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ON THE THEORY OF COHERENT INSTABILITIES DUE TO COUPLING BETWEEN A DENSE COOLED BEAM AND CHARGED PARTICLES FROM THE RESIDUAL GAS

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Introduction

Trapped ions continue to limit the stack intensity in the AA[1-3]. The present paper aims to provide some theoretical understanding of the effects observed and of the cures which have helped to push the stack intensity above $8.5\cdot10^{11}$ $\bar{p}$. Ion-$\bar{p}$ instabilities of any mode type can be analyzed using the theory of Koshkarev d Zenkevich[4], except that $\bar{p}$-$\bar{p}$ and ion-ion forces are glected in their work. These are included in ref. 5 which, however, treats only dipolar stability. We extend this calculation to quadrupolar modes, recently observed in the AA[3], including $\bar{p}$-$\bar{p}$ space-charge forces which can prevent Landau damping in cooled beams. We then discuss ways to circumvent the instability by proper choice of the $\bar{p}$ working point. Finally we analyze the ion dynamics in the presence of driven $\bar{p}$ oscillations and derive conditions for efficient clearing by $\bar{p}$ shaking.

Equations of Motion

For the motion of an individual $\bar{p}$ (y) and ion (y_i) we write:

$$\frac{1}{\Omega^2}\ddot{y} + Q_0^2G_0y - Q_{sc}^2G_{sc}(y-\bar{y}) + Q_c^2G_c(y-\bar{y}_i) = Fe^{i\omega t}$$
$$\frac{1}{\Omega^2}\ddot{y}_i - q_{sc}^2g_{sc}(y_i-\bar{y}_i) + q_c^2g_c(y_i-\bar{y}) = 0. \quad (1)$$

H--e y : transverse (h or v) deviation from nominal it, $\bar{y}$: deviation of beam center, Ω : $\bar{p}$-revolution equency, Q_0^2 : square of betatron wave number due to external focusing, $Q_{sc}^2 = 4Nr_pR/2\pi a(a+b)\beta^2\gamma^3$: $\bar{p}$-$\bar{p}$ space-charge, $Q_c^2 = 4N_ir_pRZ_i/2\pi a_i(a_i+b_i)\beta^2\gamma$: ion space-charge on $\bar{p}$-s, $q_{sc}^2 = 4N_ir_pR(Z_i^2/A_i)/2\pi a_i(a_i+b_i)\beta^2$: ion-ion space-charge and $q_c^2 = 4Nr_pRZ_i/2\pi a(a+b)\beta^2A_i$: the $\bar{p}$ space-charge on the ion; N_i, Z_i, A_i are the total number, charge state and mass number of the ions, a_i (in the direction of y) and b_i the effective transverse radii of the ion cloud; N, a, b the corresponding quantities of the $\bar{p}$ beam; $r_p = 1.54\cdot10^{-18}$ m; $2\pi R$ the orbit circumference. In the $\bar{p}$ equation the dot denotes the total[4-5] $d/dt = \partial/\partial t + R\Omega(\partial/\partial s)$. Smooth approximation has been used replacing localised external and space-charge by ring averaged forces. Space-charge image forces have been neglected.

The functions G and g express the nonlinearity, G=g=1 for linear forces; G_0 depends on the external, the others on the space-charge fields. With a parabolic density distribution[6] of both $\bar{p}$ and ions:

$$G_{sc} = G(y-\bar{y}),\ G_c = G(y-\bar{y}_i),\ g_{sc} = G(y_i-\bar{y}_i),\ g_c = G(y_i-\bar{y})$$

where

$$G(y_1-\bar{y}_2) = 1-\frac{1}{6}\frac{2a_2+b_2}{a_2^2(a_2+b_2)}(y_1-\bar{y}_2)^2-\frac{1}{2}\frac{1}{b_2(a_2+b_2)}(z_1-\bar{z}_2)^2.$$

For more general distributions higher powers in $(y_1-\bar{y}_2)$ and $(z_1-\bar{z}_2)$ - the deviation in the other transverse plane - appear. On the r.h.s. of Eq. 1, the integrated electric field $E_0\Delta se^{i\omega t}$ of the shaking kicker (assumed to be a δ function in azimuth) enters as

$$\frac{eE_0\Delta se^{i\omega t}}{m_p\gamma\Omega^2}\delta(s) = \frac{eE_0\Delta s}{m_p\gamma\Omega^2\ 2\pi R}\sum_{n=-\infty}^{\infty} e^{in(s/R)+i\omega t}.$$

Only the resonant harmonic with $(n+(\omega/\Omega)) \approx Q_0$ is retained, such that

$$F = \frac{eE_0\Delta se^{in(s/R)}}{m_p\gamma\Omega^2\ 2\pi R}. \quad (2)$$

Dipole Mode

One solves Eq. 1 (without shaking i.e. F = 0) assuming constant beam size and small oscillations of the beam center $\bar{y} = \hat{y}\exp(in(s/R)-i\nu\Omega_0 t)$ for both ions and $\bar{p}$.[4-5] Nonlinearities are neglected except for the calculation of the frequency spreads. Results are[4-5]: instability can occur in a band where the ion bounce frequency q is close to one of the $\bar{p}$ sideband frequencies (n-Q): $|q - (n-Q)| < \delta Q$; $\delta Q \approx q_cQ_c/\sqrt{q_cQ_0}$; $Q^2 = Q_0^2 + Q_c^2 - Q_{sc}^2$; $q^2 = q_c^2 - q_{sc}^2$. The fastest growth rate occurring in the band center is $1/\tau \approx (\Omega/2)\delta Q$.

For realistic frequency distributions Landau damping inside this band imposes three necessary conditions on the spreads $\Delta_{\bar{p}}$ and Δ_i in the frequencies (n-Q) Ω/Ω_0 and q, respectively[5]:

$$\Delta_{\bar{p}} > |Q_{sc}^2/Q|;\quad \Delta_i > |q_{sc}^2/q| \text{ and } \Delta_{\bar{p}}\cdot\Delta_i > |q_c^2Q_c^2/qQ|. \quad (4)$$

The first two are the well known[7] single beam conditions requiring a frequency spread larger than the modulus of the frequency shift. The third is the two-beam condition of ref. 4. When the beam is cooled, the space-charge term Q_{sc}^2 increases and the spread $\Delta_{\bar{p}}$ dedecreases simultaneously, thus preventing (full) Landau damping as the first condition is violated. For typical AA parameters (3-Q mode, $Q \approx 2.25$, $q_c = 0.75$, $\Delta_{\bar{p}} = 3\cdot10^{-3}$, $\Delta_i = 0.2$ requiring $Q_c^2 < 2\cdot10^{-3}$ from the last Eq. 4, the two-beam condition would suggest stability up to a ring average neutralization of a few 10^{-3} but damping is upset as in the cool beam $Q_{sc}^2/Q = 2\Delta Q_{Laslett} > 3\cdot10^{-3}$. This may explain why feedback has to be used in the AA, even with good clearing, to cure dipolar modes.

Quadrupolar Modes

Both transverse planes (y, z) have now to be treated jointly due to the coupling by space charge. To obtain beam envelope equations one inserts[4] $\bar{y} = \bar{z} = 0$, $y = ae^{i\theta}$ with $\dot{\theta}\alpha(1/a^2)$. Linearizing for small deviations(ξ, η) from a stable solution (a_0, b_0) one finds,

$$\frac{1}{\Omega^2}\ddot{\xi} + 4Q^2\xi + \kappa_1Q_{sc}^2\bar{\xi} + \kappa_2Q_{sc}^2\bar{\eta} - \kappa_1Q_c^2\bar{\xi}_i - \kappa_2Q_c^2\bar{\eta}_i = 0\ ; \quad (5)$$

a similar equation for ξ_i (with $Q \to q$, etc.) and a similar system for the other transverse (η) envelopes (with $a_0 \to b_0$ in the calculations of Q_{sc}, etc.).

The factors $\kappa_1 = (2a_0+b_0)/(a_0+b_0)$ and $\kappa_2 = a_0/(a_0+b_0)$ are $3/2$ and $1/2$ for equal beam radii $a_0=b_0$, to be assumed hereafter for simplicity. Close to the diagonal $Q_y = Q_z$ a symmetric and an antisymmetric mode with $\xi = \pm\eta$ occur. In this case Eq. 5 may be written as:

$$\frac{1}{Q^2}\ddot{\xi} + 4Q^2\xi + 4pQ_{sc}^2\bar{\xi} - 4pQ_c^2\bar{\xi}_i = 0$$
$$\frac{1}{Q^2}\ddot{\xi}_i + 4q^2\xi_i + 4pq_{sc}^2\bar{\xi}_i - 4pq_c^2\bar{\xi} = 0 \qquad (6)$$

with $p = 1/2$ in the symmetric, $p = 1/4$ in the antisymmetric ($\bar{\xi} = -\bar{\eta}$) case. For largely different $Q_y \neq Q_z$ one obtains a more complicated set of equations which may be reduced to the form (6) in other limiting cases, e.g. that of a ribbon beam ($b_0 \gg a_0$) for which one finds Eq. 6 with $p = 1/4$.

The system (6) may be treated in full analogy to the dipole case (Eq. 1). One finds that the worst case growth rate is smaller by 2p and the Q width of the unstable band by p compared to the dipole case. Landau damping conditions (4) become

$$\Delta_{\bar{p}} > p\left|\frac{Q_{sc}^2}{Q}\right| \;,\quad \Delta_i > p\left|\frac{q_{sc}^2}{q}\right| \;,\quad \Delta_{\bar{p}}\cdot\Delta_i > p^2\left|\frac{Q_c^2 q_c^2}{qQ}\right|$$

where $\Delta\bar{p}$ and Δ_i are the spreads of $((n/2)-Q)\Omega/\Omega_0$ and q (in the case of $Q_y \approx Q_z = Q$ both Q spreads contribute). Thus the required spreads are smaller by p and the threshold neutralization ($N_i \propto Q_c^2$) is higher by p^{-2}. This is consistent with the observation in the AA that quadrupole modes occurred at 2 or 3 times higher intensities.

Influence of Working Point

When the instability is caused by one short ion pocket, stable conditions can be restored by choice of the $\bar{p}$-working point such that fast and slow wave frequencies coincide. To illustrate this in a simple way we neglect frequency spreads in Eq. 1 and/or Eq. 6 (i.e. we put $\bar{y} = y$, etc.). For an ion pocket we modify the coupling term

$$Q_c^2 \to \hat{Q}_c^2\,\Delta s\,\delta(s) = Q_c^2 \sum_{n=-\infty}^{\infty} e^{in(s/R)}$$

with $\hat{Q}_c^2$ the local and $Q_c^2 = \hat{Q}_c^2(\Delta s/2\pi R)$ the ring averaged coupling. Assuming that the ions oscillate with $\bar{y}_i = \hat{y}_i e^{-i\nu\Omega_0 t}$ the $\bar{p}$ response (from the first Eq. 1) is a sum of terms with the usual $Q^2-(\nu-n)^2$ denominator ($n = 0, \pm1\ldots$). Retaining the two terms with $(\nu-n_2) \approx Q$ and $(\nu-n_1) \approx -Q$ closest to resonance we have

$$\bar{y} \approx -\bar{y}_i(Q_c^2/2Q)[1/(\nu-(n_1-Q)) - 1/(\nu-(n_2+Q))]\;.$$

Thus if the frequencies (n_1-Q) and (n_2+Q) coincide (with a tolerance given by the width of the unstable band, $\delta Q \approx 10^{-3}$ in the AA) then the coupling is strongly reduced by "fast wave/slow wave cancellation".

For dipolar modes this requires half-integer tunes Q which are excluded. For quadrupolar modes similar consideration - applied to Eqs. 5, 6 - suggest fast wave/slow wave cancellation for quarter integer tunes such that $(n_1-2Q) \to (n_2+2Q)$. Tuning the AA to $Q \to 2.25$ - such that $5-2Q \to -4+2Q$, $6-2Q \to -3+2Q$, etc. - quadrupolar instability was successfully suppressed. This indicates at the same time that an ion pocket preponderated. In fact, for an extended cloud or several pockets the ions follow the pattern $\exp[i(ns/R-Q\Omega t)]$ around the ring and coupling of different modes is impossible or improbable.

Theory of Shaking

The purpose of shaking is to decrease Q_c^2, especially in those places in the ring where the clearing system cannot influence ions properly. We shake the $\bar{p}$ beam with the help of an rf electric field and the $\bar{p}$ beam shakes the ions in any place we are concerned with. This is most efficient if the shaking frequency $\omega = \nu\Omega$ is close to one of the $\bar{p}$ frequencies $(Q-n)\Omega$ and close to the ion frequency $q_c\Omega_0$.

A nonlinearity will be taken into account only for the ion movement. For the case we are interested in, $\hat{y}_i \gg \hat{y}$ and $\hat{y}_i \gg \hat{z}_i$; so the frequency of the ion is:

$$\tilde{q}_c^2 = q_c^2(1 - (2\Delta\tilde{q}_c/q_c)),$$
$$\Delta\tilde{q}_c/q_c = \alpha\hat{y}_i^2 > 0, \qquad (7)$$
$$\partial\tilde{q}_c/\partial\hat{y}_i = 2\Delta\tilde{q}_c/\hat{y}_i.$$

For the parabolic distribution $\alpha = (2a+b)/16a^2(a+b)$.

Passing through Resonances

The frequency q_c depends on the location of the ions, $q_c = q_c(s)$, so for a given ion, q_c varies in time because of its longitudinal movement. If $dq_c/dt = (\partial q_c/\partial \vec{s})\vec{v}_i \equiv q_c v_i/\lambda$ is big enough, the ion crosses the resonance $\nu = \tilde{q}_c(s)$; $\lambda = q_c/(\partial q_c/\partial s)$. The influence of the nonlinearity depends on the sign of dq_c/dt: the nonlinearity helps to cross the resonance when $dq_c/dt<0$ (q_c decreases along the ion's trajectory) and resists it in the case of $dq_c/dt > 0$. The average result of a single passing is

$$\overline{\Delta y_i^2} = \frac{\pi}{4}\hat{y}^2\frac{\nu^2\Omega}{|q_c v_i/\lambda| \pm |(\partial\tilde{q}_c/\partial\hat{y}_i)(\partial\hat{y}_i/\partial t)|}\;;\; \frac{dq_c}{dt} \lessgtr 0 \quad (8)$$

$$|q_c v_i/\lambda| > |(\partial\tilde{q}_c/\partial\hat{y}_i)(\partial\hat{y}_i/\partial t)|,\; \partial\hat{y}_i/\partial t \sim \tilde{q}_c\Omega\hat{y}.$$

Quasi-Linear Transverse Oscillations of Ions

In the conditions $dq_c/dt = (\partial q_c/\partial\vec{s})\vec{v}_i < 0$ and $|q_c v_i/\lambda| \ll |(\partial\tilde{q}_c/\partial\hat{y}_i)(\partial\hat{y}_i/\partial t)|$, i.e. slow ions moving in the direction of the q_c decrease, we can consider the ions as motionless. From Eqs. 1 and 7, we have coherent oscillations $\bar{y} = \hat{y}\cos(\omega t+ns/R)$, $\bar{y}_i = \hat{y}_i\cos(\omega t+ns/R)$,

$$\hat{y}_i = F\tilde{q}_c^2/D\;;\quad \hat{y} = F(\tilde{q}_c^2 - \nu^2)/D\;;$$
$$D = (\nu^2 - \tilde{q}_c^2)[(\nu+n)^2 - Q_0^2 - Q_c^2] - q_c^2Q_c^2\;; \qquad (9)$$
$$\hat{y}_i/\hat{y} = \tilde{q}_c^2/(\tilde{q}_c^2 - \nu^2) = \tilde{q}_c^2/[(q_c^2 - \nu^2) - 2q_c^2(\Delta\tilde{q}_c/q_c)].$$

In the purely linear case, $\alpha = 0$, the best regime for the shaking is $\nu = q_c$, $\hat{y} \approx 0$, $\hat{y}_i = -F/Q_c^2$. If $\alpha \neq 0$ and $\nu = q_c$ (a pure resonance for small ion oscillations), some stable amplitudes $\hat{y}_i, \hat{y}$ will be established after a time $t \sim \pi/\Omega\Delta\tilde{q}_c \sim [2\pi/(\Omega d\tilde{q}_c/dt)]^{1/2}$

$$\hat{y} = -2\alpha\hat{y}_i^3 \; ; \; \hat{y}_i = \frac{1}{\sqrt{\alpha}}\left(\frac{\Delta\tilde{q}_c}{q_c}\right)^{1/2} \; ; \; \hat{y} = -\frac{2}{\sqrt{\alpha}}\left(\frac{\Delta\tilde{q}_c}{q_c}\right)^{3/2} \qquad (10)$$

(This is the result of a combination of (7) with (9)). To obtain a large $\hat{y}_i$ in the case of a small F we need some small D in (9). Since $v^2 - q_c^2 > 0$ when $v^2 - q_c^2 \approx 0$, the best regime is when

$$(v + n)^2 > (Q_0^2 + Q_c^2) \; . \qquad (11)$$

In the AA v must be higher than (Q_0-n), n = 2.

The natural condition $|\hat{y}| \ll |\hat{y}_i|$ needed during the shaking coincides with the condition of the quasi-linearity, $2\Delta q_c/q_c = 2\alpha\hat{y}_i^2 \ll 1$. For a » b and a parabolic distribution it gives $\hat{y}_i \ll 2a$.

Lock-on Effect

In the case of slow ions moving in the direction of the q_c increase,

$$|q_c v_i/\lambda| < |(\partial\tilde{q}_c/\partial\hat{y}_i)(\partial\hat{y}_i/\partial t)| \; , \quad (\partial q_c/\partial\vec{s})\vec{v}_i > 0,$$

i.e. (for a parabolic distribution),

$$v_i/q_c\lambda < 2(\alpha\hat{y}_i^2)\dot{y}/\hat{y}_i \; , \quad (\partial q_c/\partial\vec{s})\vec{v}_i > 0 \; , \qquad (12)$$

the ions are trapped into resonance:

$$\tilde{q}_c = v \; ; \; q_c(1 - \alpha\hat{y}_i^2) = v \; ; \; \hat{y}_i^2 = \frac{q_c(s) - v}{\alpha q_c(s)} \; . \qquad (13)$$

The solution for $\hat{y}_i$ is *independent* of $\hat{y}$ (in the condition of (12)).

The General Solution

Thus the fast ions will be heated by shaking when passing through resonance, according to (8). The slow ions will provide two different components of the dipolar response $\hat{y}_i = \hat{y}_{q\ell} + \hat{y}_{i\,lock}$. ($q\ell$: quasi-linear). Since $\hat{y}_{i\,lock}$ does not depend on $\hat{y}$, it simply changes the effective electric field in expressions (1) and (9)

$$F \to F_{ef} = F \pm Q_c^2\,\hat{y}_{i\,lock} \; , \; (v + n)^2 \gtrless (Q_0^2 + Q_c^2). \qquad (14)$$

$\hat{y}_{q\ell}$ and $\hat{y}$ depend now on F_{ef} and can be analyzed from (9).

We have two types of asymmetry: a longitudinal right-left asymmetry because of $(\partial q_c/\partial\vec{s})\partial\vec{v}_i \gtrless 0$ and an up-down asymmetry of v with respect to the betatron sideband (Q-n), $(v+n)^2 \gtrless (Q_0^2+Q_c^2)$.

Ion Cooling

It can be deduced from Ref. 8 that for locked-on ions the shaking creates a greater spectral density at the frequency $\tilde{q}_c = v(1-\delta)$; $\delta \sim (\hat{y}/\hat{y}_i)^{1/3}(\Delta\tilde{q}_c/\tilde{q}_c)^{2/3}$. The effective width of the spectral density (Δ) is much smaller than $v\delta$. If $(Q_0 - n) \approx v(1 - \delta)$ we obtain a resonance between free oscillations of antiprotons and ions. The dipolar resonant response of the ions will give Landau damping.

The random dipolar fluctuation of antiprotons and the response of ions are of the order of

$$\overline{\Delta y} = \sqrt{\frac{\overline{y^2}}{N_{\bar{p}}}}\, e^{i(\omega t+ns/R)},$$
$$\overline{\Delta y_i} = -i\,\frac{\pi}{2}\,\frac{v}{\Delta}\sqrt{\frac{\overline{y^2}}{N_{\bar{p}}}}\, e^{i(\omega t+ns/R)}. \qquad (15)$$

It is important that ion frequencies $\tilde{q}_c$ do not vary along s in the existence of the lock-on effect: $\tilde{q}_c \approx v$.

The damping time of the ion cooling has a natural statistical limit $\tau = \sqrt{N_{\bar{p}}}\,\tau_L$; $\tau_L \sim \pi/\Omega\Delta$. Above this limit

$$\tau = \frac{2}{\pi}\left(\frac{\Delta}{v}\right)\frac{Q_0 N_{\bar{p}}}{Q_c^2\Omega}\,f, \qquad f = \begin{cases} \tau_m/\tau_L \; , & \tau_m > \tau_L \\ 1 \; , & \tau_m < \tau_L \end{cases}$$

where τ_m is the mixing time for betatron cooling.

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