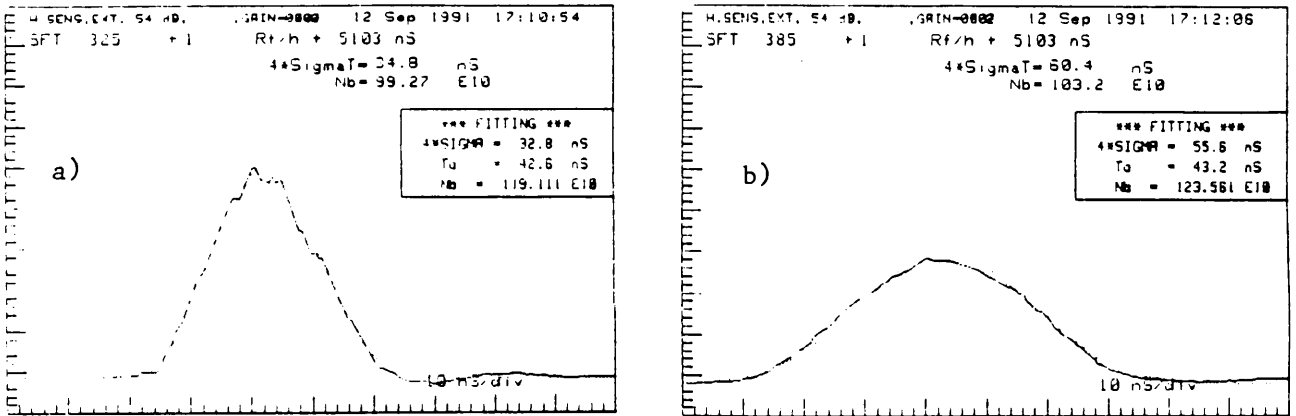


Figure 2: (a) Initial and (b) final bunches with the operational blow-up.

The ratio $k_0/l_0 = \Omega/\omega_{s0} = 3.96$, which corresponds to $k_0 = 4$ and $l_0 = 1$.

Due to the non-integer value of N , the successive bunches have different values of θ . The phase for bunch number n is given by

$$\theta_n = \frac{13}{20} 2\pi(n-1) + \theta_1. \quad (9)$$

Consequently, different bunches should have different degrees of blow-up. However, as shown in Table 1, there is no large difference between the cases $\theta = 0$ and $\theta = \pi/2$ for $k_0 = 4$. This fits with the observation that there is no significant difference between bunches.

The value $\alpha = \pi$ corresponds to the first maximum of the Bessel function of order 2. So only modes with $m = 1, 2$ or 3 contribute significantly to the blow-up.

The initial bunch length $4\sigma_t = 32.8$ ns corresponds to $r_b = 0.46$. This implies that harmonics with $(k_0 m) \geq 18$ cannot affect the bunch (see condition(7)). When $k_0 = 4$ and $m = 1, 2$ the excitation tends to blow-up the bunch centre as desired. By comparison, if k_0 were equal to 6, then for some bunches the first non-zero term would be $m = 2$, which is proportional to $J_{12}(2Nr)$, and the effect would be maximum at the bunch boundary, causing tails to appear.

To summarize, the parameters used operationally for controlled longitudinal blow-up in the PS appear to be close to optimal in terms of the resonance theory.

3.3 Effect of θ and α on the Blow-up for Non-integer N

Using parameter list 1 of Table 2, we investigated the dependence of blow-up on the phase offset, θ , between the two RF systems. To maximize the difference in blow-up between bunches, we chose an odd number for k_0 ($k_0 = 3$) while keeping all other parameters as above.

After applying a voltage with zero phase modulation ($\alpha = 0$) during 60 ms in the HF cavities, the bunches had a negligible variance in length as illustrated in Fig.3.

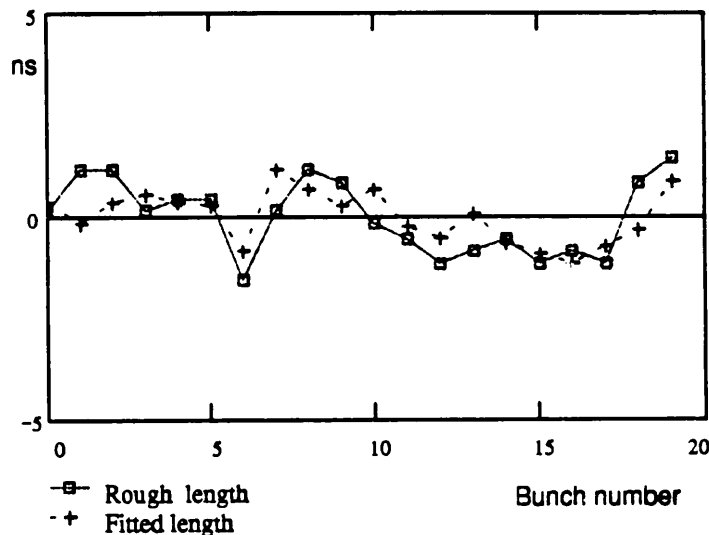


Figure 3: Measurement for $N = 21 + 13/20$ and zero phase modulation. Difference between actual and mean bunch lengths versus bunch number over one machine turn.

Whereas, with $\alpha = \pi$, a characteristic modulation pattern of the bunch length is observed (see Fig.4).

If the synchrotron frequency is the same for all bunches ($\Delta\omega_s/\omega_s = 0$ in (8)), the resonance condition, $\Omega = k_0\omega_s$, is met for the same phase modulation frequency, Ω . In this case, as

follows from (4) for odd values of k_0 and $l_0 = 1$, \dot{r} is proportional to $\cos \theta$. The increase of bunch length is, in fact, a function of $|\cos \theta|$. This means that we can expect a modulation of length from bunch to bunch which has a periodicity in θ of π . According to (9) this corresponds to 26 periods over one machine turn or to 6 observable periods because of the sampling process. This is qualitatively presented in Fig.5 for $\theta = \pi/2$.

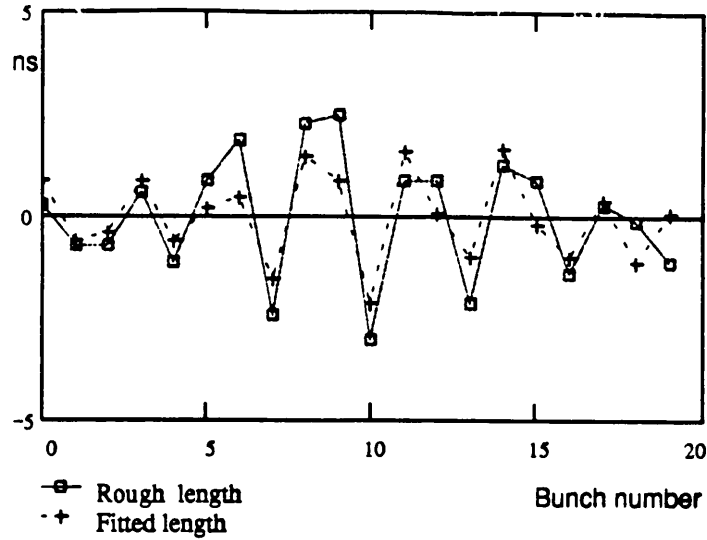


Figure 4: Measurement for $N = 21 + 13/20$ and $\alpha = \pi$. Difference between actual and mean bunch lengths versus bunch number over one machine turn.

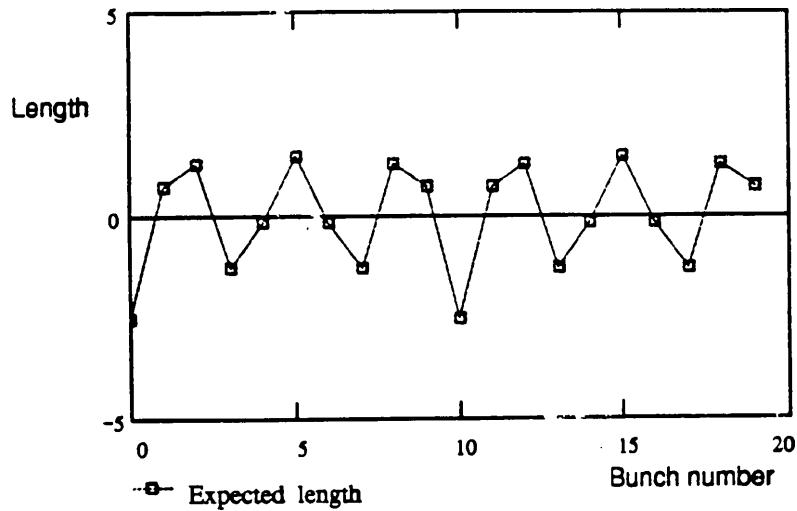


Figure 5: Expected bunch length modulation over one machine turn in arbitrary units for non-zero phase modulation, $N = 21 + 13/20$ and $\Delta\omega_s/\omega_s = 0$.

However, in the present experiment, $\alpha = 3$ and consequently $\Delta\omega_s/\omega_s \neq 0$ and a periodicity in θ of π is no longer expected. Instead, the periodicity in θ becomes 2π , which means 13 periods over one machine turn or 7 observable periods because of the sampling process.

The frequency spectrum of the bunch length modulation of Figs.4 and 5, derived by Fast Fourier Transformation, is shown in Fig.6(a), (b). As expected, harmonic 7 is dominant in the experimental data of Fig.6(a) and harmonic 6 is dominant in Fig.6(b).

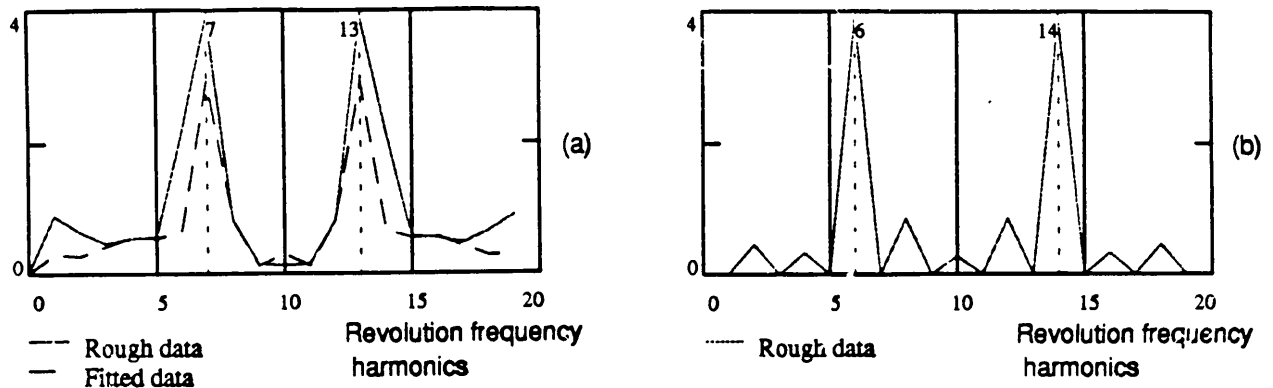


Figure 6: Frequency spectrum (a) of the experimental bunch length modulation over one machine turn for bunches with $\Delta\omega_s/\omega_s \neq 0$ and (b) of the expected bunch length modulation over one machine turn for $\Delta\omega_s/\omega_s = 0$.

3.4 Effect of θ , α and Ω on the Blow-up for Integer N

Using parameter list 2 of Table 2, we investigated the effect of the phase, θ , of the HF cavity and of the amplitude, α , and frequency, Ω , of the phase modulation.

Since N is an integer, the data taking is simplified because all bunches experience the same blow-up. Measurement confirmed this within the limit of accuracy of the acquisition system, so only a single bunch was measured during the rest of the experiment.

First we investigated the dependence of bunch length on the phase offset, θ , between the two RF systems for $k_0 = 3$ and $l_0 = 1$. With $\alpha = 2.8$, the results presented in Fig.7(a) show the expected periodicity in θ of 2π (see previous section) due to $\Delta\omega_s/\omega_s$ being non-zero. While, with $\alpha = 2.48$, the predicted periodicity in θ of π is qualitatively observed (see Fig.7(b)) because of the absence of any θ -dependent synchrotron frequency shift (see equation (8)).

The effect of the modulation frequency was measured for bunches with a difference in θ of approximately $\pi/2$ with a modulation amplitude, $\alpha = 2.48$. Fig.8 shows the bunch length as a function of the normalized modulation frequency, Ω/ω_{s0} .

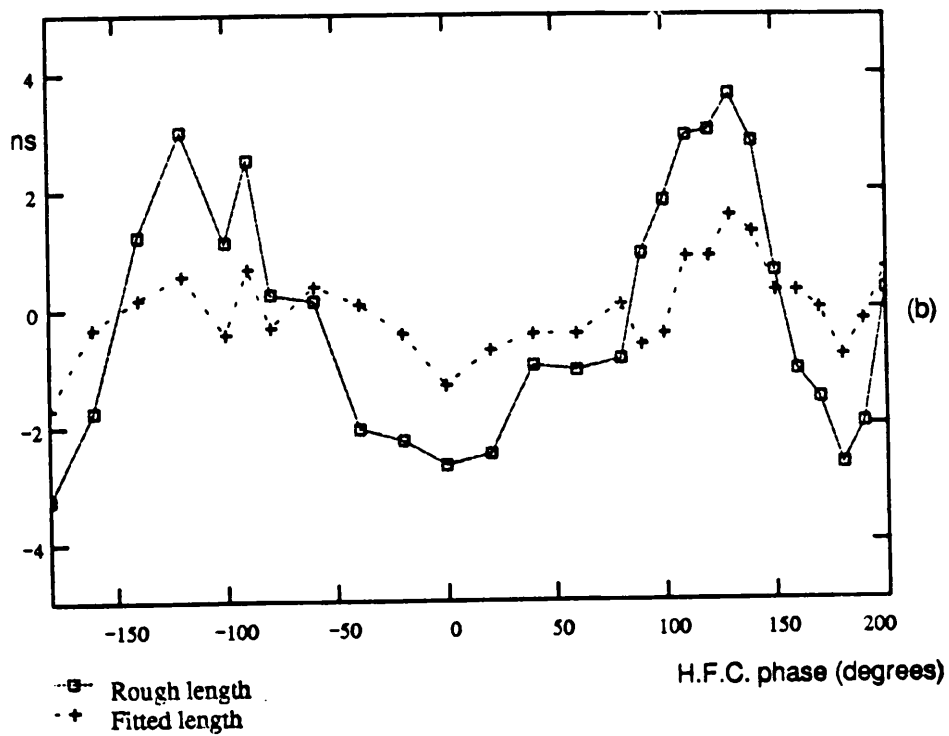
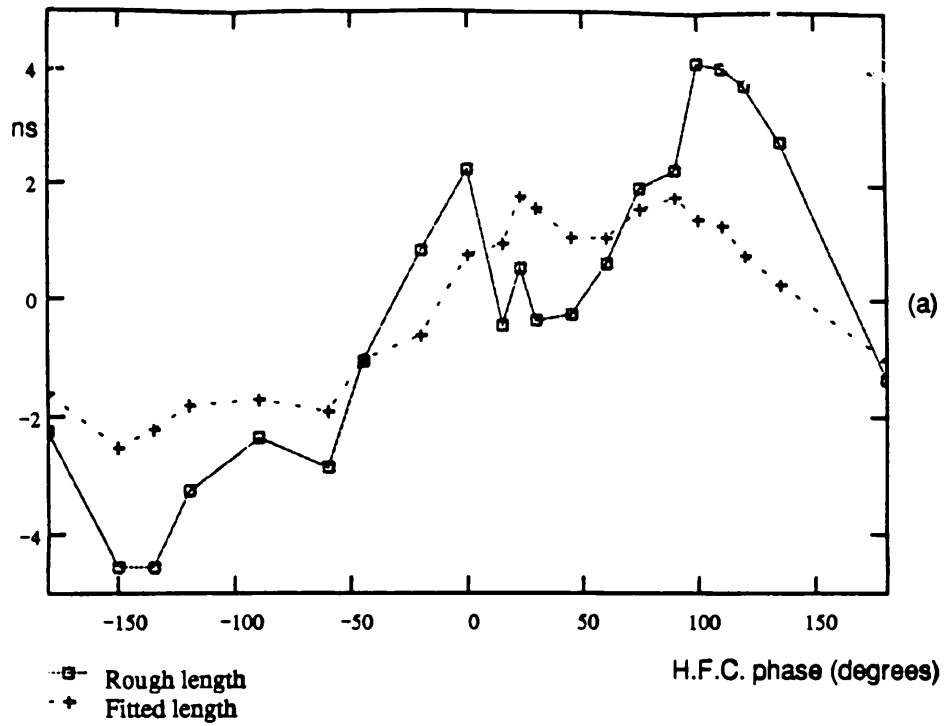


Figure 7: Measured difference (in nanoseconds) between actual and mean bunch lengths as a function of the constant phase offset, θ , between the two RF systems (a) for phase modulation amplitude, $\alpha = 2.8$ and (b) for $\alpha = 2.48$. Theory predicts a period of 360° in case (a) and 180° in case (b).

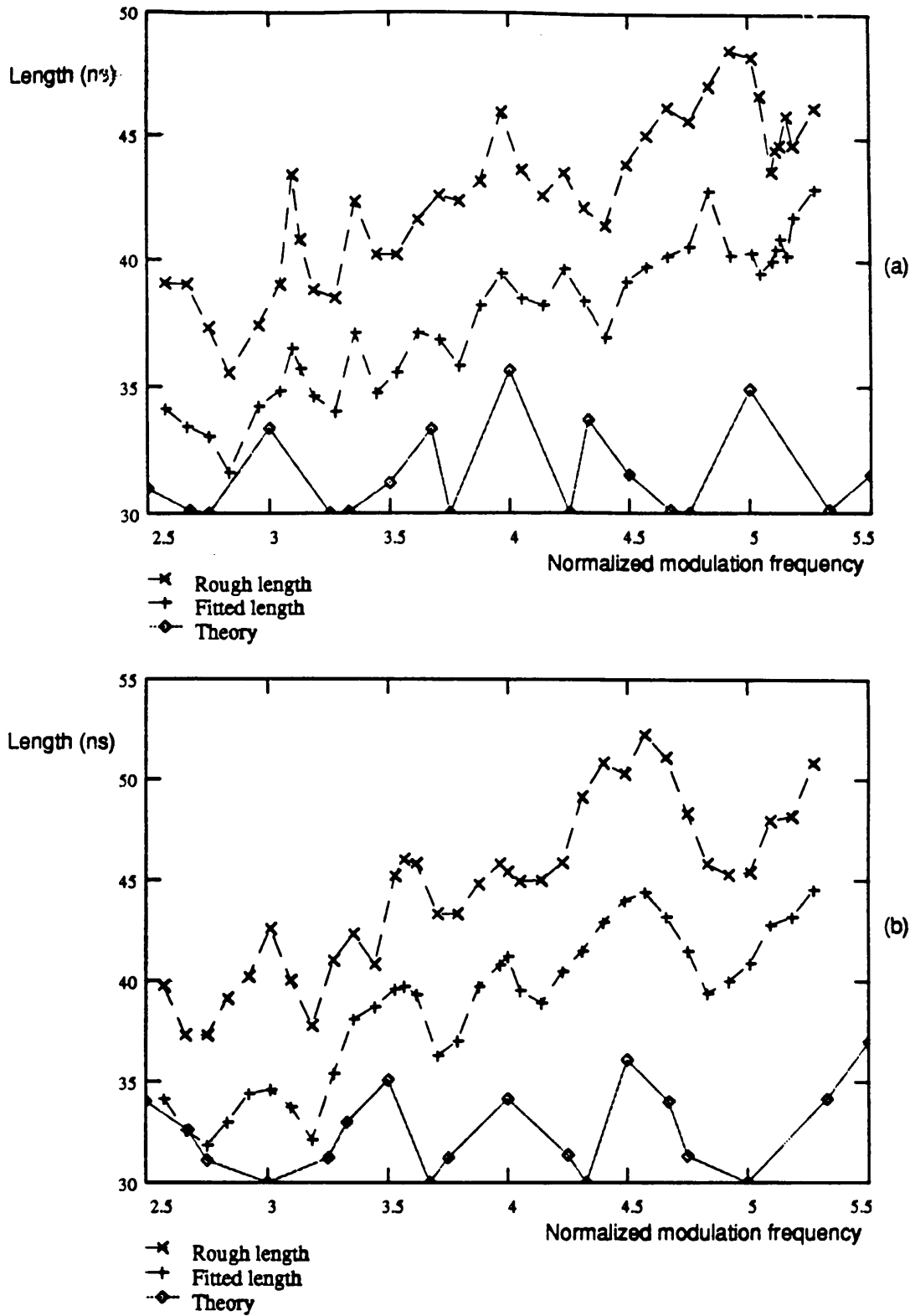


Figure 8: Bunch length as a function of the normalized modulation frequency, Ω/ω_{s0} , for $\alpha = 2.48$ with (a) $\theta = 20^\circ$ and (b) $\theta = 120^\circ$. Diamonds indicate the relative amplitude (in arbitrary units) of the major resonances predicted by theory.

As expected, the blow-up process is clearly resonant in nature. The relative amplitude of the resonance with $l_0 = 1, 2, 3$ and $k_0 = 2, \dots, 16$ (see Table 1, where the maximum value of the first term in the summation of (4) inside the bunch is given as a function of k_0/l_0) is also shown in Fig.8 (diamonds). The similarity between theoretical and experimental results is rather good, especially in Fig.8(b). Analysis shows that bunches with different phase offset, θ , for odd l_0 and even k_0 ($k_0/l_0 = 2, 4, 6$) or for even l_0 and odd k_0 ($k_0/l_0 = 2.5, 3.5, \dots$) should have their resonances at the same modulation frequency. While, for a bunch with $\theta = \pi/2$, resonances with odd values of k_0 and l_0 should be suppressed. A comparison of the experimental data with theoretical estimations is made in Fig.9.

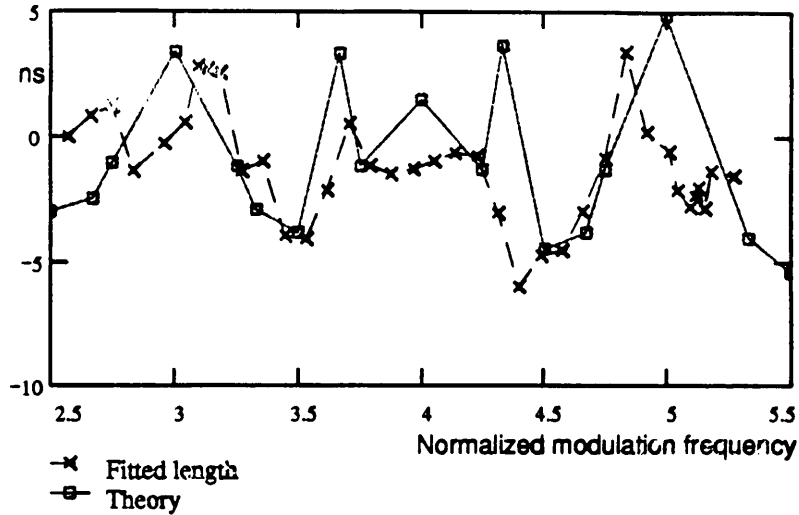


Figure 9: Comparison of expected and measured difference in length between two bunches with a difference in phase offset, $\theta \sim \pi/2$ as a function of the normalised modulation frequency for phase modulation amplitude, $\alpha = 2.48$.

It is important to note that, even for conditions under which resonances were suppressed, we never observed bunches with zero blow-up. We believe that this can be explained by the non-zero width of the resonances due to the spread of synchrotron frequencies ($\omega_s(r)$) inside the bunch.

3.5 Effect on Bunch Shape

Bunch length is only one aspect of the effect of the controlled longitudinal blow-up on the longitudinal density in phase space. Bunch shape also is strongly dependent upon parameters such as Ω , α and θ .

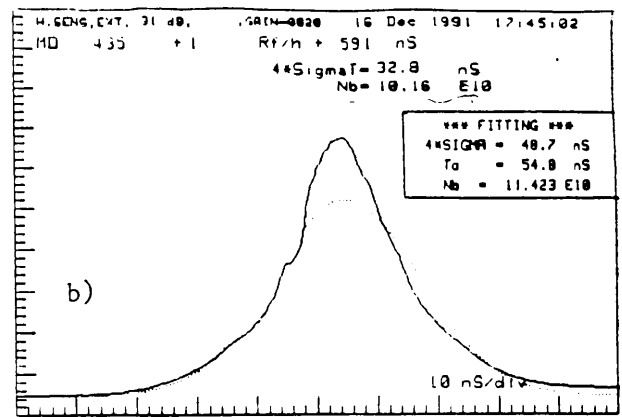
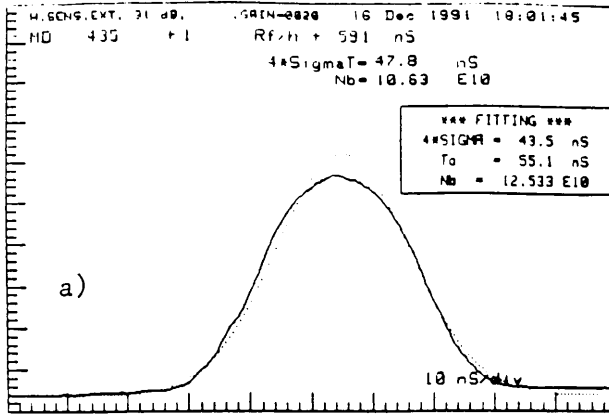


Figure 10: Bunch shape (a) for $\Omega = 3.6$ kHz ($k_0 \sim 3$) and (b) for $\Omega = 6.14$ kHz ($k_0 \sim 16$, $l_0 \sim 3$).

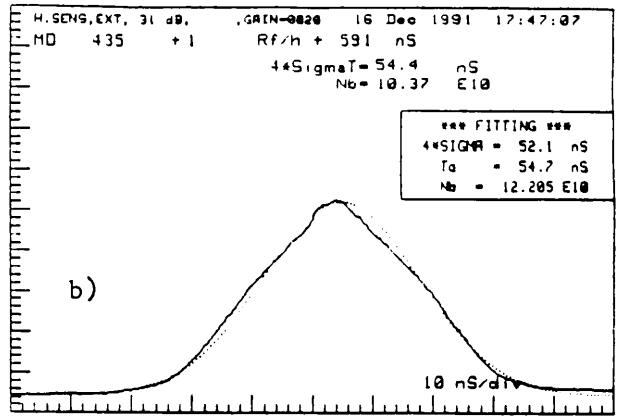
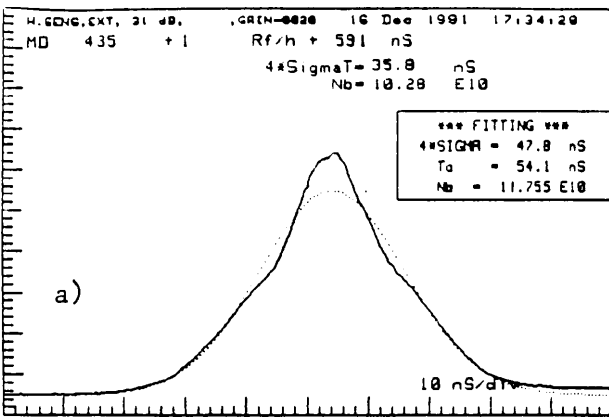


Figure 11: Bunch shape (a) for $\Omega = 5.74$ kHz and (b) for $\Omega = 5.84$ kHz.

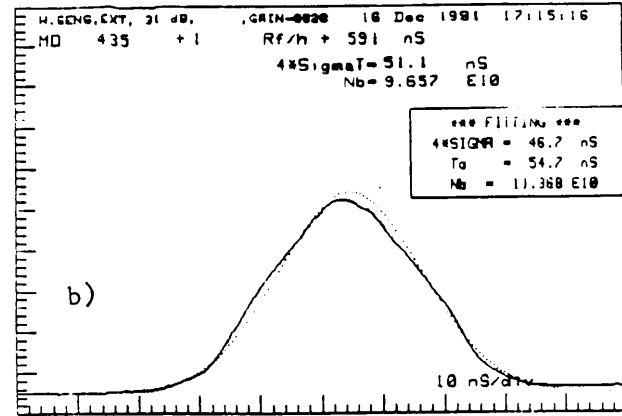
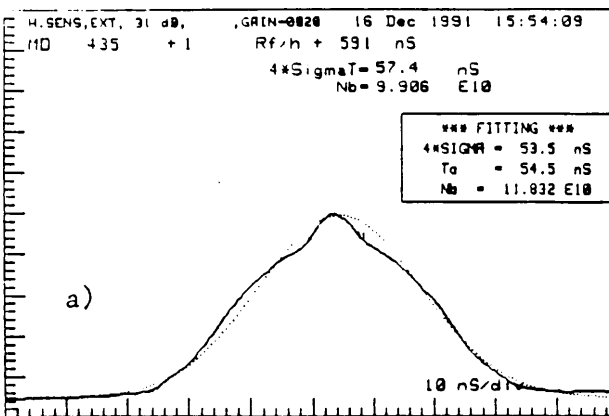


Figure 12: Bunch shapes for different signs of the synchrotron frequency modulation.

In general, when the modulation frequency Ω increases (so that k_0 also increases), resonances near the bunch centre are less efficiently driven since $J_{mk_0}(2Nr_0 \sim 0)$ decreases when k_0 increases (see Table 1). On the contrary, resonances which are further from the centre are driven more efficiently and tails appear (compare Fig.10(b) with Fig.10(a)).

Relatively small changes in Ω can also strongly affect the bunch shape by shifting the position, r_0 , of the resonance inside the bunch since $\omega_s(r_0) = \Omega (l_0/k_0)$ (from equation (6)). Fig.11 shows the consequence of a 1.7% change in the modulation frequency.

In general, the HF cavity system strongly modifies the synchrotron frequency of the particles (equation (8)), especially near the bunch centre (see Fig.1). This effect can be used to establish different resonant conditions (k_0, l_0) for the central part of the bunch as compared to the rest.

Fig.12(a) shows the resultant bunch for $\Omega/2\pi = 4.64$ kHz, which corresponds to $k_0 = 4$ for the unperturbed longitudinal motion (HF cavity off). With the parameters used in this experiment, the synchrotron frequency is lowered at the centre of the bunch and only higher order resonances can be excited there ($k_0/l_0 > 4$). Since k_0/l_0 is larger, these resonances have a smaller growth rate (see Table 1). The contribution from resonances with $k_0 = 5, 6$ and $l_0 = 1$ is also small because $J_1(\alpha) = J_1(3.8) \simeq 0.01$ (see Table 1). Consequently, we do not expect the bunch core to be modified by the blow-up process. This is the reason for the central bump in the bunch of Fig.12(a). The length of the bump (~ 8 ns) corresponds closely to the region ($r < 0.1$) of reduced synchrotron frequency in Fig.1.

By changing the phase offset, θ , (or by changing the modulation amplitude, α), the synchrotron frequency can be increased instead of being decreased at the centre of the bunch (see Fig.1). In this way the resonance $k_0 = 3$, which is most effective at small values of r , could be excited without changing the value of Ω . The central particles are then affected more strongly by the blow-up process and bumpless bunches are observed (see Fig.12(b)).

4 Conclusions

A number of predictions of the resonant theory summarized in Section 2 have been observed experimentally:

- The resonant nature of the blow-up process has clearly been shown (see Figs.8 and 9) and the mode numbers correspond reasonably well with theory.

- Some predicted effects dependent the amplitude, α , of phase modulation due to the variation of the synchrotron frequency inside the bunch (see Fig.1) have been qualitatively observed (see Fig.7 and Fig.12).

- The importance of the phase, θ , of the HF cavity has been verified and, for the first time, a pattern of bunch length modulation over one machine turn has indeed been observed (see Figs.4, 5 and 6).

To summarize, the resonance theory [4] clearly provides new, good qualitative insight into the mechanism of blow-up. It is an important contribution to the understanding of longitudinal beam dynamics which will be very helpful in the design of new accelerators.

Acknowledgement

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Appendix A: Experimental Set-up

The measurements of the bunch shape were made with the PS Bunch Observation System. Essentially, this system comprises a wide-band resistive wall pick-up connected to a fast transient digitizer controlled by a desktop computer (see Fig.A.1).

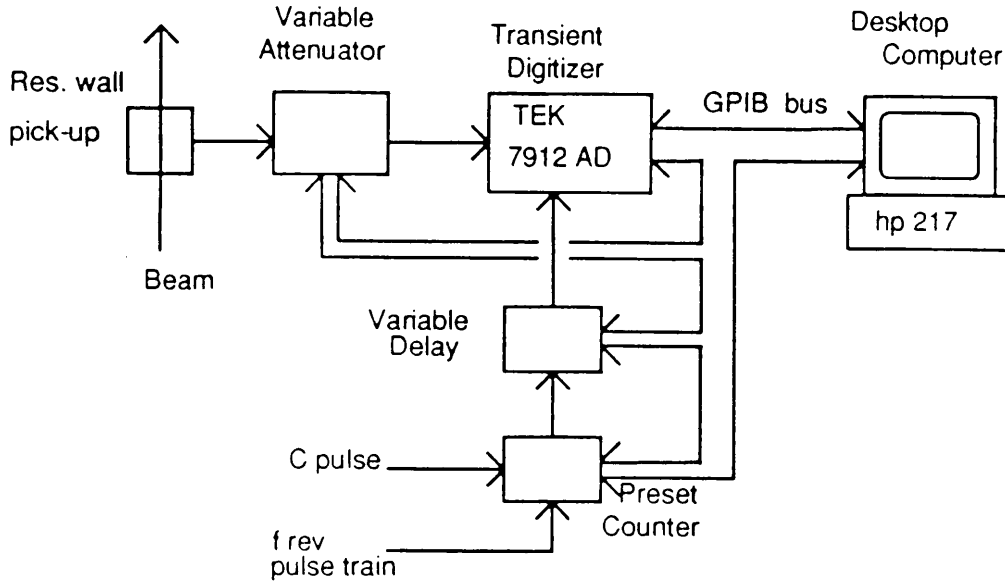


Figure A.1: Lay-out of the PS Bunch Observation System.

The resistive wall pick-up has a bandwidth ranging from 0.1 to 2000 MHz with a sensitivity, $R = 12$ Ohms. The transient digitizer (Tektronix Model 7912AD) has an analogue bandwidth of 700 MHz, an amplitude resolution of 9 bits and has 512 points in the (horizontal) time axis. To avoid signal degradation the digitizer is located as close as possible to the pick-up (within ~ 20 m). Such a system, foreseen to measure much shorter lepton bunches, can be considered practically perfect for measuring bunches > 20 ns (i.e., with a frequency spectrum < 100 MHz). In order to synchronize it with the beam, the trigger pulse is derived from the revolution frequency pulse train ($= f_{RF}/h$). The selection of the bunch number is achieved with a variable digital delay. All the instruments are remotely controlled from the Main Control Room with an HP 917 desktop computer through a GPIB link.

The data are processed and displayed together with two automatic bunch length computations, viz.,

1. A quick, $4\sigma_t$ estimation obtained by measuring simply the full width at half maximum height (for a Gaussian distribution, $4\sigma_t = 1.698$ fwhm).
2. A Gaussian least squares fit to the measured bunch shape. The fitted Gaussian curve is plotted as a dotted line (see Fig.A.2).

A large discrepancy between these two lengths indicates a bunch shape which is far from Gaussian (e.g. parabolic).

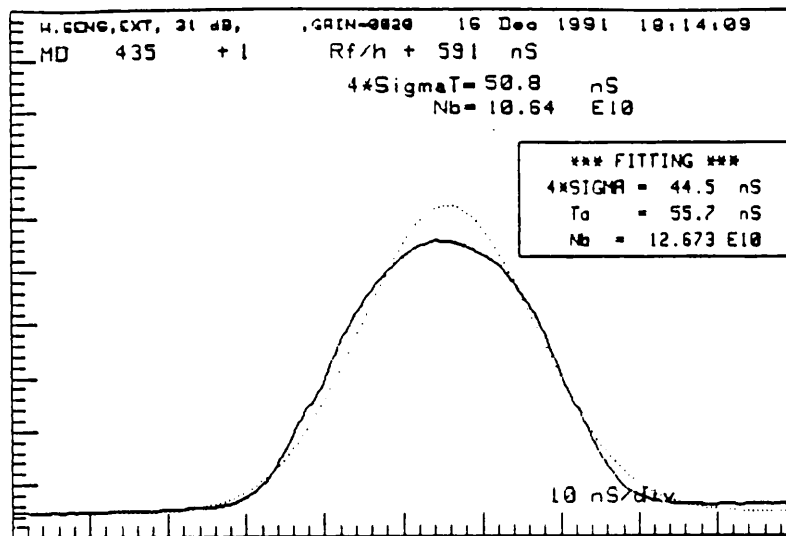


Figure A.2: Bunch current as acquired by the Bunch Observation System

Appendix B: RF Hardware Lay-out in the PS

The standard hardware for excitation of the HF cavities was used during the experiments. A simplified drawing of the actual lay-out is shown in Fig.B.1.

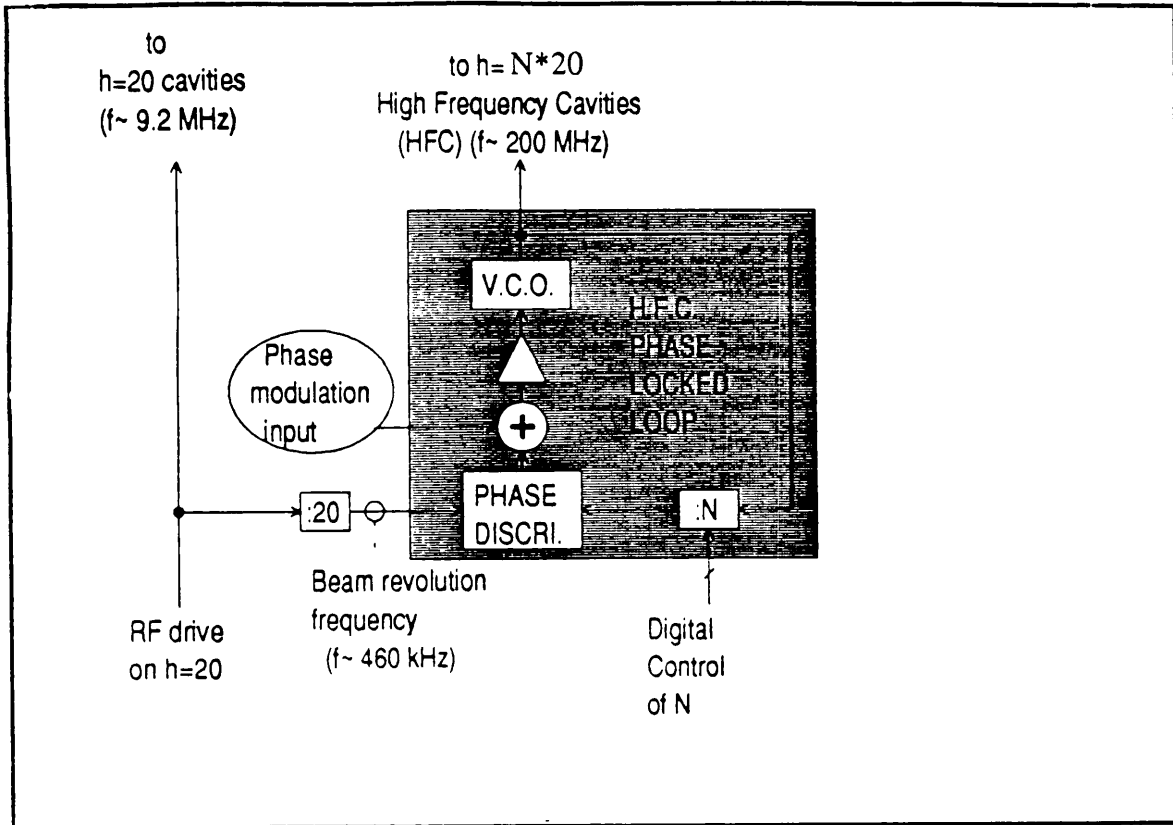


Figure B.1: Hardware lay-out for excitation of the HF cavities.

The low-level RF drive for the $h = 20$ cavities is divided by 20 to give the beam revolution frequency. This signal is then multiplied by N in the HF cavity phase-locked loop. The optimum value of N is programmed in real time during the cycle to ensure that the output is always the harmonic frequency nearest the tune of the HF cavities. In the cases considered here, N is a constant in each experiment since both the cavity tune and the magnetic field are fixed.

The phase modulation input signal, which is added to the output of the phase discriminator, controls the phase of the HF cavity drive signal with respect to that of the main, $h=20$ cavities. This input is fed by a sinusoidal, low-frequency signal to provide the required phase modulation.