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ON THE MEASUREMENT OF THE EFFECTIVE BURST LENGTH AND HIGH-FREQUENCY DUTY FACTOR

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1. INTRODUCTION

It is well known that the flux of the slow ejected beam is not constant, the burst has more-or-less a structure which is problematic for the experiments because of the change in the proton flux during the burst and because of chance coincidences. The involved frequencies reach from the KHz region (for example due to ripples on power supplies) up to several MHz (due to incomplete debunching or rebunching).

A quantitative measure for structures is obtained by the "effective burst length". The basic formulae in this connection are developed by D. Bloess, D. Dekkers and G. Shering [1] and in the following considerations we will shortly repeat the results, where it seems useful, and use the same notation wherever possible. The effective burst length T_e is the "time over which the same number of protons, evenly spread out in time, would give the same number of chance coincidences" [1] and is defined as

$$T_e = \frac{\left[\int_0^{T_s} m(t)dt\right]^2}{\int_0^{T_s} m^2(t)dt} \qquad (1)$$

where $m(t)$ describes the proton flux in time of the slow ejected burst.

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To illustrate this, three basically different burst shapes are shown in Fig. 1, which all have the <u>same</u> effective burst length:

a) high-frequency ripple

b) low-frequency fluctuation

c) no structure.

These few examples show that T_e is more-or-less sensitive to all mentioned types of burst shapes.

It is demonstrated in (1) that the global effective burst length T_e can be written as a product

$$T_e = T_l D_r \tag{2}$$

where T_l is the effective burst length disregarding high frequencies and D_r a high-frequency duty factor. The assumption for this is that $m(t)$ can be represented also as a product

$$m(t) = l(t) \cdot r(t) \tag{3}$$

where $l(t)$ represents the low frequency structure modulated by a fast oscillating function $r(t)$.

For the moment T_l is computed out of the structure $l(t)$ supplied by a Čerenkov counter looking at a target in the slow ejected beam [2,3]. In a second device the global effective burst length T_e is calculated by the formula derived in (1)

$$T_e = \frac{LN^2}{f} \tag{4}$$

which uses the countingrate N and the chance coincidences f measured in a telescope counter looking at the target. Thus the ratio of the measured T_e and T_l should give the high-frequency duty factor. The problem of this method is, of course, that T_e and T_l are measured by two completely different devices, which can introduce systematic errors into the result.

To achieve more consistent and precise information about the high-frequency structure in connection with debunching and re-bunching problems, it is important to measure the high-frequency duty factor directly with high precision. In the following, a set-up is proposed which allows that measurement of D_r and T_l simultaneously with the same device.

2. HIGH-FREQUENCY DUTY FACTOR AND LOW-FREQUENCY EFFECTIVE BURST LENGTH

For the following considerations we make the assumption that the high-frequency structure is of a kind of cos-modulation, which will be mostly a good approximation, apart from some pathologic cases, which will be discussed later.

$$r(t) = 1 + H \cos t \, , \quad H: \text{ relative amplitude} \tag{5}$$

On the other hand, it can easily be shown that T_e can be approximated by

$$T_e = \frac{\left[\int_0^{T_s} l(t)\ r(t)dt\right]^2}{\int_0^{T_s} l^2(t)\ r^2(t)dt} \approx \frac{\left[\int_0^{T_s} l(t)dt\right]^2}{\int_0^{T_s} l^2(t)dt} \cdot \frac{T}{\int_0^{T} r^2(t)dt} = T_l D_r \tag{6}$$

where T is a short time compared to the burst length T_s, during which $l(t)$ is considered to be constant. Within this limitation we can choose

$$T = 2\pi$$

and obtain for the high frequency duty factor

$$D_r = \frac{2\pi}{\int_0^{2\pi} (1 + H \cos t)^2 dt} = \frac{1}{1 + \frac{H^2}{2}} \qquad (7)$$

and

$$T_l = \frac{\left[\int_0^{T_s} l(t)dt\right]^2}{\int_0^{T_s} l^2(t)dt} \qquad (8)$$

To find a measurable quantity out of which the relative amplitudes H and D_r can be extracted, we calculate the number of accidentals measured by delayed coincidences in a set-up shown in Fig. 2 below.

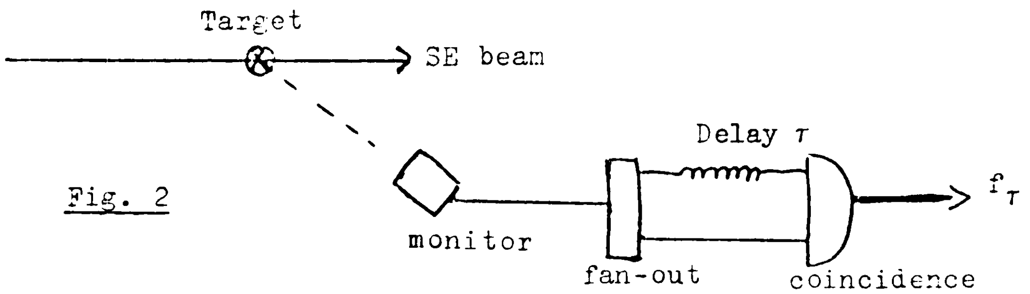


Fig. 2

The number of accidentals f measured with a delay τ is given by the following equation:

$$f_\tau = k^2L \int_0^{T_s} m(t)\, m(t+\tau)dt \approx k^2L \int_0^{T_s} l(t)\, l(t+\tau)dt\, \frac{1}{T} \int_0^{T} r(t)\, r(t+\tau)dt \qquad (9)$$

k : proportional factor, resulting from the fact that the telescope counting rate is

$$N = k \int_0^{T_s} m(t)\, dt$$

L : time resolution of the coincidence.

If τ corresponds to the length of one period of the modulation we are looking for (for example 105 nsec for debunching, 420 nsec for rebunching), we get

$$r(t)\, r(t+\tau) = (1 + H \cos t)(1 + H \cos (t+\tau)) = (1 + H \cos t)^2 \qquad (10)$$

and as assumed

$$l(t) \cdot l(t+\tau) \approx l^2(t) \qquad (11)$$

Thus we obtain

$$f_\tau = k^2 L \int_0^{T_s} l^2(t)dt \, \frac{1}{\tau} \int_0^{\tau} (1 + H \cos t)^2 dt \qquad (12)$$

and with

$$k = \frac{N}{\int_0^{T_s} m(t)\, dt}$$

$$f_\tau = N^2L\left(1+\frac{H^2}{2}\right)\frac{\int_0^{T_s} l^2(t)dt}{\left[\int_0^{T_s} l(t)dt\right]^2} = \frac{LN^2\left(1+\frac{H^2}{2}\right)}{T_1} \qquad (13)$$

On the other hand, if we measure accidentals simultaneously w... . second delay line of the length $\tau/2$ (i.e. 52.5 nsec or 210 nsec), we obtain

$$r(t)\ r(t+\tau/2) = (1 + H\cos t)(1 + H\cos(t+\tau/2) = (1 - H^2\cos^2 t) \qquad (14)$$

$$f_{\tau/2} = \frac{LN^2\left(1-\frac{H^2}{2}\right)}{T_1} \qquad (15)$$

The ratio of the so measured accidentals (13) and (15) becomes

$$R = \frac{f_\tau}{f_{\tau/2}} = \frac{1+\frac{H^2}{2}}{1-\frac{H^2}{2}} \qquad (16)$$

and thus we get an expression for the modulation amplitude H and the high-frequency duty factor D:

$$H = \sqrt{\frac{2(R-1)}{R+1}} \qquad (17)$$

$$D = \frac{1}{1 + \frac{H^2}{2}} = \frac{R + 1}{2R} \tag{18}$$

In this way it is possible to calculate H and D with the help of the measured ratio of the τ- and $\tau/2$- accidentals.

It is also possible to find an equation for the "low-frequency burst length" T_1 by measuring the average F of f_τ and $f_{\tau/2}$.

$$F = \frac{f_\tau + f_{\tau/2}}{2} = \frac{1}{2}\left\{ \frac{LN^2\left(1 + \frac{H^2}{2}\right)}{T_1} + \frac{LN^2\left(1 - \frac{H^2}{2}\right)}{T_1} \right\} = \frac{LN^2}{T_1} \tag{19}$$

or

$$T_1 = \frac{LN^2}{F} \tag{20}$$

being independent of the modulation amplitude H.

Thus to obtain T_1 we have to measure additionally the counting rate N, since f_τ and $f_{\tau/2}$ are already measured for D and H.

If we recalculate for a check T_e out of T_1 and D, we obtain

$$T_e = T_1 D = \frac{LN^2}{F} \frac{R+1}{2R} = \frac{LN^2}{f_\tau}$$

which is exactly the formula (4) given in (1) and by which T_e is measured presently.

It should be stressed that T_l measured in the way discussed is independent of a structure of the period τ, but is still sensitive to any modulation with a period longer than τ. If several modulations are present, the considerations above can easily be extended under the same assumptions. For example, for three different modulations we get

$$m(t) = l(t).q(t).r(t).s(t) \tag{21}$$

and obtain in the same way as above

$$T_e = T_l D_{qrs} = T_{lq} D_{rs} = T_{lqr} D_s \tag{22}$$

The indices are the modulations to which T or D are sensitive. It must be

$$\begin{gathered} D_{qrs} \leqslant D_{rs} \leqslant D_s \\ T_l \geqslant T_{lq} \geqslant T_{lqr} \geqslant T_{lqrs} \end{gathered} \tag{23}$$

i.e. the more structure is neglected the longer the effective burst length becomes.

3. THE APPLICABILITY AND LIMITS OF THE ASSUMPTIONS

One assumption was that the involved modulation frequencies are significantly different from each other:

$$\tau_q \gg \tau_r \gg \tau_s \tag{24}$$

so that $q(t) \approx q(t+\tau_r)$ and $r(t) \approx r(t+\tau_s)$ is justified. In reality we are investigating revolution times of 105 nsec, 420 nsec and 2090 nsec, i.e.

$$m(t) = l(t)\ (1 + H_s \cos 20t)\ (1 + H_r \cos 5t)\ (1 + H_q \cos t) \qquad (25)$$

Theoretically all the involved integrals can be solved.

For simplicity the special case of 105 nsec and 420 nsec modulation was calculated:

$$m(t) = l(t)\ (1 + H_s \cos 4t)\ (1 + H_r \cos t) \qquad (26)$$

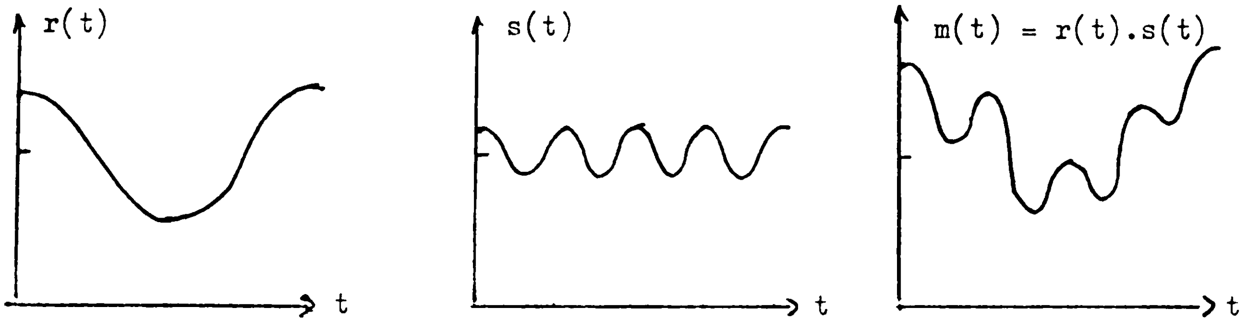


Fig.3

Here only the results of the rather lengthy calculations are given:

$$T_e = \frac{\left[\int_0^{T_s} m(t)\,dt\right]^2}{\int_0^{T_s} m^2(t)\,dt} = \frac{\left[\int_0^{T_s} l(t)\,dt\right]^2}{\int_0^{T_s} l^2(t)\,dt} \circ \frac{1}{\left(1 + \frac{H_s^{\,2}}{2}\right)} \circ \frac{1}{\left(1 + \frac{H_r^{\,2}}{2}\right)} \qquad (27)$$

i.e. the overall HF-duty factor is the product of the individual HF-duty factors.

The measured ratio

$$R_{420} = \frac{f_{420}}{f_{210}} = \frac{\left(1 + \frac{H_r^2}{2}\right)}{\left(1 - \frac{H_r^2}{2}\right)} \qquad (28)$$

is independent of the 105 nsec oscillation, while

$$R_{105} = \frac{f_{105}}{f_{52.5}} = \frac{\left(1 + \frac{H_s^2}{2}\right)}{\left(1 - \frac{H_s^2}{2}\right)} \frac{1}{\left(1 + \frac{H_r^2}{\sqrt{2}\,2}\right)} \tag{29}$$

is slightly dependent upon the 420 nsec oscillation. Of course it is always possible to calculate the correction since we know H_r from (28).

Generally in cases where condition (24) is no longer well fulfilled, the HF-duty factor depends also slightly on frequencies in the neighbourhood of the measured one.

To reduce the influence of an infrastructure $s(t)$ on measurements regarding lower modulations $r(t)$, one can choose a coincidence gate length $L_r \approx \tau_s$ as shown in Fig. 3 above.

Thus the coincidence rate f_r becomes independent of $s(t)$:

$$f_r \propto \int r^2(t)\, \bar{S}^2(t)\, dt \approx \int r^2(t)\, dt \tag{30}$$

since

$$\bar{S} = \frac{1}{\tau_s} \int_0^{\tau_s} (1 + H_s \cos t)dt = 1$$

The second assumption was the cos-form of the structure. In Fig. 4 rather extreme non-cos-shapes are illustrated.

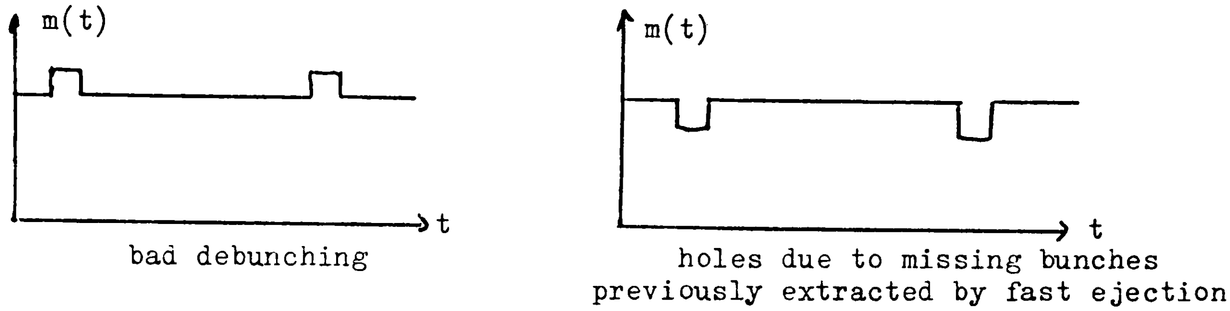


Fig. 4

In these cases the developed formulae are no longer strictly true. The ratio $f_T/f_{T/2}$ is less sensitive to the structure since the peak intensity (or bump intensity) is small compared to the intensity between the peaks. In the case shown above it is roughly a factor 3 less sensitive compared to a cos-structure with the same H. The effective burst length again is rather independent of the structure; for example with an $H = 50\%$ the error in T_1 is only 1%. Thus in extreme non-cos cases the measured D is no longer the quantitative exact RF-duty factor, but it still remains a suitable qualitative quantity to investigate the RF modulations.

4. THE REQUIRED PRECISION AND THE EXPERIMENTAL SET-UP

In Fig. 5 it is shown how the RF-duty factor D and the modulation H depend on the measured ratio $R = f_T/f_{T/2}$. The maximum possible case is $R = 3$. It shows that for the detection of low modulations a rather high precision for R is required. For example if a modulation of $H = 10\%$ shall still be measurable, the error of R must be smaller than 1%. The actual reason why it is impossible to obtain a 10% effect at a modulation of 10% is that the phase of the modulation is lost (this is also the case in the measurement of the auto-correlation function: Sandel's meter and time-to-pulse-height arrangements).

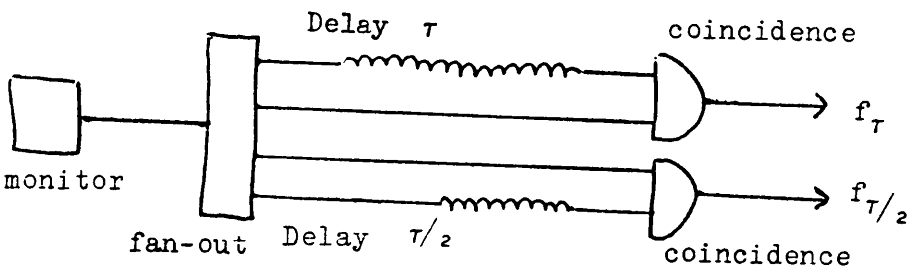
To measure R with an error of 1% one needs about 10^4 chance coincidences, or at a coincidence resolution of 20 nsec a counting rate of 10^6 per burst, which should be possible without considerable difficulties. These high counting rates could easily be achieved by bigger scintillators in the s_5 telescope. The resulting error of D is

$$\Delta D/D = \frac{2}{R+1}\frac{1}{\sqrt{F}} \leq \frac{1}{\sqrt{F}} \tag{31}$$

so that $\Delta D/D$ would be less than 1% with the above assumed counting rates.

The basic arrangement for the measurement is shown in Fig.6:

Fig. 6



For the simultaneous observation of three different modulations (105 nsec, 420 nsec, 2090 nsec) one can of course set up the same device three times, but a more economical arrangement is shown in Fig. 7:

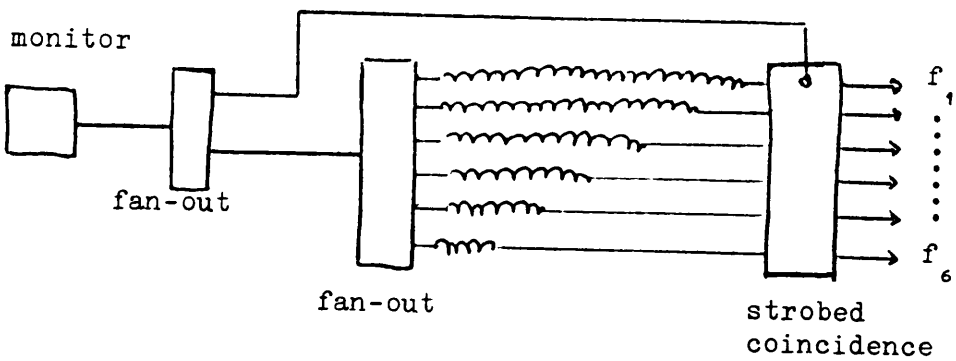


Fig. 7

The main circuit is a strobed coincidence, i.e. all the six inputs are put into coincidence with one master input. (Commercially available from Fa. EG&G, SEN.) Thus it is assured that all the six branches have the same characteristics (coincidence efficiency, resolution) which is important for the relevance of the equations in chapter 2.

The acquisition of the counting rates (for $N = 10^6$ a prescaler possibly has to be used) and the display of the values for the effective burst lengths and duty factors for each frequency could be done in a similar way to the existing arrangement. At last it might be useful to divide the slow ejected burst into, let us say, five parts of 80 msec and to do the same acquisition separately for each part. Thus, for example, the development of the debunching and rebunching in time could be observed.

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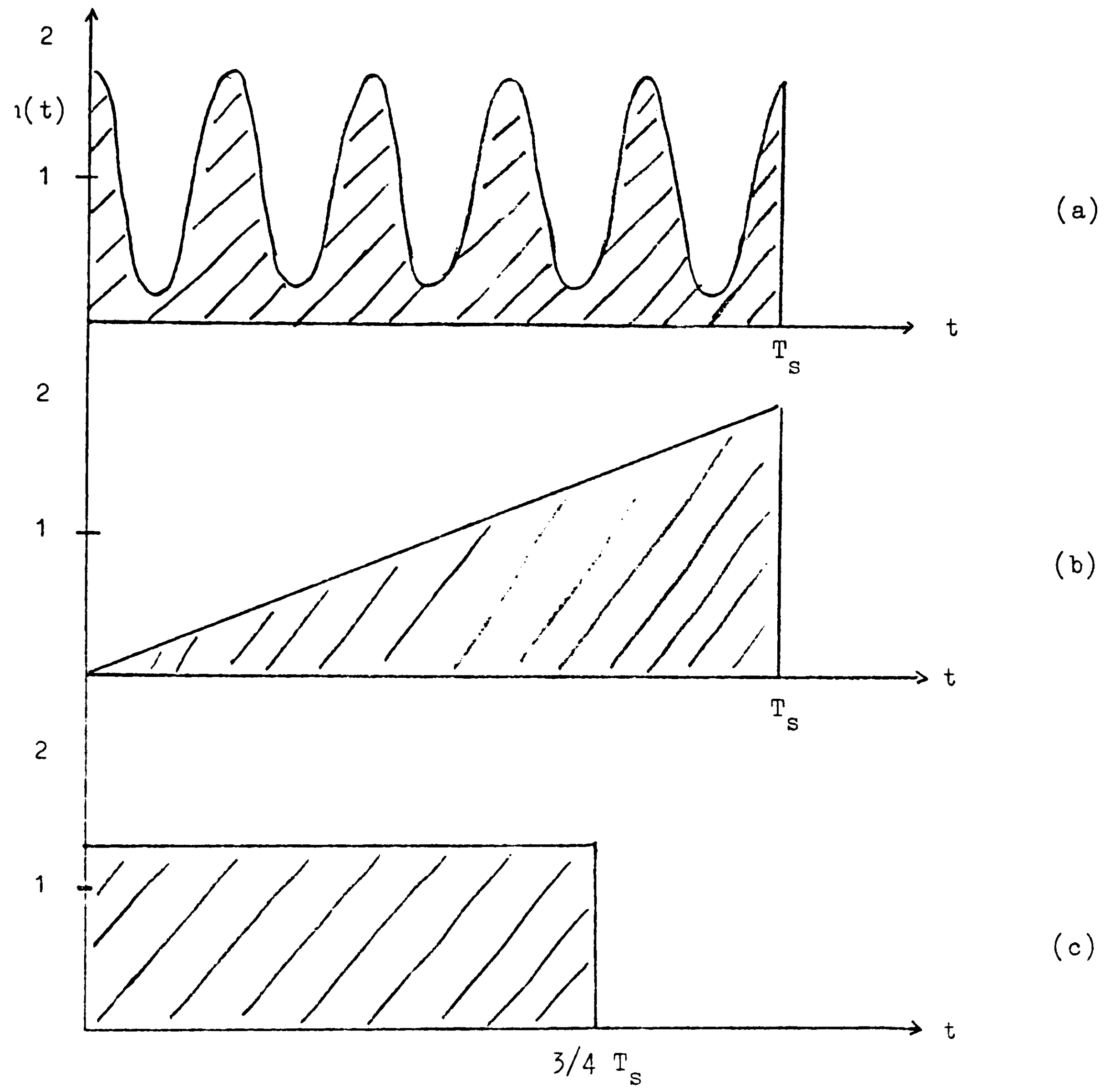

Normalization: $$\int_0^{T_s} m(t)\,dt = T_s$$

$$T_e = \frac{3}{4} T_s$$

Fig. 1 Three characteristically different burst structures with the <u>same</u> effective burst length T_e:

(a) high frequency structure

(b) low frequency structure

(c) no structure

Fig. 5 The dependence of the high-frequency duty factor D and amplitude H on the ratio of the measured accidentals f_τ and $f_{\tau/2}$