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Charmonium Production at the Tevatron

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Abstract

We present in this work a study of large-p_T charmonium production in hadronic collisions. We work in the framework of the factorization model of Bodwin Braaten and Lepage, thereby including the color octet production mechanism, and extract the values of the necessary nonperturbative parameters from a comparison with the most recent data from the Fermilab 1.8 TeV $p\bar{p}$ hadron collider. We extend the calculation to 630 GeV, and compare the results with data published by the UA1 Collaboration. The global agreement is satisfactory, indicating that the largest components of the production mechanisms for charmonium production at high p_T have been isolated.

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1. The production of heavy quarkonium states in high energy processes has recently attracted a lot of theoretical and experimental interest. The successful operation of vertex detectors in hadronic colliders has allowed to disentangle the genuine charmonium yields from the large background due to production and decay of b quarks [1]. The ability to detect the soft photons from χ_c decays has recently allowed the independent measurement of the χ_c contribution to the J/ψ rate [2, 3, 4]. On the theoretical side, charmonium production provides quite stringent tests of our understanding of QCD on the very border between perturbative and nonperturbative domains. The detailed measurements of differential cross sections for production of J/ψ, ψ' and χ_c states can be confronted with theoretical calculations. Starting from the very intuitive Colour Singlet Model [5], these have recently been improved with the inclusion of the mechanism of production of a parton with large transverse momentum, followed by the fragmentation into charmonium states [6]. The inclusion of the fragmentation mechanism has brought the theoretical predictions closer to the observed prompt [2] J/ψ production rate [7]. However the very large discrepancy, by more than an order of magnitude, between the theoretical predictions and the data for the case of the ψ', clearly demands for new mechanisms dominating the production process.

Several proposals have recently been put forward to solve this discrepancy. Among these, the possible existence of higher P-wave or D-wave states which decay into the ψ', or of new metastable or hybrid charmonium states [8]. In this paper we shall concentrate on a third proposal [9], namely the contribution to the fragmentation function of colour octet states, which subsequently evolve nonperturbatively into the ψ' plus soft light hadrons.

More recently, new data on the measurement of the χ_c/ψ fraction [3, 4] confirm that a similar problem exists for J/ψ's not coming form χ_c decays. The aim of the present paper is to make a thorough re-analysis of the full matter, trying to find a coherent picture which possibly accounts for the large J/ψ and ψ' production cross sections, including the new information available on the χ_c production data. The general framework is provided by the analysis of Bodwin, Braaten and Lepage [10], which allows a consistent treatment of short and long distance effects. We will first review the main ingredients of this formalism, and then proceed to our phenomenological analysis.

2. For the reader's convenience and to fix our notation, we briefly review some models which have been suggested in connection with charmonium production. We do not include a discussion of the color evaporation model, for which phenomenological reviews have appeared recently [11]. We start instead presenting the Color Singlet Model (CSM) [5] (for a recent review, see also [12]). The dominant mechanism is assumed to be the short-distance production of a color singlet $Q\bar{Q}$ pair with the same spin and angular momentum quantum numbers of a given quarkonium state H. All the nonperturbative (long-distance) effects that lead to the formation of the bound state are factored into a single phenomenological parameter. Hence the cross section for the production of a state $H = n\,^{2S+1}L_J$ takes the form

$$\sigma[n\,^{2S+1}L_J] = P_{nL}\,\sigma[Q\bar{Q}(n\,^{2S+1}L_J)], \tag{1}$$

where the nonperturbative parameter P_{nL} can then be expressed in terms of the radial wave

[2] Here and in the following we use the term "prompt" to refer to all sources excluding b-decay contributions.

function or its derivatives, and calculated either within potential models (see for example ref.[13]) or extracted from experimental data.

This simple factorization fails in the calculation of the production of P states, for example via $q\bar{q}$ annihilation or in B mesons decays. In fact an infrared singularity appears, associated to a final state soft gluon, and at least a second nonperturbative parameter has to be invoked to absorb it, spoiling the simple minded picture of the CSM.

A rigorous framework for treating quarkonium production and decays has been recently developed by Bodwin, Braaten and Lepage [10]. Their so-called "factorization model" expresses the cross section for quarkonium production as a sum of terms each of which contains a short-distance perturbative factor and a long-distance nonperturbative matrix element, as

$$\sigma[H] = \sum_n \frac{F_n(\Lambda)}{m^{\delta_n - 4}} \langle 0|\mathcal{O}_n^H(\Lambda)|0\rangle \tag{2}$$

F_n are short-distance coefficients which can be calculated in perturbative QCD (pQCD) as power expansions in $\alpha_s(m)$ (the quark mass m being large, $\alpha_s(m)$ is expected to be small enough to allow the perturbative expansion). Λ is a scale which separates short and long distance effects. The Λ dependence of F_n cancels against that of the matrix elements $\langle 0|\mathcal{O}_n^H(\Lambda)|0\rangle$, leaving a cross section independent of Λ. The above matrix elements can be rigorously defined in Non Relativistic QCD (NRQCD) [10]. They absorb the nonperturbative features of the process and can either be extracted from data or calculated on the lattice. Finally, the δ_n are related to the dimension of the operator $\mathcal{O}_n^H$.

The main difference between the factorization model and the CSM is that not all the operators are now related to the production of color singlet $Q\bar{Q}$ pairs. The factorization approach explicitly takes into account the complete structure of the quarkonium Fock space. Therefore the quarkonium H is no more assumed to be simply a $Q\bar{Q}$ pair, but rather a superposition of states:

$$\begin{aligned} |H = n\,^{2S+1}L_J\rangle &= O(1)|Q\bar{Q}(n\,^{2S+1}L_J, \underline{1})\rangle \\ &+ O(v)|Q\bar{Q}(n\,^{2S+1}(L \pm 1)_{J'}, \underline{8})g\rangle \\ &+ O(v^2)|Q\bar{Q}(n\,^{2S+1}L_J, \underline{8})gg\rangle + \dots , \end{aligned} \tag{3}$$

where the labels $\underline{1}$ and $\underline{8}$ refer to the colour state of the $Q\bar{Q}$ pair. Higher order components are suppressed by powers of v, the average velocity of the heavy quark in the quarkonium rest frame. v can be estimated through the relation

$$v \simeq \alpha_s(mv)\,, \tag{4}$$

which for the charmonium yields a value $v^2 \simeq 1/4$. The CSM is recovered by taking only the lowest order term in eq. (2).

The production of the state H in the factorization approach can then proceed via any of the Fock components in eq. (3). Higher order components become important when their short distance coefficients F_n are suppressed by fewer powers of α_s relative to lower order ones. Therefore the contribution of various terms to the production of H depends in general on both

the $\alpha_s(m)$ expansion of F_n and the v^2 expansion of the matrix elements: it is a two parameter problem.

Similar expressions hold within the factorization model also for the fragmentation functions of a parton k into the state H, evaluated at a scale μ larger than the heavy quark mass:

$$D_k^H(z,\mu) = \sum_n \frac{d_n^k(z,\mu,\Lambda)}{m^{\delta_n-6}} \langle 0|\mathcal{O}_n^H(\Lambda)|0\rangle. \tag{5}$$

The d_n^k are, again, short distance coefficients. They can be calculated in pQCD at a scale μ_0 of the order of the quarkonium mass and then evolved to higher scales. After evolution, the cross section is given by the usual convolution:

$$\sigma[H] = \int F^i F^j \sigma_{ij\to k} D_k^H \tag{6}$$

the F's being the parton distribution functions in the colliding hadrons and $\sigma_{ij\to k}$ the kernel cross sections describing the inclusive parton-parton scattering.

3. Within the factorization approach it is possible to relate the matrix elements of the leading operator in the v expansion to those entering the factorization formulae for quarkonium decays. They can therefore be extracted by comparing the measured decay widths to those calculated. In the case of the operators relative to higher components of the Fock space expansion, no simple relation exists in general between decay and production matrix elements [10]. So they should either be calculated (e.g. in lattice QCD), or be measured directly in some production process. We discuss here shortly the cases of interest for our study. More detailed expressions and observations can be found in [10].

In the case of χ_c production, we have the following expressions for cross sections and fragmentation functions to leading order in v^2:

$$\sigma[\chi_J] = \frac{F_1(^3P_J)}{m^4} \langle 0|\mathcal{O}_1^{\chi_J}(^3P_J)|0\rangle + \frac{F_8(^3S_1)}{m^2} \langle 0|\mathcal{O}_8^{\chi_J}(^3S_1)|0\rangle \qquad J=0,1,2 \tag{7}$$

$$D_g^{\chi_J} = \frac{d_1^g(^3P_J)}{m^2} \langle 0|\mathcal{O}_1^{\chi_J}(^3P_J)|0\rangle + d_8^g(^3S_1) \langle 0|\mathcal{O}_8^{\chi_J}(^3S_1)|0\rangle \qquad J=0,1,2 \tag{8}$$

The presence of the color octet matrix elements represents the natural extension of the CSM results, and allows the absorption of the infrared divergences which appear in the short distance coefficients of the color singlet part. Both terms are needed to give a consistent description of χ_c production at this order in v^2.

The matrix elements of color singlet operators can be related to those entering the decay processes $\chi_{J=0,2} \to \gamma\gamma$ and $\chi_{J=0,2} \to$ light hadrons. The color octet production matrix element, however, cannot be related to the corresponding decay one [14]. We have to resort to a production process to measure it. In [14] it was suggested to use the χ_c production in B decays. The results have been reported in the literature in terms of the nonperturbative parameters H_1 and H_8' for the color singlet and color octet parts respectively. They are related to the NRQCD matrix element as follows:

$$H_1 = \frac{1}{m^4} \frac{\langle 0|\mathcal{O}_1^{\chi_J}(^3P_J)|0\rangle}{2J+1} \tag{9}$$

$$H_8' = \frac{1}{m^2} \frac{\langle 0|\mathcal{O}_8^{\chi_J}(^3S_1)|0\rangle}{2J+1} \tag{10}$$

In ref.[14] H_1 was obtained fitting the $\Gamma(\chi \to \text{light hadrons})$, whereas in ref.[9] H'_8 was extracted from the CLEO measurement of $BR(B \to \chi_{J=1,2} + X)$ [15]. The values found are:

$$H_1 \approx 15 \text{ MeV} \tag{11}$$

$$H'_8 = 1.4 \pm 0.6 \text{ MeV} \tag{12}$$

Let us now consider ${}^3S_1$ states, i.e. J/ψ and ψ'. We will collectively indicate these as ψ. The cross section and fragmentation function for producing a ψ to leading order in v^2 are simply given by the CSM results:

$$\sigma[\psi] = \frac{F_1({}^3S_1)}{m^2}\langle 0|\mathcal{O}_1^{\psi}({}^3S_1)|0\rangle \tag{13}$$

$$D_k^{\psi} = d_1^k({}^3S_1)\langle 0|\mathcal{O}_1^{\psi}({}^3S_1)|0\rangle \tag{14}$$

The matrix element appearing in the above equations can be shown to be related to the standard nonrelativistic wave function $\overline{R_\psi}$ as follows [10]:

$$\langle 0|\mathcal{O}_1^{\psi}({}^3S_1)|0\rangle \simeq \frac{9}{2\pi}\left|\overline{R_\psi}\right|^2 \tag{15}$$

They can therefore be extracted from the measurement of the leptonic decay width of the ${}^3S_1$ states, or can be calculated within potential models [13].

While no color octet contribution appears at order v^2, it has recently been argued by Braaten and Fleming [9] that yet higher order terms can however be significantly enhanced since their short distance coefficient appears at lower orders in the α_s expansion. An example of this is gluon fragmentation to ψ. To leading order in v^2 the fragmentation proceeds through a color singlet ${}^3S_1$ state, and starts to order ${\alpha_s}^3$:

$$D_g^{\psi} = {\alpha_s}^3 \hat{D}_1 \langle 0|\mathcal{O}_1^{\psi}({}^3S_1)|0\rangle \tag{16}$$

This is because production of the $c\bar{c}$ pair in a color singlet state requires emission of two perturbative gluons. On the other hand, the fragmentation process where the gluon goes into a color octet state, although suppressed by v^4, starts at order α_s:

$$D_g^{\psi} = \alpha_s \hat{D}_8 \langle 0|\mathcal{O}_8^{\psi}({}^3S_1)|0\rangle \tag{17}$$

It can therefore be numerically relevant when compared to (16). No decay process is known which is dominated by the color octet component, and therefore it is not possible to extract the relative matrix elements from decay widths. One could get a crude estimate of their values by rescaling the color singlet matrix elements by the appropriate powers of v. We prefer here to take their value as a free parameter, to be fitted to the Tevatron production data. We will verify at the end that the results are consistent with the expected v^4 suppression.

4. We now present a comparison between experimental results and the calculations illustrated above. We first concentrate on results from the Tevatron collider, relative to high-p_T production of J/ψ, ψ' and χ_c states. Since we will present results for prompt production, we will only make use of the CDF data, for which the b-decay background has been removed [4]. It is

important to stress, nevertheless, that there is perfect agreement between the CDF and the D0 data when all sources of J/ψ are included [3]. We will extract the values of the nonperturbative parameters H_8', $\langle \mathcal{O}_8^{\psi}(^3S_1)\rangle$ and $\langle \mathcal{O}_8^{\psi'}(^3S_1)\rangle$ from fits to the experimental data. We will then use these values to "predict" the inclusive J/ψ p_T distributions at the energy of 630 GeV, where data are available from the measurements performed by the UA1 experiment [17].

For the calculation of large p_T charmonium production in $p\bar{p}$ collisions at the Tevatron energy ($\sqrt{s} = 1800$ GeV) we include the following contributions:

1. Direct production of charmonium states. The matrix elements were calculated in Ref. [5]. As previously noted in literature [7], this contribution is very small compared to the fragmentation one.

2. Production via fragmentation of gluons and charm quarks. These contributions were considered in ref.[7], where it was shown that they greatly enhance the cross sections with respect to the direct terms. We use the fragmentation functions of gluon to ψ [6], charm to ψ [18] and gluon to χ [19].

3. Production via fragmentation into color octet states.

We will also show separately the contributions to J/ψ and χ_c production coming from the decay of b quarks, evaluated at the next-to-leading order [20] using a choice of renormalization and factorization scales which provides the best fit to the Tevatron data [21].

All the charmonium cross sections are evaluated at leading order with the MRSA [22] parton distribution set. The renormalization/factorization scale is set at $\mu = \sqrt{{p_T}^2 + M_\psi^2}$. We have checked that using for μ the p_T of the fragmenting parton produces differences which are typically of order 10-20%, therefore definitely less than the other uncertainties involved.

Fig. 1 shows the comparison between theory and CDF data [4] for prompt ψ' production. In this and in the following figures we have not shown the band due to theoretical uncertainties due to, for example, the choice of the renormalization, factorization and fragmentation scales [7]. This is because these uncertainties mostly affect the overall normalization of the curves, and not their shape. As a consequence, their effect would be hidden by a rescaling of the fitted value of the nonperturbative parameters.

As was already shown in the work of Braaten and Fleming [9], the theoretical curve agrees well with the shape of the data. The old theoretical prediction from pure color singlet fragmentation was known to fall a factor of 30 below the CDF data, as shown in the figure. The addition of the color octet mechanism reconciles theory and data. The value we extract for $\langle \mathcal{O}_8^{\psi'}(^3S_1)\rangle$ from a best χ^2 fit is 4.3×10^{-3} GeV^3, close to what derived in [9].

Fig. 2 shows the inclusive p_T distributions of J/ψ *not* coming from χ_c decays. Both prompt and b-decay contributions are shown separately. The value we extract for $\langle \mathcal{O}_8^{\psi}(^3S_1)\rangle$ from a best χ^2 fit to the prompt data is 15×10^{-3} GeV^3. Without inclusion of the color octet components, the disagreement between theory and data would be similar that that previously noted for ψ''s, namely a factor of the order of 30.

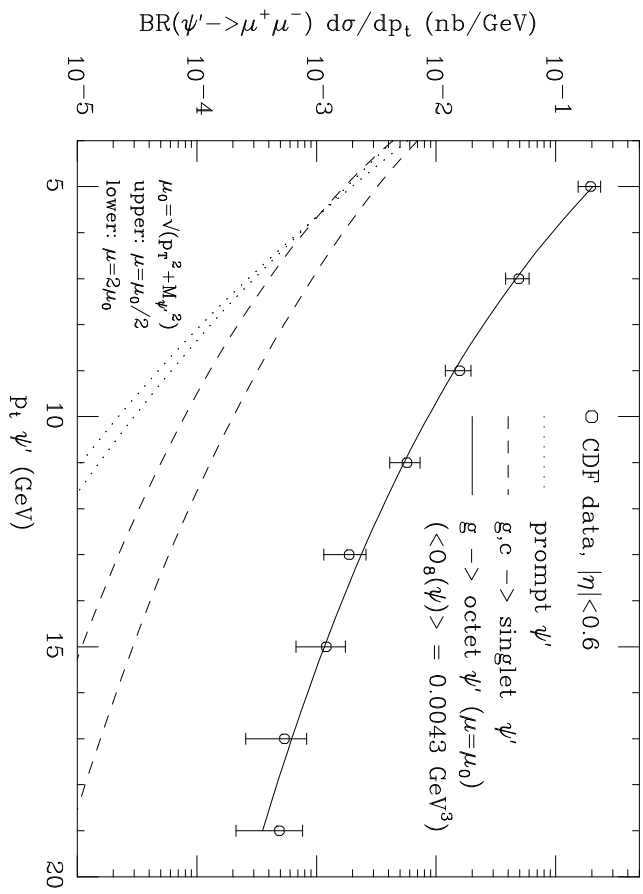

Figure 1: Inclusive prompt ψ' p_T distribution. CDF data versus theory. The contribution from different sources is shown.

The ratio $\langle \mathcal{O}_8^{\psi}(^3S_1)\rangle / \langle \mathcal{O}_8^{\psi'}(^3S_1)\rangle$ is about 3, which is consistent with the ratio of the values of the color singlet wave functions at the origin. In other words, the values one extracts from the two independent sets of data are not unnatural within the color octet scheme. An intrinsic uncertainty in the comparison of these numbers comes from the ambiguity present in the choice of the mass scales. For example, using M_ψ or $M_{\psi'}$ as opposed to $2m_c$ in the coefficient function, can lead to variations up to a factor of 2 in the fit results, and in their ratios. As for the absolute value of the matrix elements, these are consistent with a suppression of the order of $v^4 \simeq 0.06$ relative to the color singlet ones.

A precise prediction of the color octet mechanism, however, is that for sufficiently large p_T the ratio of the J/ψ and ψ' cross sections should be a constant. Current data do not fully support this expectation. It is in fact possible to obtain a good fit to the data shown in Figs. 1

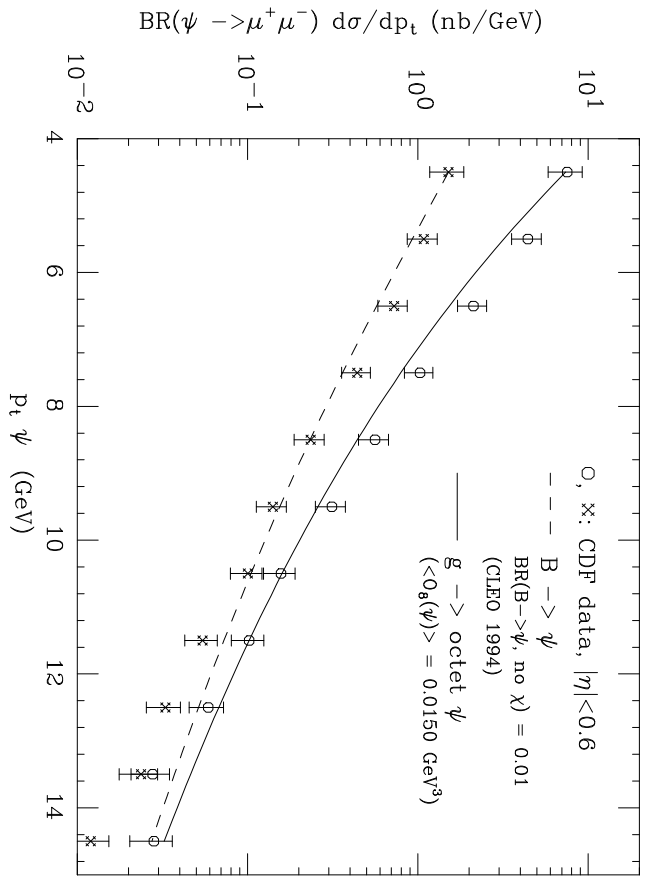

Figure 2: Inclusive ψ p_T distribution. Upper curves and data points correspond to prompt ψ's, after subtraction of the χ_c contribution. Lower ones correspond to the b decay contribution. CDF data versus theory.

and 2, in the common range $4 < p_T < 15$ GeV, using the following parametrizations [23]:

$$\frac{\mathrm{d}\sigma(J/\psi)}{\mathrm{d}p_T} = 1773 \exp(-p_T/1.65) \tag{18}$$

$$\frac{\mathrm{d}\sigma(\psi')}{\mathrm{d}p_T} = 384 \exp(-p_T/1.79) \tag{19}$$

These fits predict a ratio which is rising with p_T, varying from 0.2 to 0.4 (after removing the BR's) in the p_T range currently accessible. Taking into account the experimental uncertainties, this is not inconsistent with a constant ratio. If however future improved statistics should confirm this trend, this would be a clear indication that yet more mechanisms, such as multiple decays of higher charmonium resonances, are at work.

To conclude the survey of charmonium production at the Tevatron, we present in Fig. 3 the p_T distribution of J/ψ's coming from χ_c decays. The theoretical curves include the effect of p_T

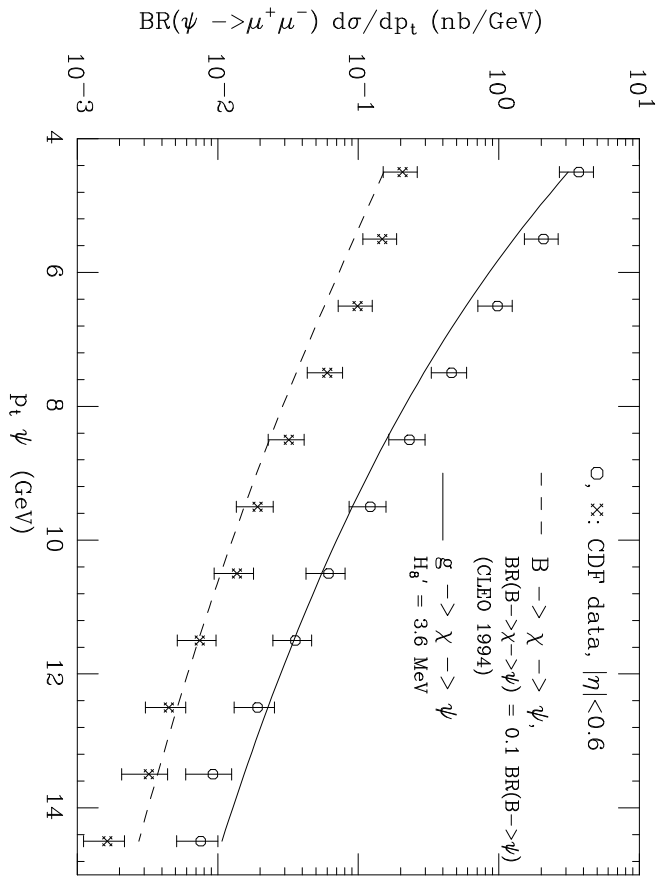

Figure 3: Inclusive p_T distribution of ψ's from χ_c production and decay. Upper curves and data points correspond to the prompt component. Lower ones correspond to the b decay contribution. CDF data versus theory.

smearing due to the $\chi_c \to J/\psi$ decays. Both theory and data use the recent determination of $BR(b \to \chi_{cJ} + X)$ from CLEO [15] to extract the $b \to \chi_c \to \psi\,\gamma$ contribution.

The best χ^2 fit to the prompt data gives a value of H_8'=3.6 MeV. The shapes of theory and data are consistent with each other, although the agreement is not as good as in the case of ψ'. The value of H_8' is larger by a factor of 2 relative to that measured by CLEO using $b \to \chi_c$ decays, Eq. 12. It should be kept in mind that the values extracted from the fits to the hadron collider data are directly sensitive to the perturbative K factors due to higher order corrections to the hard process matrix element, and to the fragmentation functions. As a reference, the NLO K factor for the production of a large p_T gluon was evaluated to be approximately 1.5 in the work of Cacciari and Greco, Ref. [7].

Having fixed the values of the nonperturbative parameters using the Tevatron data, it is possible to use them to make predictions for different energies and different beam types. We

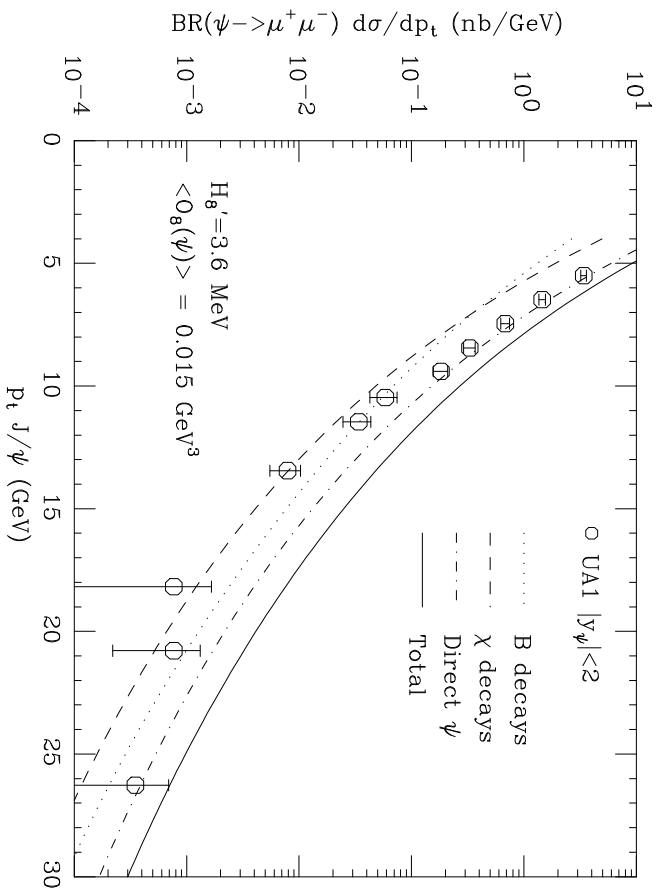

Figure 4: Inclusive p_T distribution of ψ's at 630 GeV. All sources of ψ production are here included. UA1 data versus theory. The parameters of the theoretical calculation take the values fitted on the Tevatron data.

consider here data published by the UA1 Collaboration [17], relative to interactions at the 630 GeV CERN $S\bar{p}pS$ Collider. We expect that at this energy and at the p_T values measured by UA1 the production mechanisms should be exactly the same as those active at the Tevatron. This would be true even in presence of additional processes, such as for example production and decay of higher charmonium resonances.

We present these data in Fig. 4, together with the theoretical predictions. UA1 measured the J/ψ p_T spectrum inclusive of all contributions from b- and χ_c-decays. The contribution of ψ' decays, once convoluted with $BR(\psi' \to J/\psi)$ and with the decay spectrum, amounts to much less than 10% of the total, and was neglected here. The comparison shows that theory predicts now approximately a factor of two more J/ψ's than are observed. In view of what was said few lines above, this is contrary to our expectations. We only see one possible explanation for this discrepancy, leaving out experimental systematics. Namely the significant difference in perturbative K factors at the two energies. It has been observed since a long time that b

production cross sections at 1.8 TeV are systematically higher than theory. Agreement with NLO QCD can be found only by choosing extreme values of the renormalization scale, or choosing values of α_s larger than the input parton distribution sets prescribe, in addition to fixing the b mass to the relatively low value of 4.5 GeV[21]. By choosing as input parameters for the theoretical evaluation of the b cross section at 630 GeV the same values that fit the normalization of the Tevatron data, one finds a result which is approximately 30-40% higher than the UA1 data [24]. A justification for such a discrepancy can be found in the study of small-x effects in heavy quark production at high energy [25]. It is expected that such effects should be larger for production of c quarks, although no detailed estimate exists. If this were indeed the case, however, the relative discrepancy of a factor of 2 between charmonium production at the 1800 and at 630 GeV, as found without inclusion of small-x effects, would be consistent with the similar discrepancy by a factor of 30-40% found in the case of b production.

5. We considered in this paper the large p_T production of J/ψ, ψ' and χ_c states via gluon fragmentation into the leading color singlet and color octet components of their wave function. The result of a previous study by Braaten and Fleming of ψ' production extends to the case of the J/ψ, showing that these effects can explain the unexpectedly large rate of prompt J/ψ and the small χ_c/ψ production ratio observed at the Tevatron. The values of the nonperturbative parameters needed to parametrize these production processes turn out to be consistent with what naively expected.

The extension of these calculations to the case of inclusive J/ψ production at 630 GeV results in rates which are approximately a factor of 2 larger than measured by UA1. We attribute this discrepancy to a larger K factor at the higher energy, due to more important small-x effects.

Current data from the Tevatron and the residual theoretical uncertainties cannot exclude the presence of yet additional production mechanisms, such as the production and decay of higher resonances. Dominance of the production via fragmentation into the color octet component of the ${}^3S_1$ states strictly predicts $\psi'/J/\psi$ to be a constant, at least for $p_T \gg M_\psi$. Current data do not support this conclusions, although the statistical uncertainty is still large.

This formalism cannot be directly applied to the calculation of total cross sections, or to the region $p_T < M_\psi$. This is because in this region the fragmentation approximation is not justified. The effect of color octet production, however, can be calculated including the full set of relevant Feynman diagrams. After this work was completed, we received a paper by Cho and Leibovich [26] in which this calculation has been performed, and applied to charmonium and small-p_T Υ production at the Tevatron. The results of their work are consistent with ours over most of the p_T range covered by the charmonium data. Their value of H_8' is smaller than what we find, presumably because of the the absence in their calculation of the *negative* color singlet contribution to χ_c production [19]. Their value of $\langle \mathcal{O}_8^{\psi'}({}^3S_1) \rangle$ is larger than ours, presumably because their calculation correctly incorporates the small p_T decrease in rate relative to the fragmentation approximation. The values of the nonperturbative parameters extracted from the fits to the Tevatron data can then be used, in association to the matrix elements evaluated in [26], to perform more precise predictions of total cross sections at fixed target energies, where a large amount of data is available.

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