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Graphs and Formulae for Calculating Coulomb Scattering

1. Introduction

The effects of Coulomb scattering interest both experimental physicists engaged in nuclear experiments and engineers designing particle separators, vacuum systems for secondary beams, etc. In this paper, an attempt is made to present, in a condensed form, all necessary information to enable both groups to make the more standard calculations without any other reference. To illustrate the use of the graphs and formulae given a few examples of calculations are presented in the Appendix.

2. Theories of Scattering

When a charged particle traverses a piece of matter of finite thickness it undergoes a number of small angular deflections. Each of these deflections is caused by an electromagnetic interaction between the particle and the Coulomb field of the nucleus modified by the screening effect of the electrons of the scatterer and at large angles by the form factor of the nuclei. As a consequence a beam of particles emerges from the scatterer with a certain angular distribution, an increased diameter and a larger momentum spread. For the practical purposes under discussion theory is mainly required to yield with sufficient precision :

a) The mean projected angle of scattering of the emergent particle (arithmetic mean of the projections of the angles between tangents to the particle trajectories before and after the scatterer onto a plane containing the initial trajectory).

b) The corresponding mean displacement in a plane perpendicular to the beam axis (beam blow-up).

c) The distribution function of the projected angle of scattering and of the spatial displacement.

A list of references to the treatments most commonly used is given on page 19. Most theories make simplifying assumptions. They assume in general that :

(i) The energy lost by the particle is negligible (traversing a scatterer of thickness equal to 1% of the range increases the mean spatial angle by less than 1.5 %, see however ref. (3) p. 68 and (9)).

(ii) The angle of scattering is at most about 50°.

(iii) The scatterer is infinitely wide (Øveras (9) has worked out solutions for a number of cases of practical interest in which the scatterer has limited width).

(iv) The effect of the spin and of the magnetic moment of the scattered particle can be neglected.

(v) Relativistic effects of the screening electrons (due to their appreciable speed in the c.m. system in case of energetic particle beams) may be neglected.

Depending mainly on the assumptions made for the detailed distribution of the screening electrons and on the method of integrating the individual scattering processes the resulting formulae yield more or less accurate results. Highest accuracy is obtained with the theory due to Molière (4)(5)(6). For the **scattering in a fairly thick object** the simpler formulae worked out by Rossi (3) may, however, in many cases yield satisfactory results. A recommended compromise is to correct the value of the mean projected angle obtained according to Rossi with the aid of Molière's theory as summarized on page 6.

The probability of particles being scattered by more than a given angle or lateral displacement may be calculated most accurately with the graphs and formulae due to Snyder and Scott (7)(8) reproduced on pages 13 and 16.

It seems that there exists no precise experimental verification of any of these theories, for high-energy particles. In view of this, and the limitations explained above, if an accuracy better than a few percent is required, it is suggested that the problem be studied in greater detail than is possible with the condensed information given here.

3. Formulae according to Rossi

a) Mean projected angle of scattering for negligible energy loss in scatterer.

The arithmetic mean of the absolute value of the projected angle of scattering Θ_y of a particle of charge ze given by*

$$< \Theta_y >_{[rad]} = \frac{12\ [MeV]\ z}{pc\beta\ [MeV]} \sqrt{\frac{x\,[gr/cm^2]}{X_o\,[gr/cm^2]}} \qquad (1a)^{**}$$

where

Θ_y	:	Mean projected angle of scattering in rad
$pc\beta$	:	In MeV; numerically equal to p in MeV/c times β
x	:	Thickness of scatterer in gr/cm^2 (thickness in cm x density ρ)
ρ	:	Density of scatterer in gr/cm^3 (for air and H_e, see p. 4)
X_o	:	Radiation length in gr/cm^2 (see p. 4)

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* In view of their later correction according to Molière (p. 6) Rossi's Equ. 2-16-8 and 2-16-9

$$\Theta_s^2\, x = \left(\frac{Es}{\beta cp}\right)^2 \frac{x}{X_o} \quad \text{with } E_s = 21 \text{ MeV}$$

have been transformed into (1a) by using the relation between the R.M.S. spatial angle of scattering and the mean projected angle given on p. 5 (Equ. 5).

**Equ. (1a) is a simplified version (see Ref.(3) p. 67) of

$$< \Theta_y >^2 = \frac{1}{\pi} \Theta_s^2\, x \qquad (1b)$$

with

$$\Theta_s^2 = 16\, \pi\, N \frac{Z^2}{A}\, r_e^2 \left(\frac{m_e c^2}{pc\beta}\right)^2 \ln\left[196\ (ZA)^{-1/6}\right]$$

where

Θ_s^2	:	R.M.S. spatial angle of scattering per unit thickness in gr/cm^2
π	:	3, 14159
N	:	Avogadro's number = 6,024 10^{23} molecules/gr. mole
Z	:	Atomic number of scatterer) mixtures see footnote
A	:	Atomic weight of scatterer) on page 5
$m_e c^2$	:	Rest energy of electron = 0,51079 MeV
$pc\beta$	:	In MeV; numerically equal to p in MeV/c times β
r_e	:	Radius of electron = 2,8176 10^{13} cm

Equ. (1b) is applicable as long as $pc_{[MeV]} \geq \frac{143,3}{A^{1/3}}$ which is normally the case in experiments with the CPS.
Equ. (1a) equals Equ. (1b) under the assumption that $196\left(\frac{Z}{A}\right)^{1/6} = 183$.
It may be seen that the resulting difference of $< \Theta_y >$ is less than 1 ‰, which is negligible in comparison with the other approximations of the theory.

3a) con't.

Density ρ of air and H_e at various pressures at 20° C

The values of ρ were calculated as follows :

Air : (Ref. (10) p. 2137)

$$\rho_{[gr/cm^3]} = 1.58497\ 10^{-6}\ P_{[mm\ Hg]}$$

He : (Ref. (10) p. 3188)

$$\rho_{[gr/cm^3]} = 0.2190 \times 10^{-6}\ P_{[mm\ Hg]}$$

P [mm Hg]	ρ Air [gr/cm^3]	He [gr/cm^3]
760 mm Hg	1.205 10^{-3}	1.664 10^{-4}
1 mm Hg	1.585 10^{-6}	
10^{-1} mm Hg	1.585 10^{-7}	
10^{-3} mm Hg	1.585 10^{-9}	
10^{-6} mm Hg	1.585 10^{-12}	

Radiation lengths for various scattering materials: (From ref. (11) p. 266)

Scatterer			Z	A	X_o (gr/cm^2)
H			1	1	58
H_e			2	4	85
C			6	12	42,5
Air	N 76,9 %) 0 21,8 %) A 1,3 %)	by weight	7,37	14,78	36,5
Water	H 11,1 %) 0 88,9 %)	by weight			35,9
Normal concrete		- See CPS User's Handbook U 10			27,0
Al			13	27	23,9
Fe			26	55,84	13,8
Cu			29	63,57	12,8
Baryte concrete		- See CPS User's Handbook U 10			12,7
Pb			82	207,2	5,8

b) Relation to other angles

The distribution of the projected angle of scattering θ_y and of the displacement y in a plane perpendicular to the beam are represented in Rossi's Theory by a Gaussian function. Therefore

$$2 < \theta_y^2 > = \pi < \theta_y >^2 \tag{2}$$

$$2 < y^2 > = \pi < y >^2 \tag{3}$$

Furthermore one has

$$< \theta_s^2 > = \frac{4}{\pi} < \theta_s >^2 \tag{4}$$

$$< \theta_y^2 > = \frac{1}{2} \; \theta_s^2 \; x \tag{5}$$

From $< \theta_y >$ one can therefore calculate

i) the R.M.S. spatial angle of scattering of a particle passing through a thickness of x [g/cm^2] of scatterer

$$\sqrt{\theta_s^2 \; x} = \sqrt{\pi} < \theta_y > \tag{6}$$

ii) the R.M.S. of the projected angle of scattering on a plane containing the initial trajectory

$$\sqrt{\theta_y^2} = \sqrt{\frac{\pi}{2}} < \theta_y > \tag{7}$$

c) R.M.S. value of lateral displacement

The R.M.S. value of the lateral displacement in a plane perpendicular to the beam $\sqrt{< y^2 >}$ is given by

$$\sqrt{< y^2 >} = \sqrt{\frac{\theta_s^2 (x)^3}{6}} = \sqrt{\frac{\pi}{6}} < \theta_y > x \tag{8}$$

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* (To page 3)

For a mixture of i kinds of atoms $\frac{Z^2}{A}$ becomes $\frac{\sum_i n_i Z_i^2}{\sum_i n_i A_i}$ where n_i is the proportion of nuclei of the i-th kind in one mole of mixture.

4. Molière's Theory

a) Mean projected angle

Graphs M 10 p. 5-8 represent the corrections ε to be applied to the value of the mean projected angle of scattering, calculated from Equ. (1a) p. 3 in order to obtain Molière's more accurate value *.

The values of ε are given as a function of $\frac{x}{X_o}$ with β/z, the ratio of particle speed and charge as parameter, and for scatters with Z = 1, 6, 29 and 82.

According to ref. (12) one has

$$\langle \theta_y \rangle_{\text{Molière}} = \frac{12\,[\text{MeV}]\,z}{p c \beta\,[\text{MeV}]} \sqrt{\frac{x}{X_o}}\,(1+\varepsilon) = \langle \theta_y \rangle_{\text{Rossi}}\,(1+\varepsilon) \qquad (9)$$

x : Thickness of scatterer in gr/cm^2 (ρ for air and He see page 4)

X_o : Radiation length (values p.4)

ze : Charge of particles

pcβ : In MeV; numerically equal to p in MeV/c times β

W.H. Barkas gives for ε the relation (cp. ref. 6 and 10).

$$\varepsilon = \left\{ \frac{1}{15} \sqrt{0{,}157\, X_o \frac{Z(Z+1)}{A}} \sqrt{\frac{B}{2}} \left[1 + \frac{0{,}982}{B} - \frac{0.117}{B^2} \right] \right\} - 1 \qquad (10)$$

$$\text{with B}\ldots \quad \ln B = \ln \left[\frac{6680\, Z^{1/3} (Z+1)\, X_o}{A \left(\frac{\beta}{z}\right)^2 + \left(\frac{Z}{76}\right)^2} \; \frac{x}{X_o} \right] \qquad (11)$$

and 4,5 < B < 20. (For B outside these limits the curves on graphs M 10 pages 5 to 8 have been extended in dotted lines).

b) Relations to other angles

The distribution function of θ_y underlying Molière's theory is not Gaussian. However, the general relation:

$$\langle \theta \rangle = \frac{\pi}{2} \langle \theta_y \rangle \quad \text{remains valid.}$$

$\langle \theta \rangle$: Mean spatial angle

$\langle \theta_y \rangle$: Projected angle on a plane containing the initial trajectory.

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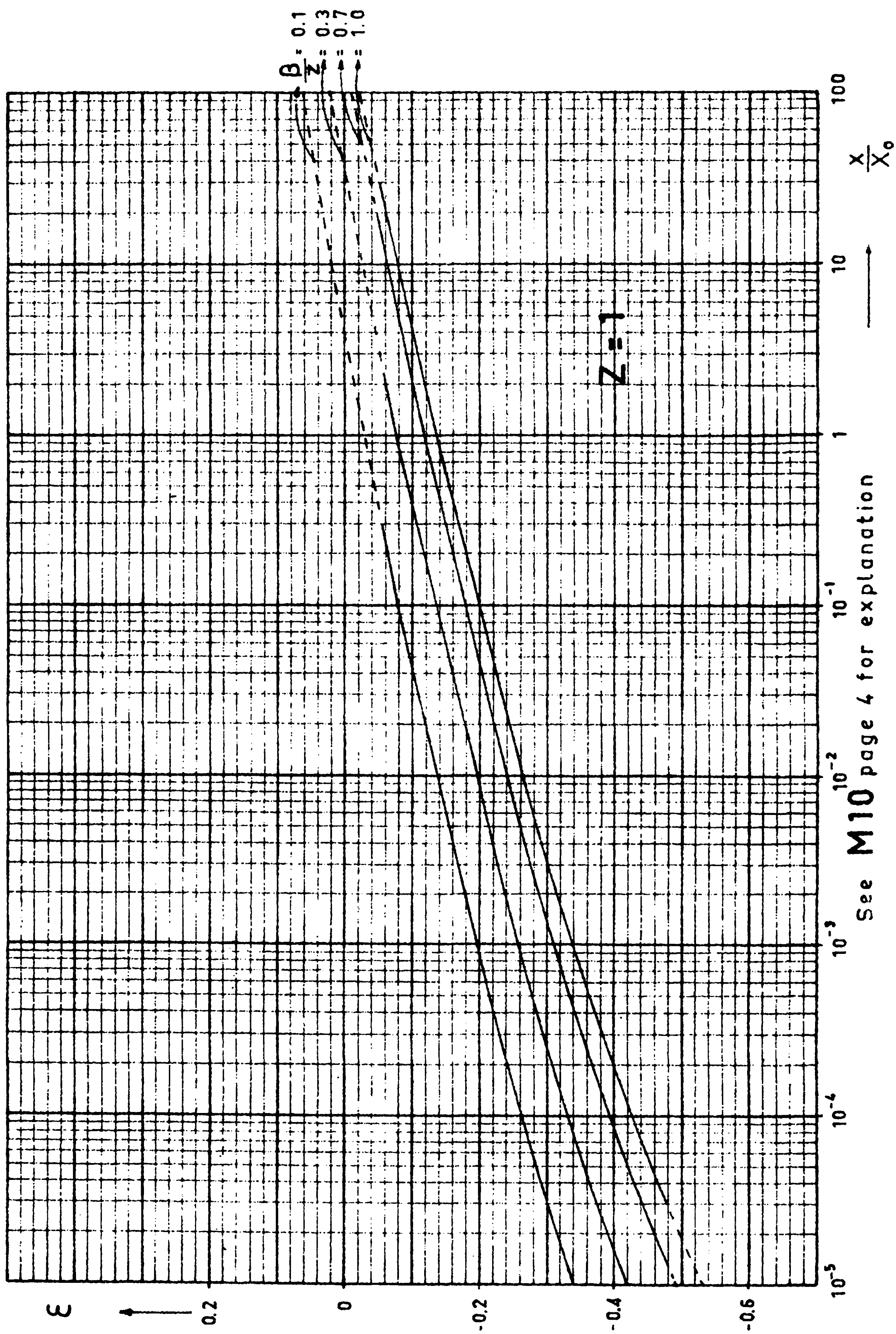
* These graphs are due to W.H. Barkas of UCRL and are reproduced here with his kind permission.



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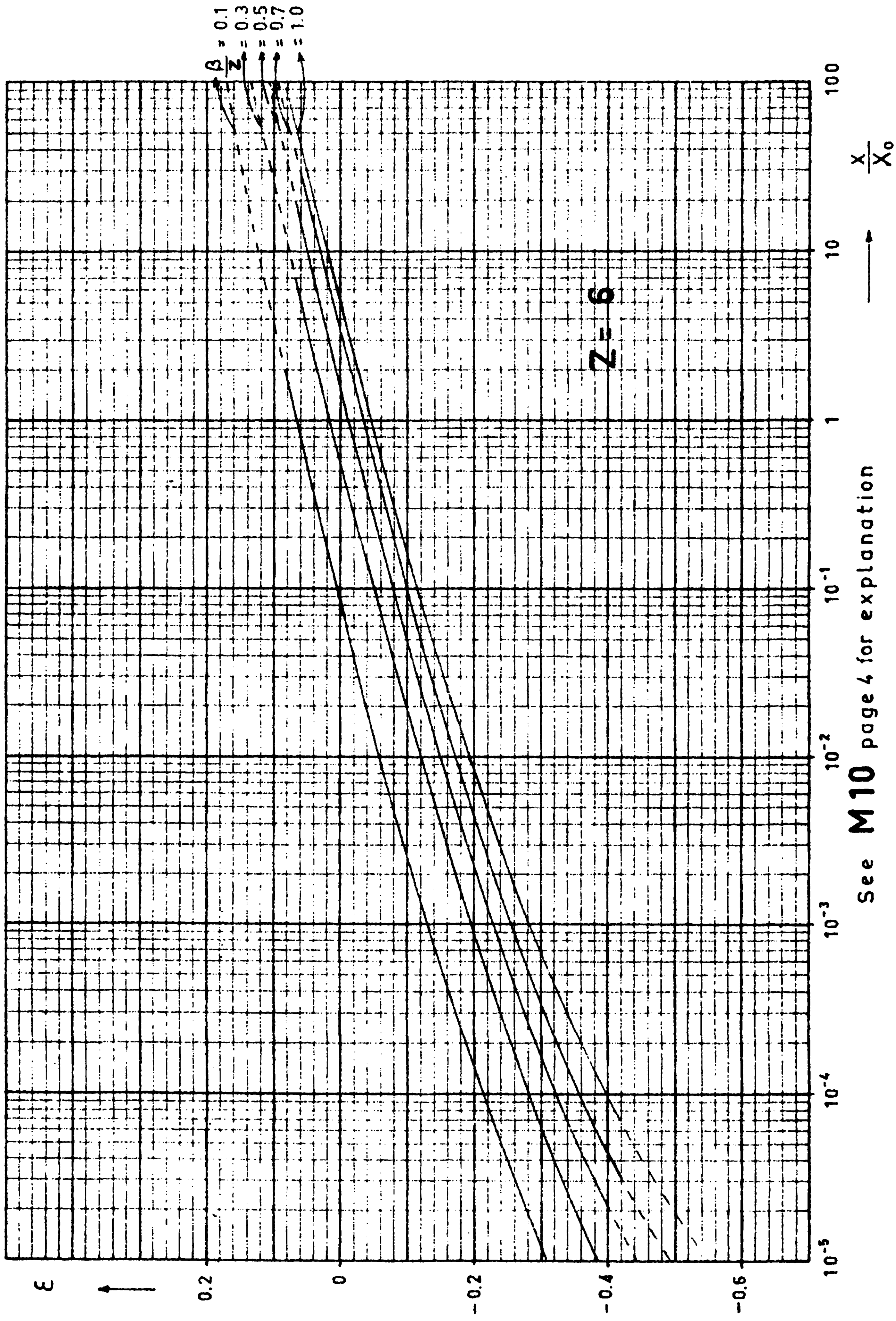
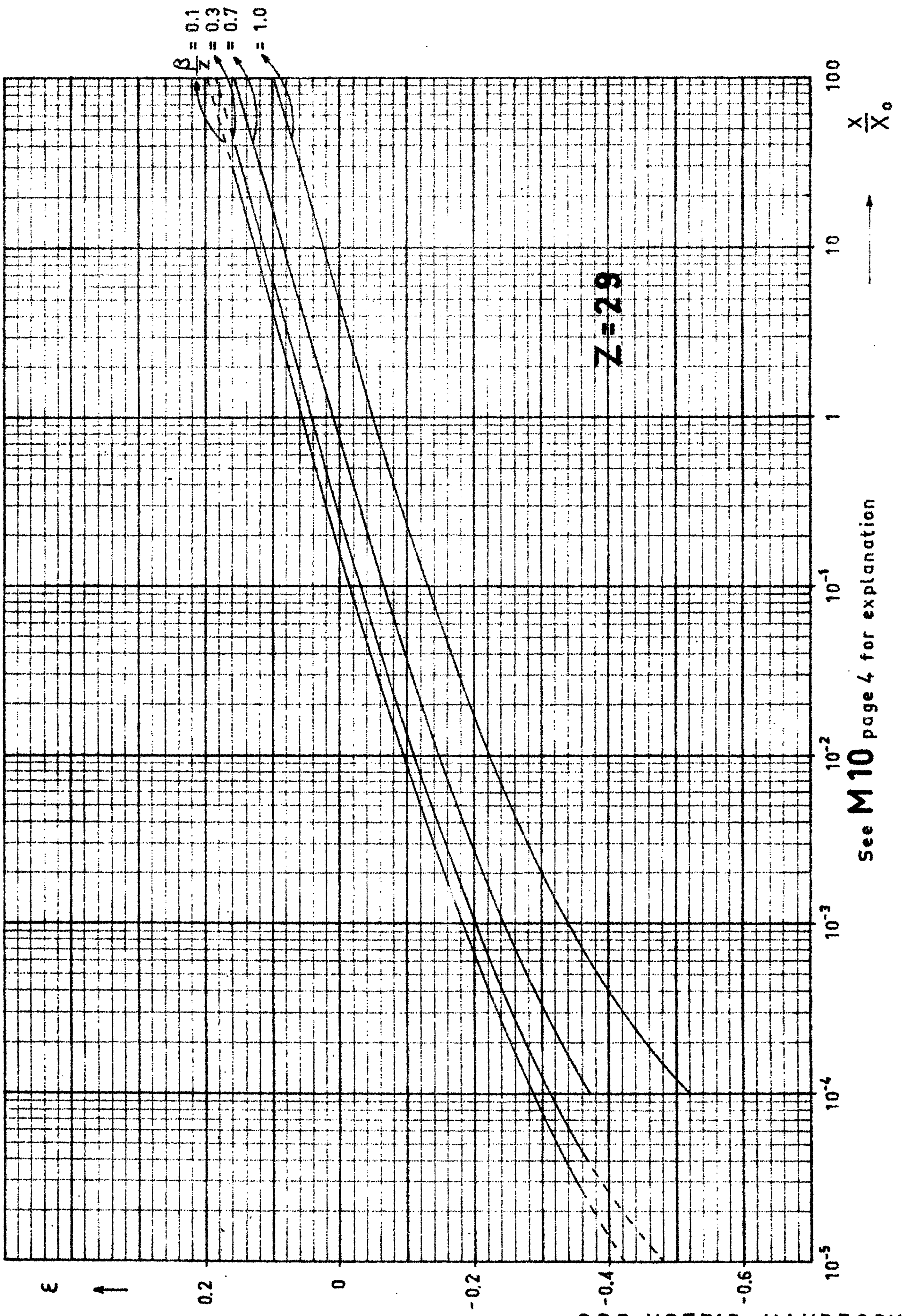


β/Z = 0.1
= 0.3
= 0.5
= 0.7
= 1.0
Z = 6
x/X₀
ε
0.2
0
-0.2
-0.4
-0.6
10⁻⁵
10⁻⁴
10⁻³
10⁻²
10⁻¹
1
10
100
See M10 page 4 for explanation



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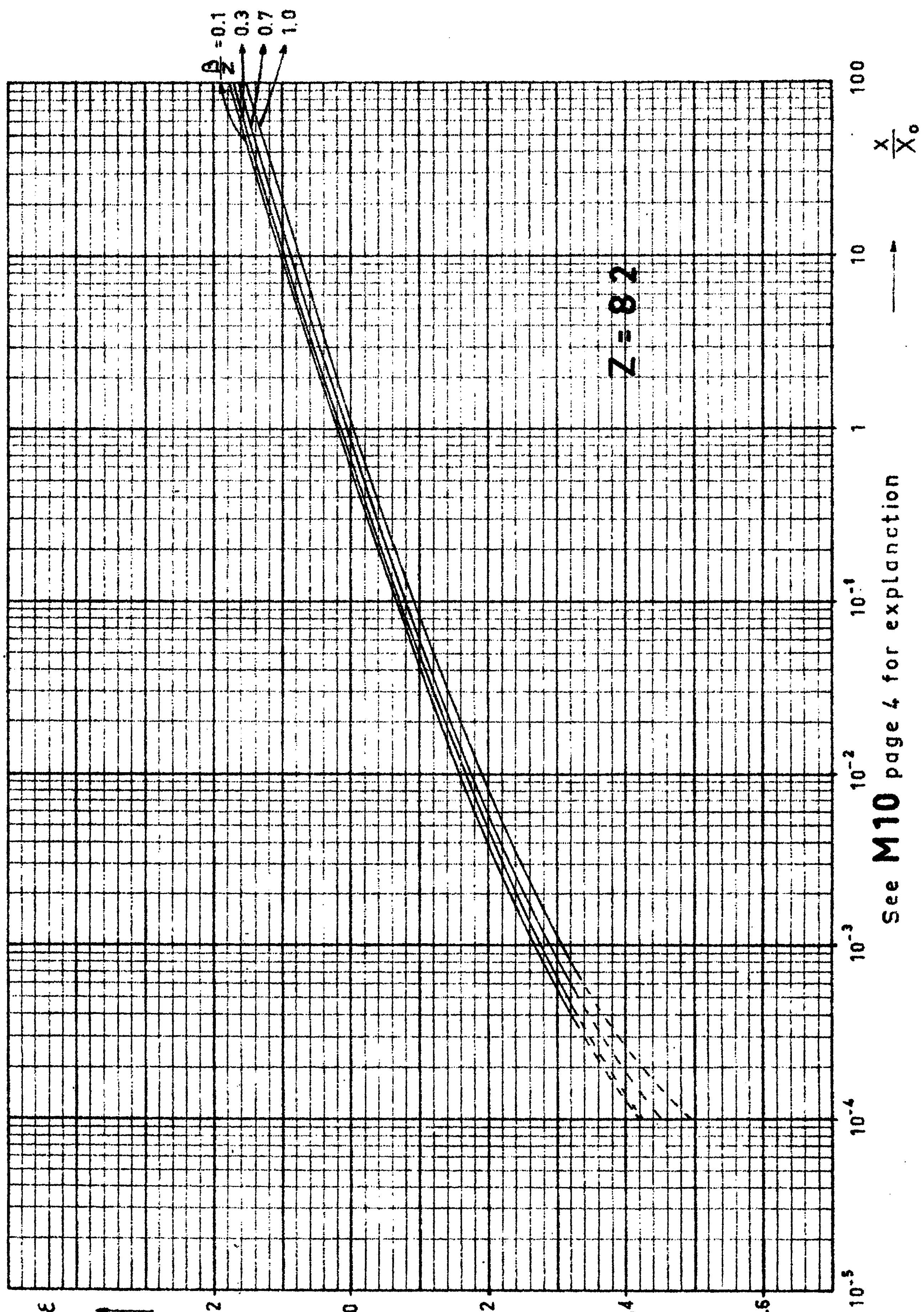

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5. Distribution Function (according to Snyder and Scott (8))

Snyder and Scott's graphs* of the distribution function are given here because i) the same graph can be used for all kinds of scatterers and particles

ii) the curves are very smooth and easy for extrapolation and interpolation

iii) the precision is more or less the same as in Molière's theory.

a) Angular deflection

Graph M 10 page 11 represents the probability of obtaining a projected angular deflection greater in absolute value than a given angle δ as a function of the particle track length z' in the scatterer.

The unit for measuring thicknesses is the "scattering length" λ [gr/om^2]

$$\frac{1}{\lambda} = \frac{4\pi N Z^{4/3} r_e^2}{\alpha^2 A \beta^2} \quad \text{(for mixtures of i kind of atoms } \frac{1}{\lambda} = \sum_i \frac{1}{\lambda_i}\text{)} \qquad (12)$$

where $\alpha = 1/137$ and the other letters have the same meaning as on p. 3.

The normalized thickness becomes then

$$z' = \frac{z}{\lambda} = \frac{z}{\beta^2} K_1 \qquad (13)$$

z : Particle track length in gr/cm^2 **

z' : Track length in terms of λ

K_1 : Constant for a given scatterer (see p. 14 for selected values)

$$K_1 = \frac{4\,(137)^2 \pi N Z^{4/3} r_e^2}{A} = 11.29\ 10^3 \frac{Z^{4/3}}{A} \qquad (14)$$

* Reproduced with kind permission of H. Snyder.

** z is approximately equal to the geometrical thickness y of the scatterer. For $y/R_o < 0.5$, one has (ref. (9)) $\frac{\rho \langle \varepsilon' \rangle}{y} < 0.32 \langle \theta^2 \rangle_{\text{Molière}}$

For 500 MeV/c protons in 50 m air at atmospheric pressure one obtains $\frac{\rho}{y} \langle \varepsilon' \rangle < 0.12\,\%$, which is negligible.

Notations:

Ref. (9)	This paper	Meaning	Units
R_o	R_o	range of the particle in the scatterer	gr/cm^2
$\ddot{\varepsilon}$	$\langle \varepsilon' \rangle$	mean increase of the track length due to multiple scattering	cm
ρ	ρ	density of scatterer	gr/cm^3
$\alpha_o\ \rho^{-1}$	$\frac{\langle \theta^2 \rangle_M}{y}$	Molière's mean square spatial scattering angle per unit thickness	rad^2 gr^{-2} cm^4
x	y	geometrical thickness of scatterer	gr/cm^2

The unit of angular measurement is η_o

$$\eta_o = \frac{m_e c^2 Z^{1/3}}{137 \; p \; c} \quad \text{(for mixtures of i kind of atoms } \frac{\eta_o^2}{\lambda} = \sum_i \frac{\eta_{oi}^2}{\lambda i} \text{)} \qquad (15)$$

$m_e c^2$: Rest mass of electron = 0.51079 MeV

pc : In MeV; same numerical value as p in MeV/c

Z : Atomic charge of scatterer

One defines : $\eta' = \frac{\eta}{\eta_o}$ (16)

η : Angular displacement in radians

η' : Angular displacement in terms of η_o

Instead of η'

$$\delta = \frac{\eta'}{\sqrt{z'}} = \frac{\eta}{\eta_o}\sqrt{\frac{\lambda}{z}} = \frac{\eta \; p \; c \; \beta}{\sqrt{z}} K_2 \qquad (17)$$

is used as parameter in the graph where K_2 is a constant for a given scatterer (see p. 14).

$$K_2 = \sqrt{\frac{A}{4\pi N}} \; \frac{1}{m_e c^2 \; Z \; r_e} = 2.53 \frac{\sqrt{A}}{Z} \qquad (18)$$

and all letters have the same meaning as before.

For $z' \begin{smallmatrix} < 100 \\ > 100\,000 \end{smallmatrix}$ probability values $\mathcal{B}$ can be found from the asymptotic formula

$$\mathcal{B} = \frac{1}{2\delta^2}\left\{1 + \frac{3}{4\delta^2}\left[\ln\delta\sqrt{z'} - 0.6340\right]\right.$$

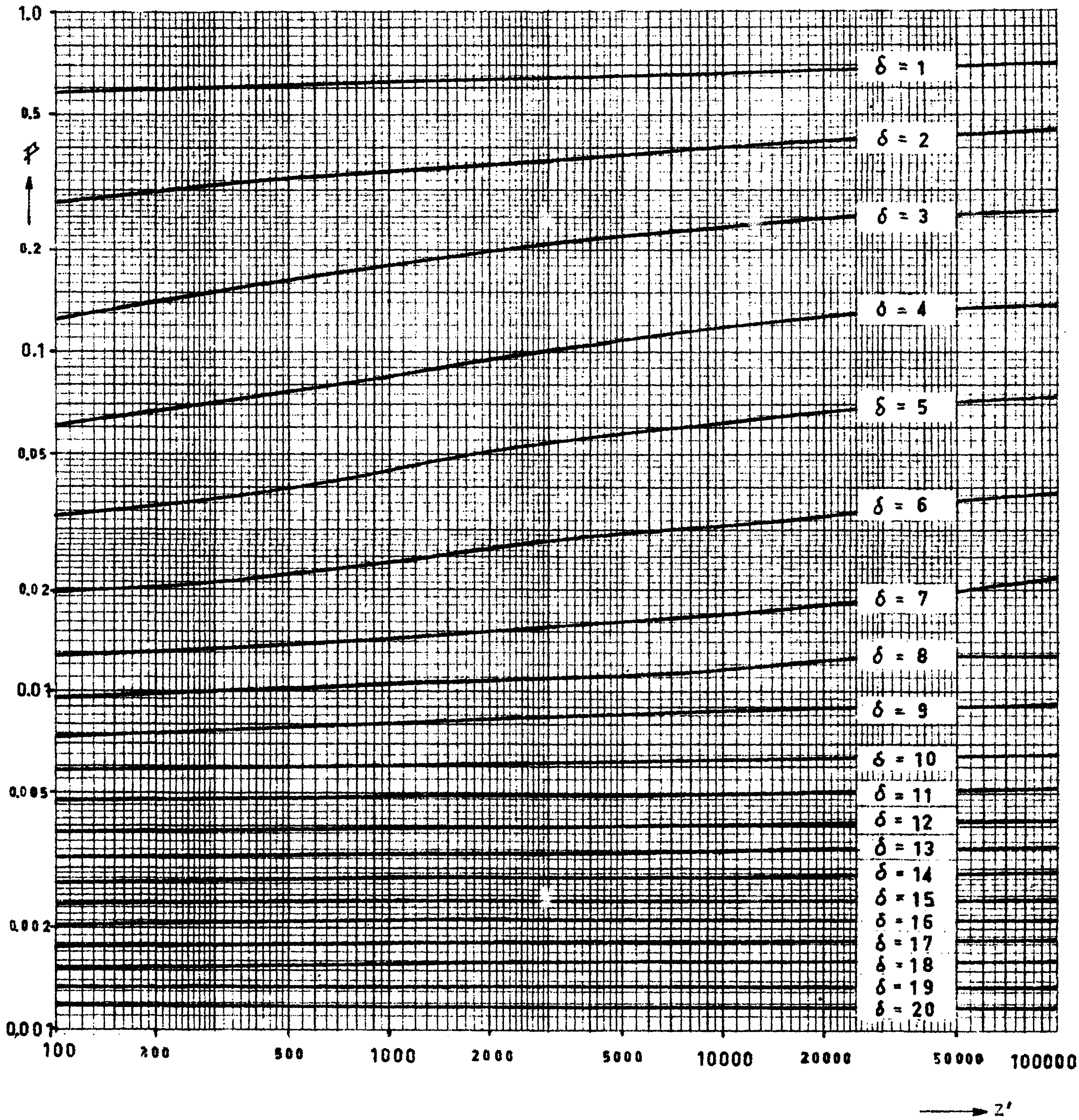
$$\left. + \frac{15}{4\delta^4}\left[\ln^2\delta\sqrt{z'} - 2.28667 \ln\delta\sqrt{z'} + 0.95952\right]\right\} \qquad (19)$$

provided $\frac{6 \, z' \, (\ln \eta' - 0.8840)}{\eta'^2} < 0{,}2$ (20)

The inequality (20) occurs for

$z' = 1$	$\delta \geq 3$
$z' = 10$	$\delta \geq 9$
$z' = 100$	$\delta \geq 11$
$z' = 100\,000$	$\delta \geq 16$
$z' = 1000000$	$\delta \geq 17$

The accuracy of Equ. (19) is about 5% to 10% (judged by comparing the values calculated from this equation with Snyder and Scott's value for z' = 100).



PROBABILITY P OF EXCEEDING AN ANGULAR DISPLACEMENT η' PLOTTED AS A FUNCTION OF THE PARTICLE TRACK LENGTH z' FOR VALUES OF δ FROM 1 TO 20.

(See M10 page 9 for explanation.)

Constants K_1 and K_2 for different scatterering materials :

Material	K_1	K_2	Z	A
H	11.19 10^3	2.54	1	1.008
He	7.10 10^3	2.53	2	4.003
C	10.24 10^3	1.46	6	12.01
Air	10.95 10^3	1.32	7.37	14.78
Al	12.78 10^3	1.01	13	27
Cu	15.82 10^3	0.69	29	63.5
Pb	19.39 10^3	0.44	82	207.2

b) Lateral displacement

M10 p.14 represents the probability of obtaining a lateral displacement in a plane perpendicular to the incident particle greater, in absolute value, than a given value x (here x is the lateral displacement in gr/cm^2 instead of the thickness of the scatterer in gr/cm^2 used by Rossi) as a function of the particle track length z' in the scatterer. The unit to measure thickness is the same as before (p. 11). The lateral displacement in a plane perpendicular to the incident particle is expressed by $\phi' = \frac{x}{z\eta_o}$

x : Lateral displacement in gr/cm^2

z : Particle track length in gr/cm^2 (see second footnote on p. 11)

η_o : As before (p. 12)

Instead of ϕ'

$$\varepsilon = \frac{\phi'}{\sqrt{z'}} = \frac{x}{z\eta_o\sqrt{\frac{z}{\lambda}}} \qquad (21)$$

is used as parameter in graph M 10 p. 14. With Equ. (17), Equ. (21) becomes

$$\varepsilon = \frac{xpc\beta}{z^{3/2}} K_2 \qquad (22)$$

where all letters have the same meaning as on p. 11 and 12.

PS/2925

For $z' \begin{smallmatrix} < 100 \\ > 100\,000 \end{smallmatrix}$ probability values β can be found from the asymptotic form

$$\beta = \frac{1}{6\varepsilon^2} \left\{ 1 + \frac{1}{\varepsilon^2} \left[\ln \varepsilon \sqrt{z'} - 0.19695 \right] \right.$$

$$\left. + \frac{5}{3\varepsilon^4} \left[\ln^2 \varepsilon \sqrt{z'} - 1.61984 \ln \varepsilon \sqrt{z'} - 0.10282 \right] \right\} , \qquad (23)$$

provided $\frac{2}{\varepsilon^2} \left[\ln \varepsilon \sqrt{z'} - 0.44691 \right] < 0.2$ (24)

The equality (24) is fulfilled for $\begin{cases} z' = 1 & \varepsilon \geqslant 1.4 \\ z' = 10 & \varepsilon \geqslant 5 \\ z' = 100 & \varepsilon \geqslant 6.5 \\ z' = 100\,000 & \varepsilon \geqslant 13 \end{cases}$

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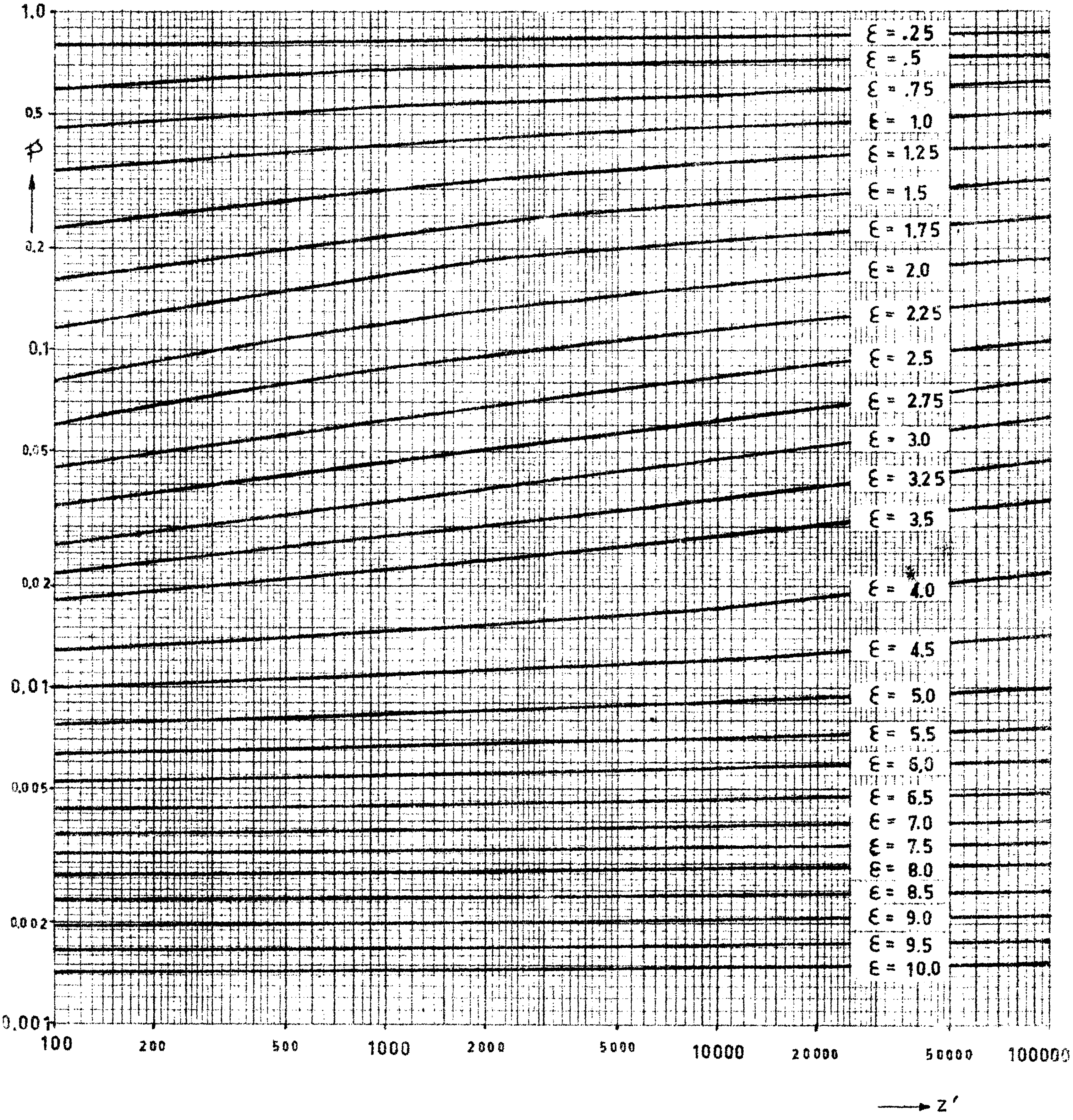

PROBABILITY $\mathcal{P}$ OF EXCEEDING A LATERAL DISPLACEMENT x PLOTTED AS A FUNCTION OF THE PARTICLE TRACK LENGTH z′ FOR VALUES OF ε FROM 0,25 TO 10.

(See **M 10** page 13 for explanation.)

EXAMPLES

1. Protons of 9 GeV/c passing through a foil of Mylar 6µ thick

a) What is the value of the probability of obtaining particles leaving the scatterer with an angle greater in absolute value than 10^{-5} rad ?

It is assumed that Mylar is carbon of density 1.1 gr/cm^3.

From Equ. (13) and (17) (Snyder and Scott)

$$z' = 6.10^{-4} \times 1.1 \times 10.24\ 10^3 = 6.75$$

$$\delta = \frac{10^{-5}}{\sqrt{6\ 10^{-4} \times 1.1}} \times 9\ 10^3 \times 1.46 = 5.13$$

One finds from extrapolation of the graph on page 13 that the probability is less than 3 ‰.

An upper limit for the increase of track length is, in this case, according to the relations of the footnote on page 11

$$\frac{\rho(\varepsilon')}{y} < 0.32 \langle \theta^2 \rangle_M \quad \text{and} \langle \theta^2 \rangle_M \fallingdotseq \frac{4}{\pi} \langle \theta \rangle_M^{2*} = \frac{4}{\pi}\frac{\pi^2}{4} \langle \theta_y \rangle^2 \quad \text{(see 4b) page 6)}$$

$$\frac{\rho(\varepsilon')}{y} < 0.32 \quad \pi\ (2.50)^2\ 10^{-12} \qquad = 6.3\ 10^{-12} \qquad \text{(see below for } \theta_y \text{)}$$

Thus the substitution of the thickness y for the track length x is entirely justified.

b) The mean projected angle of scattering

From Equ. (1a)

$$\langle \theta_y \rangle_R = \frac{12}{9\ 10^3} \sqrt{\frac{6.6\ 10^{-4}}{42.5}} = 5.2 \quad 10^{-6} \text{ rad,}$$

if Rossi's theory is to be used.

For applying Molière's correction one finds from graph M 10 p

$$\varepsilon = -0.52.$$

Hence $\langle \theta_y \rangle_M = (1 - 0.52) \langle \theta_y \rangle_R = 2.50\ 10^{-6}$ rad.

The energy loss may be neglected (see p. 2), $\frac{z}{R_o}$ being $\frac{6.6\ 10^{-4}}{4.3\ 10^3} < 10^{-2}$

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* $(\theta^2)_M = \frac{4}{\pi} (\theta)_M^2$ is true if θ_s has a Gaussian distribution, which we may assume in first approximation

c) The probability of obtaining particles coming out with an angular deflection greater than 2.10^{-5} rad ?

The answer may be found from Equ. (19) because (see Equ. 20)

$$z' = 6.75$$
$$\delta = 2 \times 5.13 = 10.26$$

One obtains

$$\beta = \frac{1}{2(10.26)^2}\left\{1 + \frac{3}{4(10.26)^2}\left[\ln 10.26\sqrt{6.75} - 0.6340\right] + \frac{15}{4(10.26)^4}\left[\ln^2 10.26\sqrt{6.75} - 2.28667 \ln 10.26\sqrt{6.75} + 0.95952\right]\right\}$$
$$= 4.85\ 10^{-3}$$

This is in good agreement with the result of extrapolation of the graph p. 13.

2. <u>K mesons of 1 GeV/c passing through 30 m of</u>

a) air at atmospheric pressure

b) air at 10^{-1} mm Hg pressure

c) He at atmospheric pressure

What is the probability of obtaining particles with a lateral displacement greater than 5 cm ?

From Equ. (13) and (21)

a)
$$\begin{cases} z' = \dfrac{1.205\ 10^{-3} \times 3000 \times 10.95\ 10^{3}}{0.8836} = 44\ 800 \\[2ex] \varepsilon = \dfrac{1.205\ 10^{-3} \times 10^{3} \times 5 \times 0.94 \times 1.32}{(1.205\ 10^{-3} \times 3000)^{3/2}} = 1.09 \end{cases}$$

From the graph on page 16 one finds that the probability is 44%.

b)
$$\begin{cases} z' = \dfrac{1.585\ 10^{-7} \times 3000 \times 10.95\ 10^{3}}{0.8836} = 5.9 \\[2ex] \varepsilon = \dfrac{1.585\ 10^{-7} \times 5\ 10^{3} \times 0.94 \times 1.32}{(1.585\ 10^{-7} \times 3000)^{3/2}} = 95.0 \end{cases}$$

From the graph on page 16 one sees that the probability is certainly less than 0.1%, which is negligible.

c)
$$\begin{cases} z' = \dfrac{1.664\ 10^{-4} \times 3000 \times 7.1\ 10^{3}}{0.8836} = 4000 \\[2ex] \varepsilon = \dfrac{1.664\ 10^{-4} \times 5 \times 10^{3} \times 0.94 \times 2.53}{(1.664\ 10^{-4} \times 3000)^{3/2}} = 5.61 \end{cases}$$

The probability is in this case 0.6 %.

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mean projected angle of scattering $\langle \theta_y \rangle$

mean spatial displacement in a plane perpendicular to the beam $\langle y \rangle$

distribution function of projected angle of scattering

distribution function of spatial displacement in a plane perpendicular to the beam; these two distributions are Gaussian).

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increase of the track length)

10. Handbook of Chemistry and Physics, forty-first ed. (1960)

11. Segre - Experimental Nuclear Physics - Volume I p. 287

12. UCRL 8030

SYMBOLS

A	:	Atomic weight of scatterer	[——]
α	:	$\frac{1}{137}$	
β	:	$\frac{v}{c}$ $\frac{\text{velocity of the particle}}{\text{velocity of light}}$	[——]
c	:	Velocity of light	[cm/sec]
δ	:	$\frac{n'}{\sqrt{z'}}$ (see page 12)	
ε	:	Correction to Rossi's theory to obtain Molière value	
ε	:	$\frac{\phi}{\sqrt{z'}}$ (see page 14)	[——]
$\langle \varepsilon' \rangle$	:	Mean increase of the track length of the particle in the scatterer	[cm]
ϕ'	:	$\frac{x}{z\eta_0}$ (see page 14)	[——]

Here

x	: Lateral displacement of the particle after passing through the scatterer	[gr/cm^2]
z	: Track length of the particle in the scatterer; (in practice equal to the thickness of the scatterer as shown on p. 11)	[gr/cm^2]
K_1	: Constant for a given scatterer (see p. 14)	
K_2	: Constant for a given scatterer (see p. 14)	
λ	: Scattering length (see p. 11)	[gr/cm^2]
$m_e c^2$	: Rest energy of electron = 0,51079	[MeV]
η_o	: Unit of angular measurement in Snyder and Scott's theory	[rad]
η	: Angular displacement of the particle after passing through the scatterer	[rad]
η'	: Same angular displacement in terms of η_o	[------]
π	: 3,14159	
p	: Particle momentum	[MeV/c]
P	: Pressure of gaseous scatterer	[atm or mm Hg]
ρ	: Density of scatterer	[gr/cm^3]
R_o	: Range of the particle in the given scatterer (see for instance UCRL 2726)	[gr/cm^2]
r_e	: Classical radius of electron = 2,8176 10^{-13}	[cm]
$\Theta_s^2 \equiv \langle \Theta_s^2 \rangle$	: Mean square spatial angle of scattering per unit thickness of scatterer	[rad^{2}gr^{-2}cm^4]
$\langle \Theta_s^2 \rangle_R$	: Rossi's value, $\langle \Theta^2 \rangle_M / x$: Molière's value (here: x = thickness of scatterer)	
$\langle \Theta_y \rangle$	: Mean projected angle of scattering	[rad]
$\langle \Theta_y \rangle_M$	: Molière's value	
$\langle \Theta_y \rangle_R$	: Rossi's value	

x	:	Thickness of scatterer (Rossi's theory)	$[gr/cm^2]$
x	:	Lateral displacement (SS's theory)	$[gr/cm^2]$
X_o	:	Radiation length	$[gr/cm^2]$
$y = \langle y \rangle$	:	Mean lateral displacement (Rossi's theory)	$[gr/cm^2]$
z	:	Particle track length in the scatterer (in most cases equal to the thickness of the scatterer; see p. 11)	$[gr/cm^2]$
z'	:	Particle track length in terms of λ ($z' = \frac{z}{\lambda}$)	[———]
ze	:	Charge of scattered particle (z times the electron charge)	