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FAST HIGH-INTENSITY SINGLE-BUNCH TRANSVERSE COHERENT INSTABILITY IN SYNCHROTRONS DUE TO A BROAD-BAND RESONATOR IMPEDANCE

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Abstract

The instability rise-time is computed when it is faster than the synchrotron period, using the mode-coupling formalism. The case is treated of a bunch interacting with a broad-band resonator impedance, and whose length is greater than the inverse of twice the resonance frequency. The formula is compared to the one obtained by Brandt and Gareyte in a beam break-up approach, and to the one first obtained by Ruth and Wang in a fast blow-up theory, and later re-derived by Kernel et al. in a post-head-tail formalism. Stabilisation by synchrotron oscillation is also discussed.

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1 INTRODUCTION

The Beam Break-Up (BBU) theory has been developed to explain the beam emittance growth and the transverse instabilities observed in linear accelerators. The mechanism of cumulative BBU in linacs can be stated as follows [1]. If a bunch in a pulse is displaced from the central axis of the linac for some reason, it excites a transverse deflecting mode in an RF cavity. The following bunches feel this field in that cavity and are deflected, even if they are on axis. These deflected bunches create fields of the same type in the cavities in the rest of the linac, which further deflect the following bunches, leading to emittance growth and subsequent beam loss.

In circular accelerators, the interaction between the beam and its surroundings is described in terms of head-tail modes through Sacherer's formalism [2]. Below a threshold intensity, these standing-wave patterns can be treated independently. This leads to instabilities where the head and the tail of the bunch exchange their roles (due to synchrotron oscillation) several times during the rise-time of the instability. The important point here is that the betatron phase varies linearly along the bunch (from the head) and attains its maximum value at the tail. The total betatron phase shift between head and tail is the physical origin of the head-tail instability. As the bunch intensity increases, the different modes, separated by the synchrotron frequency for zero intensity, can no longer be treated separately. Indeed, above a threshold intensity, the wake fields couple the head-tail modes together and a travelling-wave pattern is created along the bunch. This is the Transverse Mode Coupling (TMC) instability described by Kohaupt [3] in terms of coupling of head-tail modes. This extended to the transverse motion, the theory proposed by Sacherer [4] to explain the longitudinal microwave instability through coupling of the longitudinal coherent bunch modes.

It has been known for some time, using a two-particle model, that the TMC instability is the manifestation in synchrotrons of the BBU mechanism observed in linacs [5,6]. The only difference comes from the synchrotron oscillation, which stabilises the beam in synchrotrons below a threshold intensity by swapping the head and the tail continuously. This effect disappears close to transition energy or more generally when the instability rise-time is much faster than the synchrotron period. In this case, it is usually said that the concept of head-tail modes loses its meaning and that it is appropriate to use the BBU formalism to describe the interaction between the beam and its surroundings. It is shown in this paper that using the mode-coupling formalism, for the case of a bunch interacting with a broad-band resonator impedance, and whose length is greater than the inverse of twice the resonance frequency, the same formula is obtained in both theories (for zero chromaticity) to within a numerical factor. The formula, for any positive chromatic frequency, is then compared to the one first obtained by Ruth and Wang [7] and later re-derived by Kernel et al. [8]. It can be seen that the same result is obtained only for short bunches. The stabilising mechanism, i.e. synchrotron oscillation, is also discussed. The intensity threshold is found to be equal to the one of Kernel et al. times a numerical factor.

2 INSTABILITY MECHANISM

2.1 Transverse Mode Coupling

Considering the case where two adjacent head-tail modes (m and m+1) undergo a coupled motion, the stability of a high-intensity single-bunch beam can be discussed using the following determinant, e.g. for the vertical plane [9,10]

$$\begin{vmatrix} \omega_c - \omega_{y0} - m\omega_s - \Delta\omega^y_{m,m} & -\Delta\omega^y_{m,m+1} \\ \Delta\omega^y_{m,m+1} & \omega_c - \omega_{y0} - (m+1)\,\omega_s - \Delta\omega^y_{m+1,m+1} \end{vmatrix} = 0\,, \qquad (1)$$

with

$$\Delta\omega^y_{m,n} = \left(|m|+1\right)^{-1} \frac{j\,e\,\beta\,I_b}{2\,m_0\,\gamma\,Q_{y0}\,\Omega_0\,L}\left(Z^{eff}_y\right)_{m,n}\,, \qquad (2)$$

$$\left(Z^{eff}_y\right)_{m,n} = \frac{\sum_{k=-\infty}^{k=+\infty} Z_y\left(\omega^y_k\right) h_{m,n}\left(\omega^y_k - \omega_{\xi_y}\right)}{\sum_{k=-\infty}^{k=+\infty} h_{m,m}\left(\omega^y_k - \omega_{\xi_y}\right)}\,, \qquad (3)$$

$$\begin{aligned} h_{m,n}(\omega) = {} & \frac{\tau_b^2}{\pi^4}\left(|m|+1\right)\times\left(|n|+1\right)\times F^n_m \\ & \times\left\{(\omega\tau_b/\pi)^2-\left(|m|+1\right)^2\right\}^{-1}\times\left\{(\omega\tau_b/\pi)^2-\left(|n|+1\right)^2\right\}^{-1}, \end{aligned} \qquad (4)$$

$$F^{n\ even}_{m\ even} = (-1)^{(|m|+|n|)/2}\times\cos^2[\omega\tau_b/2]\,, \qquad (5)$$

$$F^{n\ odd}_{m\ even} = \frac{(-1)^{(|m|+|n|+3)/2}}{2\,j}\times\sin[\omega\tau_b]\,, \qquad (6)$$

$$F^{n\ even}_{m\ odd} = \frac{(-1)^{(|m|+|n|+1)/2}}{2\,j}\times\sin[\omega\tau_b]\,, \qquad (7)$$

$$F^{n\ odd}_{m\ odd} = (-1)^{(|m|+|n|+2)/2}\times\sin^2[\omega\tau_b/2]\,. \qquad (8)$$

Here, ω_c is the coherent angular frequency to be determined, $\omega_{y0} = Q_{y0}\,\Omega_0$ is the unperturbed betatron angular frequency with Q_{y0} the unperturbed tune and $\Omega_0 = 2\pi f_0$

the revolution angular frequency, $\omega_s = 2\pi f_s$ is the synchrotron angular frequency, $j = \sqrt{-1}$ is the imaginary unit, e is the elementary charge, β and γ are the relativistic velocity and mass factors, $I_b = N_b e f_0$ is the current in one bunch with N_b the number of protons in the bunch, m_0 is the proton rest mass, $L = \beta c \tau_b$ is the full (4σ) bunch length (in metres) with c the speed of light, Z_y is the coupling impedance, $\omega_k^y = (k + Q_{y0})\Omega_0 + m\omega_s$ with $-\infty \le k \le +\infty$, $\omega_{\xi_y} = 2\pi f_{\xi_y} = (\xi_y / \eta) Q_{y0} \Omega_0$ is the chromatic angular frequency, with $\xi_y = (\Delta Q_y / \Delta p)(p_0 / Q_{y0})$ and $\eta = \gamma_{tr}^{-2} - \gamma^{-2} = (\Delta T / T_0)/(\Delta p / p_0)$ the chromaticity and slippage factor, where p is the momentum and T the revolution period of a particle, and $h_{m,n}$ describes the cross-power densities of the mth and nth line-density modes. Considering the case of a driving broad-band resonator, the coupling impedance is given by

$$Z_y(\omega) = \frac{\omega_r}{\omega} R_r \Big/ \left[1 - j Q_r \left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r} \right) \right], \tag{9}$$

where $\omega_r = 2\pi f_r$ is the resonance angular frequency, Q_r the quality factor and R_r the shunt impedance. Equation (1) leads to the following solutions for ω_c

$$\begin{aligned} \omega_c^{\pm} = & \frac{1}{2} \times \left[2\omega_{y0} + (2m+1)\omega_s + \Delta\omega_{m,m}^{y} + \Delta\omega_{m+1,m+1}^{y} \right] \\ & \pm \frac{1}{2} \sqrt{\left(\omega_s + \Delta\omega_{m+1,m+1}^{y} - \Delta\omega_{m,m}^{y} \right)^2 - \left(2\Delta\omega_{m,m+1}^{y} \right)^2} \, . \end{aligned} \tag{10}$$

If the mode-coupling term $\Delta\omega_{m,m+1}^{y}$ is negligible, then the result for the de-coupled head-tail modes m is recovered, $\omega_c = \omega_{y0} + m\omega_s + \Delta\omega_{m,m}^{y}$.

Consider first the case of a long bunch (i.e. $\tau_b >> 0.5 / f_r$) with $\xi_y = 0$ (i.e. the de-coupled modes are stable), near transition (i.e. $\eta \approx 0$). It is represented in Figure 1 for $\omega \ge 0$, knowing that $h_{m,m}$, $h_{m+1,m+1}$ and $\mathrm{Im}(Z_y)$ are even functions of ω, whereas $h_{m,m+1}$ and $\mathrm{Re}(Z_y)$ are odd ones. One is interested in modes whose spectra lie in the vicinity of the resonance frequency, since the resistive impedance is maximum there. As can be seen from Figure 1, $|\Delta\omega_{m,m+1}^{y}| >> |\Delta\omega_{m,m}^{y}|$ and $|\Delta\omega_{m,m+1}^{y}| >> |\Delta\omega_{m+1,m+1}^{y}|$. Furthermore, $Z_y(\omega_k^y)$ can be removed from the summation, as it is almost a constant equal to $Z_y(\omega_r)$, and $\omega_s \approx 0$ close to transition, since it is given by (at constant energy)

$$\omega_s = \sqrt{\frac{|\eta| \hat{V}_{\mathrm{RF}} \, h \, \Omega_0^2}{2\pi \beta^2 (E/e)}} = \frac{|\eta| \times \left(\dfrac{\Delta p}{p_0} \right)_{\max}}{\tau_b / 2} \, . \tag{11}$$

Here, $\hat{V}_{RF}$ is the peak RF voltage, h the harmonic number, and E the total beam energy. Equation (10) then becomes

$$\omega_c^{\pm} \approx \omega_{y0} \pm j \left| \Delta\omega_{m,m+1}^{y} \right| . \tag{12}$$

The rise-time of the TMC instability is thus given by

$$\tau_{y,0}^{TMC} = \left| \Delta\omega_{m,m+1}^{y} \right|^{-1} . \tag{13}$$

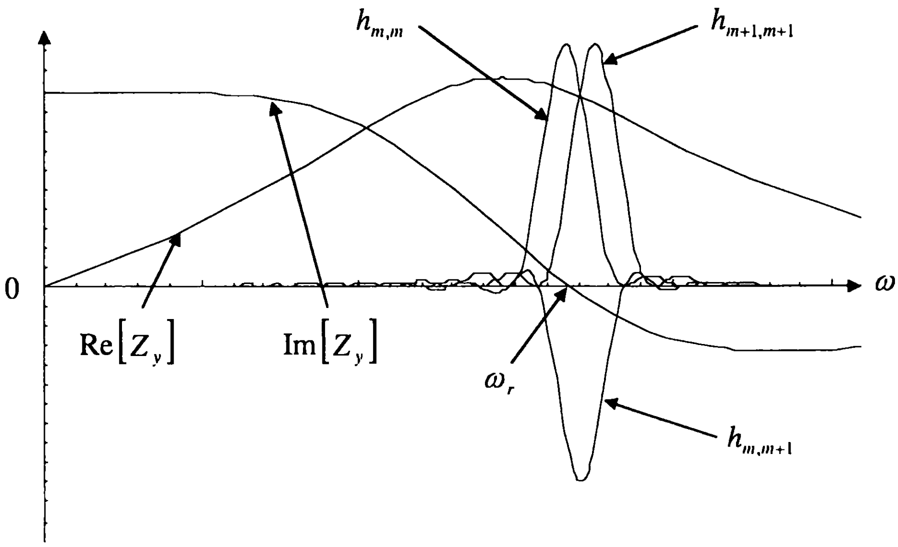

FIGURE 1. *Power spectra for the transverse modes m and m+1 of a long bunch ($\tau_b >> 0.5/f_r$), and real and imaginary parts of the driving broad-band impedance.*

Expressing $\Delta\omega_{m,m+1}^{y}$, remembering that the power spectrum of mode $|m|$ is peaked near $\omega_r \approx (|m|+1)\pi/\tau_b$, and noting that

$$F = \frac{\sum_{k=-\infty}^{k=+\infty} \left| j\, h_{m,m+1}\left(\omega_k^y - \omega_{\xi_y}\right) \right|}{\sum_{k=-\infty}^{k=+\infty} h_{m,m}\left(\omega_k^y - \omega_{\xi_y}\right)} \approx 0.6 , \tag{14}$$

Eq. (13) leads to

$$\tau_{y,0}^{TMC} = \pi \times \frac{\beta\left(E/e\right)\tau_b^2}{N_b\, e} \times \frac{b}{\beta_y \left| Z_l / p \right|} . \tag{15}$$

Here, b is the half minor (vertical) axis of an elliptical vacuum pipe (as in the CERN PS), $\beta_y = R/Q_{y0}$ is the average vertical betatron function with R the average radius of the machine, and Z_l/p is the longitudinal impedance divided by the harmonic number

$p = \omega / \Omega_0$. Furthermore, the classical formulae for the broad-band resonator model have been used, $Q_r \approx 1$, $\omega_r \approx \omega_{cut-off} \approx c/b^*$, and $R_r \approx (2R|Z_t/p|)/(b^2\beta)$.

If one now considers the case of a bunch of arbitrary length (with $\tau_b \geq 0.5/f_r$), the terms $\Delta\omega^y_{m,m}$ and $\Delta\omega^y_{m+1,m+1}$ have to be taken into account, but again the same result is obtained to within a few percent. Finally, approximately the same result is also obtained by solving numerically the (infinite) eigenvalue problem, instead of considering the coupling of only two adjacent modes.

Consider now the case of a bunch with $\xi_y \neq 0$ and $f_{\xi_y} \geq 0$ (this is the stability criterion for the head-tail mode m=0). Following the same procedure as before, the instability rise-time is given by

$$\tau_y^{TMC} = \tau_{y,0}^{TMC} \times \frac{|m|+1}{2 f_r \tau_b}. \tag{16}$$

The modes m and m+1 correspond to the most critical ones, interacting with the negative resistance peaked at $-f_r$, and are given by

$$|m|+1 = 2 f_r \tau_b \left| 1 + \frac{f_{\xi_y}}{f_r} \right|. \tag{17}$$

2.2 Beam Break-Up

Brandt and Gareyte have derived a formula for the single-bunch BBU in circular machines [6] from the theory developed by Yokoya [1]. They have approximated a bunch by a train of short bunchlets and have applied Yokoya's formula for cumulative BBU in a train of ultra-relativistic bunches, with the initial condition that every bunch in the train has the same initial position offset. Furthermore, this computation has been done in the absence of acceleration and for the smooth approximation. The time between the bunchlets is chosen to be small compared to the decay time of the considered resonator ($2Q_r/\omega_r$) and the wave period ($T_r = 2\pi/\omega_r$). They obtained the following equation, e.g. in the vertical plane, which gives the ratio between the amplitude of the bunch tail after n turns in the circular machine, and that of the whole bunch at the beginning of the instability process,

* This approximation is in fact perfectly valid for elliptical waveguides with aspect ratio $a = 2b$, since in this case the lowest cut-off frequency (i.e. of the dominant mode) is $\omega_{cut-off} \approx 0.94 \times c/b$. However, in the case of circular waveguides with $a = b$, $\omega_{cut-off} \approx 1.84 \times c/b$ [11].

$$\frac{y_n}{y_0}=\frac{1}{2\sqrt{2\pi}}\times\left(\frac{\Omega L}{c}\right)^{1/4}\times\frac{c}{L\omega_r}\times e^{\frac{-\varepsilon L}{c}+\sqrt{\frac{\Omega L}{c}}}, \tag{18}$$

with

$$\frac{\Omega L}{c}=\frac{N_b e c}{\omega_{y0}\left(E/e\right)}\times\frac{\omega_r R_r}{Q_r}\times n. \tag{19}$$

Here y_0, the initial vertical amplitude of the bunch, is the injection error for an instability which develops right after injection, or the average closed-orbit deviation for an instability which develops, for instance, near transition [5]. Furthermore, $\varepsilon=\omega_r/(2Q_r)$ is the damping characteristic of the resonator model of the coupling impedance.

It has been shown in Ref. [12] that the BBU mechanism is essentially described by the exponential term of Eq. (18). An approximate formula can be derived, which gives the time τ_y^{BBU} after which the amplitude of the bunch tail has been multiplied by exp(1), i.e. one e-folding time (which is also approximately equal to the time when the tail particles are lost). It is given by (for $\beta=1$)

$$\tau_y^{BBU}=\frac{\tau_{y,0}^{TMC}}{4}. \tag{20}$$

Therefore, the BBU and TMC approaches (for zero chromaticity) lead to the same formula except for the numerical factor 4. In the TMC formalism, real bunches are considered (parabolic longitudinal bunch distribution), whereas in the BBU derivation the bunch is a succession of bunchlets treated as point charges (uniform distribution). The numerical factor may be partially explained by this difference.

2.3 Fast Blow-Up and Post-Head-Tail Theories

Ruth and Wang have developed a theory of fast blow-up in a single bunch when the instability growth-time is smaller than the synchrotron period [7], and this result has been recently re-derived by Kernel et al. in a different approach called post-head-tail theory [8]. In fact two minor differences exist between their formulae. The intensity threshold of Ref. [7] is equal to the one of Ref. [8] times the numerical factor $\sqrt{2/3}$, and the peak impedance is used in the first reference, whereas the effective one is used in the second.

Comparing their results to ours, the instability rise-time of Eq. (16) is equal to the one of Ref. [8] times the factor $2(|m|+1)$, using the peak impedance in Eq. (16) instead of the effective one used in Ref. [8]. The difference between the two approaches comes from the term $|m|+1$ in Eq. (2), which indicates that the higher modes are more difficult to drive. This term does not appear in Ref. [8], where all the head-tail modes are considered mixed-up (their result can be deduced from the coasting-beam theory). The formulae converge to the same result only in the case where $\tau_b \approx 0.5 / f_r$, i.e. for a short bunch, since in this case the effective impedance is approximately equal to half the peak one. Comparing now the result of Kernel et al. to the one of Brandt and Gareyte, the instability rise-time of Eq. (20) is equal to the one of Ref. [8] times the factor $f_r \tau_b$. Therefore, their formulae converge to the same result also only for short bunches.

It is interesting to compare these formulae with the observations made in Ref. [12], where a high-intensity single-bunch beam was unstable at the CERN PS near transition. In Ref. [12], the time when the tail particles were lost was computed using Brandt and Gareyte's formula, and was found to be 1.2 ms. It was in good agreement with the observations made, where the tail particles were lost in about (less than) 1 ms. If one computes the rise-time of the instability described in Ref. [12] using Eq. (16) with $\xi_y = 0$, 4.6 ms are found, whereas using the formula given in Ref. [8], 27 μs are obtained ($f_r \tau_b = 42$). The first value is greater than what was observed, and the second one is much smaller (if correctly observed and modelled!). Therefore, in this experimental case, Eq. (20) seems to be the more appropriate. Or, the instability rise-time has to be computed from the de-coupled head-tail modes with $f_{\xi_y} < 0$. Note that the fastest rise-time is obtained for the mode m=0 when $f_{\xi_y} \approx -f_r$. It is given by

$$\tau_y^{HT, fastest} = \frac{\beta (E/e) \tau_b}{N_b e} \times \frac{b}{f_r \beta_y |Z_l / p|} . \tag{21}$$

This rise-time is equal to the coasting-beam one times the bunching factor $B = f_0 \tau_b$.

3 STABILISING MECHANISM

In addition to the actions that can be taken to increase the rise-time of the instability through the different parameters of Eq. (16), one mechanism can be used to prevent it. This is the synchrotron oscillation, which stabilises the beam below a threshold intensity by swapping the head and the tail continuously. From Eqs. (10) and (16), stability is obtained if the synchrotron period $T_s = 1/f_s$ satisfies

$$T_s \leq \pi \tau_y^{TMC} . \tag{22}$$

If Eq. (22) is not fulfilled, the beam is unstable and the rise-time is given by Eq. (16). This instability can develop at transition, since there the synchrotron period becomes infinite, but also far from transition if $T_s > \pi \tau_y^{TMC}$. Using Eq. (11), the stability criterion can also be written

$$|\eta| \geq \left(\frac{1}{\tau_y^{TMC}} \right)^2 \times \frac{2\beta^2 (E/e)}{\pi f_0^2 \hat{V}_{RF} h}, \tag{23}$$

or, using Eq. (16),

$$I_b \leq \pi^2 \times f_s f_0 \beta (E/e) \tau_b^2 \times \frac{b}{\beta_y |Z_l / p|} \times \left| 1 + \frac{f_{\xi_y}}{f_r} \right|. \tag{24}$$

In the case of zero chromaticity, Zotter's result for the TMC instability of long bunches is recovered [13,14]. Indeed, the threshold intensity given by Eq. (24) for zero chromaticity is equal to the one found in Ref. [13] times $\sqrt{2}$. This numerical factor may be partially explained by the fact that Hermitian modes for Gaussian bunches are used in Ref. [13], whereas sinusoidal modes for parabolic bunches are considered here. Using Eq. (11), Eq. (24) can also be written

$$I_b \leq \pi \times B \beta (E/e) \times \frac{b}{\beta_y |Z_l / p|} \times \left| 1 + \frac{f_{\xi_y}}{f_r} \right| \times |\eta| \times \left(\frac{\Delta p}{p_0} \right)_{\max} . \tag{25}$$

The intensity threshold of Eq. (25) is equal to the one of Kernel et al. times the numerical factor $2\sqrt{6}/\pi$, considering $(\Delta p / p_0)_{\max} \approx 2 (\Delta p / p_0)_{rms}$, and using the peak impedance instead of the effective one used in Ref. [8]. Therefore, the same result is obtained by Landau damping through momentum spread for the coasting-beam approach, and by stabilisation through synchrotron oscillation for the mode-coupling formalism. This fact was already observed for the longitudinal microwave instability.

Finally, consider the following two cases where (i) $\xi_y = 0$, and (ii) $\eta = 0$. If $\xi_y = 0$, stability is obtained when

$$\left(\frac{\Delta T}{T_0} \right)_{\max} \geq \frac{\tau_b}{\tau_{y,0}^{TMC}}. \tag{26}$$

Applying this formula to the case of Refs. [5,6], where f_r = 1.5 GHz and τ_b = 2.2 ns, and using Eq. (20), one obtains

$$\left(\frac{\Delta T}{T_0} \right)_{\max} \geq 0.8 \times \frac{T_r}{\tau_y^{BBU}}. \tag{27}$$

Gareyte's conjecture is thus recovered [5]: the threshold is reached when the ΔT accumulated over an e-folding time $\tau_y^{BBU} = T_0$ is equal to the wave period T_r (here, in the numerical computation, the factor 0.8 appears). If $\eta = 0$, then the stability criterion is given by

$$\varepsilon_l \geq \frac{\beta_y^2 \left| Z_l / p \right|}{b^2} \times \frac{I_b}{2 f_0 \left| \xi_v \right|} . \tag{28}$$

Here, ε_l is the longitudinal emittance (at 2σ) in eV.s, assuming an elliptic area in the longitudinal phase space.

4 CONCLUSION

Simple formulae are derived for instabilities whose rise-time is faster than the synchrotron period, using the mode-coupling formalism. The instability rise-time for zero chromaticity is found to be equal to 4 times the time after which the amplitude of the bunch tail has been multiplied by exp(1), i.e. one *e*-folding time, computed by Brandt and Gareyte using a BBU approach. This numerical factor may be partially explained by the fact that in the mode-coupling formalism, real bunches are considered (parabolic longitudinal bunch distribution), whereas in the BBU derivation the bunch is a succession of bunchlets treated as point charges (uniform distribution). The instability rise-time for any positive chromatic frequency is higher than the one of Kernel et al. by a factor equal to $2(|m|+1)$, using the peak impedance instead of the effective one used in Ref. [8]. Here, the modes m and $m+1$ correspond to the most critical ones, interacting with the negative resistance peaked at $-f_r$. Therefore, the formulae converge to the same result only in the case where $\tau_b \approx 0.5 / f_r$, i.e. for a short bunch, since in this case the effective impedance is approximately equal to half the peak one. Note that the instability rise-time of Brandt and Gareyte is equal to the one of Kernel et al. times the factor $f_r \, \tau_b$. Therefore, their formulae converge to the same result only for short bunches.

Concerning the stabilising mechanism, the intensity threshold found by stabilisation through synchrotron oscillation is equal to the one of Kernel et al. times the numerical factor $2\sqrt{6} / \pi$, using the peak impedance instead of the effective one used in Ref. [8]. Therefore, the same result is obtained by Landau damping through momentum spread for the coasting-beam approach, and by stabilisation through synchrotron oscillation for the mode-coupling formalism. This fact was already observed for the longitudinal microwave instability. In the case of zero chromaticity, Gareyte's conjecture for stabilisation by the differential streaming of particles is recovered, as well as Zotter's TMC threshold intensity for long bunches.

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