Flavor Identification of Reconstructed Hadronic Jets

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Identifying the flavor of reconstructed hadronic jets is critical for precision phenomenology and the search for new physics at collider experiments, as it allows one to pinpoint specific scattering processes and reject backgrounds. Jet measurements at the LHC are almost universally performed using the anti-k_T algorithm; however, no approach exists to define the jet flavor for this algorithm that is infrared and collinear safe. We propose a new approach, a *flavor-dressing* algorithm, that is infrared and collinear safe in perturbation theory and can be combined with any definition of a jet. We test the algorithm in an e^+e^- environment and consider the $pp \to Z + b$-jet process as a practical application at hadron colliders.

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Introduction.—The confining property of the strong interactions—described by quantum chromodynamics (QCD)—prohibits the observation of free quarks and gluons: in high-energy particle collisions, such as those at the Large Hadron Collider (LHC), they give rise to collimated sprays of hadrons inside the detector, denoted as *jets*. A jet is defined by the associated reconstruction algorithm and plays a crucial role as the interface between experiment and theory. In this regard, a core property of any jet algorithm is infrared and collinear (IRC) safety, i.e., the insensitivity to soft (low-energy) emissions and collinear (small angle) splittings. Only if such a property is satisfied can a comparison between measurements and theoretical predictions based upon fixed-order perturbation theory be reliably carried out.

Further identifying the "flavor" of the jets is critical to pinpoint specific scattering processes and reject backgrounds. An important example is the identification of a jet which is consistent with being initiated by a heavy-flavor (charm or beauty) quark. The identification of such signatures provides a window into the interactions of heavy-flavor quarks with other fundamental particles from GeV to TeV energy scales. This in turn provides a unique opportunity to perform (flavor-specific) direct searches for new physics phenomena [1,2], test the mechanism for generating the mass of elementary particles [3–7], and probe the internal flavor structure of hadrons [8–10].

Jet measurements at the LHC are almost universally performed using the anti-k_T algorithm [11] owing to the geometrically regular shape of the jets and desirable properties that derive from it. Use of anti-k_T jets persists in identifying jet flavor, which currently follow IRC-unsafe flavor assignment procedures. As such, no robust comparison between data and the available precise fixed-order calculations can currently be carried out.

The issue of IRC safety in the flavor assignment was first pointed out in Refs. [12,13] with a solution that modifies the jet definition itself to ensure IRC safety. This algorithm, however, requires the flavor information of all particles as input, thus making an experimental realization challenging. Very recently, further approaches were proposed to assign heavy-flavor quantum numbers to jets: based on soft drop grooming techniques [14], through the alignment of flavored particles along the winner-take-all axis [15], or by modifying the anti-k_T algorithm [16]. Other prescriptions have also been proposed [17–21]. However, no approach exists that reproduces the same jets as a flavor-agnostic anti-k_T algorithm, can be applied to generic processes with multiple jets, and at the same time is IRC safe to all orders.

In this Letter, we propose a new approach which allows us to assign heavy-flavor quantum numbers to a set of flavor-agnostic jets. This algorithm has the following properties: (i) it is IRC safe to all orders in perturbation theory and can therefore be applied in fixed-order predictions, (ii) it can be combined with any IRC-safe definition of a jet, such as anti-k_T jets, as the flavor assignment procedure is factorized from the jet reconstruction, and (iii) the flavor assignment can be applied at the level of quarks, heavy-flavor hadrons, or with proxy particles that can be reconstructed in an experimental environment [such as secondary vertices (SVs)]. The procedure we propose can therefore be directly applied in an experimental or theoretical setting to arbitrary scattering processes, enabling more direct and precise comparisons between theory and data.

In the following we present the algorithm, perform tests of IRC safety in an e^+e^- environment, and focus on $pp \to Z + b$-jet production as a case study.

Inputs to the flavor dressing.—The proposed algorithm provides a way of assigning a specific flavor quantum number $\mathfrak{f}$ to jets. The required inputs to the algorithm are a set of m flavor-agnostic jets which have been obtained from an IRC-safe jet definition, denoted by $\{j_1, \ldots, j_m\}$; the set of all "flavored clusters" in the event $\{\hat{f}_1, \ldots, \hat{f}_n\}$, where $\hat{f}_i$ arise from a flavored particle $f_i \in \{f, \bar{f}\}$ after an appropriate aggregation of surrounding radiation (to be specified below); a criterion for associating the flavored clusters with jets; and a flavor accumulation (counting) criterion. The flavor dressing then proceeds by sequentially assigning flavored clusters to the jets. Before presenting the algorithm, we first discuss these various inputs and comment on their potential differences in a theoretical or experimental setting. (i) *Flavor-agnostic jets*. This set of inputs should be obtained with an IRC-safe jet algorithm, and depending on the association criterion (see below) could require injecting ghost particles or retaining the constituent information of the jets. In the following, this set is considered to be composed of resolved "analysis" jets (either in exclusive or inclusive modes) that have passed a fiducial selection criterion. (ii) *Flavored particles and clusters*. In a parton-level prediction, the flavored particles f are identified as all (anti)quarks in a given event with the flavor quantum number $\mathfrak{f}$, e.g., $\mathfrak{f} = c(b)$ when identifying $c(b)$-tagged jets. In a hadron-level prediction with stable heavy-flavor hadrons, the replacement $c(b) \to D(B)$ can be made. In an experimental setting, the flavored particles can be replaced by a proxy particle for the heavy flavor, such as a reconstructed SV. In the latter, the charge information of the flavored particles is likely unavailable and they may not have a definite flavor (i.e., they could have a probability of being associated to c and b heavy flavors). In all cases, the flavor-dressing algorithm requires that these flavored particles f be first combined with neighboring radiation to ensure collinear safety, with the resulting cluster denoted as $\hat{f}$. This clustering procedure ($f \to \hat{f}$) preserves flavor information but alters the momentum according to the following sequential clustering routine. (1) Initialize a set with the same objects used as input to the flavor-agnostic jet algorithm, supplemented by the flavored particles f (appropriately removing any double counting). The former and latter are labeled as flavorless and flavored objects, respectively. (2) While there are at least two objects (at least one flavored) in the set, find the pair with smallest ΔR_{ab}^2, with $\Delta R_{ab}^2 = (y_a - y_b)^2 + (\varphi_a - \varphi_b)^2$ the angular separation in rapidity (y) and azimuth (φ). If $\Delta R_{ab} > R_{\mathrm{cut}}$, terminate the sequential clustering and move to step 3. Otherwise, three possible cases must considered. (a) Both a, b are flavorless: Remove a, b from the set and replace them with a single flavorless object that carries their combined momentum. (b) Only one of a, b is flavored: Remove the unflavored object from the set and test the criterion [22]:

$$\frac{\min(p_{T,a}, p_{T,b})}{(p_{T,a} + p_{T,b})} > z_{\mathrm{cut}} \left(\frac{\Delta R_{ab}}{R_{\mathrm{cut}}}\right)^{\beta}. \tag{1}$$

If it is satisfied, update the momentum of the flavored object to include that of the flavorless one. (c) Both a, b are flavored: The accumulation into f_a and f_b is complete, and they are removed from the set. (3) The momentum of each flavored cluster $\hat{f}_i$ is defined as the momentum accumulated by f_i in step 2. The choice of values for the parameters appearing in Eq. (1), and which are used in the remainder of this Letter, are $z_{\mathrm{cut}} = 0.1$, $R_{\mathrm{cut}} = 0.1$, and $\beta = 2$. (iii) *Association criterion*. For each flavored cluster $\hat{f}_i$ in the event, determine whether it can be associated to a jet. This criterion is important (although not unique) as only those jets that have at least one associated $\hat{f}$ can be assigned nonzero flavor. From the point of view of a parton-level prediction, an obvious choice is to associate $\hat{f}_i$ with j_k if the corresponding flavored particle f_i is a constituent of a jet j_k. Other sensible options to make this association are the requirement $\Delta R(\hat{f}_i, j_k) < R_{\mathrm{tag}}$, or to include the $\hat{f}$ as ghost particles in the reconstruction to determine in which jets they are clustered [23–25]. A discussion on the experimental feasibility of these approaches is given in the Supplemental Material [26]. We emphasize that a jet flavor assignment based solely on this association criterion is *not* IRC safe. (iv) *Accumulation criterion*. In an ideal situation, both the flavor ($\mathfrak{f}$) and charge (f versus $\bar{f}$) information of flavored particles (and hence clusters) is known. In such a scenario, one considers f ($\bar{f}$) to carry a positive (negative) flavor quantum number, and an object is then considered flavored if it is assigned an unequal number of f and $\bar{f}$. If the charge information is not available, one possibility is to instead consider an object to be flavored if it has been assigned an odd number of flavored clusters $\hat{f}$.

Flavor-dressing algorithm.—With this information at hand, the flavor-dressing algorithm to identify whether a reconstructed jet can be assigned the flavor quantum number $\mathfrak{f}$ proceeds as follows. (1) Initialize empty sets $\mathrm{tag}_k = \varnothing$ for each jet j_k to accumulate all flavored clusters assigned to it. (2) Populate a set $\mathcal{D}$ of distance measures based on all possible pairings. (a) For each unordered pair of flavored clusters $\hat{f}_i$ and $\hat{f}_j$, add the distance measure $d_{\hat{f}_i \hat{f}_j}$. If the charge information is available, the pairings can be restricted to only compatible quantum numbers, i.e., $(f, \bar{f})$ but not (f, f). (b) If the flavor cluster $\hat{f}_i$ is associated to jet j_k, add the distance measure $d_{\hat{f}_i j_k}$. In a hadron-collider environment, the beam distances $d_{\hat{f}_i B_\pm}$ should be added if $\hat{f}_i$ is not associated to any jet. (3) While the set $\mathcal{D}$ is

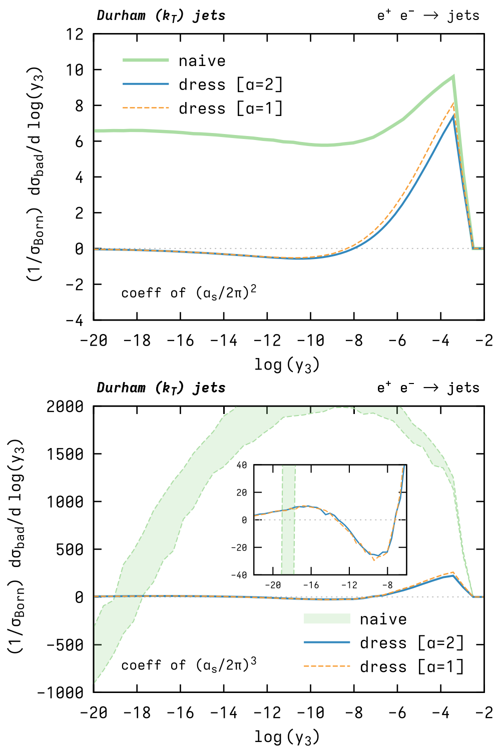

FIG. 1. Behavior of the flavor misidentified (“bad”) cross section in $e^+e^- \to$ jets production as a function of the y_3 resolution variable. Comparison of a naive flavor assignment (green) with the flavor-dressing approach (blue and orange lines) at the second (upper) and third (lower) order in α_s.

nonempty, select the pairing with the smallest distance measure. (a) $d_{\hat{f}_i\hat{f}_j}$ is the smallest: the two flavored clusters “annihilate” and all entries in $\mathcal{D}$ that involve $\hat{f}_i$ or $\hat{f}_j$ are removed. (b) $d_{\hat{f}_i j_k}$ is the smallest: assign the flavored cluster $\hat{f}_i$ to the jet j_k, $\mathrm{tag}_k \to \mathrm{tag}_k \cup \{\hat{f}_i\}$, and remove all entries in $\mathcal{D}$ that involve $\hat{f}_i$. (c) $d_{\hat{f}_i B_\pm}$ is the smallest: discard flavored cluster $\hat{f}_i$ and remove all entries in $\mathcal{D}$ that involve $\hat{f}_i$. (4) The flavor assignment for jet j_k is determined according to the accumulated flavors in tag_k. For the distance measure between two final-state objects a and b (flavored clusters or jets) we use

$$d_{ab} = \Delta R_{ab}^2 \max(p_{T,a}^{\alpha}, p_{T,b}^{\alpha})\ \min(p_{T,a}^{2-\alpha}, p_{T,b}^{2-\alpha}), \quad (2)$$

For lepton colliders, this measure can be adjusted through a suitable replacement of variables [27,33]. The distance between a flavored cluster and a hadron beam in the direction of positive (+) or negative (−) rapidity is

$$d_{\hat{f}B_\pm} = \max[p_{T,\hat{f}}^{\alpha}, p_{T,B_\pm}^{\alpha}(y_{\hat{f}})]\ \min[p_{T,\hat{f}}^{2-\alpha}, p_{T,B_\pm}^{2-\alpha}(y_{\hat{f}})],$$

$$p_{T,B_\pm}(y) = \sum_{k=1}^{m} p_{T,j_k}[\Theta(\pm\Delta y_{j_k}) + \Theta(\mp\,\Delta y_{j_k})e^{\pm\Delta y_{j_k}}], \quad (3)$$

with the rapidity separation $\Delta y_{j_k} = y_{j_k} - y$ and $\Theta(0) = 1/2$. The distance measures in Eq. (2) are inspired by the flavor-k_T algorithm [12], with a parameter α that can be chosen in the range $0 < \alpha \leq 2$. This choice of measure ensures that soft pairs of flavored particles are recombined early on, thus avoiding a sensitivity to infrared physics. A hierarchical tagging of flavors can also be applied, e.g., by running the algorithm for $\mathfrak{f} = b$ and then $\mathfrak{f} = c$ and requiring that c jets must not have a b-flavor assignment.

Test of IRC safety in $e^+e^- \to$ jets.—In order to test the IRC safety of the flavor-dressing algorithm, a resolution variable is introduced that allows us to probe the fully unresolved regimes, i.e., restricting all emissions to be only soft and/or collinear. In this limit, the probability of a misidentification of flavors (a “bad” tag) must vanish for any IRC-safe procedure of identifying jet flavor.

For the $e^+e^- \to$ jets process, the correct flavor assignment in the unresolved limit is determined by the underlying Born-level scattering reaction, $e^+e^- \to f\bar{f}$, and therefore corresponds to two jets with a net flavor tag. Jets are defined using the k_T (or “Durham”) algorithm [34], which is not IRC safe in the case of a naive flavor assignment, i.e., simply accumulating the flavors of the jet constituents. A suitable resolution variable for this process is given by the parameter y_3, which determines the transition between identifying an event as a 2-jet or a 3-jet configuration in the Durham algorithm. As such, it allows us to probe the fully unresolved region by inspecting the limit $y_3 \to 0$.

In Fig. 1 we perform a comparison between different prescriptions of assigning flavor to the jets as a function of the y_3 resolution variable. For simplicity, the test is performed by considering all (anti)quarks to carry a single quantum number f ($\bar{f}$). These comparisons are provided for the perturbative coefficients of the cross section up to third order, i.e., $\mathcal{O}({\alpha_s}^3)$, using the calculation of Refs. [35,36]. At first order (not shown), the $e^+e^- \to f\bar{f}g$ process is not yet exposed to the subtleties of flavor creation that jeopardizes IRC safety and also the naive prescription is thus IRC safe. Starting from the second order, however, the naive prescription develops a soft singularity, which manifests itself by the associated curve (solid green) in the upper figure approaching a nonvanishing value in the $y_3 \to 0$ limit. At third order, the IRC-unsafe behavior of the naive prescription becomes more severe, as can be seen in the lower plot; the IRC singularities in this case are no longer confined to the $y_3 \to 0$ regime, but the entire spectrum is ill defined as indicated by the width of the green band that corresponds to varying the internal technical cutoff parameter of the calculation. The flavor-dressing approach (solid blue and dashed orange curves), on the other hand, correctly approaches zero in the limit $y_3 \to 0$ at all considered orders, confirming the IRC safety of the procedure.

The IRC sensitivity of the algorithm to universal all-order effects was also tested in a pp environment and is reported in the Supplemental Material [26].

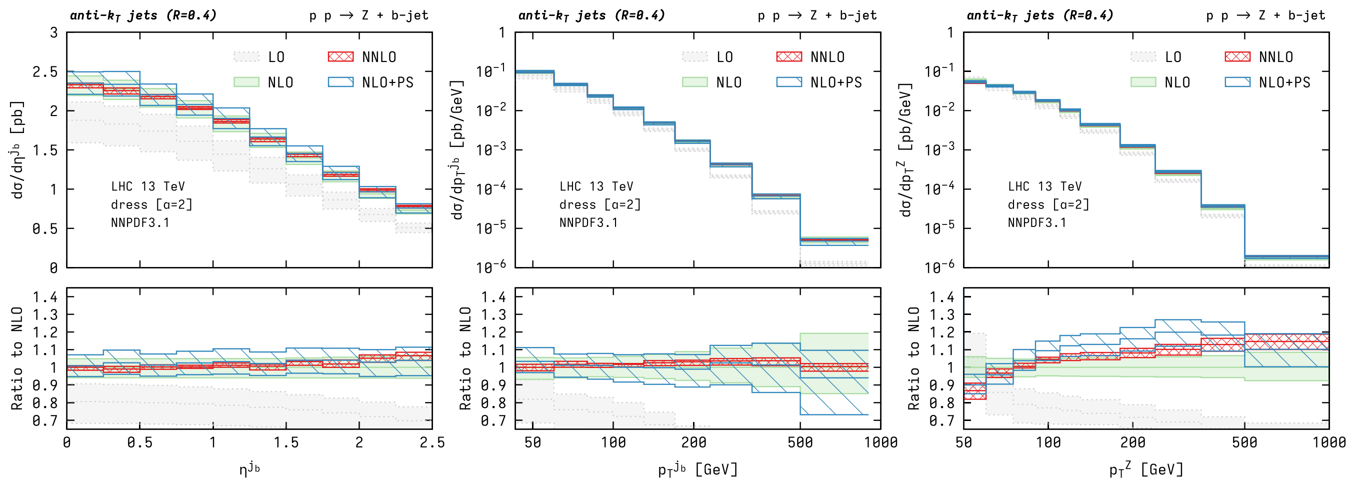

FIG. 2. Comparison between fixed-order and NLO + PS descriptions of the leading flavored jet pseudorapidity (left), transverse momentum (center), and the Z boson transverse momentum (right). Upper panels show the absolute cross-section distributions, while those in the lower panels are normalized to the central NLO prediction. Scale uncertainties are shown in each case.

Application to $Z + b$-jet production.—Beyond the test of IRC safety discussed so far, it is also important to apply and test the flavor-dressing algorithm in realistic scenarios. To do so, we consider the process $pp \to Z + b$ jet, and compare theory predictions based on fixed order (parton level) with those obtained by matching fixed-order predictions with a parton shower (PS) Monte Carlo generator. Comparisons of these predictions, for a range of differential observables, demonstrate the potential sensitivity of the algorithm to universal all-order effects and nonperturbative corrections.

The fixed-order parton-level predictions are obtained up to next-to-next-to-leading order (NNLO) [37], and are compared to hadron-level NLO + PS accuracy generated with MADGRAPH5_AMC@NLO [38] interfaced to PYTHIA8.3 [28] (in total 200 M events were generated). For the parton distribution functions (PDFs), the neural-network PDF set NNPDF3.1 [39] at NNLO with $\alpha_s(M_Z) = 0.118$ and $n_f^{\max} = 5$ is used throughout as provided by the LHAPDF [40] library. The complex-mass and G_μ input schemes are adopted using the values as quoted in Ref. [41]. The central prediction is obtained for the central scale $\mu_0 \equiv E_{T,Z}$, and uncertainties due to the variation of the factorization (μ_F) and renormalization (μ_R) scales by a factor of 2 around μ_0, with the constraint $1/2 \le \mu_F/\mu_R \le 2$, are shown as shaded areas in Fig. 2. The following fiducial selection is applied: $m_{\ell\bar{\ell}} \in [71, 111]$ GeV, $p_{T,\ell(j)} > 27(40)$ GeV, $|\eta_{\ell/j}| < 2.5$, and $\Delta R(\ell, j) > 0.4$. The set of $R = 0.4$ anti-k_T jets passing this fiducial selection are then used as an input to the flavor-dressing algorithm described in this Letter. In the final selection, we additionally require the leading jet to be flavored.

The results of the comparison (for the choice $\alpha = 2$) are shown in Fig. 2 for the pseudorapidity (η_b) and transverse momentum of the leading flavored jet ($p_{T,b}$) and Z boson ($p_{T,Z}$). A good agreement between the fixed-order and hadron-level predictions is found, both in terms of overall normalization and shape of the distributions. At NLO accuracy the agreement is typically within 2%, demonstrating that the algorithm is robust with respect to both nonperturbative (hadronization corrections) and universal all-order effects. The latter is again verified by the fact that the NNLO accurate distribution lies within the uncertainty estimate of both the NLO and NLO + PS predictions. In the case of $p_{T,Z}$ some sensitivity to higher-order corrections is observed, which is indicated by the poorer agreement between NLO and NLO + PS predictions (although typically compatible within uncertainties). This effect can be traced back to flavor creation through gluon emission with a subsequent $g \to f\bar{f}$ splitting, which cannot yet be accessed in a fixed-order NLO prediction. Indeed, we observe that at NNLO the description is greatly improved. As a further test of the IRC safety of the algorithm, it was checked that the NNLO calculation was independent of the internal technical cutoff parameter.

Conclusions and outlook.—In this Letter, a novel approach is proposed for assigning (heavy) flavor quantum numbers to arbitrary flavor-agnostic jets that is IRC safe to all orders. The *flavor-dressing* algorithm accomplishes this by fully disentangling the kinematic reconstruction of jets from the flavor assignment, giving rise to a simple yet very generic approach that can be applied universally to the study of all physics processes that involve flavored jets. While specific choices were made both in the flavor cluster definition and their association with jets, alternatives (some of which we noted) can be considered in view of experimental feasibility [26].

The property of IRC safety is imperative for a robust theoretical definition of flavored jet observables and was explicitly tested for e^+e^- and pp environments—up to order $\mathcal{O}(\alpha_s{}^3)$. The all-orders IRC safety of the proposed algorithm emerges from the use of flavored clusters in combination with the dressing algorithm itself: the former ensures collinear safety of flavored particles with respect to QCD radiation, without introducing sensitivity to soft physics; soft and collinear issues related to flavor creation or annihilation are instead treated by the dressing algorithm and the associated IRC safety properties derive from the use of flavor-k_T inspired distance measures [12]. While the issues of IRC safety are deeply linked to the use of massless quarks in the calculation, such a setup is the basis for the resummation of potentially large mass logarithms $\ln(Q^2/m_q^2)$ to all orders that are otherwise only accounted for to a finite order when quark masses m_q are retained in the calculation. Moreover, an IRC-unsafe prescription introduces a direct sensitivity to such mass logarithms that are not power suppressed and can thus potentially spoil the reliability of the calculation.

As an explicit example, the flavor-dressing algorithm was applied to the process $pp \to Z + b$ jet highlighting its robustness with respect to nonperturbative hadronization effects and higher-order corrections as modeled by parton showers. Allowing for a theoretically rigorous flavor tagging of anti-k_T jets, the flavor-dressing algorithm further resolves the main mismatch between theory and data, paving the way for precision phenomenology using flavored jets.

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[1] A. M. Sirunyan *et al.* (CMS Collaboration), J. High Energy Phys. 10 (2019) 244.
[2] G. Aad *et al.* (ATLAS Collaboration), J. High Energy Phys. 05 (2021) 093.
[3] M. Aaboud *et al.* (ATLAS Collaboration), Phys. Lett. B **786**, 59 (2018).
[4] A. M. Sirunyan *et al.* (CMS Collaboration), Phys. Rev. Lett. **121**, 121801 (2018).
[5] M. Aaboud *et al.* (ATLAS Collaboration), J. High Energy Phys. 05 (2019) 141.
[6] G. Aad *et al.* (ATLAS Collaboration), Eur. Phys. J. C **81**, 178 (2021).
[7] G. Aad *et al.* (ATLAS Collaboration), Phys. Lett. B **816**, 136204 (2021).
[8] G. Aad *et al.* (ATLAS Collaboration), J. High Energy Phys. 05 (2014) 068.
[9] R. Aaij *et al.* (LHCb Collaboration), Phys. Rev. Lett. **128**, 082001 (2022).
[10] A. Tumasyan *et al.* (CMS Collaboration), Eur. Phys. J. C **82**, 1094 (2022).
[11] M. Cacciari, G. P. Salam, and G. Soyez, J. High Energy Phys. 04 (2008) 063.
[12] A. Banfi, G. P. Salam, and G. Zanderighi, Eur. Phys. J. C **47**, 113 (2006).
[13] A. Banfi, G. P. Salam, and G. Zanderighi, J. High Energy Phys. 07 (2007) 026.
[14] S. Caletti, A. J. Larkoski, S. Marzani, and D. Reichelt, Eur. Phys. J. C **82**, 632 (2022).
[15] S. Caletti, A. J. Larkoski, S. Marzani, and D. Reichelt, J. High Energy Phys. 10 (2022) 158.
[16] M. Czakon, A. Mitov, and R. Poncelet, arXiv:2205.11879.
[17] A. Buckley and C. Pollard, Eur. Phys. J. C **76**, 71 (2016).
[18] D. Goncalves, F. Krauss, and R. Linten, Phys. Rev. D **93**, 053013 (2016).
[19] P. Ilten, N. L. Rodd, J. Thaler, and M. Williams, Phys. Rev. D **96**, 054019 (2017).
[20] S. Caletti, O. Fedkevych, S. Marzani, D. Reichelt, S. Schumann, G. Soyez, and V. Theeuwes, J. High Energy Phys. 07 (2021) 076.
[21] O. Fedkevych, C. K. Khosa, S. Marzani, and F. Sforza, Phys. Rev. D **107**, 034032 (2023).
[22] A. J. Larkoski, S. Marzani, G. Soyez, and J. Thaler, J. High Energy Phys. 05 (2014) 146.
[23] M. Cacciari and G. P. Salam, Phys. Lett. B **641**, 57 (2006).
[24] M. Cacciari, G. P. Salam, and G. Soyez, J. High Energy Phys. 04 (2008) 005.
[25] ATLAS Collaboration, Proposal for particle-level object and observable definitions for use in physics measurements at the LHC, Technical Report CERN, Geneva, 2015, http://cds.cern.ch/record/2022743/files/ATL-PHYS-PUB-2015-013.pdf.
[26] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.130.161901 for additional details on the IRC sensitivity of the proposed algorithm in a hadron-collider environment as well as its experimental feasibility, which includes Refs. [12,27–32].
[27] S. Catani, Y. L. Dokshitzer, M. H. Seymour, and B. R. Webber, Nucl. Phys. **B406**, 187 (1993).
[28] C. Bierlich *et al.*, SciPost Phys. **8** (2022).10.21468/SciPost-PhysCodeb.8
[29] R. Aaij *et al.* (LHCb Collaboration), J. Instrum. **10**, P06013 (2015).
[30] G. Aad *et al.* (ATLAS Collaboration), J. Instrum. **11**, P04008 (2016).
[31] A. M. Sirunyan *et al.* (CMS Collaboration), J. Instrum. **13**, P05011 (2018).
[32] R. Aaij *et al.* (LHCb Collaboration), J. Instrum. **17**, P02028 (2022).
[33] S. D. Ellis and D. E. Soper, Phys. Rev. D **48**, 3160 (1993).
[34] S. Catani, Y. L. Dokshitzer, M. Olsson, G. Turnock, and B. R. Webber, Phys. Lett. B **269**, 432 (1991).
[35] A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover, and G. Heinrich, Comput. Phys. Commun. **185**, 3331 (2014).

[36] T. Gehrmann, E. W. N. Glover, A. Huss, J. Niehues, and H. Zhang, Phys. Lett. B **775**, 185 (2017).

[37] R. Gauld, A. Gehrmann-De Ridder, E. W. N. Glover, A. Huss, and I. Majer, Phys. Rev. Lett. **125**, 222002 (2020).

[38] J. Alwall, R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, O. Mattelaer, H. S. Shao, T. Stelzer, P. Torrielli, and M. Zaro, J. High Energy Phys. 07 (2014) 079.

[39] R. D. Ball *et al.* (NNPDF Collaboration), Eur. Phys. J. C **77**, 663 (2017).

[40] A. Buckley, J. Ferrando, S. Lloyd, K. Nordström, B. Page, M. Rüfenacht, M. Schönherr, and G. Watt, Eur. Phys. J. C **75**, 132 (2015).

[41] M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018).