Quenched B_K-parameter with the Wilson and Clover actions at $\beta = 6.0$

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We present results for the kaon B parameter obtained by APE-100 from a sample of 100 configurations using the Wilson action and 200 configurations using the Clover action, on a $18^3 \times 64$ lattice at $\beta = 6.0$. We have also compared the results for B_K using two different values for the effective coupling constant g. We observe a strong dependence of B_K on the prescription adopted for g in the Wilson case, contrary to the results of the Clover case which are almost unaffected by the choice of g.

1. Simulation.

We have computed the light fermion propagators on an ensemble of 100 configurations with the Wilson action and 200 configurations with the Clover action, on a $18^3 \times 64$ lattice at $\beta = 6.0$, using the 6-gigaflop version of APE-100. The light quark masses correspond to the hopping parameter $K = 0.1530, 0.1540, 0.1550$ for the Wilson action and $K = 0.1425, 0.1432, 0.1440$ for the Clover case. With these values, the meson masses are in the range $600 - 900$ MeV. We have considered only pseudoscalar and vector mesons consisting of quarks degenerate in mass. Moreover, the two- and three-point functions have been evaluated for mesons with several momenta. In the three-point functions this corresponds to six different values of the product $(p \cdot q)$, (p, q) being the momenta of the two mesons. All the propagators are thinned. All the errors are jacknife.

2. The B_K parameter.

The kaon B-parameter is a measure of the deviation from the vacuum saturation approximation of the matrix element:

$$\langle \bar{K}^0 \mid \hat{O}^{\Delta S=2} \mid K^0 \rangle = \frac{8}{3} f_K^2 m_K^2 B_K \tag{1}$$

where $\hat{O}^{\Delta S=2} = \bar{s}\gamma_\mu^L d \bar{s}\gamma_\mu^L d$. On the lattice the matrix element (1) is usually parameterized as

$$\begin{aligned} \langle \bar{K}^0 \mid \hat{O}_{latt}^{\Delta S=2} \mid K^0 \rangle &= \alpha + \beta m_K^2 + \gamma (p \cdot q) + \\ +\delta m_K^4 + \epsilon m_K^2 (p \cdot q) &+ \zeta (p \cdot q)^2 + \dots \end{aligned} \tag{2}$$

where $\gamma = \frac{8}{3} f_K^2 B_K$. The parameters α, β are lattice artefacts that should vanish in the continuum limit. Following [1], in order to reduce the mass dependence of the different parameters, we have adopted as fit variables:

$$\begin{aligned} m_K^2 \to X &= \frac{8}{3} \frac{f_K^2}{Z_5 Z_A^2} m_K^2 \\ (p \cdot q) \to Y &= \frac{8}{3} \frac{f_K^2}{Z_5 Z_A^2} (p \cdot q) \end{aligned}$$

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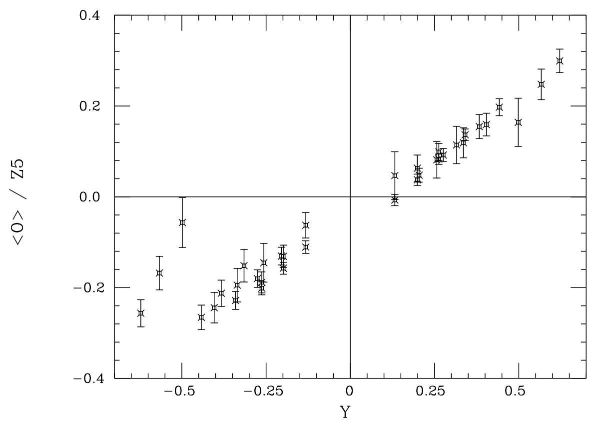

Figure 1. Dependence of $\langle \bar{K}^0 \mid \hat{O}_{\Delta s=2} \mid K^0 \rangle$ on mass and momentum. The points refer to the Wilson action data.

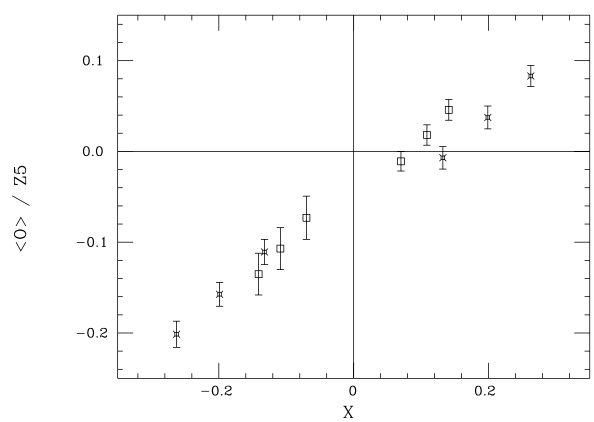

Figure 2. Comparison between Clover (squares) and Wilson (crosses) data for mesons at rest.

where $Z_5 = < 0|\bar{\psi}\gamma_5\psi|K^0 >$ and Z_A is the axial current renormalisation constant. Up to quadratic terms, with $\alpha, \beta, \ldots, \zeta$ appropriately redefined, (2) becomes

$$\frac{1}{Z_5}\langle \bar{K}^0 \mid \hat{O}_{latt}^{\Delta S=2} \mid K^0 \rangle = \alpha + \beta X + \gamma Y + +\delta X^2 + \epsilon XY + \zeta Y^2 \quad (3)$$

Fitting in X and Y substantially reduces higher order terms in $(p \cdot q)$ and m_K^2 (see [1]). When fitting eq.(3), we obtain values for α, β and γ which differ by about 20% to those obtained from the linear fit:

$$\frac{1}{Z_5}\langle \bar{K}^0 \mid \hat{O}_{\Delta s=2}^{latt} \mid K^0 \rangle = \alpha + \beta X + \gamma Y \quad (4)$$

This systematic effect is currently under study. Here we only present results from eq.(4). In Fig.(1) the Wilson results are given for different mass and momentum values. The set of points at negative values of Y represents the matrix element $\langle 0 \mid \hat{O}_{\Delta s=2}^{latt} \mid K^0 K^0 \rangle$. We have not used these points in the measure of B_K, due to the final state interaction [2]. Points corresponding to mesons with high momenta (i.e. the three points on the leftmost side of the figure) receive large contributions from the lattice artefacts, which are to be understood better. In Fig.(2) we present a comparison between Wilson and Clover data for mesons at rest. Some improvement of the chiral behaviour of the matrix element is obtained by using the Clover action instead of the Wilson one.

3. Results.

We present our Wilson (W) and Clover (SW) results where, in the perturbative calculation of the mixing coefficients used in the renormalisation of $\hat{O}^{\Delta S=2}$ [3], we have used either the "naive" coupling or two different "boosting" procedures for the coupling constant á la Lepage-Mackenzie [4]:

1. $g_1^2 = (8K_c)^4 g_0^2 \simeq 2.49\ (W);\ 1.84\ (SW)$
2. $g_2^2 = \frac{1}{\langle \mathrm{Tr}\Box \rangle} g_0^2 \simeq 1.68\ (W);\ 1.68\ (SW)$

As can be seen, the effective coupling depends quite significantly on the prescription in the Wil-

Table 1
Clover(SW) and Wilson(W) results with different choices of effective coupling (see text); $B_K = \gamma/Z_A^2$.

Choice of g	α	β	γ	$B_K(a^{-1} = 2.06GeV)$
Unboosted (SW)	−0.08(2)	0.3(2)	0.65(15)	0.58(13)
Boosting Proc. 1 (SW)	−0.07(2)	0.3(2)	0.65(15)	0.58(13)
Unboosted (W)	−0.09(1)	0.11(8)	0.58(8)	0.77(11)
Boosting Proc. 1 (W)	−0.031(5)	0.04(3)	0.22(3)	0.48(6)
Boosting Proc. 2 (W)	−0.064(9)	0.08(6)	0.42(5)	0.69(8)

son case, while the variation in the Clover (SW) case is small. Our results at $\beta = 6.0$ are shown in the Table, for both the Clover and Wilson actions and for different boosting procedures. In the Clover case, procedures 1. and 2. give essentially the same results. It appears that the Clover value of γ is almost unaffected by the boosting procedure, while the results in the Wilson case are quite unstable and strongly depend on the choice of the effective coupling constant. The one-loop renormalisation group invariant B-parameter is defined as:

$$\hat{B}_K = \left[\alpha_s(\mu = 2GeV)^{-\frac{6}{33-2N_f}}\right]\frac{\gamma}{Z_A^2} \qquad (5)$$

In this expression we use a non-perturbative estimate of $Z_A^{SW} = 1.06$ [5], and the perturbative results $Z_A^W(g_0^2) = 0.87, Z_A^W(g_1^2) = 0.68, Z_A^W(g_2^2) = 0.78$. From (5) we obtain:

- Clover action:

$$\hat{B}_K = (0.74 \pm 0.17) \qquad (6)$$

- Wilson action:

$$\begin{cases} \hat{B}_K = (0.98 \pm 0.14) & \text{unboosted} \\ \hat{B}_K = (0.61 \pm 0.08) & \text{boosted 1} \\ \hat{B}_K = (0.88 \pm 0.10) & \text{boosted 2} \end{cases} \qquad (7)$$

These values reasonably agree with other lattice results [1], [6] (see also A. Soni and T. Bhattacharya at this conference) and are to be compared to other theoretical results, namely those from QCD sum-rules (QCD/SR)[7] and from the $1/N_c$ expansion[8].

$$\begin{cases} \hat{B}_K = (0.39 \pm 0.10) & \text{QCD/SR} \\ \hat{B}_K = (0.70 \pm 0.07) & 1/N_c \end{cases} \qquad (8)$$

In conclusion we stress that the non-linear terms in $m_K^2, (p \cdot q)$ change B_K by about 20%. Moreover, different boosting procedures drastically change the Wilson results, leaving almost unaffected the Clover ones. We need to work on larger lattices and remove thinning in order to reduce finite size effects. It might be useful to adopt some non perturbative renormalisation procedure in order to improve the chiral behaviour of the lattice matrix element [9].

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