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ON THE POSSIBLE USE OF THE AXIAL ELECTRIC FIELD OF A LASER BEAM FOR PARTICLE ACCELERATION

F. Caspers, E. Jensen

Abstract

Optical beams with finite spot size have axial field components. These must not be neglected if highly relativistic particles propagate in parallel to such a beam because the transverse field components seen by the particles are significantly reduced by the Lorentz transformation. In this note we discuss the possibility of using these axial field components for particle acceleration. Synchronism between wave and particles cannot be sustained for more than half a RF period, but due to the relativistic Doppler shift, this is increased to approximately twice the confocal length, kw^2_0 of a laser beam. For a CO_2 laser with a power density of 10^{14} W/cm^2, an acceleration in the order of 10 MV might be achieved in the confocal length.

Geneva, Switzerland

A homogeneous plane e.m. wave has only field components in the plane transverse to its direction of propagation. An optical or quasi-optical beam can often be regarded as such a plane wave in good approximation, its longitudinal fields can simply be neglected. The name "quasi TEM wave" for the Gaussian beam emphasizes this feature. But due to the finite spot size radius w_0, there is in principle a transverse variation of the fields. The axial components of both $\nabla\times\vec{E}$ and $\nabla\times\vec{B}$ do not vanish, and according to Maxwell's equations, this leads to axial field components. For a lowest order Gaussian beam which has a field maximum on its axis of symmetry, this is true only off axis, for higher transverse order modes axial fields might even occur on the axis.

These axial field components are normally very small as compared to the transverse components; their ratio is in the order of λ/w_0, where λ is the wavelength. But if we look at highly relativistic particles propagating in axial direction, and Lorentz transform the e.m. fields into the frame of these particles, we find that the transverse fields are reduced basically proportional to the factor γ, whilst axial fields are not affected. Hence, for particle energies in the order of 1000 times their rest energy, i.e. some GeV for electrons, the axial field components must not be neglected anymore. This Lorentz reduction of transverse fields also seems to set fundamental limits to transverse field – axial accelerators (the mechanism described in [1], the inverse FEL, the cyclotron autoresonance accelerator, and the scheme proposed in [2]).

Also, in addition to the small axial field components, the axial phase velocity of the Gaussian beam is slightly above c. These two effects are connected to each other, and might be explained from the distribution of the waves around the axial direction (expanding the Gaussian beam in homogeneous plane waves). When regarding the possibility of using the small axial field components for acceleration, one finds that the increase of the phase velocity acts against the possibility of synchronism between the e.m. field and even highly relativistic particles, and seems to make a *net* acceleration impossible because the particles cannot be kept in the accelerating phase of the e.m. field over a longer period of time. This is in full agreement with a general statement made by Sessler [3] that "for a relativistic particle, which moves with essentially constant speed and in a straight line, there is no net acceleration" far from a guiding structure.

These two counterrunning effects, namely the rise of axial field components of an optical beam with finite spot size, and the increase of the phase velocity of the associated mode above c, are not of the same order. The Gaussian beam might be regarded as consisting of an infinite number of homogeneous plane waves propagating at different angles α against the axis. For illustration, consider as a simple model just one plane wave with its wave vector tilted against the axis by an angle α. In this case, the axial field component $E_\parallel$ is given by

$$\frac{E_\parallel}{E_\perp} = \tan\alpha \,. \tag{1}$$

The relative deviation of the phase velocity in axial direction v_φ from c is in this case

$$\frac{v_\varphi}{c} - 1 = \frac{1}{\cos\alpha} - 1 \,. \tag{2}$$

Equation (1) is of first order in α, while (2) is of second order.

The Lorentz transformation affects the e.m. fields as seen in the frame of the particles in two manners. Besides the above mentioned reduction of the transverse fields, there is the relativistic Doppler shift, i.e. the modification of the frequency. Let βc denote the particles' axial velocity, and $x \equiv 1 - \beta$. In the highly relativistic limit $x \ll 1$, x can be approximately calculated from the particle energy in the laboratory frame by the formula

$$x \approx \frac{1}{2\gamma^2} . \tag{3}$$

In addition, let y denote the relative deviation of the waves phase velocity from c, $y \equiv v_v/c - 1$. From (2) we can deduce that for small α, which corresponds to a large spot radius in an optical beam, that also $|y| \ll 1$. Under these assumptions we get for the relativistic Doppler shift:

$$\frac{f'}{f} \approx \frac{x+y}{\sqrt{2x}} \tag{4}$$

where f denotes the laboratory frame frequency, f' the frequency in the particle frame. The nominator basically accounts for the non-relativistic Doppler shift, while the denominator is due to the modification of the proper time.

Because of the missing synchronism, the particles will slip out of phase after half a period of this reduced frequency f'. If we transform back to the laboratory frame, the time dilatation has to be taken into account again. In the highly relativistic limit, this is again the factor $\sqrt{2x}$ which cancels out the one in the denominator of (4). Thus, the half wavelength will be stretched by a factor of $1/(x+y)$ in the laboratory frame, i.e. the particle may travel $\lambda/(2(x+y))$ before the axial field acting on it reverses its direction.

Let us now refine our above equations (1) and (2) for a more realistic, lowest transverse order Gaussian beam [4]. One finds that such a beam has a maximum of the axial electric field at its waist radius w_0 divided by $\sqrt{2}$. The ratio of this axial field to the maximum transverse electric field on the axis is approximately given by

$$\frac{E_{\parallel max}}{E_{\perp max}} \approx \frac{e^{-0.5}}{\sqrt{2}\pi} \frac{\lambda}{w_0} \approx 0.136 \frac{\lambda}{w_0} . \tag{5}$$

If the beam broadens to $\sqrt{2}w_0$ in half a meter (like in a confocal resonator of 1 m length), and at $\lambda = 10\,\mu$m, e.g., the value of w_0/λ is 126, resulting in a ratio $E_{\parallel max}/E_{\perp max}$ of approximately 10^{-3}.

If we look at the deviation of the axial phase velocity from c, we find for this beam approximately

$$y \approx \frac{1}{8\pi\left(\frac{w_0}{\lambda}\right)^2 - 1} . \tag{6}$$

For the same example as above, y will be in the order of $2.5\ 10^{-6}$. As can be seen from (5) and (6), the angle α in (1) and (2) corresponds well to the ratio λ/w_0. For high γ, the half wavelength would be stretched for this example by a factor of approximately $4\ 10^5$ which results in a length of 2 m. Hence, the confocal length of 1 m corresponds to the stretched quarter wavelength.

A change of particle energy changes x. This would of course modify the proper time, but the factor $1/(x+y)$ by which the wavelength is stretched, would scarcely be affected since it is dominated by y for high particle energies. As long as y is significantly greater than $1/\sqrt{2|y|}$ (which is about 450 in our example), x has not to be considered in this expression.

How much energy could a particle gain using this scheme? The transverse electric field can be calculated from the laser power density S by

$$\frac{E_{\perp max}}{\mathrm{V/m}} \approx 27.45 \sqrt{\frac{S}{\mathrm{W/m^2}}}. \tag{7}$$

The axial electric field $E_{\parallel max}$ follows from (5). In the confocal length one would get a factor $2\sqrt{2}/\pi$, another form factor $k \approx 0.9$ due to the longitudinal distribution of the Gaussian beam. The total energy increase W would then be given by

$$\frac{W}{e\mathrm{V}} \approx \frac{E_{\perp max}}{\mathrm{V/m}} \frac{E_{\parallel max}}{E_{\perp max}} \frac{2\sqrt{2}}{\pi} 0.9 \frac{\lambda/4}{\mathrm{m}} \frac{1}{x+y} \approx 0.759 \sqrt{\frac{S}{\mathrm{W/m^2}}} \frac{w_0}{\mathrm{m}} \left(\frac{\lambda}{w_0}\right)^2 \frac{1}{x+y} \tag{8}$$

with x and y from (3) and (6), respectively. For an assumed laser power density of $S = 10^{14}\ \mathrm{W/cm^2}$ on a spot size of $1\ \mathrm{mm^2}$ (1 J, 1 ps[1]), this results in an accelerating gradient of 54 MeV/m for electrons over a total length of 20 cm. To first order, the total energy gain could only be increased with the applied laser power because W scales proportional to $\sqrt{S}\, w_0$.

In [5], a plane wave model of the focused laser beam was used to study electron acceleration. For an electron traversing the laser beam in an oblique angle, the same results concerning the usable acceleration length were obtained. The focal ratio F used in [5] is $\sqrt{\pi/2}\, w_0/\lambda$ in our nomenclature. Also the obtained overall energy gain is in the same order of magnitude, from which we conclude that the influence of the axial field of a laser beam may in fact be modelled by a small inclination of plane waves.

The proportionality between the acceleration and the square root of the laser power is a remarkable difference between this scheme and the one proposed by Scheid and Hora in [2] where the acceleration is directly proportional to the laser power. Equation (50) of [2] becomes for $N=1$: $W/e\mathrm{V} \approx 8.39\ 10^{-15}\ S/(\mathrm{W/m^2})$ while we get for the example considered above: $W/e\mathrm{V} \approx 1.08\ 10^{-2} \sqrt{S/(\mathrm{W/m^2})}$. The trade off point between these two effects is in the order of $10^{24}\ \mathrm{W/m^2}$; for the power level of our example the acceleration due to [2] is negligible.

The above estimation treats the lowest order Gaussian beam which has axial fields only off axis. In order to have an axial field on the axis, one could use the TEM_{01} mode. This mode has favorable symmetry properties which cannot be modelled by inclined plane waves. This mode would also have

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[1] The laser pulse could in fact be very short because it travels with the particles.

zero transverse field on the axis, thus the transverse fields could be completely neglected in this case. The particles moving on the axis would experience no transverse accelerating forces at all. Hence there would be no radiation due to transverse acceleration. These radiations are thought to be crucial for the above mentioned transverse field – axial acceleration schemes as well. Also the risk of beam defocusing by transverse field would thus be diminished. Another advantage of the TEM_{01} mode is its axial field maximum on the axis; this might possibly have a stabilizing effect on the beam. Furthermore, the total stretched half wavelength could be used in the confocal length.

In conclusion, we must state that for the time being the axial field components of laser beams do not seem to lead to competitive particle acceleration for practically available laser powers. We have given above only a rough estimate of the effect, and certainly further thought is needed.

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