EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH
ORGANISATION EUROPEENNE POUR LA RECHERCHE NUCLEAIRE

CERN - PS DIVISION

PS/RF/ Note 2001-002
CLIC Note 468

COOLING AND VIBRATION IN THE CLIC MAIN ACCELERATING STRUCTURE

W. Schnell

Abstract

The mechanical vibration induced by the flow of cooling water is discussed. The possibility of combining acceptable heat-transfer with laminar flow is investigated and an approximate formula for the effect of turbulent flow is given. The CLIC main accelerating structure is taken as example.

Geneva, Switzerland
15 January 2001

1. Introduction

Inevitably, the flow of cooling water induces vibrations. Tolerances are tightest in the quadrupole magnets but there, water-cooled coils can be (and very probably have to be) mechanically isolated from the yokes. In the accelerating structures this is impossible so that a fundamental problem may arise. The analysis given here is only semi-quantitative but believed to give a reasonable and somewhat pessimistic estimate.

2. Liquid coolants considered

Water is the obvious choice of coolant but ethylene glycol ($C_2H_6O_2$) may be considered because of its higher viscosity. The following properties are for 25^0 C:

	Water	**$C_2H_6O_2$**	
Mass density ρ	10^3	1.11×10^3	kg m^{-3}
Viscosity η	0.89×10^{-3}	16.1×10^{-3}	Ns m^{-2}
Cinematic viscosity η/ρ	0.89×10^{-6}	14.5×10^{-6}	$m^2\ s^{-1}$
Heat capacity c	4.19×10^6	2.4×10^6	Ws $m^{-3}K^{-1}$
Heat conductivity α	0.6	0.26	W $m^{-1}K^{-1}$

Water is assumed unless stated otherwise.

3. Required volume flow and velocity

The following parameters will be assumed:

Total dissipation per unit length: $\partial W/\partial l$	5 kW/m
Number of cooling channels n per structure	4
Temperature rise along a channel $\partial T/\partial l$	$10^0 Cm^{-1}$
Equivalent channel diameter: d	8 mm

With water, this requires a volume flow rate per channel of 3.0×10^{-5} m^3s^{-1} or 1.8 liters/minute and an average flow velocity of

$$u = 4\ (\partial V/\partial t)\ \pi d^{-2} = 0.593 ms^{-1}$$

4. Laminar flow

Reynold's number

$$Re = ud\rho/\eta$$

amounts to 5330 with the above parameters.

This is moderately above the onset of turbulence occurring at about Re = 2000. Laminar flow could be established by employing glycol in the 8 mm cooling channel but the heat transfer from metal to liquid would be insufficient. Instead, each cooling channel can be subdivided into subchannels. For instance, 64 such subchannels of 1 mm diameter each might be formed by extruded copper strips, each 15 mm wide and 1.5 mm thick, say, with eight semicircular grooves of 0.5 mm depth on their faces. Packed face to face into a 14 by 12 mm^2 assembly and sealed around the circumference, they can form cooling *bars*, which may be brazed to the side of the accelerating structure. With this and water-cooling, *Re* is reduced to 666. Employing glycol cooling would reduce it to 71 (at the cost of a 5 bar/m pressure drop) if really necessary.

The cooling surface is 0.2 m^2 per meter of length in such a cooling bar. By conduction alone, the temperature gradient across a 0.2 mm layer of water is 2^{0}C (5^0 for glycol) for the 1.25 kW assumed heat throughput and thus quite acceptable. Even with nominally laminar flow some transverse motion will occur due to surface roughness and mechanical tolerances. This is likely to be harmless, however, as can be seen from the subsequent treatment of fully turbulent flow.

5. Turbulent flow

With turbulent flow, the pressure-drop along a circular tube of length l and diameter d is given by the semi-empirical formula

$$\Delta p = \frac{\rho u^2}{2} \lambda \frac{l}{d}$$

The coefficient λ decreases very slowly with increasing *Re*. It can be found from published diagrams such as ref. [1] or (for smooth pipes) from empirical formulas. The one best suited for moderate turbulence is Blasius' formula

$$\lambda = 0.316\, Re^{-1/4}$$

An appropriate value here is $\lambda = 0.04$, including a small margin for surface roughness.

In the turbulent regime the pump power $\Delta p \partial V/\partial t$ is almost entirely converted to irretrievable kinetic energy of turbulent motion. If υ is the local and instantaneous velocity seen from a frame moving with u, the volume density of turbulent kinetic energy thus created is $\frac{1}{2}\rho\upsilon^2$. It departs from the length of pipe in which it was created with the same volume flow rate $\partial V/\partial t$. Thus, the pressure drop Δp equals the average fraction of hydrostatic energy density p converted to irrecoverable turbulent kinetic energy density. It follows that

$$\overline{\upsilon^2} = 2\frac{\Delta p}{\rho} = \overline{\upsilon_x^2} + \overline{\upsilon_y^2} + \overline{\upsilon_z^2} = \lambda \frac{l}{d} u^2$$

is the mean-square velocity of turbulence. Assuming isotropy, one finds a local momentum density in vertical direction, say, given by

$$\rho \upsilon_y^{rms} = \rho u \sqrt{\lambda \frac{l}{3d}}$$

Note that the length l refers to the total length of cooling pipes of diameter d. This length should be kept to a minimum – namely the length of the structure to be cooled. Therefore, the n channels of a structure should be fed in parallel and laminar flow should be imposed on the input and output pipes (with tapered transitions to the actual cooling channels). That is what is assumed here.

From here on a rough but probably pessimistic approximation will be made. Its validity is discussed in section 6 below. It is assumed that the kinetic energy is concentrated in cells of average coherence-length equal to half the pipe diameter, *d/2*. This seems justified by the fact that *Re* equals only 2.5 times the turbulence limit. The assumption is believed to give a pessimistic estimate of vibration, since any momentum shared by smaller cells will average out better in time and in distance, while larger cells, if existing, would only favour longitudinal momentum transfer. With this, the vertical momentum contained in a coherence-cell of volume *Ad/2* (where *A* is the cross-section) is given by

$$P_{ycell}^{rms} = \frac{Ad\rho}{2}\upsilon_y^{rms} = \frac{Ad\rho}{2}u\sqrt{\lambda\frac{l}{3d}}$$

There are *2l/d* such cells in a structure of length *l* and their momenta must be added in quadrature, thus adding a factor $(2l/d)^{1/2}$ to the above equation. The total rms momentum transmitted from the turbulent water to the structure is therefore

$$P_{ytot}^{rms} = Al\rho u\sqrt{\frac{n\lambda}{6}} = mu\sqrt{\frac{n\lambda}{6}}$$

where m is the mass of the water in a cooling channel and *n* their number (four in this example).

If the accelerating structure including its support (as far it is free of internal mechanical resonance), can be expressed by an equivalent rigid mass M, its vertical rms velocity induced by the turbulence of the cooling water is, therefore, given by

$$\upsilon_{ytot}^{rms} = u\frac{m}{M}\sqrt{\frac{n\lambda}{6}}$$

The assumption of a coherence-length implies the existence of a dominant frequency

$$\omega = 2\pi\frac{u}{d}$$

due to coherence-cells of length *d/2*, passing a fixed location at velocity *u* with the assumed parameters $\omega/2\pi$ = 74 Hz. Clearly this is not a discrete frequency but merely the average of a range in which most of the energy is localized. Combining the last two equations gives a rms vibration amplitude

$$s_y^{rms} = \frac{1}{2\pi}\sqrt{\frac{n\lambda}{6}}\frac{m}{M}d$$

This is a surprising result: The dependence on *u* is reduced to the very slowly decreasing coefficient $\sqrt{\lambda} \propto Re^{-1/8}$. Apart from this, the vibration amplitude equals the width of the cooling channel scaled down by the obvious mass ratio m/M, times a small collection of (uncertain) numerical factors. With the parameters chosen the mass m of one water column is 50 g/m. If M is the mass of the structure alone, taken as a copper cylinder of 40 mm diameter, M amounts to 11 kg/m and

$$s_y^{rms} = 0.94\mu m$$

Note that an isolated accelerating structure, taken as a copper rod of 40 mm diameter and 0.5 m length, has its lowest bending mode at 540 Hz ($\propto$ diameter/length^2), the nodes being situated at 0.224 *l* from the ends. It will not be easy, therefore, to increase M much beyond the mass of the bare structure by anchoring it to a heavy support.

6. Discussion

At first sight, rapid gain seems to come from *reducing* the width of the cooling channel, thus *increasing* the flow velocity and Reynold's number, since, on face value, the vibration amplitude scales with $md \propto d^{-3}$. This would, however, stretch characteristic frequencies into a range where structural resonances are inevitable.

In fact, it is found [2,3] that for Reynold's numbers far above the critical one (a situation characteristic of aerodynamics) the spectral density of turbulent kinetic energy decreases with the $-5/3^{th}$ power of frequency between two corner frequencies. The lower one is essentially the characteristic frequency given above, associated with coherence lengths roughly equal to the channel radius. The upper one is due to minimum-size coherence cells within which Reynold's number has decreased to the turbulence limit, so that they do not break up further. In the example considered above, the two frequencies span roughly an octave, so that it is justified (and slightly pessimistic) to consider the lower frequency only. With higher flow velocities, turbulent frequencies will span structural resonances, which may be difficult to damp. A high-velocity solution may, however, be appropriate for water-cooled magnet coils where the inevitable vibrational isolation will be easier for smaller input-amplitudes and higher frequencies.

In conclusion it appears that the problem is not fatal but that it does require experimental study and careful design.

References:

[1]: K. Gieck, Technische Formelsammlung. Gieck Verlag, Heilbronn D (1984)

[2] L. Landau, E.M. Lifschitz, Lehrbuch der theoretischen Physik (1991)VI §33

[3] C.F. v. Weizsäcker, Z. Physik **124** (1948) p. 614, W. Heisenberg ibid. p. 628.