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ACHROMATIC BEAM OPTICS FOR LOW VELOCITY PARTICLES

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1. INTRODUCTION

Several antiproton experiments at LEAR require very low momenta, well below 100 MeV/c (for example PS200: $p \sim 60$ MeV/c, PS189: $p \sim 20$ MeV/c).

The possibility to use either electric or magnetic field for the transport of these low velocity particles led us to consider a combination of the two.

Computation of the effect of crossed fields on a particle beam leads to an important result: it is possible to obtain a dispersionless bending device which could make the beam transport systems much simpler.

We first explain the principle in the case of small deflections. Then we compute the transfer matrix of such a device and finally, we give the results of a Monte-Carlo program which allows us to follow a beam of given emittance $(x,x',\Delta p/p)$ through such a system in the general case.

In the following, we consider only the motion of particles submitted to perpendicular electric and magnetic fields applied over the same path.

2. CASE OF SMALL DEFLECTIONS

A particle of mass m, velocity $v = c\beta$, momentum $p = mv$ and charge q moves in the z direction (Fig. 1). We assume that the main component of the initial velocity is following the z axis with the additional condition of small deflections in the system. In this case, in fact close to that of the mass spectrometer, it is easily shown (Simon, 1970) that the angular deflection in the (y,z) plane is given by

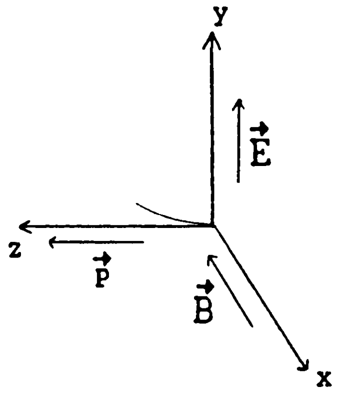

Fig. 1

$$\theta \simeq \frac{q}{cp}\left(\frac{E}{\beta} - cB\right)z = \theta_E - \theta_B \qquad (1)$$

where

$$\theta_E = \frac{qE}{cp\beta} z = \text{electric deflection}$$

$$\theta_B = \frac{qB}{p} z = \text{magnetic deflection}$$

Differentiation of (1) with the momentum gives

$$\frac{d\theta}{dp} \simeq -\frac{1}{p}\left[\theta_E(2-\beta^2)-\theta_B\right] \qquad (2)$$

We see that the system is achromatic to a first order if: $d\theta/dp = 0$, i.e.

$$\theta_E = \frac{\theta_B}{2-\beta^2} \qquad (3)$$

From equations (1) and (3) we easily deduce the values of the electric and magnetic fields which give the achromatism condition for the deflection θ.

In our case of low velocity antiprotons, equation (3) is simplified because β becomes negligible and we obtain the simple condition:

$$\theta_E = \frac{1}{2}\,\theta_B \quad \text{with } \beta^2 << 2$$

It is interesting to compare the chromatic dispersions in the pure "electric" and "magnetic" cases. From equation (2) we get also

$$\frac{d\theta}{\theta} = -\frac{dp}{p}\left(1 + \frac{\theta_E}{\theta}(1-\beta^2)\right).$$

Then, for a pure magnetic deflection (E = 0):

$$\frac{d\theta}{\theta} \propto \frac{dp}{p} \quad .$$

And for a pure electric deflection (B = 0):

$$\frac{d\theta}{\theta} \propto 2\,\frac{dp}{p} \qquad (\beta^2 << 1)$$

$$\frac{d\theta}{\theta} \propto \frac{dp}{p} \qquad (\beta \simeq 1)$$

We conclude that electric deflections are twice dispersive as magnetic deflections at very low energy.

3. GENERAL CASE: LARGE DEFLECTING ANGLE

$\vec{B}$ is on the z axis and $\vec{E}$ is on the radius of curvature as shown in Fig. 2
The central trajectory is defined by:

$$r_0 = \frac{m}{qB_0}\,v_0$$

v_0 = central velocity
B_0 = magnetic field required for a pure magnetic deflection (E = 0).

The equations of motion are the following:

$$m(\ddot{r} - r\dot{\theta}^2) = qE - qr\dot{\theta}B \qquad (4)$$

$$m(2\dot{r}\dot{\theta} + r\ddot{\theta}) = q\dot{r}B \qquad (5)$$

With $\dot{r} = dr/dt$, $\dot{\theta} = d\theta/dt$. If the

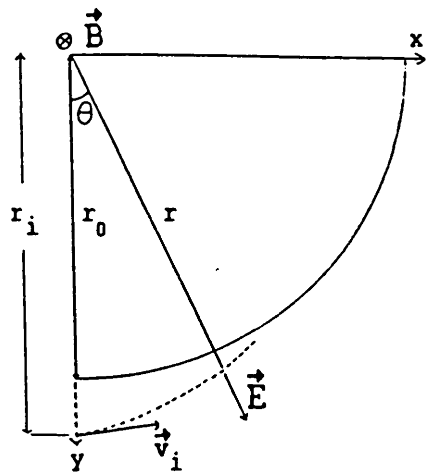

Fig. 2

radial acceleration is small ($\ddot{r} \ll r\dot{\theta}^2$), the equation (4) gives a first-order achromatism condition:

$$\frac{dr}{dv} = \frac{mv}{q} \frac{vB-2E}{(E-vB)^2} = 0$$

$$\Rightarrow E = \frac{1}{2} vB. \qquad (6)$$

The central trajectory has to be kept constant. This implies that if an electrical force qE is applied as shown on Fig. 2, the magnetic field B_0 must be increased in such a way that:

$$qE = qv_0 dB \quad \text{with } dB = B-B_0 .$$

Then, equation (6) becomes:

$$B = 2B_0 , \qquad E = v_0 B_0 \qquad (7)$$

Taking again the equations of motions (4), (5), keeping the condition (7) and assuming small abberations

$$r = r_0(1+X) \quad \text{with } X^2 \ll 1$$

equations (4) and (5) reduce to:

$$X'' + \frac{2}{r_0^2} X = 0 \qquad (8)$$

with

$$X'' = \frac{d^2X}{dS^2} = \frac{1}{v_0^2} \ddot{X} \quad \text{and } S = r_0\theta.$$

Solutions of equation (8) give a transfer matrix in the bending plane (x,x'):

$$\begin{pmatrix} X \\ X' \end{pmatrix} = M_\theta \begin{pmatrix} X_i \\ X_i' \end{pmatrix} \qquad X_i,\ X_i' \text{ are initial conditions}$$

$$M_\theta = \begin{pmatrix} \cos\sqrt{2}\theta & \frac{r_0}{\sqrt{2}} \sin\sqrt{2}\theta \\ -\frac{\sqrt{2}}{r_0} \sin\sqrt{2}\theta & \cos\sqrt{2}\theta \end{pmatrix}$$

4. RESULTS OF A MONTE-CARLO PROGRAM

The equations of motion (4) and (5) have been analytically integrated in Part 3 with a few simplifying assumptions. In order to obtain exact results, a program has been written to solve these equations numerically. However, it is interesting to note some important relations. A first integration of equation (5) gives:

$$\dot{\theta} = \frac{qB}{2m} - \left(\frac{qB}{2m} - \frac{v_{i//}}{r_i}\right) \frac{r_i^2}{r^2} \qquad (9)$$

$v_{i//}$ is the initial tangential velocity and r_i is the initial radius as shown on Fig. 2. Then, integration of equation (4) gives:

$$\dot{r}^2 = \frac{2qE}{m}(r-r_i) + v_i^2 - (r\dot{\theta})^2 \qquad (10)$$

Equation (10) gives the relation between the velocity v and the radius r:

$$v = \sqrt{\dot{r}^2+(r\dot{\theta})^2} = \sqrt{v_i^2 + \frac{2qE}{m}(r-r_i)}\ .$$

Those equations permit to find the value of the angle x':

$$x' = d\theta \ \alpha \ \arccos\left(\frac{r\dot{\theta}}{v}\right)$$

The program generates an initial emittance (x_i, x_i', $\Delta p/p$) and uses the Runje-Kutta method to find the trajectories.

Figure 3 shows the results for a 1 m long magnet bending antiprotons by 90°, p = 20 MeV/c and $\Delta p/p = \pm 10$ %. Figure 3a shows the initial emittance, Fig. 3b final emittance in the normal conditions (B ~ 0,1 T, E = 0) and Fig. 3c final emittance when the achromatic condition (7) is fulfilled (B ~ 0,2 T, E ~ 6,7 kV/cm).

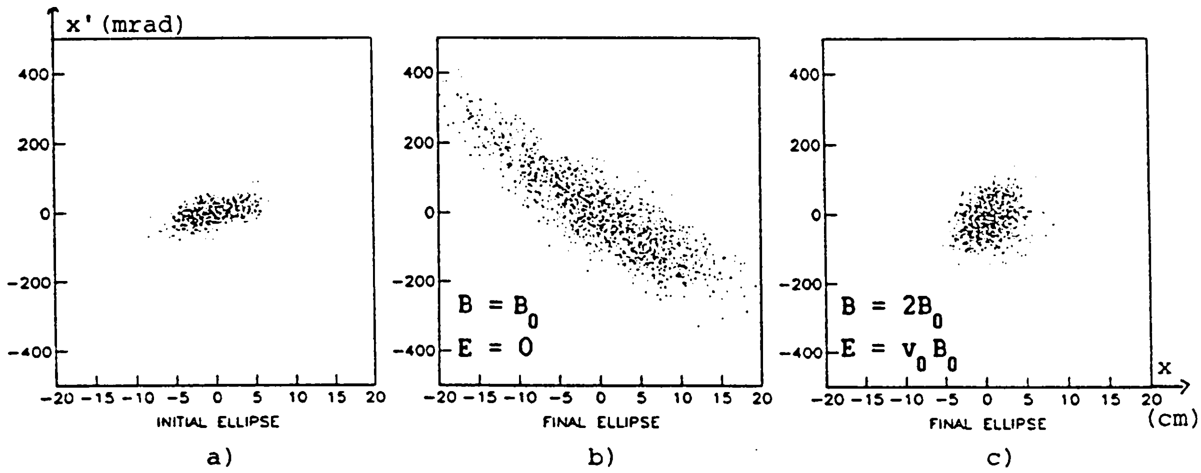

Fig. 3

Usual $\bar{p}$ beams from LEAR have much smaller momentum bites ($\Delta p/p \simeq \pm 0.1$ % in the ultra-slow extraction mode and $\Delta p/p \simeq \pm 0.3$ % in the fast extraction mode). Nevertheless, this achromaticity would be very interesting in some special cases [see for example the new s5 line, (Danloy et al., 1987)].

CONCLUSION

It is possible to conceive achromatic bending devices for antiprotons which require reasonable electric fields (E = 6.7 kV/cm at p = 20 MeV/c).

More work has still to be devoted to edge effects and behaviour in the vertical plane.

The condition of achromatism described in this paper could also be applied to quadrupoles but this application would give some technological difficulties.

REFERENCES

Danloy, L. et al., The LEAR experimental areas. New layout for the coming AAC-period - This workshop

Simon, D.J., CERN/MPS-MU/EP 70-1