EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

MPS/CCI/Note 74-42
8th August 1974

NUMERICAL ANALYSIS OF ELECTRON COLLECTION IN THE IBS

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CONTENTS

1. INTRODUCTION

The choice of crossed-field electron optics for the Ionization Beam Scanner (IBS) arose from a study of the various ways in which a combination of electric and magnetic fields could be used to collect ionization electrons from the residual gas, giving a projected image of the proton beam and at the same time scanning this image across the entrance slit of a sensitive detector[1].

This arrangement of fields gives high scanning speed since only the electric field is swept, the magnetic induction, B, remains constant. Beam disturbance is relatively low since B is along the beam direction, and for low beam currents the theoretical spatial resolution is very high (a resolution of 0.1 mm has been demonstrated in the laboratory[2]). Although E and B are made very uniform inside the IBS, this is not true at the entrance and exit (along the z axis) where fringing fields are produced in 3 dimensions. In order to define a symmetric region from which electrons are collected, the non-uniformity of B must be tailored empirically by the use of trimming coils[3]. On the other hand, with the intensities reached nowadays in accelerators, the self fields associated with the proton beam can be of great concern for the proper functioning of an IBS. In fact, in the case of the CPS and the PSB the magnetic field associated with the beam remains less than one percent of the IBS focusing field, B; in addition, due to its symmetry, it does not affect, to first order, the IBS performance and so it can be disregarded. However, the electric field of the beam is superposed on the IBS electric field, distorting the equipotential contours and thus causing a deformation of the electron density distribution as a function of potential which in turn distorts the IBS signal.

The last effect is the subject of this report. Since the bunch length is always long compared to the IBS, this space-charge field is treated in two dimensions, but a complete study is made of the effect on the IBS signals of bunched beams which create a time-dependent space charge field.

Computer simulation of the IBS trajectories has enabled us to show that for an unbunched beam, the IBS signal can be corrected by applying a scaling law for IBS electric field, beam current and dimensions. The

dimensions can be taken from the IBS signals themselves, in which case an iterative procedure is adopted (Section 5.1).

For bunched beams the situation is much more complex and there we are only able to indicate errors in beam size measurements for various sets of IBS and beam parameters, in particular for beam current and bunching factor, which give the strongest effects (Section 5.2).

In the course of this work it has been necessary to compute electric field maps within the IBS as a function of the potentials applied to the full-size electrode structure. This computational facility was used to optimize the choice of electrode potentials for the new Booster IBS and the results of this optimization study are described and presented in Section 6.

2. POISSON'S EQUATION : SOLUTIONS IN TWO DIMENSIONS

The entire analysis is made in the x,y plane, perpendicular to the IBS magnetic field, which is assumed to be uniform and constant in time. The IBS electric field, while always orthogonal to the magnetic field, is shaped in the x,y plane to effect the beam scanning. This is done by varying the potentials on the field electrodes (Figure 6). In pratice there is only one variable, the a.c. voltage applied in parallel to the two main field electrodes.

An additional perturbing electric field comes from the beam itself, (note that, due to the action of the IBS in pulling out the ionization electrons, there is never any neutralization of the beam). This field also varies in time due to changes in proton current and distribution. For us, the most important time variation comes from the bunches at frequencies from 3 MHz to 10 MHz. These are two to three orders of magnitude greater than the usual scan frequency.

The problem of calculating the large number of instantaneous field maps, which we require for the numerical analysis, is simplified by composing them from linear combinations of the fields without beam and those due to beam space charge alone. An example is presented in Section 6 (Figure 18). More details of the computational procedures are given below.

2.1. SPECIAL CASE : NO BEAM

The solution to the Laplace equation in two dimensions is obtained by a relaxation method for each set of electrode potentials, which corresponds to a given position of the scanning zero equipotential as it moves across the IBS aperture. Boundary potentials are assigned in each case to the points marked in bold in Fig. 6, using a 2 mm mesh. Details of the computer programme which performs this calculation are given in Appendix I.

2.2. SPECIAL CASE : WITH BEAM ONLY

With beam, the electric field in the IBS due to the proton space charge alone is calculated for a certain linear charge density and proton distribution by putting all electrodes at zero potential and solving the two-dimensional Poisson equation by two-fold Fourier analysis[4]. A programme which does this for any charge density and distribution is described in Appendix I. In order to study the effects of a bunched beam we are most interested in fixing the spatial charge distribution, $f(x,y)$, and varying, according to chosen time functions, $\lambda(t)$, the linear charge density. Note that a bunch in the PS is, except at transition, never shorter than 3.5 m, whereas the IBS length is 0.3 m. Thus, to a first order approximation, we consider the instantaneous linear charge density to be uniform in the z direction.

2.3. GENERAL CASE : LINEAR COMBINATION OF 2.1 AND 2.2

The instantaneous electric field map is then obtained from a linear combination of the solutions to the two special cases 2.1 and 2.2. Within the IBS, the electric potential is defined at any time t by the equation :

$$P(I,J,t) = P_1(I,J) + \lambda(t)\, P_2(I,J) \qquad (1)$$

where, I,J are indices at any mesh point and $\lambda(t)$ is the linear charge density of the proton beam. The interpolation between mesh point is described in Section 3.3.

3. ELECTRON DYNAMICS

Knowing the potential function, P(x,y,t), the magnetic induction, B, and making some assumptions about the initial energy distribution of the ionization electrons, we have sufficient information to calculate the motion of the electrons in the two dimensions x,y. The full analytical solution to the equations of motions is given in Appendix II. The electron trajectories are epitrochoids, which may be decomposed into circular motion plus translation. We shall first be concerned with the full solution and then with the solution for the motion of the center of rotation.

3.1. FULL SOLUTION

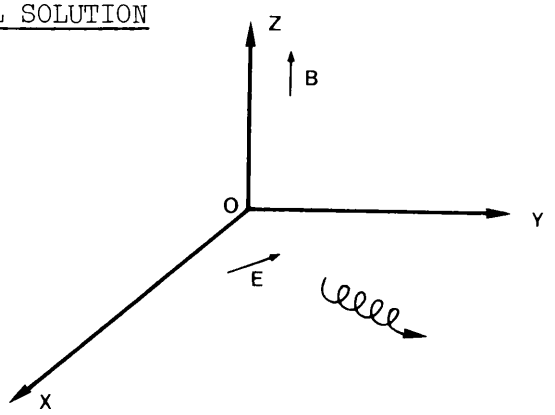

Figure 1

In Figure 1, B is the magnetic induction, the electric field E is in the plane x,y. $E_x = -\frac{\partial P}{\partial x}$, $E_y = -\frac{\partial P}{\partial y}$.

The equations of motion are :

$$m\ddot{x} = eE_x + eB\dot{y} \qquad (2)$$

$$m\ddot{y} = eE_y - eB\dot{x} \qquad (3)$$

$$m\ddot{z} = 0 \qquad (4)$$

(e is negative for electrons)

The general solution to these equations for an electron liberated at x_o, y_o with initial velocity components $\dot{x}_o, \dot{y}_o$ is :

$$x(t) = \frac{1}{\omega}\left[A \sin \omega t - A'(1 - \cos \omega t)\right] + x_o + v_x t \qquad (5)$$

$$\dot{x}(t) = - A' \sin \omega t + A \cos \omega t + v_x \qquad (6)$$

$$y(t) = \frac{1}{\omega}\left[A' \sin \omega t + A(1 - \cos \omega t)\right] + y_o - v_y t \qquad (7)$$

$$\dot{y}(t) = A \sin \omega t + A' \cos \omega t - v_y \qquad (8)$$

where : $\omega = \frac{eB}{m}$, $\frac{e}{m}$ = charge to mass ratio of electron

$v_x = E_y/B$

$v_y = E_x/B$

$A = \dot{x}_o - v_x$

$A' = \dot{y}_o + v_y$

From these equations one sees that the motion is the sum of a drift velocity v given by :

$$v = E/B \qquad (9)$$

and a rotation with angular velocity and of radius R given by :

$$R = \frac{\sqrt{(\dot{x}_o - v_x)^2 + (\dot{y}_o + v_y)^2}}{\omega} \qquad (10)$$

Note : i) if E,B are constant, v is constant

ii) for electrons of zero initial energy R becomes :

$$R = \frac{m}{e}\frac{E}{B^2} \qquad (11)$$

iii) the direction of v is perpendicular to E and B.

3.2. CENTER OF ROTATION SOLUTION

In the IBS the values of R are small compared with the distances travelled by the electrons in the direction of v. Hence, for the purpose of tracking an electron an approximate idea of its position may be obtained by following the center of rotation of the epitrochoid.

Consider the moving frame :

$$\xi = x - v_x t$$

$$\eta = y + v_y t$$

Equations (5), (7) become :

$$\xi(t) = \frac{\dot{\xi}_o}{\omega} \sin \omega t + \frac{\dot{\eta}_o}{\omega} \cos \omega t - \frac{\dot{\eta}_o}{\omega} + \xi_o \tag{12}$$

$$\eta(t) = \frac{\dot{\eta}_o}{\omega} \sin \omega t - \frac{\dot{\xi}_o}{\omega} \cos \omega t + \frac{\dot{\xi}_o}{\omega} + \eta_o \tag{13}$$

This should lead to a pure rotation around a fixed point in the moving frame, say (a,b), and will have the form :

$$\xi - a = (\xi_o - a) \cos \omega t - (\eta_o - b) \sin \omega t \tag{14}$$

$$\eta - b = (\xi_o - a) \sin \omega t + (\eta_o - b) \cos \omega t \tag{15}$$

These two pairs of equations are equivalent if a and b are given the values :

$$a = \xi_o - \frac{\dot{\eta}_o}{\omega} = x_o - \frac{\dot{y}_o + v_y}{\omega} \tag{16}$$

$$b = \eta_o + \frac{\dot{\xi}_o}{\omega} = y_o - \frac{\dot{x}_o - v_x}{\omega} \tag{17}$$

which is a simple parametric equation for the motion of the centre of rotation (a,b) in the original frame.

3.3. ELECTRIC FIELD INTERPOLATION

As shown in subsection 2.3 the electric potential is defined at any time, t, by equation (1)

$$P(I,J,t) = P_1(I,J) + \lambda(t) P_2(I,J)$$

In order to know the potential, and by derivation the field component at the point (x,y), an interpolation is necessary.

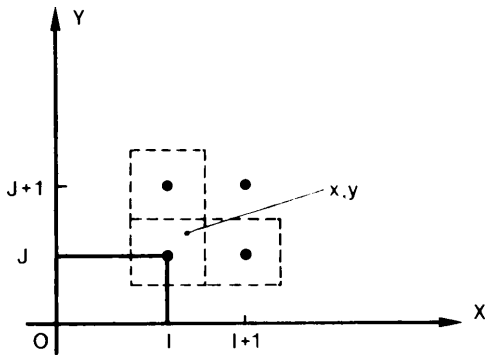

Figure 2

We use an interpolation with 9 points :

$$P(x,y) = P_o + ax + cx^2 + (b + ex + fx^2)y + (d + gx + hx^2)y^2 \tag{18}$$

which gives the field components :

$$\left.\begin{aligned} -E_x(x,y) &= \frac{\partial P}{\partial x} = a + ey + 2cx + 2fxy + gy^2 + 2hxy^2 \\ -E_y(x,y) &= \frac{\partial P}{\partial y} = b + ex + 2dy + 2gxy + fx^2 + 2hx^2y \end{aligned}\right\} \tag{19}$$

When (x,y) passes from one dashed square to another (see Fig. 2) there is a jump from one polynomial to another and continuity is assured neither for P(x,y) nor for its derivatives. In some cases this fact might be considered as unacceptable since such discontinuities of the fields could affect the long term stability of our solution. But provided the number of integration steps is not high (around one hundred in our case) this method is safe.

When a mathematical continuity of the polynomial and some of its first and second derivatives is needed, a much more cumbersome bicubic spline method can be adopted [5]. In our case remember that the continuity in time is already disregarded when a jump is made over a finite step Δt.

In order to balance,in terms of computer time, the interpolation and the solution of the electron dynamics we restrict the fields given by equation (19) to first order in x and y.

3.4. INTEGRATION METHOD

The center of the epitrochoid has the following motion (see equation (9)) :

$$dx = - \frac{E_y(x,y,t)}{B} \, dt \qquad (20)$$

$$dy = \frac{E_x(x,y,t)}{B} \, dt \qquad (21)$$

where E_x and E_y are functions of space and time for a bunched beam.

The electric field along the trajectory can be considered as a funciton of t alone : $E_x \left[x(t), y(t), t\right] = E_x(t)$.

Using the expansion :

$$E_x(t) = E_x(t_o) + \frac{dE_x(t_o)}{dt} (t - t_o) + \ldots \qquad (22)$$

one can integrated for a finite step Δt :

(23)

$$\Delta y = \int_{t_o}^{t_o + t} \frac{E_x(t)}{B} \, dt = \frac{1}{B} \left[E_x(t_o) \; t + \tfrac{1}{2} E'_x(t_o) \, \Delta t^2 + \ldots \right]$$

and with constant steps, Δt, one has :

$$\Delta y = \frac{1}{2B} \left[3 E_x t_o - E_x(t_o - \Delta t) \right] \; . \; \Delta t \qquad (24)$$

Remark 1 : The total electric field $E(x,y,t)$ is a sum of the static external field applied by the electrode configuration, $E_1(x,y)$ and the space charge field $E_2(x,y,t)$.

Remark 2 : The use of variable step size Δt is not recommended since $E_2(t)$ and $E'_2(t)$ are wild functions in the case of a bunched beam (see Figure 3).

Remark 3 : During intervals of time where $E_2(t)$ vanishes, the integration may be done in one step if one uses a model with uniform external field $E_1(x,y) = E_x$ (c.f. Section 4.3).

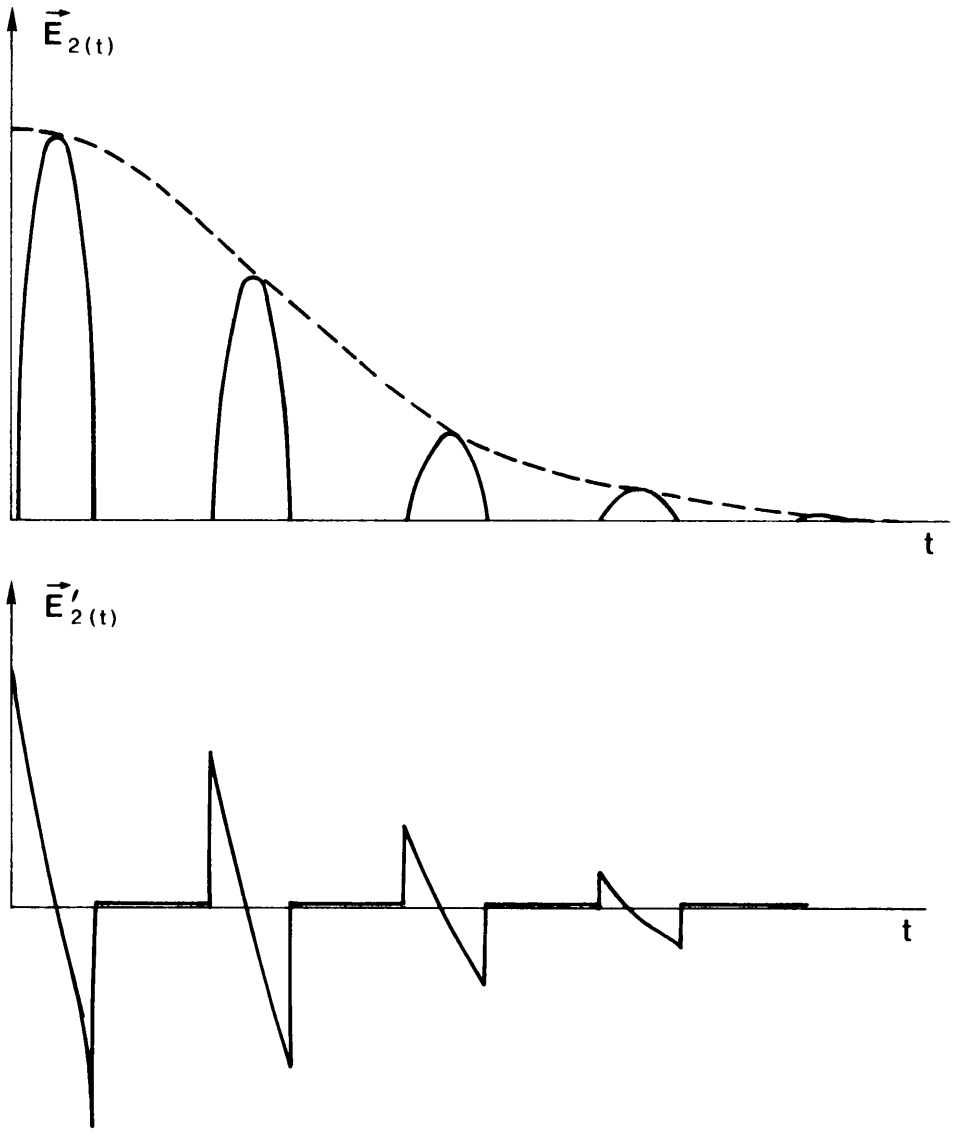

Figure 3

Remark 4 : The use of higher order derivatives in Equation (22) would not lead to any better solution since the continuity of E is not assumed by the interpolation method (see Section 3.3).

4. SIMULATION OF ELECTRON COLLECTION

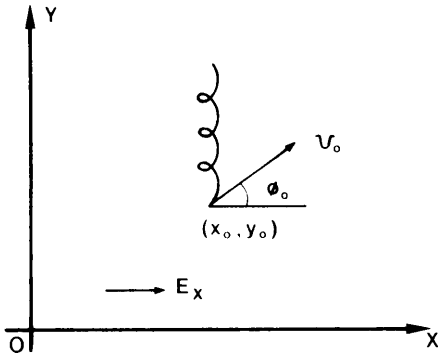

Figure 4

Having shown how to find by computation and then interpolation, the electric field at any point in space and time within the IBS, and having derived the equations of motion of the electron and also of the centre of the epitrochoid, we are now almost in a position to describe the IBS simulation. We need first to choose distribution functions for five random variables, viz : starting position of electron (x_o, y_o); initial velocity, $\vec{v}_o$ (magnitude, v_o, and direction, ϕ_o); time of creation, t_o, (see Fig. 4).

4.1. PROTON DISTRIBUTION

We assume that protons are distributed in space according to

$$g(x,y,z) = f(x,y) \cdot \lambda(t)$$

where $f(x,y)$ is a two dimensional distribution with elliptical equidensity contours inside an ellipse with semi-axes x_{max}, y_{max}. Various distributions have been tested namely those with $n = 4, 6, 8$ (from Ref. 7, equation (14), where we put $a^2 = (x/x_{max})^2 + y/y_{max})^2$), and $\lambda(t)$, the linear charge density of the bunches, is taken as a parabola.

4.2. VELOCITY DISTRIBUTION

As the exact form of the velocity or energy distribution function for primary ionization electrons in a gas is not known at low energies, we use the approximate distribution shown in Figure 5. $\frac{dN}{d\mathcal{E}}$ is constant from $\mathcal{E}$ = 0 to 10 eV and then falls off as $1/\mathcal{E}^2$ with a cut-off at 100 eV corresponding to the energy beyond which electrons, because of their large radius trajectories, cannot enter the IBS collector.

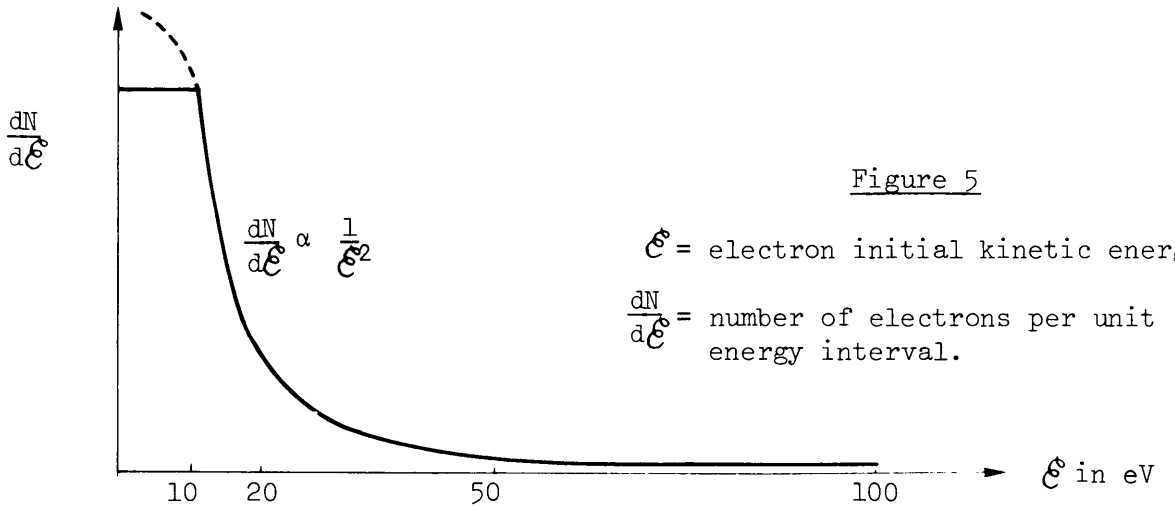

Figure 5

$\mathcal{E}$ = electron initial kinetic energy.

$\frac{dN}{d\mathcal{E}}$ = number of electrons per unit energy interval.

This arbitrary distribution, chosen because it contains equal numbers of electrons above and below 10 eV, will give pessimistic results since in reality the distribution is thought to be more heavily populated on the lower side of 10 eV, (e.g. broken line) (Ref. 6).

The corresponding velocity distribution is :

$$\frac{dN}{dv_o} = k'v \quad , \quad \text{below} \quad 10 \text{ eV}$$

(k', k", constants)

$$\frac{dN}{dv_o} = \frac{k''}{v^3} \quad , \quad \text{above} \quad 10 \text{ eV}$$

The angular distribution of velocities in the x,y plane is taken as isotrope, $\frac{dN}{d\phi_o}$ = constant.

4.3. COMPUTATIONAL METHODS

Within the framework described in the foregoing sections, we have written a set of computer programmes to analyse various aspects of

the IBS performance. These are outlined below. Further information and examples appear in Appendix I.

"IBSPOT" is a programme which computes and plots the electric equipotentials within the IBS for a given set of external potentials applied to the full-size electrode geometry shown in Figure 1. An off-shoot of the main study, this programme was used to optimize the final choice of electrode potentials for the new Booster IBS. Further reference to it is made in Section 6.

"IBS" computes the space charge potentials due to a proton beam within a rectangle having zero potential at its periphery. These potentials are superimposed on a uniform field within the rectangle, and this is done at the location of each of the protons simulating the beam, giving for each proton a potential V. Since electrons created at the proton locations will follow equipotentials, for an unbunched beam a comparison between the spatial distribution in x of all protons and the potential distribution in V will give a measure of the difference between the true beam profile and the IBS signal (see pages A_1 to A_3).

"IBIS" is the most comprehensive programme. Individual electrons are tracked from starting positions within the beam region up to a certain level in the IBS just short of the collector mouth. The IBS scanning process is ignored, the entire computation being done for one configuration only of the IBS field. To be clear on this point, we are interested in knowing the deformation of the electron density distribution between the beam region, where it corresponds to the proton distribution, and the region just before the collector. This deformation is attributed to : i) the effect of the beam space charge when and where the electron is created; ii) the finite velocity with which they are created; and iii) the effect of the beam space charge during their collection. During the scanning process, the IBS samples the electron distribution at a speed which is slow compared to the bunch frequency, and so the IBS signal (assuming perfect electron collection) will be truly represented by the time-averaged electron distribution as a function of potential in this region just before the collector,

where it is assumed that the further effects of beam space charge can be neglected.

This rather lengthy programme was used mainly as a check on a simplified and faster simulation programme, "IBSMODL" which gave the bulk of the results described in Section 5.

"IBSMODL" is a simplification of "IBIS" designed to speed up the simulation so that many runs with varying IBS and beam parameters can be made. The Poisson equation is solved as in "IBS". The external field is uniform. The electron initial velocity is taken as zero and only the drift of the centre of rotation is computed. The last two changes are justified on the grounds that for the study of the rather strong effects due to space charge of bunched beams, the known errors due to the electron orbits can be treated apart and later added in quadrature.

This programme computes a number of parameters which define the extent of deformation between the electron distribution in space at the beam region and the distribution function in potential, V, (the IBS signal) at the collector level. The most important parameter is the apparent increase in size : $\frac{\sigma_v - \sigma_{x_o}}{\sigma_{x_o}}$ where σ_v^2 is the second moment of the distribution function in potential and $\sigma_{x_o}^2$ that of the starting distribution in the beam.

In appendix I, Table A1 gives comparative descriptions of these four programmes and their outputs.

5. RESULTS

In order to work out a realistic example we took the case of a PS beam measured at 10 GeV/c, with $I = 2 \times 10^{12}$ p. The vertical profile of this beam is shown in Figure 7, it is very similar to the mathematical distribution generated for n = 8 (see Ref. 7). In all computations we have for the IBS electrodes :

AIBS = .04 m, BIBS = .08 m, (see Figure A1)

and for the beam profile :

$$C1 = 8.1, \quad C2 = -25.7, \quad C3 = 25.8, \quad C4 = -3.4, \quad C5 = -4.9$$

(see Figure 7).

5.1. COASTING BEAM

Various cases were run with the following parameters :

B	=	200 G	magnetic field in IBS
U	=	200 to 5000 V	potential applied to electrodes
XM	=	8 to 24 mm	horizontal beam size
YM	=	4 to 16 mm	vertical beam size
n	=	0.5×10^{12} to 4×10^{13}	number of protons
R	=	25 m	accelerator radius
TI	=	T2 = 110 ns	bunch distance and length

The increase of beam size measured by the IBS is given by this simulation as :

$$\rho = \frac{\sigma_m}{\sigma} - 1 \quad ,$$

where σ is pertinent to the actual proton distribution and σ_m is its measured value.

In the range of interest log ρ is very linear with log (n/RE), as it is shown in Figure 8. Other power laws can be seen for the variations of ρ with XM and YM so that a scaling low can be written as :

$$\rho = 2.0\times10^{-14}\left(\frac{n}{R.E}\right)^{1.9} \quad XM^{-1.7} \quad XM^{0.2}$$

where E = U/AIBS is the electric field.

In practice, for a given machine, the correction to sizes measured by the IBS can be obtained with a few iterations of the following formulae :

$$\sigma_x = \sigma_{xm} \left[1 + k \left(\frac{n}{E_x} \right)^{1.9} \left(\frac{\sigma_x}{\sigma_y} \right)^{-0.2} \sigma_x^{-1.5} \right]$$

$$\sigma_y = \sigma_{ym} \left[1 + k \left(\frac{n}{E_y} \right)^{1.9} \left(\frac{\sigma_y}{\sigma_x} \right)^{-0.2} \sigma_y^{-1.5} \right]$$

where E_x and E_y are known, and n, σ_{xm} and σ_{ym} are measured values.

5.2. BUNCHED BEAM

Simulations were done for

B = 100 to 300 G
U = 200 to 5000 V
XM = 0.014 m
YM = 0.007 m
n = 0.5×10^{12} protons
R = 25 m
TI = 110 ns
T2 = 15, 30, 60 ns

The results given in Figures 9, 10, 11 show that space charge effects can be much more important than for a coasting beam, but these effects are subject to strong modulation according to various parameters. These effects are most important when the time of electron travel in the space charge potential matches the bunch duration (v_y . T2 $\sim$ 30 mm). On the other hand there is an apparent "pinch" when the electrons travel slowly enough to be somehow focused by the distorted equipotentials. This overall behaviour, in particular the shapes of the curves in Figs. 9, 10, 11 has been observed experimentally and a set of measurements, comparable to the theoretical predictions in Figure 9, is presented in Figure 12.

The conclusions which we draw from these results are that :

i) we should work with the highest possible electric field, E, and magnetic induction B.

ii) the self-correction due to the "pinch" cannot be used in practice since the value of E/B to give $\rho = 1$ varies with T2 (i.e. with bunching factor).

iii) it would be possible to gate the IBS signal on the end of a proton bunch in order to diminish the signal error [8].

iv) in the SPS IBS there will be no phase relation between the transit time of the electron and the proton bunch length, since the bunches will be short (< 1 ns). The space charge effect will be averaged over ten to twenty bunches and is therefore expected to be similar to the case for an unbunched beam.

6. CHOICE OF BOUNDARY POTENTIALS FOR THE BOOSTER IBS

The IBS electric field can be divided roughly into two regions, the uniform region through which the proton beam passes, and that near to the mouth of the collector, where the field is shaped so as to bring the scanning equipotential (zero or near to zero) into the collector. The programme "IBSPOT" developed for the solution of the two-dimensional Laplace equation (Section 2.1 and Appendix I) has been used to optimize the choice of potentials on the boundary electrodes. Both the curvature and the strength of field are taken into account. In the beam region we require :

$$\frac{dE_x}{dx} = 0$$

$$E_y = 0$$

In the remainder of the IBS we must ensure that E(x,y) stays fairly constant in the regions at either side of the scanning equipotential.

Some computer print-outs of the IBS electric field for various choices of electrode potentials are reproduced in Figures 13 to 17. This method is much more convenient than the experimental field plotting techniques

used hitherto. It has recently been applied by K. Schindl[9] to the question of how to arrange the electrodes at the bottom of the IBS (Figure 6) so as to avoid having wires at intermediate potentials (these cause problems due to secondary electron emission). The solution finally adopted was to place an earthed plate at a few cm distance from the bottom of the main electrodes. In the computation, analytic functions were used to obtain the potentials on the mesh points between this plate and the main electrodes. With this simplified electrode arrangement, the curvature of the electric field in the beam region is negligible and the field strength varies by only a few per-cent across the beam aperture.

Another useful equipotential plot is that available from "IBIS" in which the distortion of the equipotentials due to the proton beam can be seen. Figure 18 is an example for the vertical PS IBS with a circulating unbunched beam of 10^{13} protons at 10 GeV/c.

ACKNOWLEDGEMENTS

We are pleased to acknowledge the contributions to this work made by H. Koziol, R. Le Bail, K. Schindl.

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APPENDIX I : PROGRAMMES AVAILABLE

I B S *

This programme is considering a uniform electrical field over the rectangle with sides 2 AIBS and 2 BIBS, given by the voltage U (see Figure A1). The beam extends to the ellipse with semi axes XM, YM. The space charge potential is computed in solving Poisson's equation with this charge density inside the rectangle and with zero potential at the periphery, by fast Fourier transforms.

This beam density is created by a random spread of AN protons, according to the amplitude distribution with polynomial coefficients C1 ... C9 (see Fig. 7 and Ref. 7).

Potentials are finally computed at the location of each individual proton. Since electrons will be produced there in proportion to the number of protons and will be collected along equipotentials, the distribution of potentials computed above is expected to be the same as the IBS signal. This distribution in V is compared with the profile distribution in X of all protons. In the output these distributions (smoothed to reduce statistical noise) are plotted and some parameters are printed :

$$X1 = \mu_1$$

$$SIG = \sqrt{\mu_2}$$

$$SQW = \mu_3/\mu_2^{3/2}$$

$$FLT = \mu_4/\mu_2^{2} - 3$$

where μ_i are the moments of order i of the distributions (see Figure A2).

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* See also Table A1

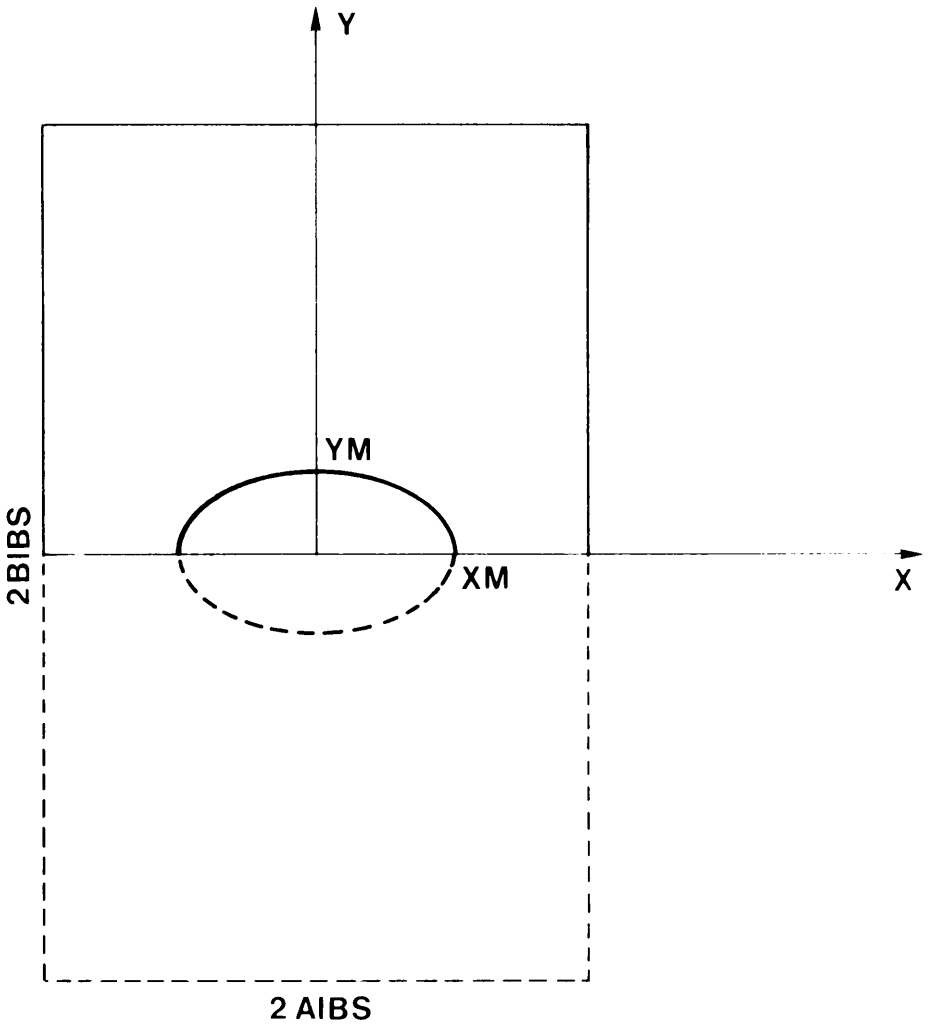
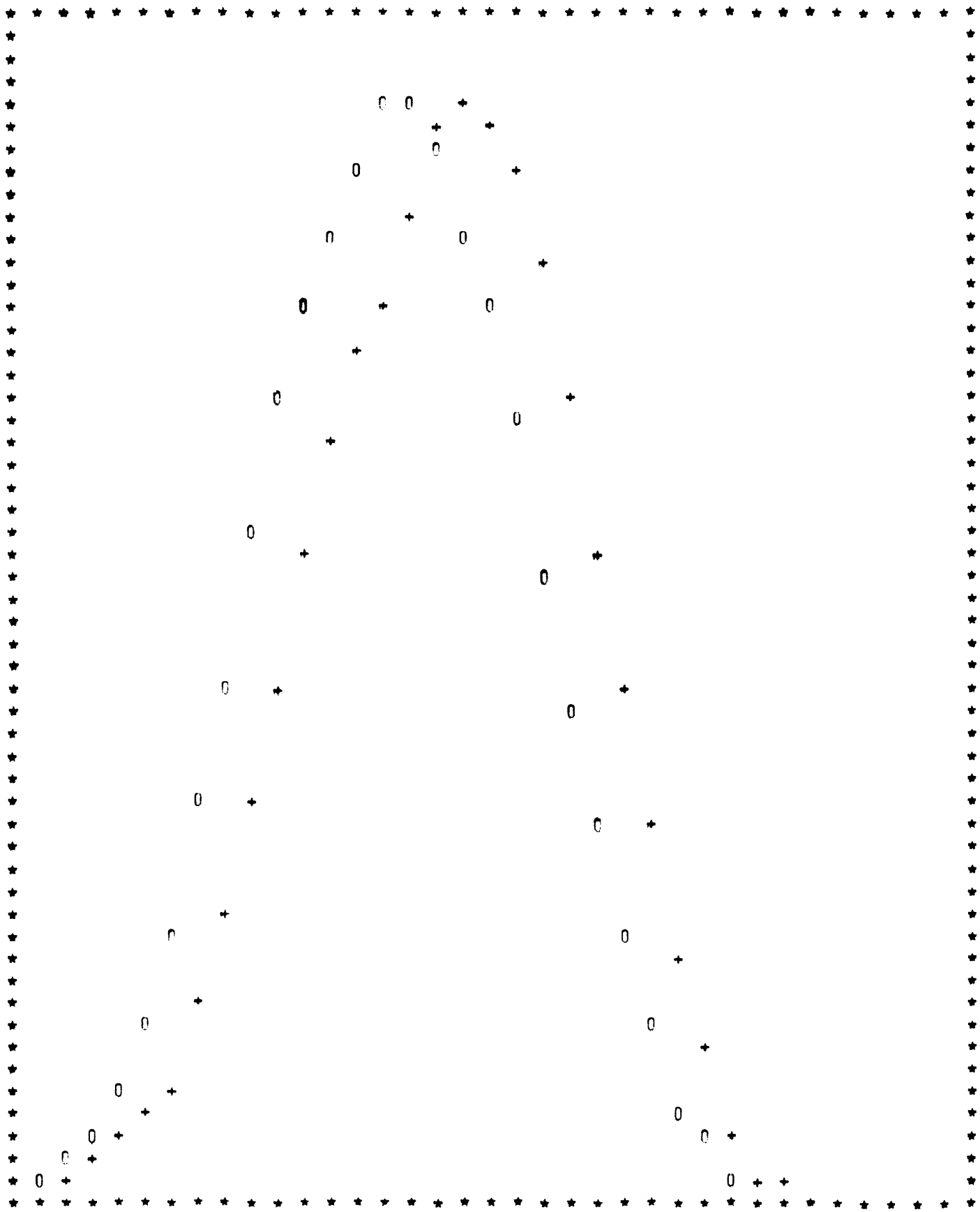


Figure A1

IBS COMPUTATION

A,B,XM,YM	4,00	7,50	,60	1,20	
V,NP,NSTAT	3000,00	1,74	10000,00		
B0,B2,B4,B6,B8	1,15996	-3,64828	3,89188	-,80741	-,63418



X1	,557	,633
SIG	,195	,196
SQW	-,009	-,170
FLT	-,327	-,706

Fig. A2

IBSPOT *

This programme is used in computing, by the relaxation method, the exact external potentials which correspond to the full size electrode configuration (see Figure 1). Potentials of the electrodes are determined by a fixed resistor chain dividing the given voltage difference, U, between the main plates. The difference of potential between the plates of the collector, V, is given separately, and the sweeping voltage applied in order to collect electrons with various positions along the X axis is determined by XSTART.

Examples of Calcomp plots are shown in Figures 13 to 17.

---

* See also Table A1

IBIS *

This programme performs the most complete treatment of electron collection simulation. Observing that the field at the centre of the IBS is very close to uniform (see results obtained in Section 6), we have reduced the net size by imposing a linear drop of potential on the y = 0 axis and a field symmetry versus this axis for small negative values of y. (This reduction is required for the FFT). In effect the space charge potential P_2 of Equation (1) is solved with the FFT method and with inversion of the capacitance matrix in order to create zero potential on the electrodes contour line. An equipotential plot corresponding to $P_1 + P_2$ can be obtained showing the distortion due to space charge effect (see Figure 18). The variation in time of the space charge potential is determined by the number of protons ANP ($\times 10^{12}$) in a machine with radius = 25 m, the RF period is given by T_1(ns) and the bunch length by T_2(ns). DT is the integration time and ANE the number of electrons to be traced.

All electrons are created in a slice with X = XSTART, but at various positions YO, times TO, initial velocity VO (and its direction ϕO). Their epitrochoid trajectories are computed until the centre of rotation has reached 85% of the distance BIBS to the collector. A characteristic output is shown in Figure A3, where TAU (ns) is the period of the epicycloid, BUNCHING is the peak versus average proton linear density, POTMAX (V) is the space charge potential at the bunch centre, XC and YC (mm) are the coordinates of the centre of rotation of the electron, T (ns) is the time at which YC has reached YMAX = 68 mm, EX, EY are the components of the electric field at the electron location.

---

* See also Table A1

```
AIBS,BIBS,B,U,V =      .0400      .0800      .0120    680     10

C1,C3,C5,C7,C9 =     8.083   -25.700     25.770    -3.370    -4.903

XM,YM,STAT =      .0140      .0070    10000

OCT LU 0000216723041501 6025

ANP,T1,T2,DT,ANE =      1.00      30.00      110.00        .10      40.00

EPS =   9.951E-04
XSTART =    10.000
TAU= 2.979
BUNCHING      5.50
POTMAX =   218.94
OCT2 =0000216723041501 6025
```

T0	Y0	V0	R0	T	XC	YC	RO	EX	EY
17.93	.21	1.686	.77	66.13	4.90	68.10	.81	-20506	-3035
1.10	-1.27	2.506	.57	51.60	7.30	68.05	.52	-21989	-2653
23.04	.28	1.946	1.38	69.04	5.17	68.07	1.42	-21383	-2922
26.01	1.58	2.041	1.51	70.51	5.00	68.17	1.54	-19980	-2914
22.91	-1.08	1.940	1.38	69.71	5.08	68.03	1.42	-20835	-2461
20.82	-1.99	2.066	.56	68.92	3.66	68.06	.65	-20527	-2949
14.65	1.30	1.861	.97	63.05	5.93	68.08	.96	-20824	-2875
16.77	3.07	.896	.80	62.67	6.26	68.14	.86	-21541	-2985
21.42	.56	1.292	.55	68.02	4.31	68.13	.51	-20618	-2740
14.74	7.16	2.733	.80	56.44	5.92	68.04	.76	-21483	-2768
22.42	.83	1.251	.30	67.82	4.20	68.06	.29	-20227	-3168
9.34	.50	2.655	.96	59.64	6.36	68.03	.98	-21590	-2969
12.28	7.04	.892	.92	55.38	7.39	68.04	.90	-22104	-3113
5.74	-2.68	.975	.50	58.04	7.18	68.04	.61	-21633	-2538
26.43	-1.11	1.938	1.47	72.73	4.88	68.10	1.46	-21324	-2861
4.81	1.75	1.933	.48	52.61	7.44	68.15	.47	-21990	-2771
22.57	-.31	4.200	2.34	70.17	5.36	68.10	2.36	-19579	-2682
16.02	.84	1.963	1.23	64.22	6.11	68.17	1.27	-20862	-3015
22.94	-.74	1.129	.57	69.94	4.14	68.04	.65	-20354	-2881
16.32	-7.11	.691	.61	69.52	3.29	68.14	.58	-20134	-3461
4.53	.63	1.701	1.01	54.73	7.81	68.17	1.04	-22496	-2120
11.48	4.92	2.088	1.42	56.38	7.83	68.07	1.40	-21511	-3039
6.76	-2.21	2.636	1.28	57.76	7.15	68.12	1.27	-21878	-3369
13.14	1.90	2.291	.77	61.04	5.88	68.13	.80	-20967	-2170
13.19	-2.33	2.145	1.07	65.09	5.24	68.13	1.08	-21007	-3238
5.76	-1.45	14.242	7.19	54.46	11.00	68.19	7.28	-22438	-6781
25.49	7.03	1.414	.18	65.89	4.09	68.03	.18	-20540	-3013
17.33	.46	17.093	7.88	59.13	2.25	69.31	7.74	-21332	-3772
18.69	.57	1.590	.62	65.19	4.98	68.15	.61	-20192	-2787
20.75	-.85	3.118	1.60	69.25	4.42	68.15	1.56	-20300	-3499
16.54	2.57	1.125	.13	62.44	5.73	68.16	.12	-21291	-2890
10.65	.29	1.942	1.23	60.75	7.05	68.14	1.28	-21441	-2459
21.54	1.37	2.535	.82	67.34	3.90	68.04	.86	-20243	-3373
18.35	-.94	1.978	.60	66.95	4.20	68.02	.64	-20188	-3230
5.55	-7.15	1.510	.82	62.95	5.26	68.02	.85	-20666	-3113
8.74	3.78	3.778	2.25	56.04	8.45	68.12	2.26	-22006	-1464
12.63	-1.00	1.738	.58	62.33	5.77	68.07	.60	-20843	-2627
7.24	-6.96	.547	.45	63.54	5.00	68.02	.44	-20529	-2845
16.55	-1.45	.885	.77	65.95	5.26	68.11	.73	-20918	-2775
22.60	3.20	.625	.68	66.30	5.02	68.10	.69	-20671	-2448

Fig. A3

IBSMODL *

This programme is designed to speed up the simulation of bunched beams done in IBIS. The external field is uniform as in IBS, and the space charge field is computed with FFT implying symmetry around the y = 0 axis.

For each electron, initial values of XO, YO, TO, are taken at random and VO is assumed zero. Only the drift of the centre of rotation is computed until $V \geq 0.85$ BIBS.

The potential at this location is recorded to make a distribution of the number of electron in this variable V. This distribution is compared with the distribution in XO. The two of them can be plotted and the space charge effect is characterized by various parameters, the most important of which is the apparent size increase :

$$\text{RATIO} = \frac{\sigma_v - \sigma_{xo}}{\sigma_{xo}}$$

All these parameters (see Figure A4) are computed in five different ways by putting the base line respectively at 0, 1%, 2%, 3%, 4% of the peak value, in order to be less sensitive to erroneous tail effects.

Tracking 50000 electrons takes only 14 s of CDC 6600.

---

* See also Table A1

```
 DATE  15/05/74            IBSMODL VERSION NO 10

AIBS,BIBS,P,U   =      .0400      .0800      .0200  3200

C1,C3,C5,C7,C9 =     10.000    -40.000     60.000   -40.000     10.000

XM,YM =      .0160      .0080

OCT LU 00000331330545343265


DATA NO

ANP,T1,T2,ANE,DT =      50.00     110.00     110.00 -50000.00        .02

TAU= 1.788
BUNCHING=    1.00
POTMAX=  2079.83
VY(MM/NS)= 4.000
OCT2=177705160400000423171

     ANE        MEAN         SIG     RATIO      SQW1      SQW2      SQW3      FLT
   49729   -.000000    .004626                .0085   -.0001   -.0005   -.4276
   49729    .022978    .005231               -.3487   -.3972  -1.3173   1.2478
            .022978    .000605    .1309      -.3572   -.3970  -1.3168   1.6754

   49477   -.000000    .004573                .0104    .0001   -.0000   -.5093
   49477    .022978    .005057               -.3668   -.3524  -1.2300    .8277
            .022978    .000484    .1058      -.3772   -.3524  -1.2299   1.3370

   49084   -.000000    .004501                .0122    .0025    .0030   -.5843
   49084    .022978    .004855               -.3918   -.2940  -1.1604    .5485
            .022978    .000354    .0786      -.4040   -.2965  -1.1633   1.1328

   48550   -.000000    .004435                .0182   -.0000   -.0001   -.6471
   48550    .022978    .004630               -.4239   -.2211  -1.0983    .3254
            .022978    .000195    .0440      -.4421   -.2211  -1.0982    .9725

   47997   -.000000    .004366                .0231    .0001    .0001   -.7006
   47997    .022978    .004420               -.4593   -.1428  -1.0492    .1635
            .022978    .000054    .0123      -.4824   -.1429  -1.0492    .8641

DTIME      13.99          P(1,2),P(1000,2) =        0.0       0.0

 TIME      60.90
```

Figure A4

Table A1

Physics Implied in the Various Programmes

Physical Treatment	IBS	IBSPOT	IBIS	IBSMODL
External potentials applied on a rectnagular contour (uniform field)	X			X
External potentials applied on the electric geometry with field symmetry around y = 0 axis			X	
External potentials applied on the electrode geometry (full size)		X		
Space charge potential solved with zero potentials on the appropriate contour	X		X	X
Full electron dynamics			X	
Centre of rotation dynamics				X
Equipotential plot		X	X	
Beam profile analysis	X			X

APPENDIX II : FULL SOLUTION OF ELECTRON DYNAMICS IN CROSSED FIELDS

The equation of motion of an electron liberated at a point x_o, y_o with initial velocity components $\dot{x}_o$, $\dot{y}_o$ can be obtained by integrating the equations :

$$m\ddot{x} = - eE_x - eB\dot{y} \tag{A1}$$

$$m\ddot{y} = - eE_y + eB\dot{x} \tag{A2}$$

which gives :

$$m\dot{x} = - eE_x t - eB_y + C_1 \tag{A3}$$

$$m\dot{y} = - eE_y t + eB_x + C_2 \tag{A4}$$

from (A1) + (A3) $\qquad \ddot{x} = - \frac{e}{m} E_x - \frac{eB}{m^2} (- eE_y t + eB_x + C_2)$

$$\ddot{x} + \left(\frac{eB}{m}\right)^2 x = - \frac{e}{m} E_x - \frac{eB}{m^2} C_2 + \left(\frac{e}{m}\right)^2 BE_y t$$

putting $\omega = \frac{e}{m} B$, we obtain : (A5)

$$x = A_1 \cos \omega t + A_2 \sin \omega t - \frac{1}{\omega}\left(\frac{E_x}{B} + \frac{C_2}{m}\right) + \frac{E_y}{B} t$$

$$x_o = A_1 - \frac{1}{\omega} \left(\frac{E_x}{B} + \frac{C_2}{m}\right) \tag{A6}$$

$$\dot{x} = - A_1\omega \sin \omega t + \omega A_2 \cos \omega t + \frac{E_y}{B} \tag{A7}$$

$$\dot{x}_o = \omega A_2 + \frac{E_y}{B} \tag{A8}$$

Thus $\qquad A_2 = \frac{1}{\omega} \left(\dot{x}_o - \frac{E_y}{B}\right)$. (A9)

Similarly :

$$y = A_3 \cos \omega t + A_4 \sin \omega t - \frac{1}{\omega}\left(\frac{E_y}{B} - \frac{C_1}{m}\right) - \frac{E_x}{B} t \qquad (A10)$$

$$y_o = A_3 - \frac{1}{\omega}\left(\frac{E_y}{B} - \frac{C_1}{m}\right) \qquad (A11)$$

$$\dot{y} = - \omega A_3 \sin \omega t + \omega A_4 \cos \omega t - \frac{E_x}{B} \qquad (A12)$$

$$\dot{y}_o = \omega A_4 - \frac{E_x}{B} \qquad (A13)$$

Thus : $$A_4 = \frac{1}{\omega}\left(\dot{y}_o + \frac{E_x}{B}\right) \qquad (A14)$$

$$\ddot{x} = - \omega^2 A_1 \cos \omega t - \omega^2 A_2 \sin \omega t \qquad (A15)$$

$$\ddot{x}_o = - \omega^2 A_1 = - \frac{eE_x}{m} - \omega \dot{y}_o \qquad (A16)$$

Thus : $$A_1 = \frac{1}{\omega}\left(\dot{y}_o + \frac{E_x}{B}\right) \qquad (A17)$$

similarly : $$A_3 = - \frac{1}{\omega}\left(\dot{x}_o - \frac{E_y}{B}\right) \qquad (A18)$$

From (A11) :

$$\frac{C_1}{m} = \omega y_o - \omega A_3 + \frac{E_y}{B}$$

$$C_1 = \omega m y_o + m \dot{x}_o = m (\dot{x}_o + \omega y_o) \qquad (A19)$$

and from (A6) $$C_2 = m (\dot{y}_o - \omega x_o)$$

Therefore :

$$x = \frac{1}{\omega}\left(\dot{y}_o + \frac{E_x}{B}\right)\cos \omega t + \frac{1}{\omega}\left(\dot{x}_o - \frac{E_y}{B}\right)\sin \omega t$$
$$- \frac{1}{\omega}\left(\frac{E_x}{B} - \omega x_o + \dot{y}_o\right) + \frac{E_y}{B} t$$

i.e. $$x = \frac{1}{\omega}\left[(\dot{x}_o - v_x) \sin \omega t + (\dot{y}_o + v_y) \cos \omega t - v_y + \omega x_o - \dot{y}_o\right] + v_x t \qquad (A20)$$

$$\dot{x} = - (\dot{y}_o + v_y) \sin \omega t + (\dot{x}_o - v_x) \cos \omega t + v_x \qquad (A21)$$

$$y = \frac{1}{\omega}\left[- (\dot{x}_o - v_x) \cos \omega t + (\dot{y}_o + v_y) \sin \omega t - v_x + \dot{x}_o + \omega y_o\right] - v_y t \qquad (A22)$$

$$\dot{y} = + (\dot{x}_o - v_x) \sin \omega t + (\dot{y}_o + v_y) \cos \omega t - v_y \qquad (A23)$$

where $v_x = \frac{E_y}{B}$, $v_y = \frac{E_x}{B}$.

Putting $A = \dot{x}_o - v_x$, $A' = \dot{y}_o + v_y$,

we have as the general solution :

$$x(t) = \frac{1}{\omega}\left[A \sin \omega t - A' (1 - \cos \omega t)\right] + x_o + v_x t \qquad (A24)$$

$$\dot{x}_o(t) = - A' \sin \omega t + A \cos \omega t + v_x \qquad (A25)$$

$$y(t) = \frac{1}{\omega}\left[A' \sin \omega t + A(1 - \cos \omega t)\right] + y_o - v_x t \qquad (A26)$$

$$\dot{y}(t) = A \sin \omega t + A' \cos \omega t - v_y \qquad (A27)$$

PROFILE DISTRIBUTIONS AND GENERATING COEFFICIENTS Ci

Ci NORMALIZED	n	BINOMIAL COEFFICIENTS
10 −40 60 −40 10	10	1 −4 6 −4 1
8 −24 24 −8	8	1 −3 3 −1
6 −12 6	6	1 −2 1

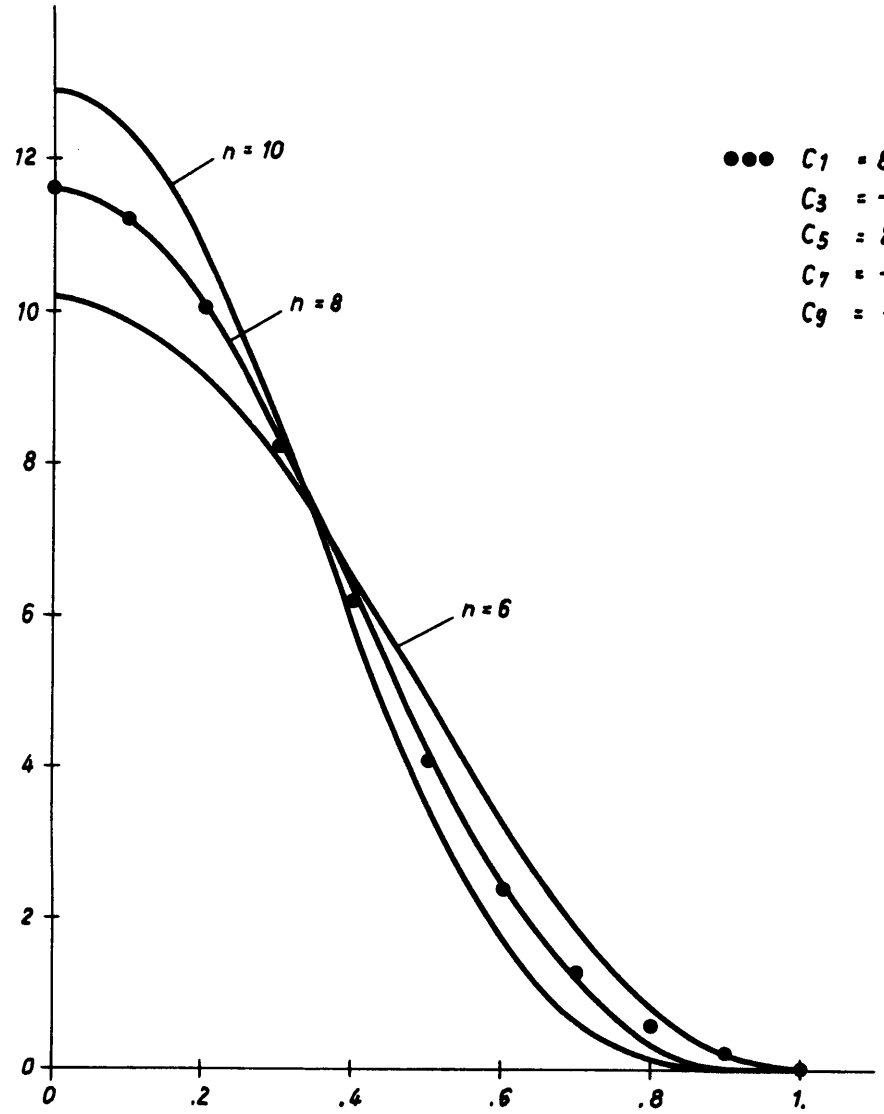


Fig. 7

31.7.74 G. Ewald

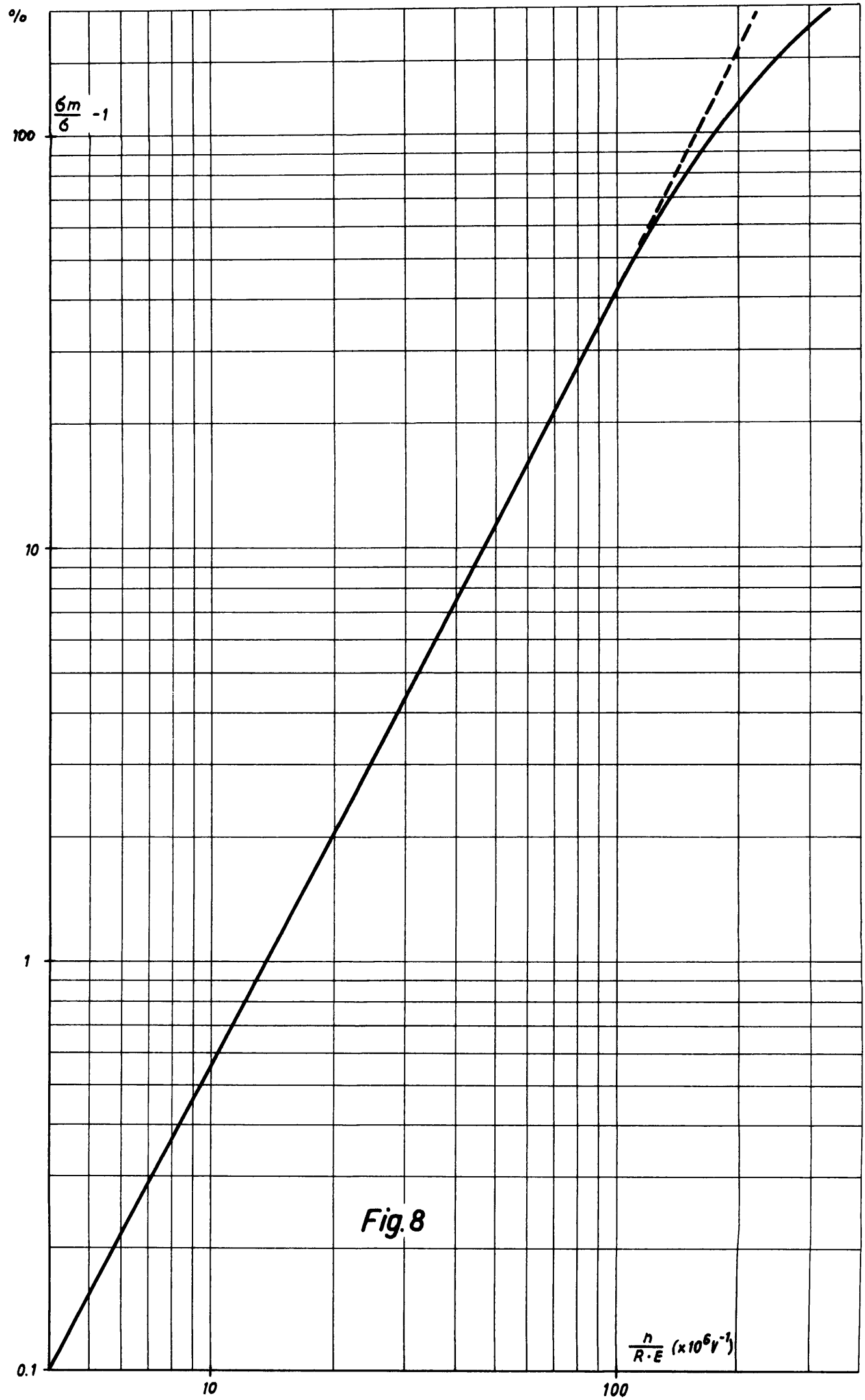


Fig. 8

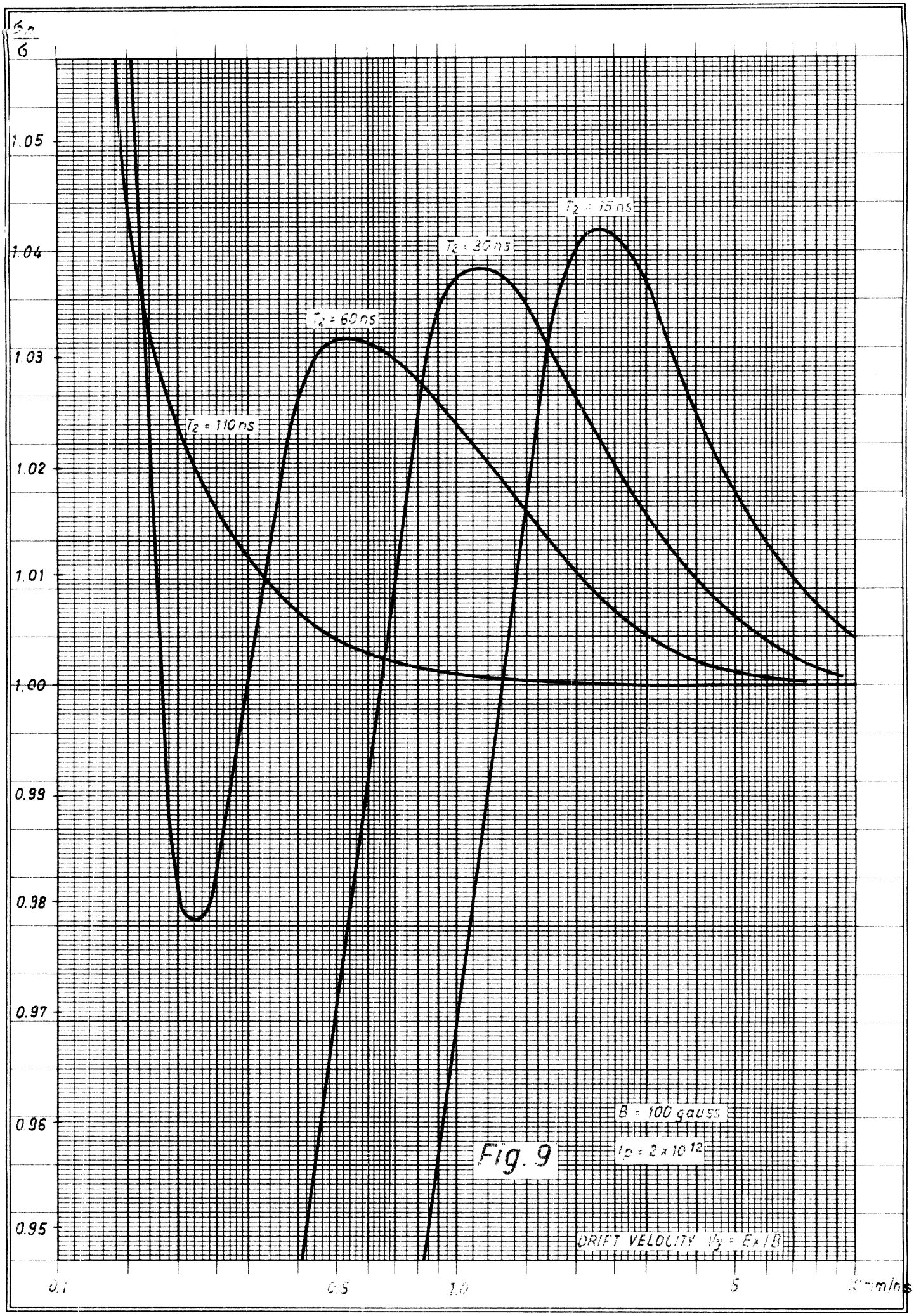


Fig. 9

1.8.74 G. Knedl

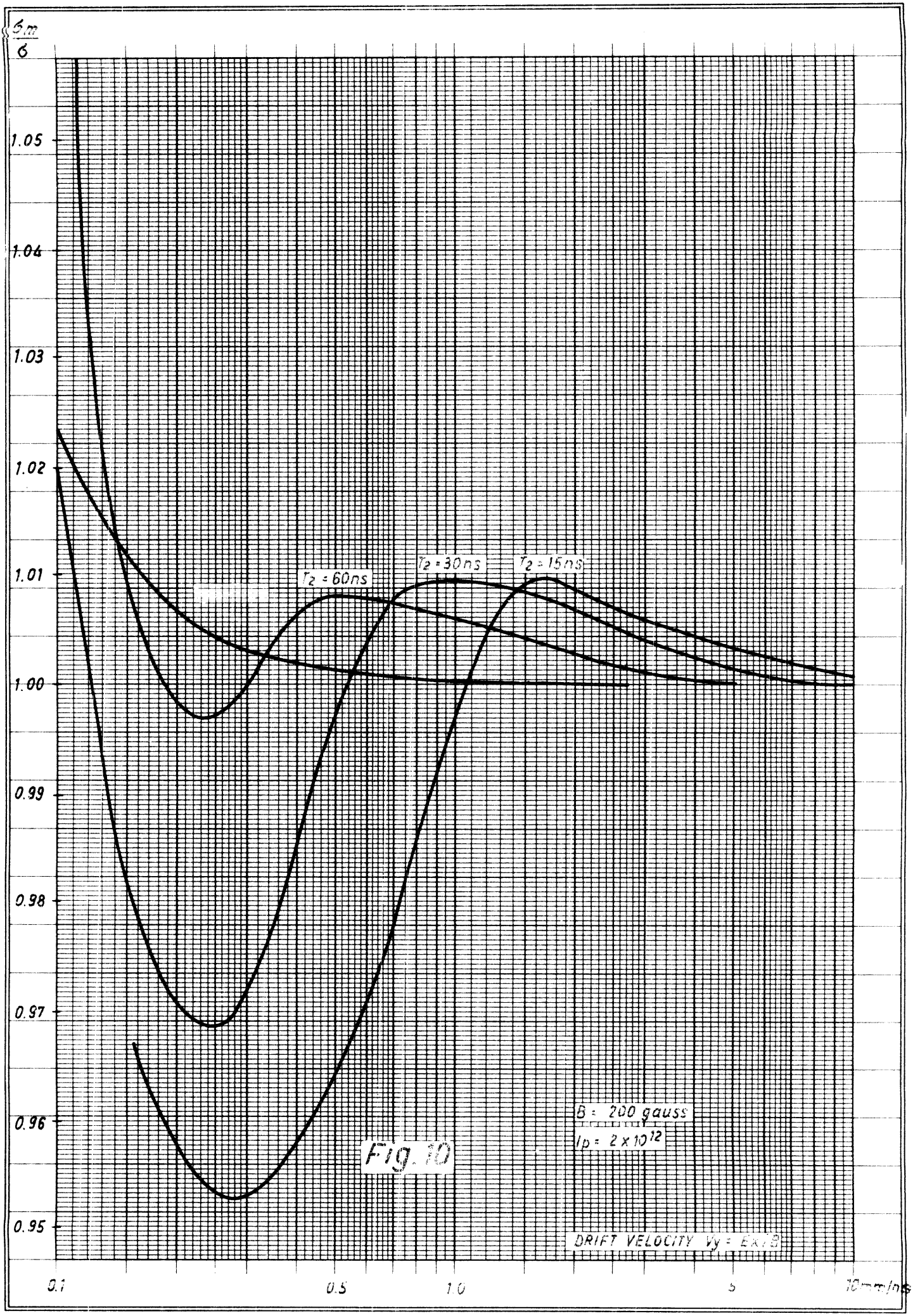

$\frac{G_m}{G}$
1.05
1.04
1.03
1.02
1.01
1.00
0.99
0.98
0.97
0.96
0.95
τ_2 = 60ns
τ_2 = 30ns
τ_2 = 15ns
B = 200 gauss
Ip = 2 x 10^{12}
DRIFT VELOCITY Vy = E x B
0.1
0.5
1.0
5
10 cm/ns

Fig. 10

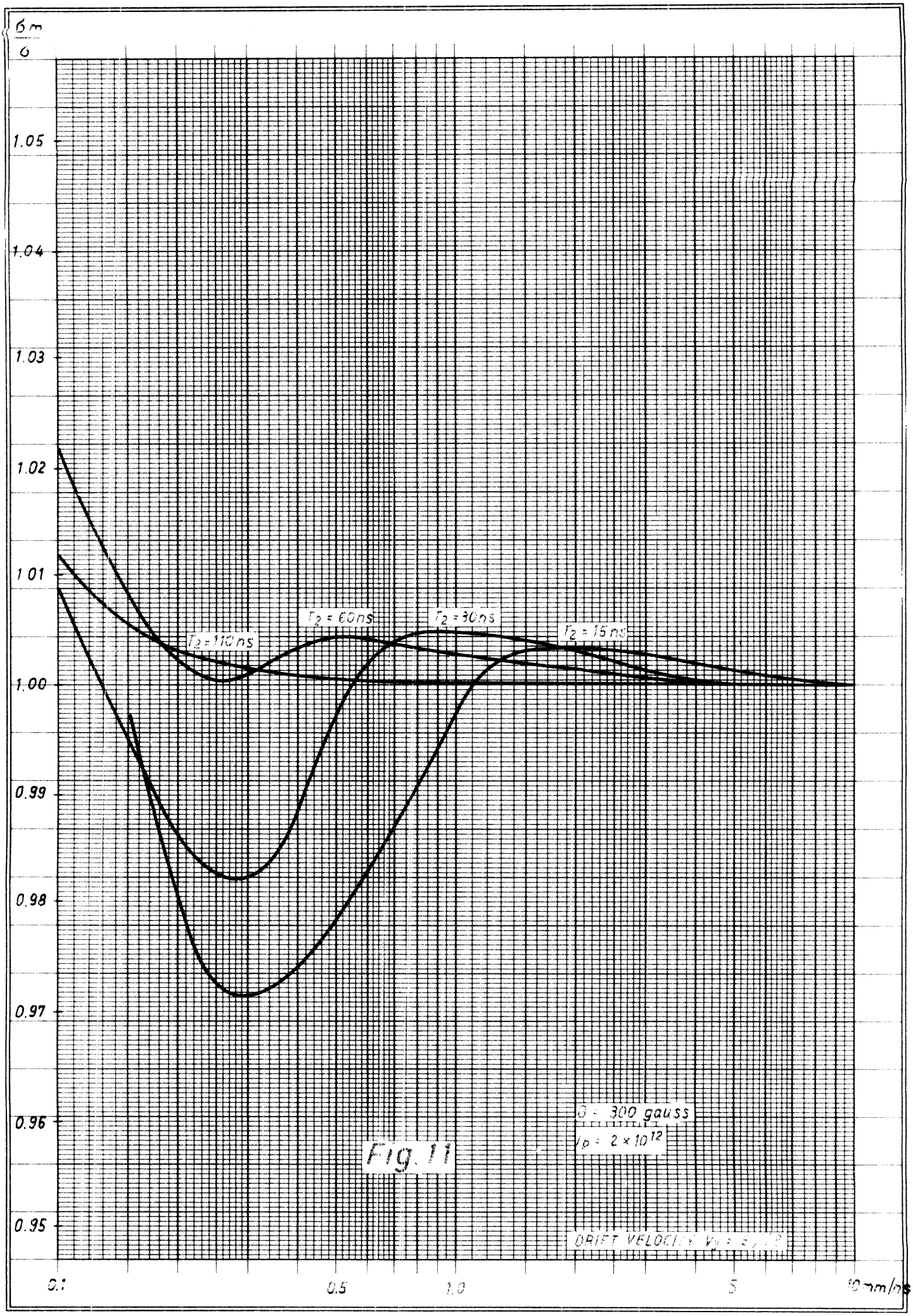

δm
δ
1.05
1.04
1.03
1.02
1.01
1.00
0.99
0.98
0.97
0.96
0.95
T_2 = 110 ns
T_2 = 60 ns
T_2 = 30 ns
T_2 = 15 ns
B = 300 gauss
N_0 = 2 × 10^{12}
0.1
0.5
1.0
5
10 mm/ns

Fig. 11

1 8.74 G.

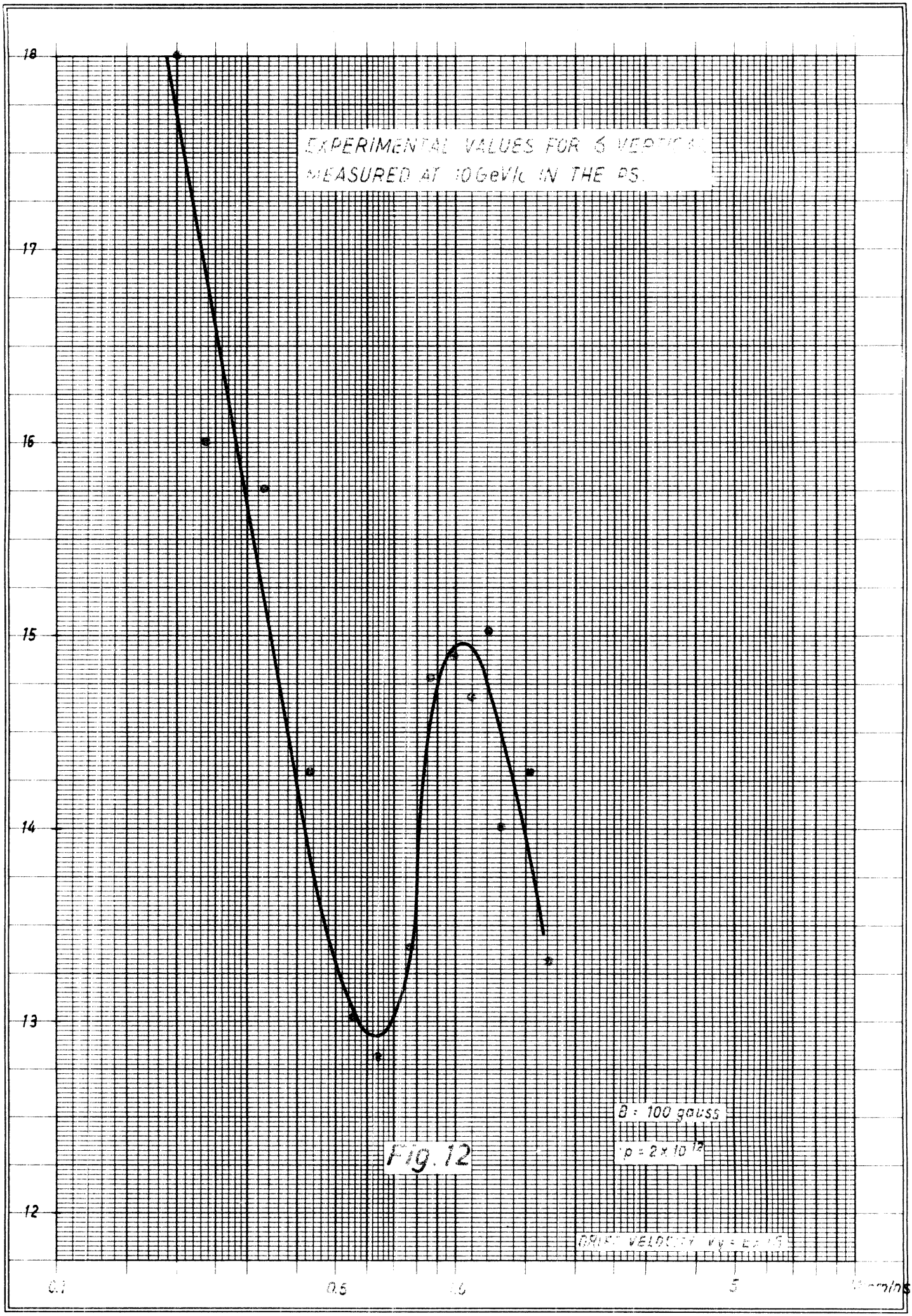
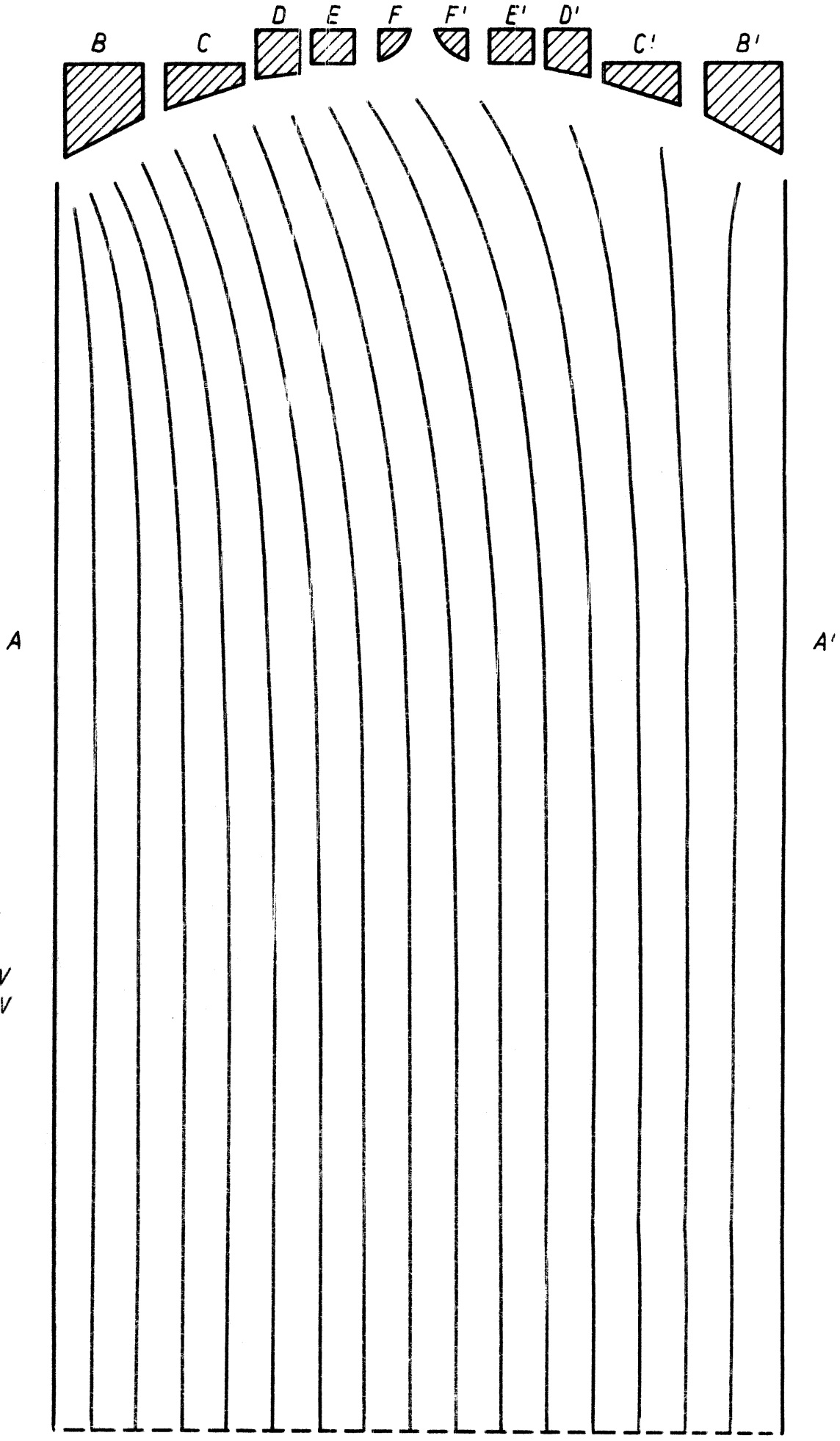


FIG. 12

1.8.74 G. Lutz

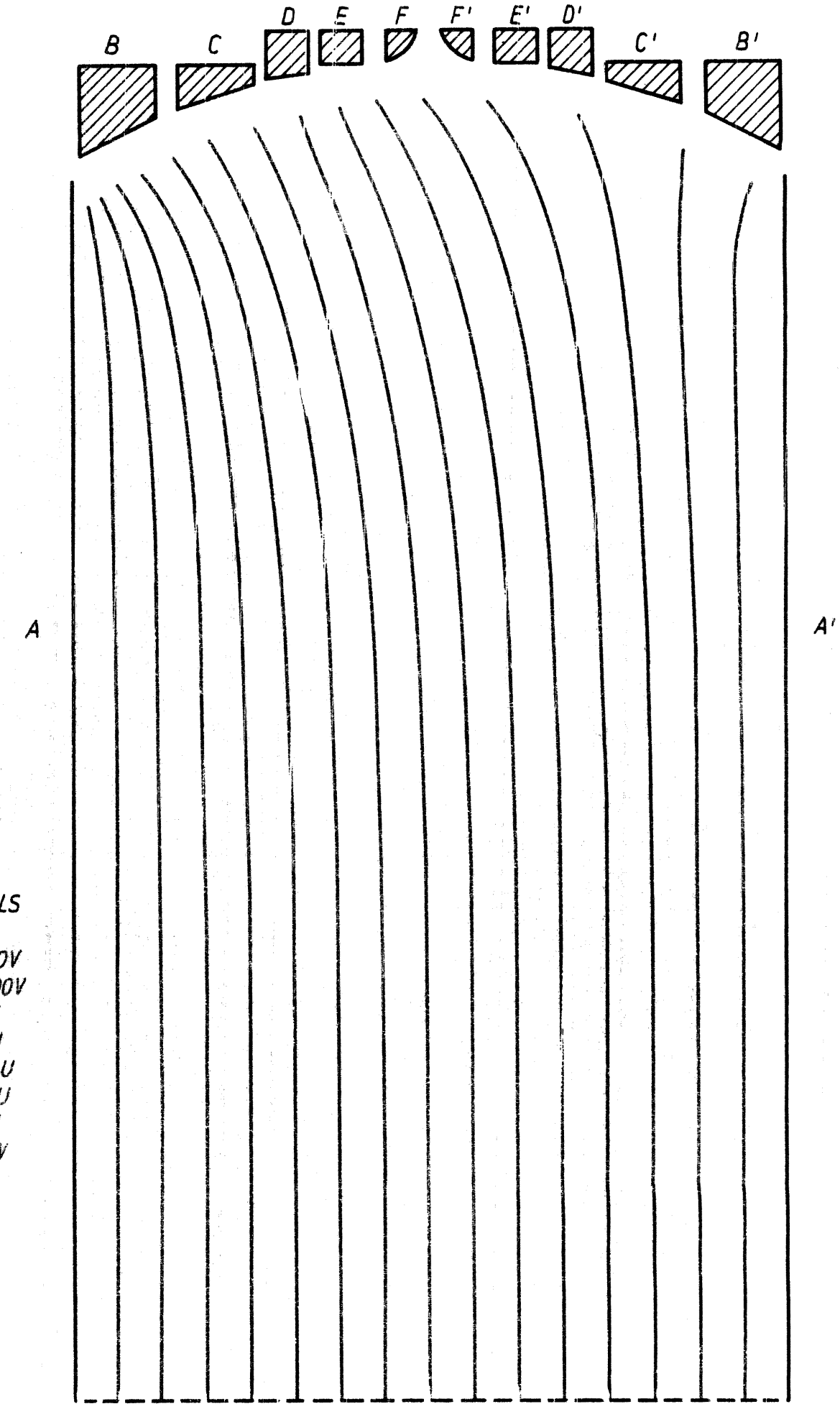


POTENTIALS

U = -1500V
U' = +4500V
A = 1.0 U
B = 0.7 U
C = 0.5 U
D = 0.35 U
E = 0.2 U
F = -100V

Fig.13

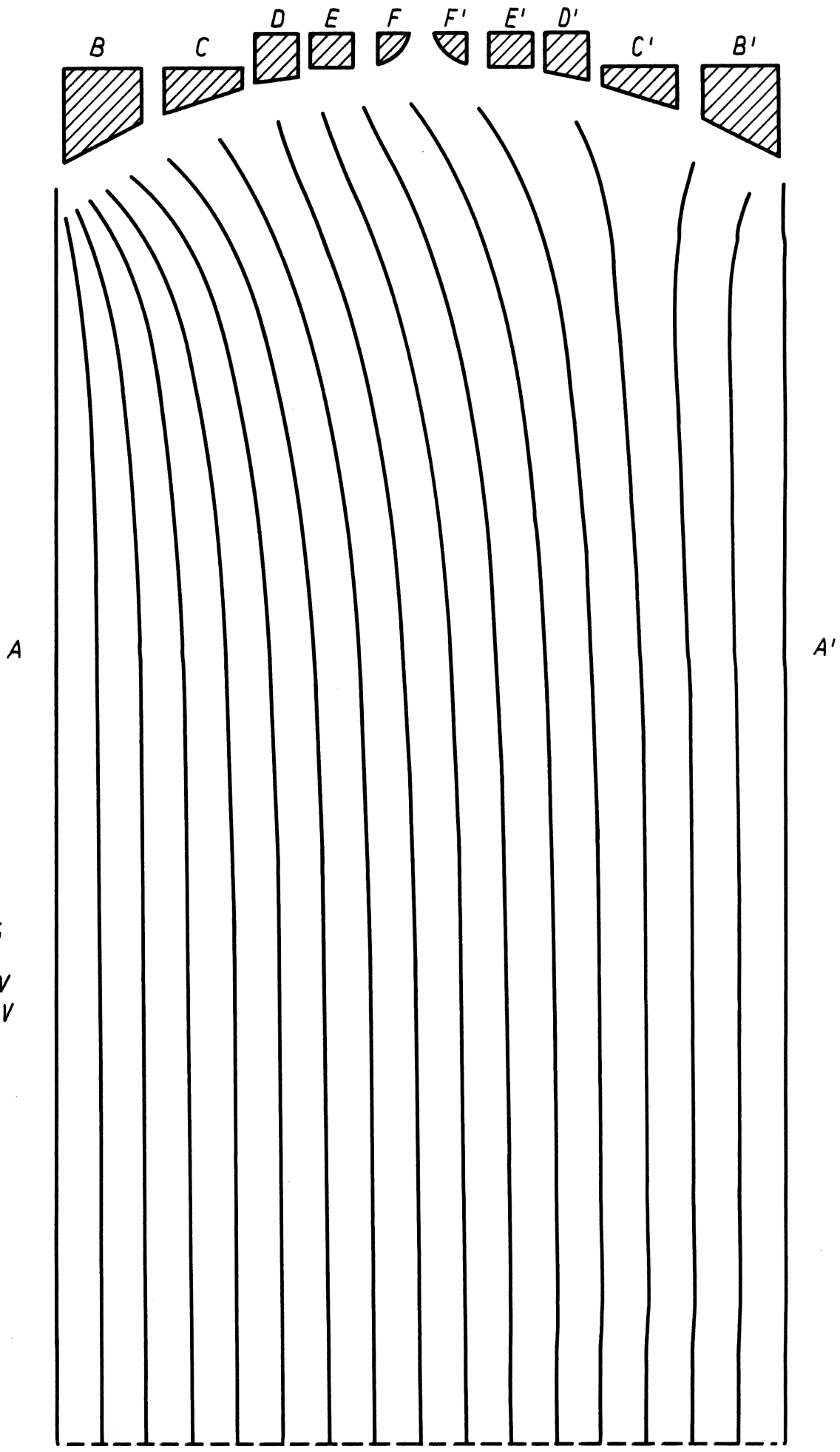
29.7.76 C Hackl



POTENTIALS

U = -1500V
U' = +4500V
A = 1.0 U
B = 0.6U
C = 0.45U
D = 0.35U
E = 0.2 U
F = -100V

Fig. 14

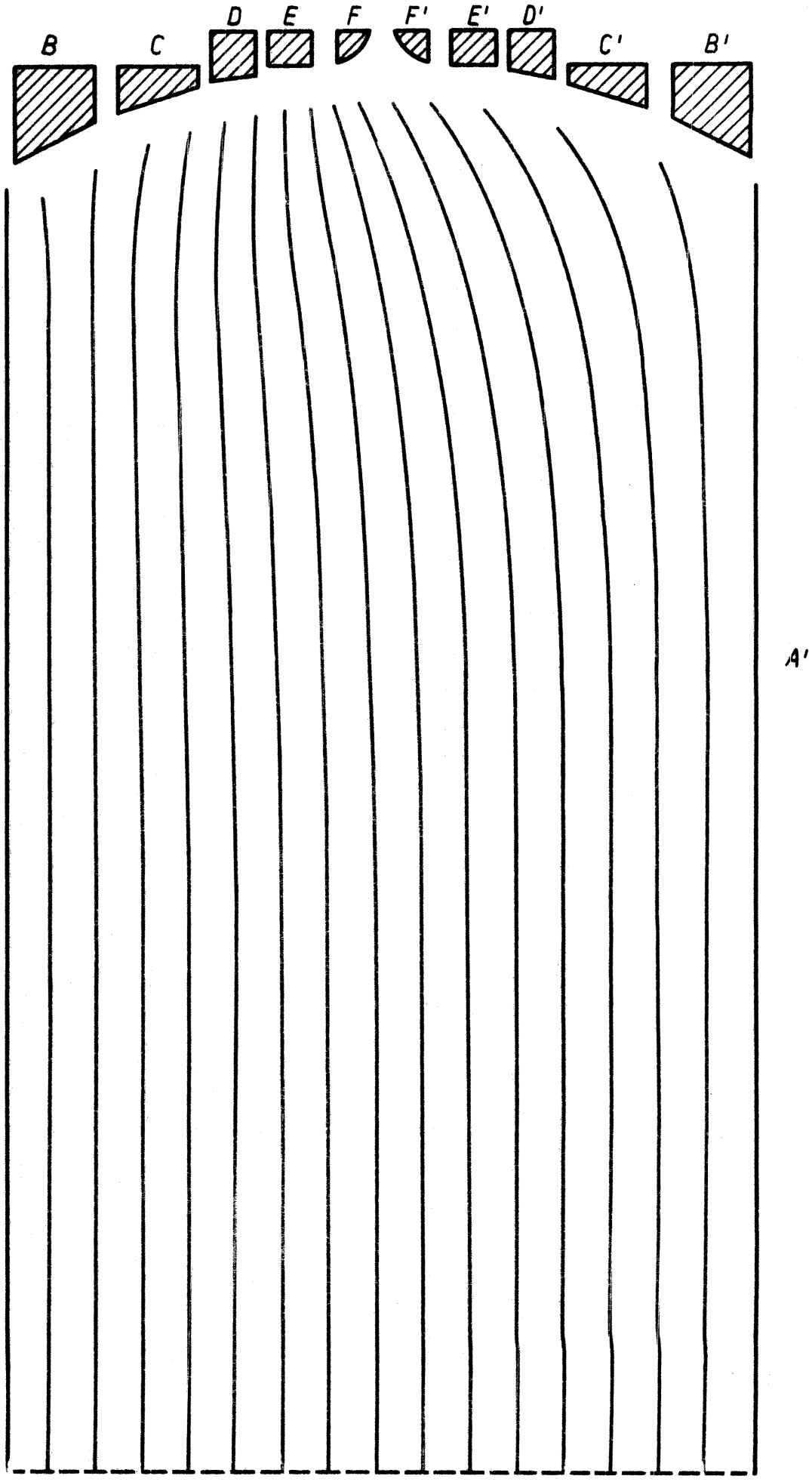


POTENTIALS

U = -1500V
U' = +4500V
A = 1.0 U
B = 0.5 U
C = 0.4 U
D = 0.35 U
E = 0.2 U
F = -100V

Fig. 15

29.7.76 C Hall



POTENTIALS

U = -1500 V
U' = +4500 V
A = 1.0 U
B = 0.9 U
C = 0.8 U
D = 0.6 U
E = 0.4 U
F = -100 V

Fig. 16

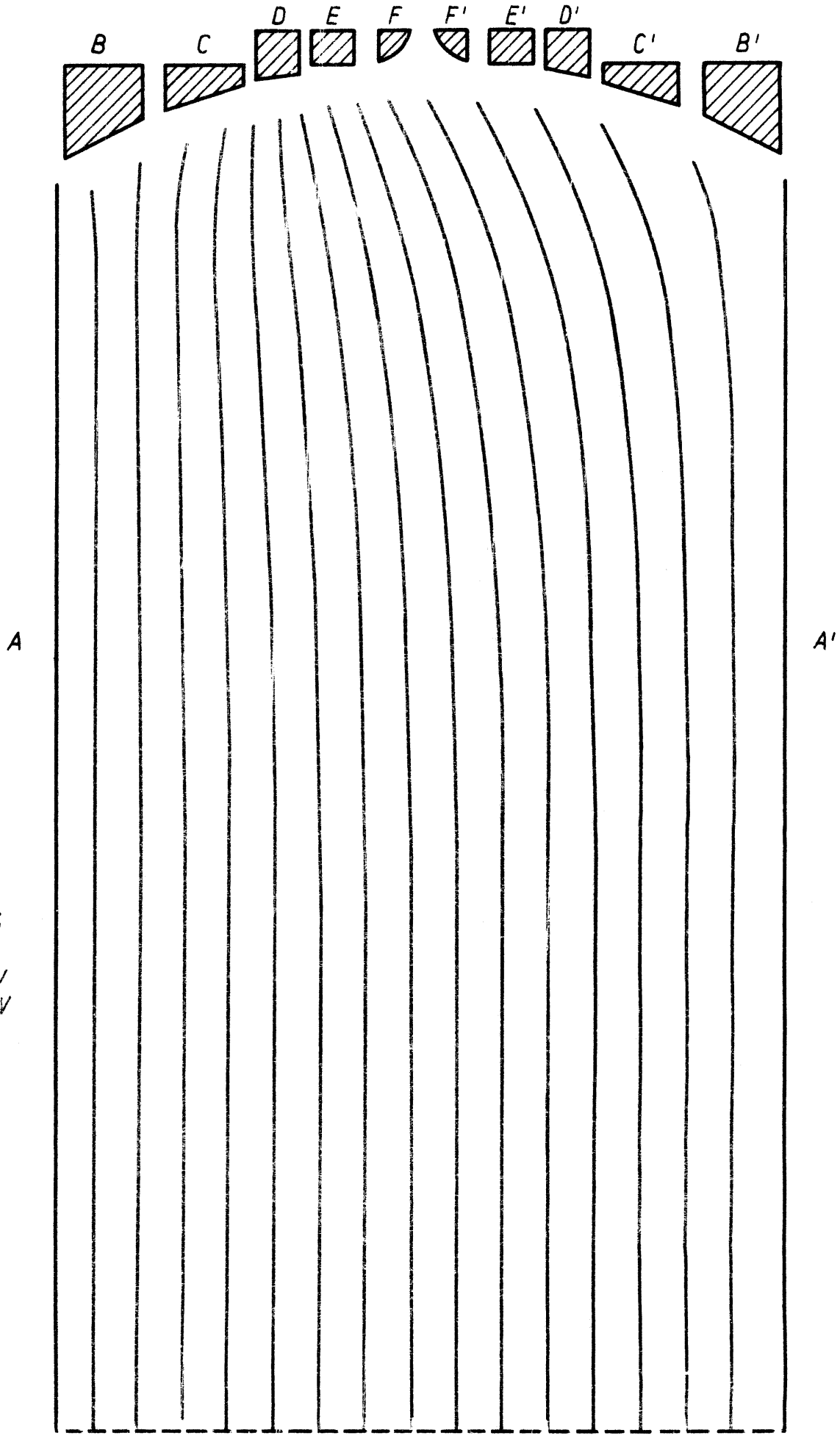


Fig. 17

29.7.74 C Hoff

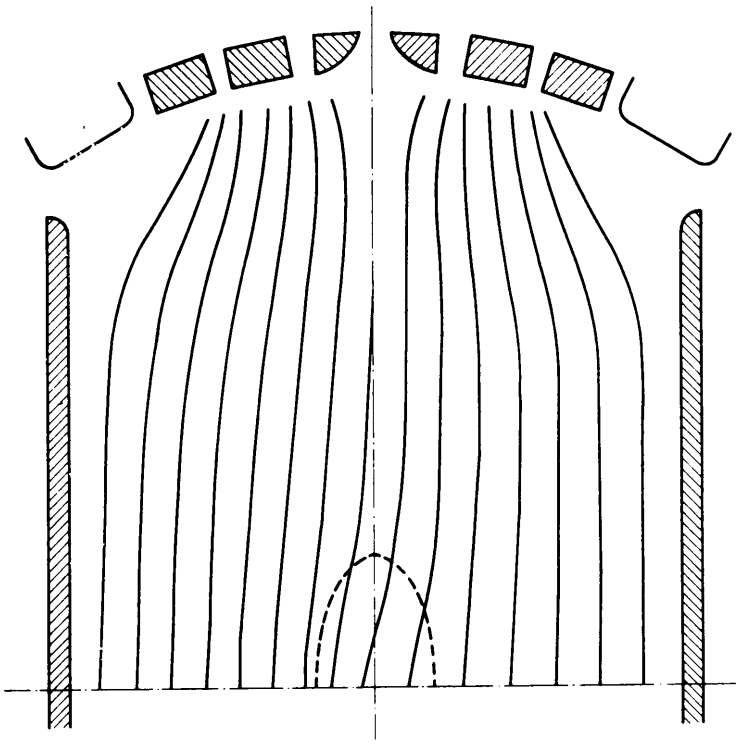


Fig. 18
IBS EQUIPOTENTIALS WITH BEAM: 10^{13} PROTONS

29.7.74 G. Hautot