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ON THE BEAM LOSSES IN MAGNETIC SPLITTER DEVICES.

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Abstract

The losses in the magnetic devices used at CERN-PS to split the ejected beams of protons (East Hall) and antiprotons (LEAR experimental area) are calculated by integration of the beam distribution in the transverse phase space over the acceptance areas. Analytical formulae for a rectangular distribution are compared with numerical evaluations of the integrals using Gaussian beam distribution. The programs developed to perform the numerical integration are described in the appendix.

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Contents

1. Introduction

The use of magnetic devices for sharing a beam of particles in two or more parts implies the introduction of some kind of septum in the beam causing part of the beam to be lost. In this note we investigate the influence of the beam parameters on these losses in splitter magnets of the types used at LEAR and East Hall areas at CERN.

The method employed is a comparison of the beam distribution with the acceptance of the magnets as represented in the phase space. After a short description of the magnets and method some simple formulae are deduced assuming a uniform rectangular particle distribution in the vertical phase plane. To get quantitatively valid results a more realistic beam distribution had to be used forcing us to develop a computer program which performs the integrations numerically. The program was further refined to take into account the emittance and acceptance in the horizontal plane as well.

2. Basic Considerations

The distribution of particles in a beam at a position z along the the beam line can be written as $B(x,x',y,y',p)$, where x,y are the displacement and $x' = dx/dz$, $y' = dy/dz$ the direction of particles relative to the reference beam trajectory. p represents the other possible parameters not taken into account here. The acceptance $A(x,x',y,y',z_1,z_2)$ can be represented in the same space of coordinates. $A = 0$ for $x...y'$ such that the particle is lost between z_1 and z_2 ,and $A = 1$ if the particle is transmitted. If B is normalized the total transmitted beam is given by $\int A \cdot B dV$ over the phase space V. In a phase plane this becomes the integral of the beam distribution over the acceptance areas.

Before applying this to the splitter magnets we give a short description of them and their way of functioning. The cross−sections are shown in figure 1a for the LEAR type and in figure 1b for the EAST HALL type.

Figure 2 shows schematically the principle of operation of splitter magnets and gives the definition of the coordinate system used below.

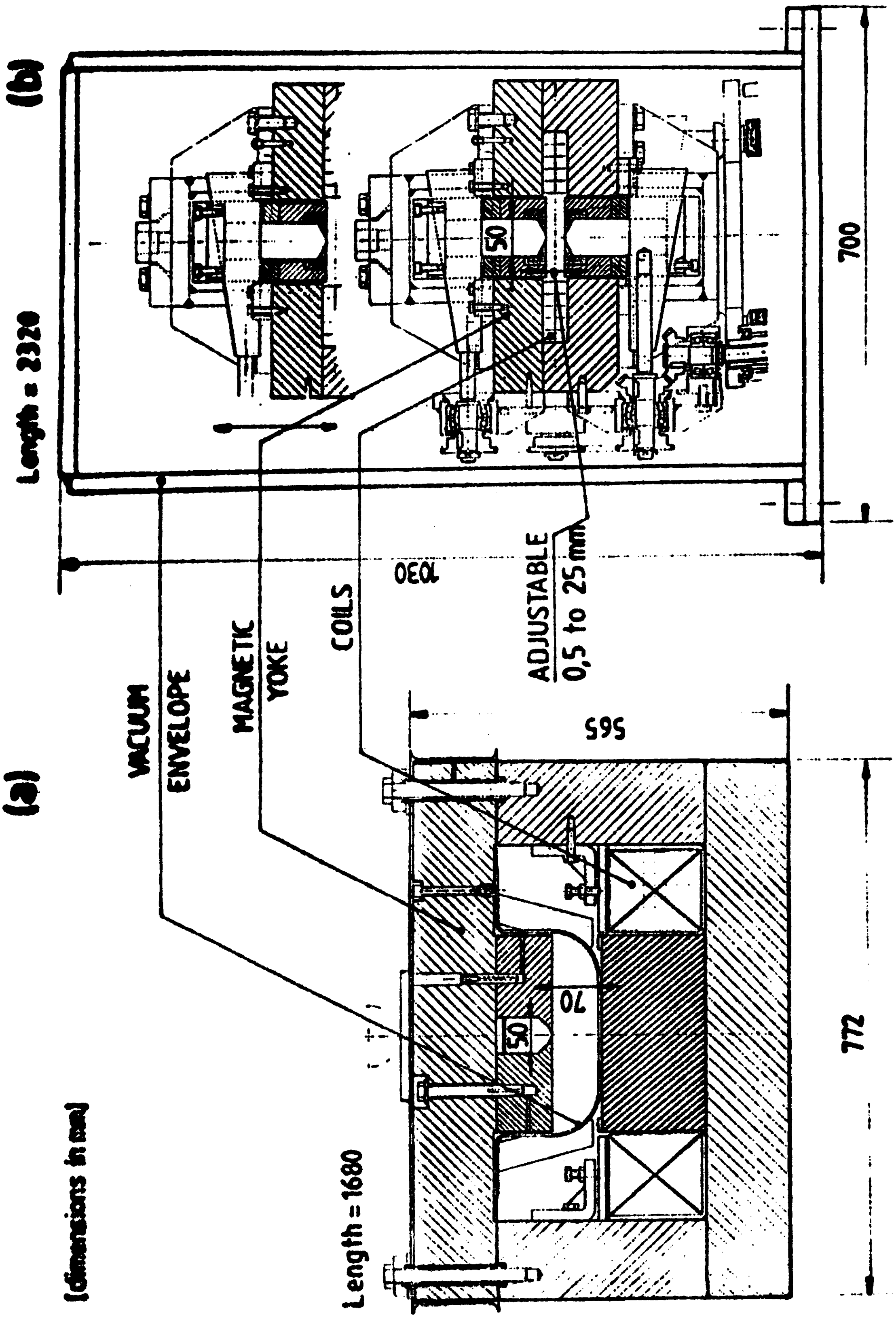

Figure 1: Cross – sections of splitter magnets.
a) The LEAR type splitter magnet. b) The East Hall type.

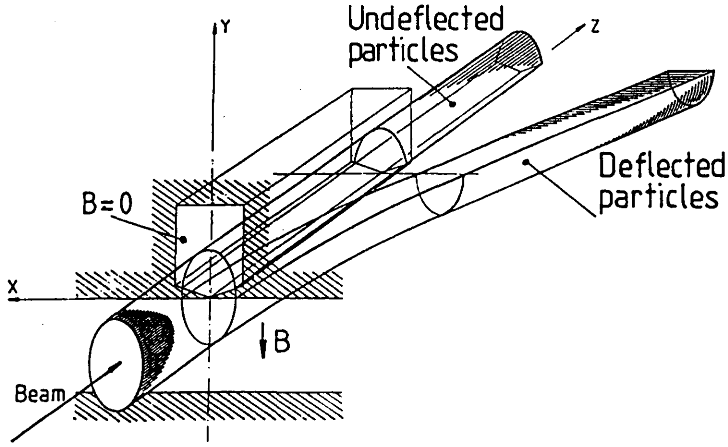

Figure 2: Principle of operation. Schematic representation of beam – sharing in a splitter magnet.

The hole in the upper yoke is practically field – free [1], hence the part of the beam entering it passes unaffected (apart from fringing field effects which are negligible). The part entering in the pole gap is deflected as in a deflecting magnet. As can be seen a minor part of the beam hits the thin "knife" of the upper pole separating the field – free region from the full field region. The part lost on the face of the knife will be referred to as **large angle losses (P_1)**. Due to the non – zero convergence of the beam also some particles are lost on the surfaces of the knife, above and below. These losses will be referred to as **small angle losses (P_2)**. The coordinate system defined in this figure will be used throughout the paper unless otherwise stated.

In figure 3 a cross – section of a LEAR magnet is given with a typical beam extension indicated. The appearence of the beam on the multi – wire proportional chamber downstream the magnet with zero and a small magnetic field is also shown.

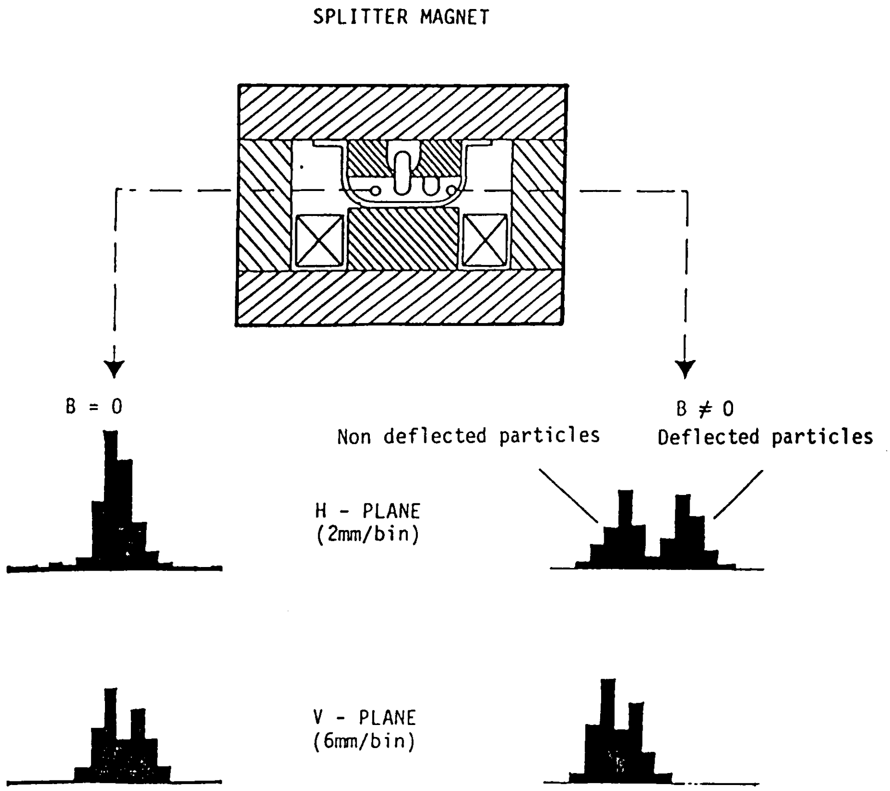

Figure 3: Example of beam splitting in the Lear experimental area.
MWPC's displays of the beam profiles downstream the first splitter magnet using 50 MeV protons are shown.

3. Uniform Rectangular Beam Distribution in the Vertical Plane.

To get a fundamental understanding we first deduce some formulae using the simplest possible distribution and the vertical acceptance of a LEAR splitter magnet (cf reference [2]). In figure 4 the vertical phase plane together with a uniformly distributed beam are shown.

The sides of the beam rectangle are given by the height Δy and divergence $\Delta y'$ of the beam. The vertical emittance is then $E_V = \Delta y \cdot \Delta y'$ and the centre is at δy, $\delta y'$. A non zero δy gives a sharing ratio different from 1, whereas $\delta y'$ defines the vertical tilt of the beam. A convergent/divergent beam is described by a tilted parallelogram. Here we assume the projection on the y − axis constant and the symmetry axis given as $y = 0$, $y' = k \cdot y$, meaning convergent beam if $k < 0$. The horizontal phase space is

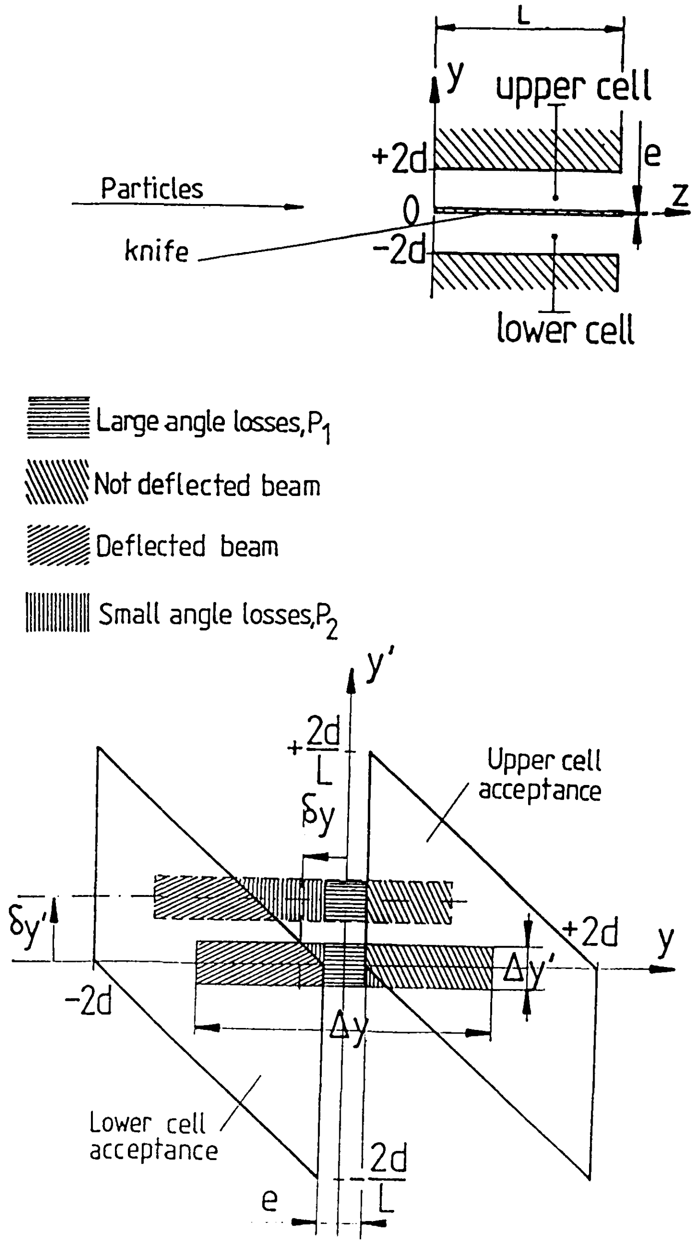

Figure 4: Vertical phase plane at the entrance of a splitter magnet.
The insertion shows schematically a vertical cut along the central line of a splitter magnet.

taken into account by defining an effective thickness e_{eff} of the knife. The formulae assume that e/L is small compared to 1.

1. $\delta y = 0$, $\delta y' = 0$, $k = 0$

$$P_{1min} = P_1 = e \cdot \Delta y' / E_V$$

$$P_{2min} = P_2 = (L/4) \cdot (\Delta y')^2 / E_V$$

2. $\delta y \neq 0,\ \delta y' = 0,\ k = 0$

P_1 and P_2 are the same as in case 1 when $|\delta y| < \Delta y/2 + L \cdot \Delta y'/2$

3. $\delta y = 0,\ \delta y' \neq 0,\ k = 0$

$P_1 = P_{1_{min}}$ of case 1

$P_2/P_{2_{min}} = 1 + 4 \cdot (\delta y'/\Delta y')^2$ if $|\delta y'| \leq \Delta y'/2$

$P_2/P_{2_{min}} = 4 \cdot \delta y'/\Delta y'$ if $|\delta y'| \geq \Delta y'/2$

4. $\delta y = 0,\ \delta y' = 0,\ k \neq 0$

$P_1 = P_{1_{min}}$

$P_2 = (\Delta y')^2/(4E_V(1 + kL))$.

The qualitative information to be gained from this approach is:

1. For a given splitter and beam the losses will only depend on the beam divergence which should be as small as possible to minimize the losses

2. The losses do not depend on the sharing ratio in this coarse model.

3. A vertically tilted beam strongly increases the P_2 − losses.

4. The influence of convergence/divergence is small but such that the losses are smaller with a divergent beam.

To get quantitative results the values of the parameters coming from the first splitter in the LEAR experimental area can be inserted; $L = 1.68$ m, $\Delta y = 45$ mm, $E_V = 20\pi$ mm·mrad, leading to $\Delta y' = 1.4$ mrad. The thickness e can be estimated to be 1.58 mm. All are ending at $P_1 = 3.52$ % and $P_{2_{min}} = 1.40$ %. The dependence of a vertically tilted beam on the small angle losses calculated with the formula of point 3, is shown in figure 5. The losses can be accepted if the tilt is less than ± 0.5 mrad which is quite a severe constraint.

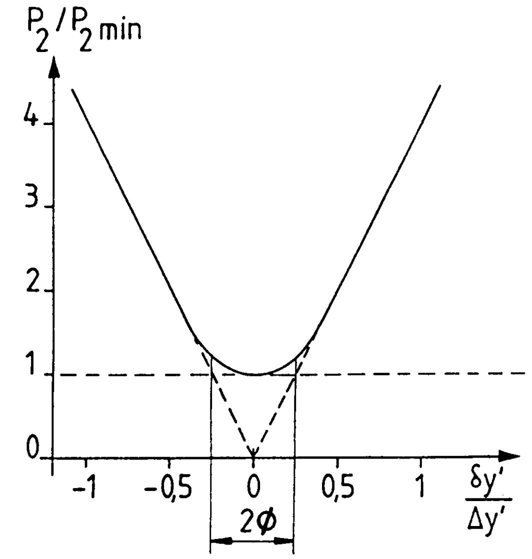

Figure 5: $P_2/P_{2min} = f(\delta y')$.
The relative small angle losses as function of the vertical tilt angle. The acceptable tilt is $\pm\phi$.

To illustrate the influence of a convergent beam a comprehensive measure of the convergence must be introduced (see below). This is not straightforward using a rectangular distribution. A realistic variation, however, will change the losses by less than 10 % of P_{2min}; the divergent beam giving the smallest loss.

4. Gaussian Beam Distribution

A more accurate calculation of the losses must use a better approximation of the distribution of particles in the phase space. Much closer to reality is the Gaussian distribution

$$f_1 = (\sqrt{2/\pi})\cdot(1/s_t)\cdot e^{-2((t-\delta t)/s_t)^2} .$$

where $s_t = 2\sigma_t$.

In the two dimensions of the vertical phase plane (y,y′) this becomes

$$f_2(y,y') = f_1(y)\cdot f_1(y')$$

or in the four dimensions of the x−y phase space (x,x′,y,y′)

$$f_4(x,x',y,y') = f_2(x,x')\cdot f_2(y,y').$$

Following the same ideas as in the previous chapter integrals like $P=\int f_4 dV$ should be integrated over different volumes. In the general case the integral cannot be separated due to couplings between the variables on the boundaries of the integration volume. This is also the case for the actual splitter magnets even if only the vertical phase plane is considered. To get solutions in all geometrical cases we decided to make the integrations numerically by computer. As can easily be seen the calculation time increases as the product of the number of steps in each coordinate. The computation time becomes really long when the integration is performed in all four coordinates. Therefore most of the calculations are made in the vertical phase plane accounting for the horizontal one by introducing an effective thickness of the knife.

5. The Computer Program for Two – Dimensional Calculations

To reduce the integration to two dimensions the influence of the horizontal beam distribution can be normalized to an effective thickness of the knife. As the beam is horizontally focused in the center of the magnet only the upstream face of the knife contributes to the losses in the horizontal plane if the beam is centered on the magnetic axis. We will assume this to be true also for small tilts. Therefore the physical extension of the knife weighted with the horizontal beam distribution at the entrance of the magnet, not including the convergence, could be taken as the effective thickness:

$$e_{eff} = \int_{-\infty}^{\infty} f_1(x)\cdot e(x)dx.$$

The central parts of the two types of splitter magnets are shown in figure 6.

Taking only the central part into account one gets $e(x) = e_0 + \tan(\gamma)\cdot|x|$ and the integral becomes $e_{eff} = e_0 + \tan(\gamma)\cdot s_x/\sqrt{2\pi}$ in the case of a centered beam. However, this can also be evaluated numerically by the program so that the horizontal beam parameters can be varied easily.

The program integrates the beam distribution f_2 over five areas in the y,y′–plane ; the full beam and four others for which the boundaries have to be given. The program steps in the coordinate system (ψ,ψ') defined by the main axis of the beam ellipse. The step length is chosen to be

$$dt = t_{min} + |t|/t_{div} \quad , \quad t = \psi, \psi'.$$

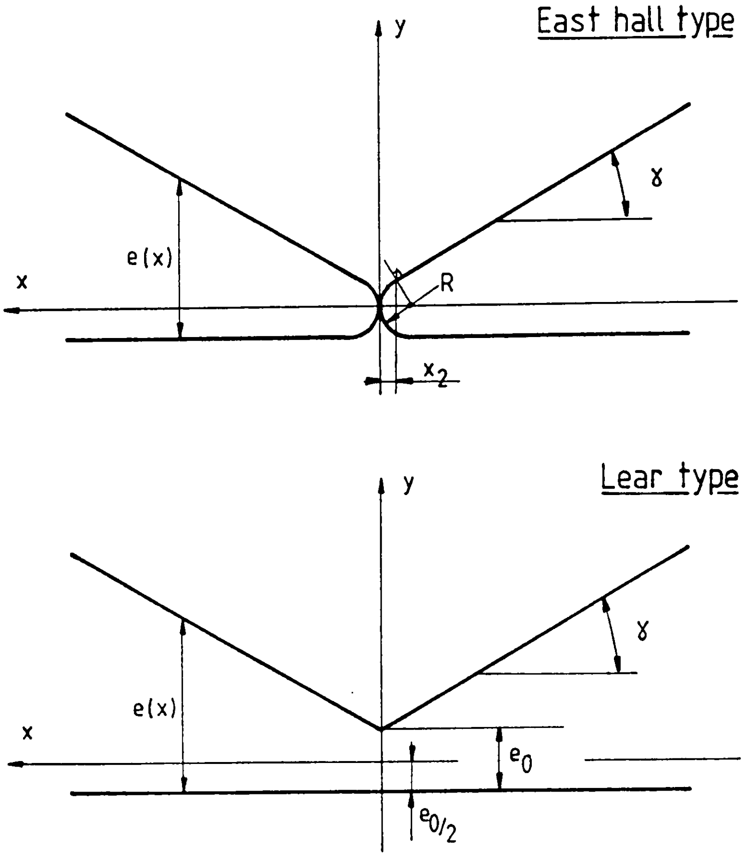

Figure 6: Pole knife shapes.
The shape of the pole knifes with coordinate system is indicated as well as the parameters used in the effective thickness calculations (see appendix).

where t_{min} and t_{div} are constants to be given to minimize the number of steps for a given accuracy. The beam is defined by s_ψ, $s_{\psi'}$, δy, $\delta y'$ and α. α being the rotation angle of the beam ellipse (ψ,ψ') as shown in figure 7.

These parameters are calculated from the input which should be E, $\sqrt{\sigma_{11}}$, $\sqrt{\sigma_{22}}$, r. $r = \sigma_{21}/\sqrt{\sigma_{11}\cdot\sigma_{22}}$. The notation follows the TRANSPORT manual [3]. From the ψ,ψ' – system the y,y' – coordinates are calculated as:

$$y = \delta y + \psi\cos\alpha - \psi'\sin\alpha$$

$$y' = \delta y' + \psi\sin\alpha + \psi'\cos\alpha.$$

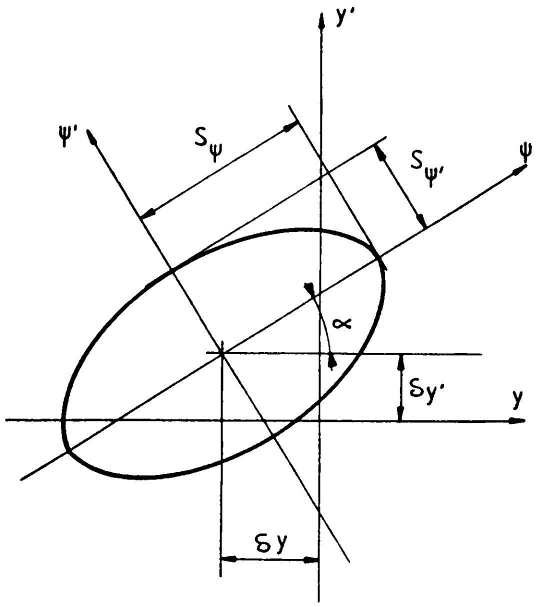

Figure 7: Coordinate systems.
The coordinates y,y′ in the vertical phase plane are defined by the reference beam and the magnet (x,y). ψ,ψ' is defined by the axis of the beam ellipse under consideration.

If this falls within a given region the corresponding sum is increased by $f_2(\psi,\psi')\cdot d\psi\cdot d\psi'$. The full area of integration is $|\psi| < 3s_\psi$, $|\psi'| < 3s_{\psi'}$.

The input and output are made to easily transform from/to practically observed quantities. As an example the appearence of the beam on a Multi−Wire Proportional Chamber (MWPC) at different positions along the beam line is plotted (see below). The details of the program is described in the appendix.

6. Results of Two − Dimensional Calculations

6.1 Splitter magnets of the LEAR type

The mechanical dimensions of a LEAR splitter magnet are shown in figure 1. Two splitters of this type are used to split the antiproton beam of .1 − 2 GeV/c in three parts. The angles of deflection are 120 mrad.

The calculations are made using the beam parameters in the following table as being the optimal beam at the center of the splitter magnet. By optimal we mean a beam with horizontal focus at the center of the splitter magnet and vertically minimally divergent, "parallel".

	splitter 1	splitter 2	
E_H	5	5	π·mm·mrad
E_V	20	10	π·mm·mrad
s_x	0.84	1.76	mm
$s_{x'}$	5.97	2.85	mrad
s_y	22.5	36.7	mm
$s_{y'}$	0.89	0.34	mrad

The horizontal beam size (half value) at the entrance of the splitter 1 (2) becomes 5.07 (3.77) mm from which the effective thickness of the pole knife is calculated as described above. The result is e_{eff} = 1.89 (1.69) mm taking only the central part, $|x| < 10$ mm, into account.

The influence on the lossses as the beam parameters varied has been investigated in the case of splitter 1. The magnitudes of the variations are set to about 3 times the value which produces an easily detectable change on the existing beam monitoring facilities, MWPC 3 and 4 , positioned 0.4 m upstream, respectively 3.1 m downstream the entrance of the splitter. The results are presented using the appearence on these monitors. The vertical tilt is given as the centroid shift and the convergence/divergence as the difference in size between MWPC 3 and 4.

The errors in the calculations are coming from the finite step length and from the addition of many numbers. The latter can be neglected as the program uses double precision variables. The first one is difficult to deduce accurately. It consists of two parts, one from the approximation of the function, the other from the step length at the borders of integration. The error from approximation of the function is small as can be seen from the full range integration. A maximum value of the "border" error is $dy \cdot f_1$, both taken at $y = e/2$ which evaluated for the optimum beam becomes .09 %. Runs with different step lengths indicate that the over-all error is several times smaller. The error in $P_1 + P_2$ is of the order of the full integration error, ≈ 0.01 %.

A summary of the calculations is presented in figures 8 – 14.

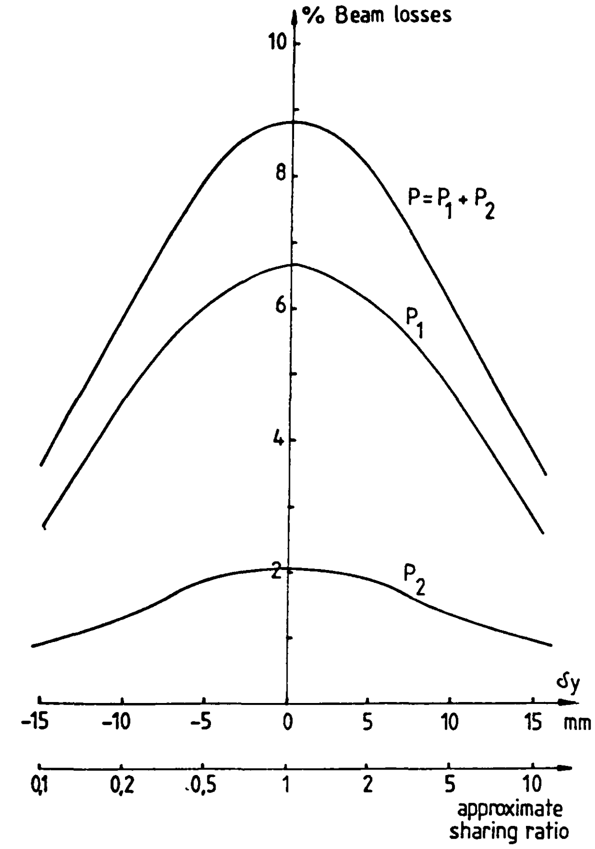

Figure 8: P = f(δy).
Losses in the first LEAR splitter magnet as function of sharing ratio.

Firstly in figure 8 the variation with sharing ratio ($\delta y \neq 0$) is shown. The optimum beam conditions as given above are assumed but the beam is vertically displaced by varying δy from -15 to 15 mm corresponding to a change in sharing ratio of about 0.1 to 10. The Gaussian distribution is directly transferred to P_1 but also P_2 is biggest at the ratio 1. **The maximum losses** are $P_1 = 6.67$ % , $P_2 = 2.09$ % and $P = P_1 + P_2 = 8.76$ %. The losses on the upper or lower pole tip are not included in these values but they enter in the sharing ratio as the transmission decreases.

The P_2 – losses of a beam not vertically coaxial with the magnet is shown in figure 9. The angle of tilt $\delta y'$ is varied from -1.5 to 1.5 mrad, δy was kept to zero and the rest of the values at the above given values. Two abscissae are given. One shows the tilt in mrad. The other gives the corresponding displacement between MWPC 3 and 4, which is the practical measure of the tilt. The resolutions are 6 mm/bin , which makes a displacement of less than 3 mm detectable, corresponding to a tilt of 0.85 mrad and an increase of P_2 from 2.1 to 4.9 %. P_1 is of course practically constant. The crosses

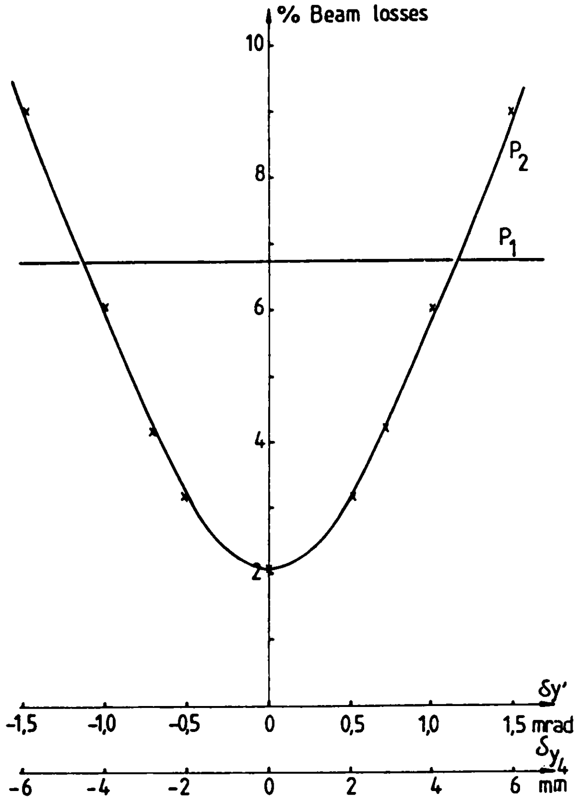

Figure 9: $P_2/P_{2_{min}} = f(\delta y')$.
Losses in the first LEAR splitter magnet as function of a vertical tilt angle. For the crosses see text.

give P_2 – losses calculated from the formula in chapter 3 point 3 (hypothesis of a uniform rectangular distribution in the vertical phase plane), normalized to the point at $\delta y' = 0$. $\Delta y'$ in this formula has been set to 1.4 mrad to keep the vertical size and the emittance equal in both the calculations. Even if this scaling is somewhat arbitrary, it can be concluded that the **strong dependence of the tilt** found in the simple approach is really valid.

A rotation of the beam ellipse in the phase space, which corresponds to a converging or diverging beam, also changes the losses as shown in figure 10. The rate of convergence is given as the difference of the vertical size in MWPC 3 and MWPC 4 with the size in MWPC 3 kept constant. The other beam parameters are those of the optimal beam with $\delta y' = 0$ and $\delta y = 0$, sharing ratio 1. This way of defining the beam gives rise to the small change of the P_1 – losses. If instead the size at the entrance is kept constant this effect disappears. The P_2 – losses however change a little but significantly. A slightly divergent beam is consequently preferable.

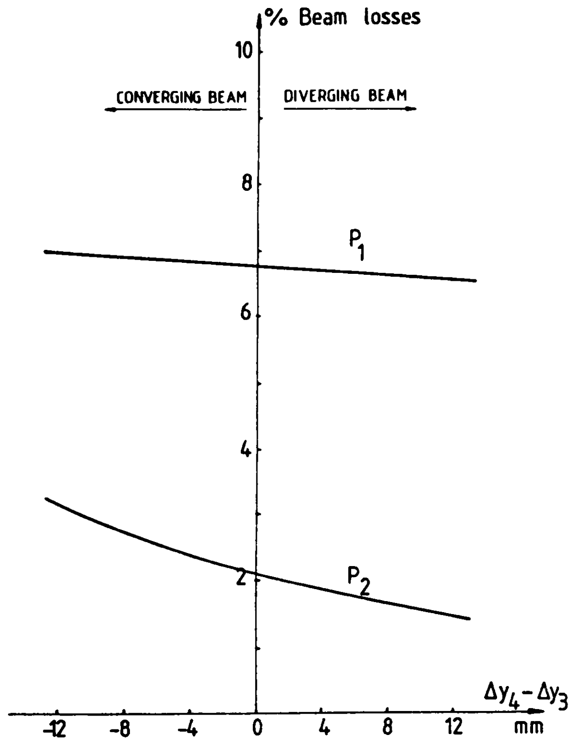

Figure 10: P = f(divergence).
Losses in the first LEAR magnet as function of beam optics in the vertical plane.

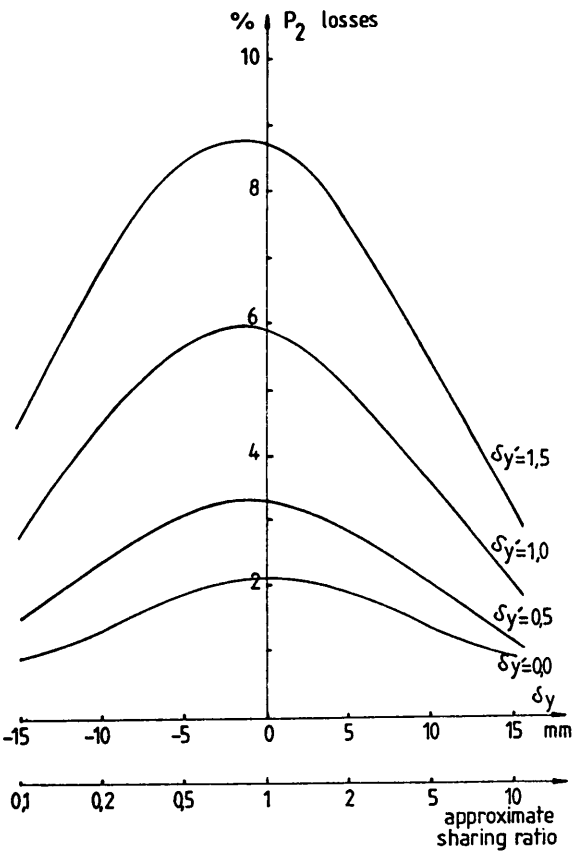

Figure 11: $P_2 = f(\delta y, \delta y')$.
Small angle losses in first LEAR splitter magnet as function of sharing ratio at different vertical tilt angles (in mrad).

The combined effect of a vertical tilt and vertical displacement of the beam for a few values of $\delta y'$ is shown in figure 11. As could be expected the influence on P_2 from the vertical displacement (sharing ratio) is increased with the tilt of the beam. The variation of P_1 with δy is not dependent on $\delta y'$.

Changes in the horizontal plane affect only the effective thickness of the knife because the horizontal convergence does not enter in this model. A horizontal tilt can be transformed to a displacement at the entrance. Therefore the only parameter to vary is the position at the entrance of the magnet. This dependence is given in figure 12.

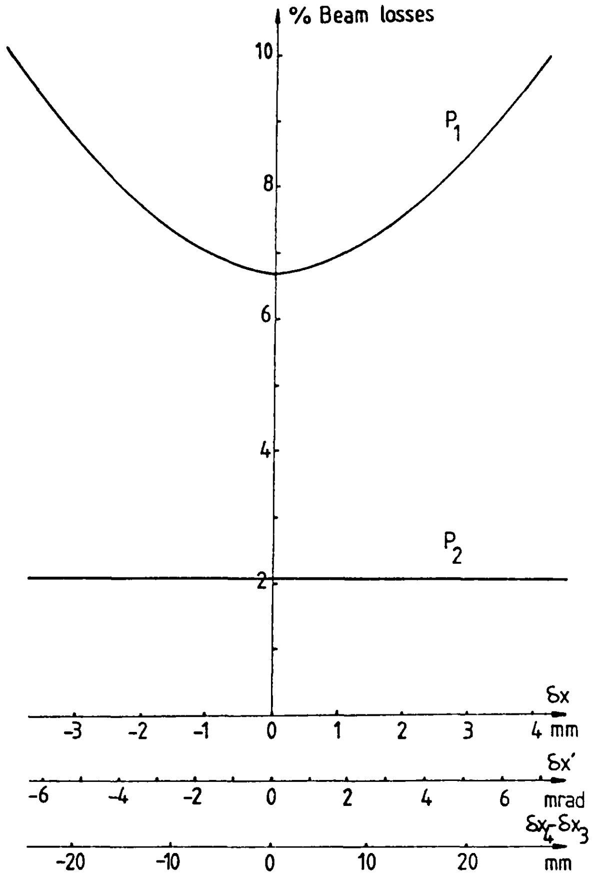

Figure 12: P = f(δx).
Losses in the first LEAR splitter magnet as function of the horizontal beam position.

These results can also be presented as the appearence of the beam on the beam monitors. An example of this is shown in figure 13. The first column gives the beam on MWPC 3, the other four on MWPC 4. In column 2 the splitter is on, the lower part deflected and the non-deflected part shown, column 3 with splitter off, all the transmitted beam. Columns 4 and 5 give the losses P_1 and P_2 respectively.

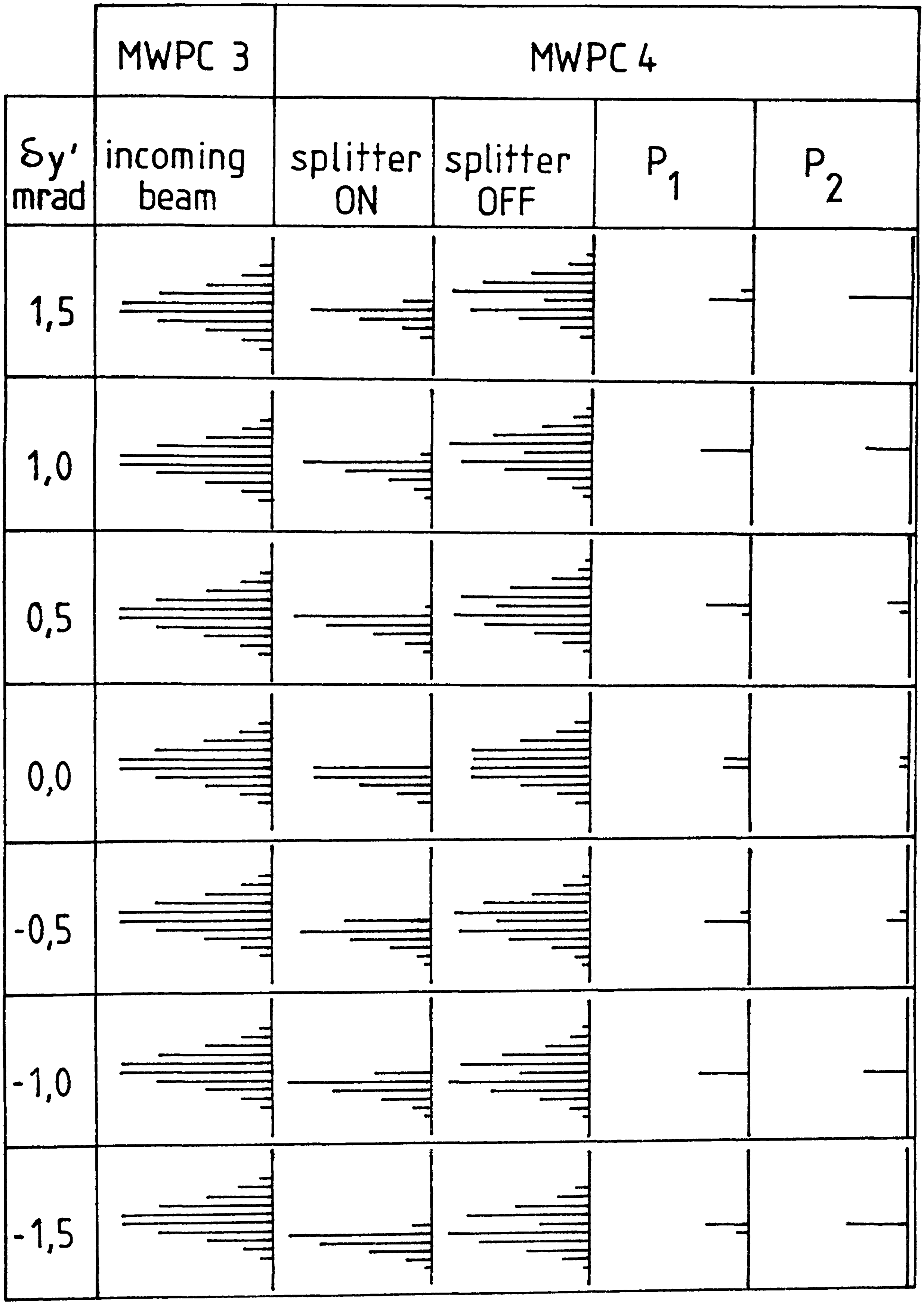

Figure 13: MWPC – simulation.
The calculated vertical distribution of the beam as seen on MWPC 3 and 4 before and after the first splitter magnet in the LEAR area at different vertical tilt angles.

These calculations have been performed using the assumed beam parameters as given above and the geometry of splitter 1 in the LEAR area. Care should be taken to interpret the results as quantitative estimates of the actual losses. Some of the points to consider are:

– Other optimal beams can be conceived. A change of the vertical size will change P_1 practically linear with the size. However, the optics focusing into the first splitter has only one extra degree of freedom to adjust the vertical size. Consequently the horizontal size changes simultaneously. The practical dependence of the vertical size will in this case be larger than in our model.

– The assumed emittances might also be wrong. A smaller vertical emittance will decrease P_2 if the vertical size is kept constant. A change of horizontal emittance renormalizes the effective thickness of the knife to be used. Depending on whether the size (A), the divergence (B), or both (C) are changed P_1 will vary as shown in figure 14.

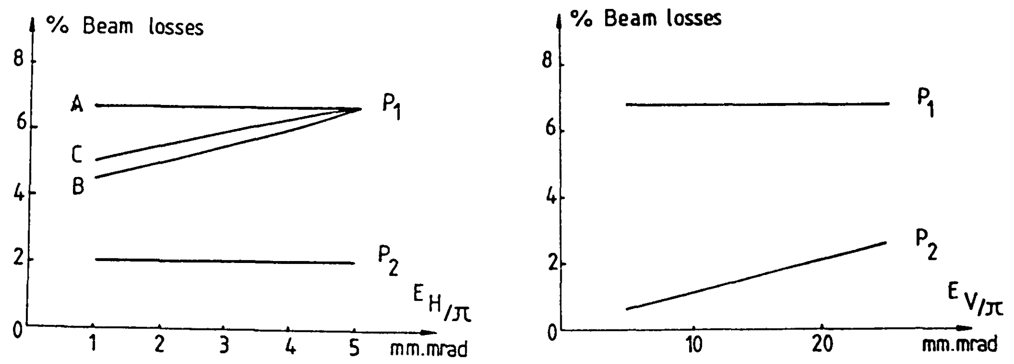

Figure 14: $P = f(E_H, E_V)$
Losses in the first LEAR splitter magnet as function of vertical and horizontal emittances. Optimal beam and sharing ratio 1 are assumed. A = beam size constant, B = beam divergence constant, C = change of both size and divergence.

The result of the optimal beam in splitter 2 becomes $P_1 = 3.70$ %, $P_2 = 0.38$ % and $P = 4.08$ %. The vertical emittance is reduced by a factor 2 as a sharing ratio of 1 is assumed in splitter 1. The optics between splitter 1 and 2 does not allow for any change of vertical size so these calculated values are only relative to the assumed optimal beam in splitter 1. Furthermore, it should be noted that the assumption of a Gaussian distribution could be bad because of the cut in splitter 1.

6.2 Splitter magnet of the East Hall type

This magnet is actually used to split the 24 GeV proton beam in two equal parts sent to two targets to produce secondary beams. The deflection angle is 18 mrad.

It has two knifes but the calculations are made assuming the use of only one (optics 1985). Following the same path as in the previous chapter the work can be summarized as below.

We assume that the beam parameters are the following:

$E_H = 3.50 \cdot \pi$ mmmrad	$s_x = 2.68$ mm	$s_{x'} = 1.31$ mrad
$E_V = 2.90 \cdot \pi$ mmmrad	$s_y = 41.42$ mm	$s_{y'} = 0.07$ mrad

The geometrical dimensions of the magnet is shown in figure 1. The shape of the pole knife (see figure 6) obliged us to make the evaluation of the effective thickness numerically. The result becomes $e_{eff} = 1.40$ mm. The length of the splitter is 2.18 m.

The results are shown in figures 15 to 17.

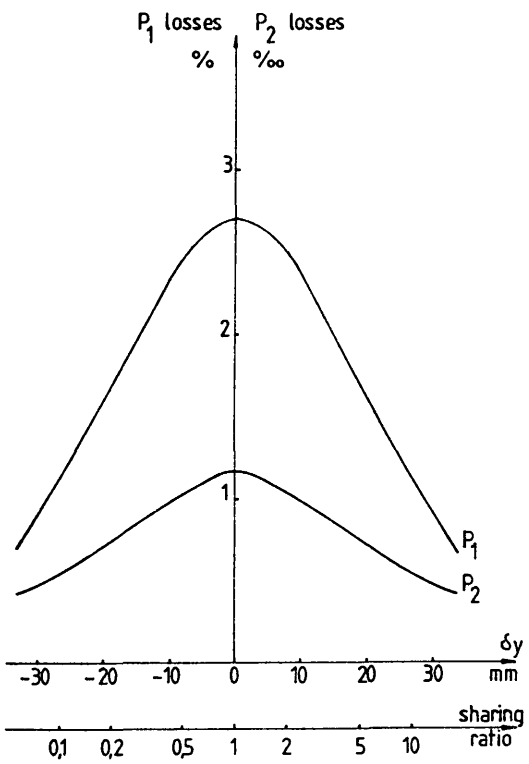

Figure 15: P = f(δy).
Losses in the East Hall splitter magnet as function of the sharing ratio.

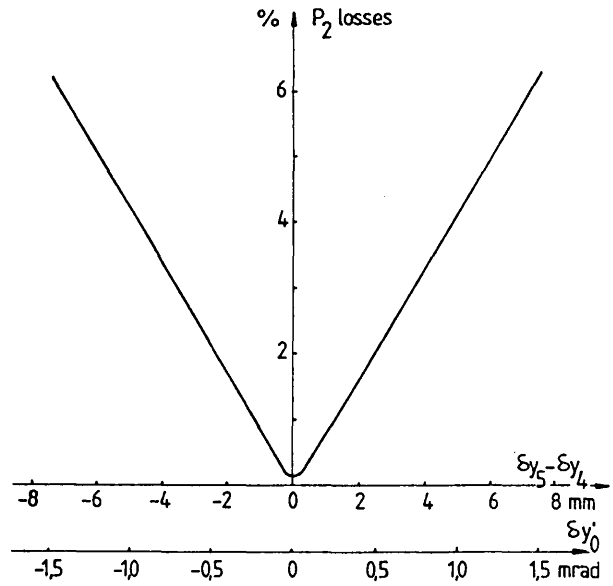

Figure 16: $P_2 = f(\delta y')$.
Small angle losses in the East Hall splitter magnet as function of vertical tilt angle.

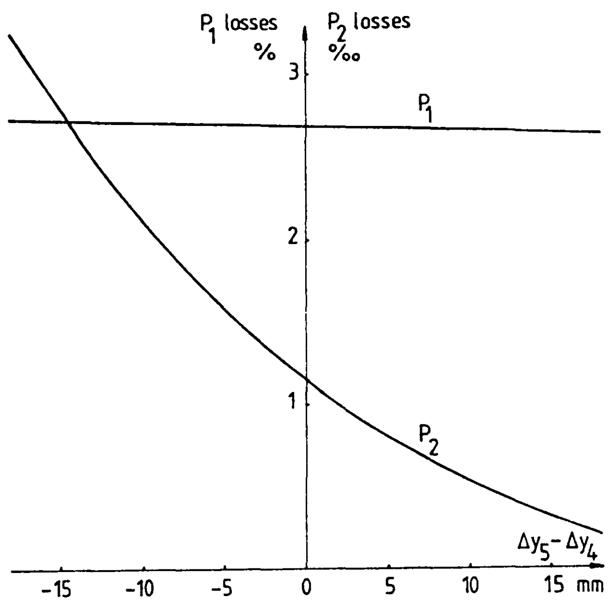

Figure 17: P = f(divergence).
Losses in the East Hall splitter magnet as function of the optics.

The losses of an optimal beam become $P_1 = 2.7\ \%$, $P_2 = 0.12\ \%$. The sizes and positions on TV 4, 0.40 m upstream the splitter entrance, and TV 5, 4.59 m downstream, correspond to those of MWPC 3 and 4.

The P_2 − losses are small as $\Delta y'$ is small, but they are also very sensitive to the tilt of the beam. It is possible to imagine the linear dependence shown in chapter 3 point 3 when $|\delta y'| > \Delta y'/2$. The relative change of P_2 with convergence is also stronger than for the LEAR splitter but the absolute values are small.

7. Calculations in Four Dimensions

A further refinement is to fully include the horizontal coordinates, by integrating in all four dimensions x,x′,y,y′. However, the computer time needed increases considerably. Furthermore the definition of integration areas becomes very complicated in the general case. In the special case of the LEAR splitter it was possible to bypass this problem by defining the interesting quantities as:

1. The upper hole transmission, non − deflected beam:

   All particles being inside the hole at entrance and at exit.

2. The pole gap transmission, deflected beam:

   All particles being in the pole gap both at entrance and at exit.

3. Large angle losses, P_1:

   All particles hitting the face of the knife.

4. Small angle losses, P_2:

   All particles entering the hole but exit below the upper surface of the knife or enter the pole gap but exit above the lower surface of the knife.

This version of the computer program had to be simplified, e.g. no simulation of MWPC outputs is made and the step length is increased. Only the optimal beam conditions are investigated. Using the mechanical dimensions and beam parameters of the LEAR magnet the result becomes:

$$P_1 = 6.63\ \%\ ,\ P_2 = 2.24\ \%\ \text{and}\ \ P_1 + P_2 = 8.87\ \%.$$

The error could be as big as 0.4 % using the estimate described above. The result is to be compared with 6.76,2.09 and 8.82 % with much less error from the two – dimensional calculations.

It can be concluded that the approximation with an effective thickness of the knife to take the horizontal beam distribution into account is good enough, at least close to optimal beam conditions.

8. Acknowledgements

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APPENDIX A

DESCRIPTION OF PROGRAMS DIM2.2 AND PROG11.

A.1 Common features

A.1.1 General

The losses of a particle beam can be estimated if the beam distribution is compared with the acceptance areas as represented in a phase plane or a phase volume. The computer programs DIM2.2 and PROG11 are based on this method to calculate beam losses. They are actually versions of the same program where DIM2.2 integrates in two dimensions but include some input/output features whereas PROG11 integrates in four dimensions and hence is much slower. The programs were developed to deal with splitter magnets but can easily be adopted to other similar problems.

A.1.2 Represention of the beam.

The beam is represented by a two – or four – dimensional Gaussian distribution

$$f_1 = (\sqrt{2/\pi})\cdot(1/s_t)\cdot e^{-2((t-\delta t)/s_t)^2}$$

where $s_t = 2\sigma_t$ and δt is the displacement.

In the two dimensions of the vertical phase plane (y,y′) this becomes

$$f_2(y,y') = f_1(y)\cdot f_1(y')$$

or in the four dimensions of the x – y phase space (x,x′,y,y′)

$$f_4(x,x',y,y') = f_2(x,x')\cdot f_2(y,y').$$

The beam is defined at a certain point along the beam line by giving E, $\sqrt{\sigma_{11}}$, $\sqrt{\sigma_{22}}$, and r (see TRANSPORT Manual) for x and y. A converging or diverging beam ($r\neq 0$) is represented by an ellipse; the axis of which makes the angle α with the coordinate axis. In the x,x′ plane the parameters of the ellipse are calculated from the given beam parameters as

$$\sigma_{12} = r\cdot(\sigma_{11}\cdot\sigma_{22})^{1/2}$$

$$\alpha = \arctan(2\sigma_{12}/(\sigma_{11}-\sigma_{22}))/2$$

$$s_\xi = (E/\pi)(\sigma_{22}\cos^2\alpha - 2\sigma_{12}\sin\alpha\cos\alpha + \sigma_{11}\sin^2\alpha)^{-1/2}$$

$$s_{\xi'} = (E/\pi)(\sigma_{22}\sin^2\alpha + 2\sigma_{12}\sin\alpha\cos\alpha + \sigma_{11}\cos^2\alpha)^{-1/2}$$

s_ξ, $s_{\xi'}$ are half the lengths of the main axis of the ellipse. A physical displacement δx or tilt $\delta x'$ give rise to a displacement of the ellipse in the phase plane by $\delta x, \delta x'$. The beam is consequently defined in the horizontal phase plane by E, $\sqrt{\sigma_{11}}$, $\sqrt{\sigma_{22}}$, r and δx, $\delta x'$, (see input I = 2) and correspondingly in the vertical phase plane. The coordinate systems are defined in figure 7 of the main text.

In the case of splitter magnets and when integrating only in two coordinates the horizontal phase plane can be included in the acceptance/loss conditions (see below), by introduction of an effective thickness of the pole knife (see below).

The values of the beam parameters at any position along the beam line can be used as input. The values used by the program, at the entrance of the splitter,are calculated assuming pure drift (see I = 7). The beam parameters are also calculated and printed at up to four different positions along the beam line.

A.1.3 Definition of acceptance/loss conditions.

The acceptance or loss conditions are defined as areas in the phase plane. The borders are the geometrical constraints, transformed to the vertical phase plane in the two – dimensional calculations. In the four – dimensional calculations they have to be given in another way as discussed below. The program reads a set of parameters (up to seven) and the number of the version to be used. Four conditions can be defined and two versions are so far implemented, 1) splitter magnet, 2) electrostatic splitter. For details see input description , I = 1. The four conditions of the splitter magnet version represents:

1. the beam passing through the upper hole
2. the beam passing through the lower hole
3. the beam lost on the edge of the pole knife (P_1)
4. the beam lost on the surfaces of the pole knife (P_2).

All input data have to be given in the coordinate system of the reference beam x,x′,y,y′,z. In the case of the splitter magnets the geometry is supposed to be such that the symmetry plane of the magnet and the middle of the pole knife is on the z−axis.

A.1.4 Integration

The integration is made by stepping in the coordinate system $(\xi,\xi',)\psi,\psi'$ defined by the axis of the beam phase ellipses (superellipse).This coordinate transformation is calculated by the program from the input as explained above. It is:

$$x = \delta x + \xi\cos\alpha - \xi'\sin\alpha$$

$$x' = \delta x' + \xi\sin\alpha + \xi'\cos\alpha$$

$$y = \delta y + \psi\cos\alpha - \psi'\sin\alpha$$

$$y' = \delta y' + \psi\sin\alpha + \psi'\cos\alpha.$$

The step length is varied during the integration as

$$dt = t_{min} + |t|/t_{div} \quad , \quad t = (\xi, \xi',) \psi, \psi',$$

where t_{div} and t_{min} are given in the input (I = 3) to optimize the number of steps for a given accuracy. The limits of integration are 0 to $\pm 3 \cdot s_t$.

The corresponding (x,x′,)y,y′ coordinates are calculated at each step, and the conditions are checked. If one is fulfilled the corresponding sum is increased by $f_{(4)2}((\xi,\xi',)\psi,\psi')\cdot(d\xi\cdot d\xi'\cdot)d\psi\cdot d\psi'$.

A.2 Program DIM2.2

A.2.1 General

This is the two−dimensional calculation program where the horizontal plane is included via the conditions. The results are visualized by a simulation of the beam appearance on a MWPC at two different positions along the beam line, (see I = 7). Also the input phase plane with condition areas indicated can be plotted, see I = 4 and I = 5.

A.2.2 Effective pole knife thickness calculation

The effective thickness of the pole knife is the geometric area weighted with the horizontal beam distribution:

$$e_{eff} = \int_{-x_1}^{x_1} e(t+\delta x)\cdot e^{-2(t/s_x)^2} dt \ ,$$

where e(x) is the actual pole thickness as function of x, $x_1 = 5\cdot s_x$. δx and x are beam parameters as defined above. The influence of the divergence is neglected. Input of parameters see I = 0.

The formula used for e(x) is (see figure 6 of the main text):

$$e(x) = e_1(x) + e_0$$

$$e_1(x) = 2(2x(R-x))^{-1/2} \qquad 0 \le x \le x_2 \ , \ x_2 = R(1-\sin\gamma)$$

$$= (2x(R-x))^{1/2} + (2x_2(R-x_2))^{1/2} + \tan\gamma\cdot(x-x_2) \qquad x_2 < x \le R$$

$$= R + (2x_2(R-x_2))^{1/2} + \tan\gamma\cdot(x-x_2) \qquad R < x$$

A.2.3 Input of DIM2.2

The input (and output) is governed by a control variable I, (given with format I1) after which follows a number of parameters Pi, (given with format F10.3) the number and interpretation of which depend on the value of the steer variable.

I

P1 P2 .. PN N = f(N)

I can take the values 0 to 9 and −8 with the following meaning:

I = 0, N = 8 special operations,optional.

P1 = definition of operation. So far only 1.0 is used.

P1 = 1 effective thickness calculation.

P2 = e_0

P3 = γ

P4 = R

P5 = s_x

P7 − P8 are not used but must be given a value.

The result $e_{eff}/2$ is transferred to the parameter P3 of geometric input , I = 1.

I = 1, N = 7 parameters for condition regions.

The meaning of P1 − P7 depends on the set of conditions which is chosen with I = 6.

1. Splitter magnet

   P1 = height of pole gap (both assumed equal)

   P2 = length of splitter

   P3 = half effective pole knife thickness

2. Double electrostatic splitter. (See reference [2]).

   P1 = gap width

   P2 = length of plates

   P3 = half thickness of central plate

I = 2, N = 7 beam parameters,start of calculations.

P1 = emittance/π

P2 = vertical size

P3 = vertical divergence

P4 = r

P5 = k

P6 = δy

P7 = δy′

As can be seen this is redundant. Therefore one of the parameters Pi, i = 1 − 4, is calculated from the other 3. The value of i is chosen by k (P5). The beam is to be given as of z − value 1, see I = 7.

I = 3, N = 4 step length definition, optional.

P1 = ydiv

P2 = ymin

P3 = y′div

P4 = y′min

In default of input the values 50,0.1,50,0.1. are used.

I = 4, N = 6 plot parameters, optional.

P1 − P6 = YPMIN,YPMAX,AK,Y′PMIN,Y′PMAX,AL

which represent the region of the phase plane to be plotted. AK, AL are the numbers of columns, lines, the plot will occupy in the output.

In default of input the values −60,60,121,−60,60,121 are used.

I = 5, N = 0 to print the phase plane plot, optional.

I = 6, N = 1 selection of condition set.

Observe!! Special input, format I1.

N1 = 1, splitter magnet

N1 = 2, double electrostatic splitter

In default of input the value 1 is used.

I = 7, N = P1 z−value of beam parameter output.

P1 = number of z−values, $2 \leq$ P1 ≤ 5

P2. .PN = z−values in meters.

I = 8, N = N = 5·16 + 3 wire setting, positions in the MWPC−simulation, optional.

Special input format 5·16I3,3F10.3.

Each of the first 5 lines should give the number of wires connected to the 16 channels, with the sum equal to 100. Line number j corresponds to setting of output column j.

On the last line three values should be given P1 − P3. where P1,P2 are the positions of the MWPCs relative to the entrance of the magnet. P3 is a magnification factor of the content in the MWPCs. The pile from the original value to the magnified is then printed with another symbol, '−' instead of '*'. P3 $\leq$ 1 gives only the basic output.

Once the above setting is done the plot can be switched off/on by I = −8/8.

I = 9, N = 0 stop of calculations.

A.2.4 Output

After printing the input the output are the following:

INPUT P1 − P5

Same as given at I = 2.

CALC P1 − P7

P1 = drift length

P2 − P4 are the calculated size, divergence and r at this drift length

P5 = distance to closest waist assuming drift only.

BEAM P1 − P7

P1 − P5 = s_ψ $s_{\psi'}$ δy $\delta y'$ α

P6,P7 are the limits of integrations.

AREA A1 − A5

SUMS S0 − S5,P1 − P3

AN is the area and SN the sum corresponding to the condition N, N = 0 represents full area and full beam.

P1 = S1/S2, sharing ratio

P2 = S0 − S1 − S2, all losses

P3 = S3 + S4, $P_1 + P_2$

MWPC − plots (optional, I = 8)

The columns corresponds to

1. full beam at z − value 1, (from input I = 8)
2. condition 1 at z − value 2, (beam passing below in splitter magnet)
3. condition 1 + 2 at z − value 2, (total transmitted beam in splitter magnet)
4. condition 3 at z − value 2,(large angle losses in splitter magnet)
5. condition 4 at z − value 2,(small angle losses in splitter magnet)

PLOT of phase plane at the z − value where the integration is made (optional, I = 5).

The printed numbers are the condition numbers. In case of overlap the highest number is printed.

EFFECTIVE length calculation.

The integrals of e·x and e are given in addition to e_{eff}.

A.3 Program PROG11

A.3.1 General

This is a developement of DIM2.2. The integration is made in four dimensions. Two dimensional calculations can however also be made, see I = 3. No coupling between the horizontal and vertical motion is assumed. Only one condition set is defined namely for a LEAR type splitter magnet.

The definition of conditions are different from DIM2.2 and are such that the position of the particle at entrance and at exit decides which condition is fulfilled. The result should correspond to those of DIM2.2 directly.

The integration time is very sensitive to the numbers of steps in each dimension, in principle to the power of four! This means that the accuracy is limited even at considerable computer times. Therefore all unnecessary computations have been left out, such as MWPC – simulation.

A.3.2 Changes in input and output of PROG11 relative to DIM2.2

Most input are the same with the following exceptions. The selection between two – or four – dimensional calculations are made by I = 6. In the case of only two coordinates the value of K used below is 1, in case of four coordinates K = 2.

I = 0 does not exist

I = 1 "

I = 2 N = K·7

The x and y parameters have to be given on seperate lines.

I = 3 N = K·4

x and y values on different lines.

I = 4,5 Does not exist

I = 6, N = 2

P1 = set of conditions as in DIM2.2

P2 = number of coordinates, 2 or 4.

I = 8 Does not exist.

The output is in principle the same as for DIM2.2 but the x and y values are put on two consecutive lines.