Fusion of Halo Nuclei

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MIT-CTP-2389 To appear in: *Nucl. Phys. A* December 1994

The sub-barrier fusion of halo nuclei such as ^{11}Be and ^{11}Li with heavy target nuclei is discussed. The couplings to the soft dipole mode as well as to the break-up channels are taken into account. At barrier energies, the fusion cross section was found to be more than one order of magnitude *smaller* than that corresponding to the uncoupled problem. At sub-barrier energies, it was shown to be *larger*.

1. INTRODUCTION

The most conspicuous feature of halo nuclei is their abnormal sizes: ^{11}Li has practically the same rms radius as that of ^{208}Pb. One question that arises in this connection is how would this abnormality affect the sub-barrier fusion of these exotic nuclei. Such a question is of great relevance to astrophysics, as several of the important nuclear reactions which take part of evolution cycles involve neutron or proton rich nuclei [1,2].

Two aspects of halo nuclei of particular relevance to sub-barrier fusion are: the existence of a very low lying giant dipole resonance, the so-called pygmy resonance, and the relative ease with which these nuclei undergo break-up, owing to the extremely small Q-value (for ^{11}Li, $Q < 1$ MeV). Whereas the coupling of the entrance channel to the pygmy resonance enhances fusion, the break-up coupling reduces it. The result of the action of these competing effects is an abnormal fusion excitation function, that shows a small dip in the barrier region [3,4].

In the following, we describe the theoretical effort to understand the fusion of halo nuclei and comment on the different approaches to the problem. We concentrate our discussion on the ^{11}Li + ^{208}Pb system and comment on the one neutron halo ^{11}Be nucleus.

2. VIBRATIONAL ENHANCEMENT vs. BREAK UP HINDRANCE

The calculation of the fusion excitation function for ^{11}Li + ^{208}Pb at above and below barrier energies, has been reported in Refs. [3] and [4]. The coupling to the pygmy resonance, treated in the sudden limit ($Q = 0$), results in a tunneling probability which is the average of two eigen-barrier tunneling probabilities. These eigen-barriers are given by $V_B \pm V_C$, where V_B is the height of the bare Coulomb barrier, and V_C represents the coupling to the pygmy resonance. The enhancement in the fusion at sub-barrier energies ensues since the penetrability for $V_B - V_C$, which dominates the average referred to above, is much larger than the bare penetrability for the bare barrier V_B.

The break-up effect is accounted for through the appropriate dynamic polarization potential, V_{bup}, whose real part, $\mathrm{Re}\{V_{bup}\}$, supplies a small repulsion and whose imaginary part, $\mathrm{Im}\{V_{bup}\}$, leads to long range absorption. The formula used in Ref. [4], can be compactly written as

$$\sigma_F = \frac{\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)\tfrac{1}{2}\left[T_\ell^f(E_+) + T_\ell^f(E_-)\right]\cdot \exp\left\{\frac{2}{\hbar}\int_{R_\ell}^{\infty}\frac{\mathrm{Im}\{V_{bup}(r)\}}{v_l(r)}\,dr\right\}, \quad (1)$$

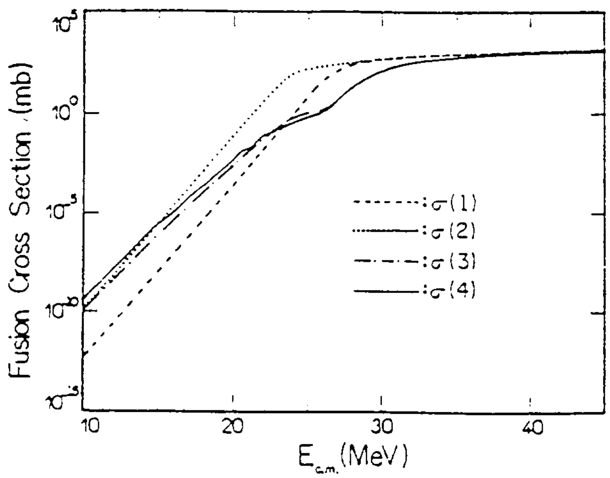

Figure 1. Fusion cross section for the ^{11}Li + ^{208}Pb system as a function of incident energy. The solid line corresponds to σ_F (calculated with the above equations), the dotted line corresponds to one dimensional tunneling through the optical potential, and the dashed line to σ_F with no break-up coupling.

where $E_{\pm} = E \pm V_C$ is the collision energy shifted by the coupling to the pygmy resonance, $v_\ell(r)$ in the unperturbed local velocity and R_ℓ is the classical turning point, determined from the condition $v_\ell(R_\ell) = 0$. The eigen-barrier tunneling transmission coefficients, $T_\ell^f(E_\pm)$ can be approximated by the Hill-Wheeler form

$$T_\ell^f(E_\pm) = \left\{ 1 + \exp\left[\frac{2\pi}{\hbar w} \left(E_\pm - \tilde{V}_B - \frac{\hbar^2 \ell(\ell+1)}{2\mu \tilde{R}_B^2} \right) \right] \right\}^{-1}, \tag{2}$$

where the potential barrier, including the contribution from the polarization potential, $\mathrm{Re}\{V_{bup}(r)\}$, has been parametrized by a parabolic shape with barrier height $\tilde{V}_B$, radius $\tilde{R}_B$ and curvature $\hbar w$.

Applying the above formulae to the fusion of ^{11}Li with ^{208}Pb, one gets the results shown in Fig. 1, which were reported in Ref. [4]. In the barrier region, the fusion cross section, σ_F, is significantly smaller than that predicted by one-dimensional tunneling through the optical potential ($V_C = V_{bup} = 0$) $\bar{\sigma}_F$. As the energy is lowered below the barrier, the break-up channel becomes effectively virtual, leading to a vanishing $\mathrm{Im}\{V_{bup}\}$ and to a small attractive $\mathrm{Re}\{V_{bup}\}$. In this case, the coupling to the pygmy resonance dominates and σ_F is enhanced over $\bar{\sigma}_F$. It would be very interesting to verify these findings experimentally. The recent measurement of the ^{11}Be + ^{238}U and the ^{11}Be + ^{208}Pb fusion [5], would help testing the above ideas.

3. FORMAL CONSIDERATIONS

A more natural formal framework in which to discuss the issue of vibration enhancement vs. break-up hindrance is to consider the effect of the coupling to a resonant state in one of the colliding nuclei. The width of the resonance is composed of a spreading width, $\Gamma^\downarrow$, plus an escape width, $\Gamma^\uparrow$. The spreading width accounts for the coupling of the resonance to fine structure states in the host nucleus, whereas the escape width measures the degree of coupling of the resonance to open decay channels (γ, particle, fission, etc.). In a recent paper [6], Hussein and Toledo Piza solved a schematic tunneling problem that contains the above features. In this section we give a short account of this work [6,7].

The resonant state is treated as an exit doorway. Therefore, the full wave function of the scattering problem can be decomposed as

$$|\Psi> = p|\Psi> + d|\Psi> + q|\Psi> + b|\Psi> \tag{3}$$

where p, d, q, and b are usual projection operators ala Feshbach [8]. Notice that p projects, onto the entrance channel while d, q and b project respectively onto the exit doorway (the giant resonance in one of the nuclei), onto the many dense excited states fed from the elastic channel through the doorway, and onto the several open exit channels to which d is coupled. The states $b|\Psi>$ represent, in general, three-body channels (we ignore γ-decay of d). Notice that all the channels, $p|\Psi>$, $d|\Psi>$, $q|\Psi>$, and $b|\Psi>$ are *open channels.* The closed channels, that represents the compound nucleus, have been eliminated in the usual manner and their effects are represented, on the average, by appropriate complex interactions (optical potentials) in the the different open channels.

The coupled channels equations describing our two-nuclei systems can be explicitly written down using the usual Feshbach projection method [8]. Eliminating the $q|\Psi>$ and $b|\Psi>$ components in favor of the exit doorway, and performing an appropriate average we find

$$\begin{aligned}\left[E - K_0 - U_0 - V_0^{pol}(b)\right]\Psi_0^{(+)} &= V_{od}\Psi_d^{(+)} \\ \left[E - K_d - U_d - V_d^{pol}(b)\right]\Psi_d^{(+)} &= V_{do}\Psi_0^{(+)} + \left(E_d - \frac{i\Gamma_d^{\downarrow}}{2}\right)\Psi_d^{(+)}\end{aligned} \tag{4}$$

where we have introduced the usual dynamic polarization potential that accounts for the coupling of $\Psi_0^{(+)}$ to $\Psi_b^{(+)}$ and $\Psi_d^{(+)}$ to $\Psi_b^{(+)}$. In deriving Eq. (4), we have employed the approximation $V_0^{pol}(b) \equiv V_{ob}G_b^{(+)}V_{bo}$ and $V_d^{pol}(b) = V_{db}G_b^{(+)}V_{bd}$ where $G_b^{(+)}$ represents propagation the three-body break up channel. Also $\Psi_o \equiv p|\Psi>, \Psi_d \equiv d|\Psi>$ and $\Gamma_d^{\downarrow}$ is the spreading width of the doorway state.

If the break-up channel is the only decay channel of the doorway, such as in ^{11}Li, then the imaginary part of the polarization potential, V_d^{pol}, would represent the escape width, $\Gamma_d^{\uparrow}$. The above discussion considering the break-up coupling effects suggests writing

$$\begin{aligned}\Phi_0 &= C_0\Psi_0 \\ \Phi_d &= C_d\Psi_d\end{aligned} \tag{5}$$

The functions C_0 and C_d, which are related to the escape width, can be chosen so as to have Φ_0 and Φ_d satisfy the break-up uncoupled equations

$$\begin{aligned}(E - K_0 - U_0)\,\Phi_0 &= V_{od}\Phi_d \\ (E - K_d - U_d)\,\Phi_d &= V_{do}\Phi_0 + \left(E_d - i\frac{\Gamma_d^{\downarrow}}{2}\right)\Phi_d\end{aligned} \tag{6}$$

Assuming, as in Ref. [3], $U_0 \sim U_d, U_{od} = V_{do} \equiv V =$ constant, Eq. (6) can be diagonalized, with an appropriate biorthogonal transformation. The functions C_0 and C_d, asymptotically are given, respectively, by

$$\begin{aligned}C_0 &= \exp\left[-\frac{i}{\hbar}\int_0^{\infty} V_0^{pol}(r(t))dt\right] \\ C_d &= \exp\left[-\frac{i}{\hbar}\int_0^{\infty} V_d^{pol}(r(t))dt\right]\end{aligned} \tag{7}$$

Putting things together, we obtain the expression for the average value of σ_F, given in Eq. (4) of Ref. [6]. The expression, not given here for lack of space, depends on $\Gamma_d^{\downarrow}$ and $\Gamma_d^{\uparrow}$ in a qualitatively different way.

It is found in [6] that if $\Gamma_d^{\downarrow} \sim \Gamma_d$, the fusion would be slightly more enhanced than the zero width case (Eq. (1)), whereas if $\Gamma_d^{\uparrow} \sim \Gamma_d, \sigma_F$ would be hindered. The reason why $\Gamma_d^{\uparrow}$ hinders fusion is ease to understand. The break-up channel involves a continuum of states with a corresponding large density. Thus the system once in this intermediate configuration, would find it very hard to find its way back to fusion. This reminds one of irreversible processes in which phase space dictates the scenario. Therefore the break-up coupling induces a reduction in fusion.

We should stress that Eq. (24) of [6] represents the average fusion cross-section, which contains the effect of the coupling to the fine structure states, on the average. Clearly there is a fluctuation contribution to σ_F. Thus in general, one has

$$\sigma_F = <\sigma_F> + \sigma_F^{\text{fluc}}\ , \tag{8}$$

where $\sigma_F^{\rm fluc}$ is expected to depend on the value $\Gamma_d^{\downarrow}/\Gamma_d$. Full account of the contribution of $\sigma_F^{\rm fluc}$, will be reported elsewhere [9].

4. COMPARISON TO OTHER WORK

In this section we make brief comments on two recent papers addressing the fusion of halo nuclei. The paper of Takigawa, Kuratani and Sagawa [10] follows basically the same discussion as that presented in Section 2 and reach similar conclusions concerning the hindrance of σ_F in the barrier region (these authors obtain a smaller reduction in σ_F). They do not, however, take into account the effect of the real part of V_{bup}, which is repulsive and, therefore, helps reducing σ_F. The authors of Ref. [10] go further and suggest that the fusion cross-section should be

$$\sigma_F = \sigma_F(^{11}\mathrm{Li}) + \sigma_F(^{9}\mathrm{Li})\,, \tag{9}$$

where $\sigma_F(^{11}\mathrm{Li})$ is the complete fusion of the intact ^{11}Li and $\sigma_F(^{9}\mathrm{Li})$ is the incomplete fusion of the ^{9}Li fragment, after ^{11}Li is broken. According to [10], the sum in Eq. (9) does not show any reduction. We stress here the distinction between complete fusion and incomplete fusion. The quantity of real relevance is complete fusion, which, as we have already seen, shows the hindrance-enhancement feature.

Quite recently, Dasso and Vitturi [11] have gone at length in criticizing the works of Refs. [3] and [10]. After claiming that they have included the effects of the couplings with the pygmy resonance and with break up states, these authors reach the conclusion that the break up coupling produces further *enhancement* in the complete fusion cross section. However, they have treated the break up states as a *single channel* (like a transfer channel) and not a *continuum of channels* as physics dictates. Therefore, we believe that their calculation does not contain the main feature of continuum coupling and their conclusions may be misleading.

ACKNOWLEDGEMENTS

Supported in part by funds provided by the U.S. Department of Energy (D.O.E.) under cooperative research agreement DE-FC02-94ER40818 and by FAPESP (Brazil).

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