Addendum: Ultraviolet behaviour of Higgs inflation models

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Abstract: This article is an addendum to [1]. We extend our computation of the on-shell scattering amplitudes in an arbitrary inflaton background $\bar{\phi}_1$. Although the effective Einstein frame cutoff for $\bar{\phi}_1 \gg M_P/\sqrt{\xi}$ turns out to be $\bar{\phi}_1$ or $\xi\bar{\phi}_1$ for the U(1) model, this is not the case for the realistic doublet Higgs model where the effective Einstein frame cutoff turns out to be the standard $M_P/\sqrt{\xi}$ for both the Palatini and metric formulations. However, as it has been pointed out in [1] the background $\bar{\phi}_1$ is the effective Jordan frame cutoff in all cases for both the Palatini and metric formalisms.

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1 Framework

We start by reminding the set-up of our computation. We consider the usual action of Higgs inflation, ignoring the potential to focus on the effects of the non minimal coupling:

$$\mathcal{S} = \int d^4 x \sqrt{-g} \left(\frac{1}{2} M_P^2 \Omega^2 R - |\partial H|^2 \right) \quad \text{with} \quad \Omega^2 = 1 + \frac{2\xi |H|^2}{M_P^2} \qquad (1.1)$$

calculations with this action are simpler in the Einstein frame, in which we go with the transformation:

$$g_{\mu\nu} \to \Omega^{-2} g_{\mu\nu}, \quad g^{\mu\nu} \to \Omega^2 g^{\mu\nu}, \quad \sqrt{-g} \to \Omega^{-4}\sqrt{-g}, \quad R \to \Omega^2 R - \underline{3/2 \Omega^{-2}(\partial \Omega^2)^2} \quad (1.2)$$

the underlined term in the transformation of R being present in the metric formalism but not in Palatini.

From now, we will only work in the Einstein frame. The action then becomes:

$$\mathcal{S} = \int d^4 x \sqrt{-g} \left(\frac{1}{2} M_P^2 R - \frac{1}{\Omega^2} |\partial H|^2 - \underline{\frac{3M_P^2}{4} \frac{(\partial \Omega^2)^2}{\Omega^4}} \right) \qquad (1.3)$$

in [1], we considered the Higgs to be a complex singlet $H = (\phi_1 + i\phi_2)/\sqrt{2}$. Although not "realistic", it seemed enough to make our point on the ultraviolet behaviour. Then, we introduced by hand a large, inflationary background $2\bar{H}^2 \gg M_P^2/\xi$, taken in the direction of ϕ_1 without loss of generality; $\phi_1 = \bar{\phi}_1 + \phi_1'$. In this notation, ϕ_1' is then the physical Higgs and ϕ_2 is a Goldstone boson. After doing so, we expanded the different terms of the action to find the following interactions between ϕ_1' and ϕ_2, using $\bar{\phi}_1^2 \gg M_P^2/\xi$. This

results into several simplifications. For instance, in the Palatini formalism we obtain the following lagrangian terms in a self-explanatory notation:

$$\mathcal{L}_{\chi_1'^3} = \frac{\sqrt{\xi}}{M_P} \chi_1' (\partial \chi_1')^2, \quad \mathcal{L}_{\chi_1' \chi_2^2} = \frac{\sqrt{\xi}}{M_P} \chi_1' (\partial \chi_2)^2, \quad \mathcal{L}_{\chi_1'^2 \chi_2^2} = -\frac{3\xi}{2M_P^2} \chi_1'^2 (\partial \chi_2)^2 + \frac{\xi}{2M_P^2} \chi_2^2 (\partial \chi_1)^2$$
(1.4)

here, $\chi_1' = M_P \phi_1'/(\sqrt{\xi}\bar{\phi}_1)$ and $\chi_2 = M_P \phi_2/(\sqrt{\xi}\bar{\phi}_1)$ are the canonically normalized counterparts of ϕ_1', ϕ_2. From these interactions, we deduce vertices, for instance, the vertex coming from $\mathcal{L}_{\chi_1'^3}$ is:

$$V_{\chi_1'^3} = -\frac{2i\sqrt{\xi}}{M_P}(p_{\chi_1',1} \cdot p_{\chi_1',2} + p_{\chi_1',1} \cdot p_{\chi_1',3} + p_{\chi_1',2} \cdot p_{\chi_1',3})$$
(1.5)

where the $p_{\chi_1',i}$ are the momenta entering the vertex. Here, the minus sign is a i^2 from the two derivatives. Using the above vertices and the propagator:

$$G_{\chi_i}(p) = -\frac{i}{p^2}$$
(1.6)

we proceed to compute the $\chi_1'\chi_2 \to \chi_1'\chi_2$ amplitude (the $\chi_1'\chi_1' \to \chi_1'\chi_1'$ vanishes due to crossing symmetry, and the same for χ_2). Ignoring graviton exchange, that do not depend on ξ in the Einstein frame, there are four graphs corresponding to this amplitude at tree level: one for the quartic vertex, one for χ_1' exchange in the t channel, two for χ_2 exchange in the s and u channels. We denote the corresponding subamplitudes $\mathcal{M}^4, \mathcal{M}_{\chi_1'}^t, \mathcal{M}_{\chi_2}^s, \mathcal{M}_{\chi_2}^u$. Computing them yields:

$$\mathcal{M}^4 = -\frac{2\xi t}{M_P^2}, \quad \mathcal{M}_{\chi_1'}^t = \frac{\xi t}{M_P^2}, \quad \mathcal{M}_{\chi_2}^s = -\frac{\xi s}{M_P^2}, \quad \mathcal{M}_{\chi_2}^u = -\frac{\xi u}{M_P^2}$$
(1.7)

here, $s = -(p_1 + p_2)^2, t = -(p_1 - p_3)^2, u = -(p_1 - p_4)^2$ are the Mandelstam variables in mostly plus convention (in [1], we used a different definition, without the minus sign, but this does not make much difference), where p_1, p_2 (resp. p_3, p_4) are the momenta of the incoming (resp. outgoing) χ_1', χ_2. Correcting a sign error in the scalar propagator made in [1], we obtain a vanishing amplitude:

$$\mathcal{M}(\chi_1'\chi_2 \to \chi_1'\chi_2) = \mathcal{M}^4 + \mathcal{M}_{\chi_1'}^t + \mathcal{M}_{\chi_2}^s + \mathcal{M}_{\chi_2}^u = 0$$
(1.8)

this computation was made in the Palatini formalism, but the same cancellation occurs in the metric formalism. As a consequence of this, demanding tree-level unitarity, it seems that the usual cutoff $\Lambda \sim M_P/\sqrt{\xi}$ of Higgs inflation in a large background should be lifted up to M_P, where graviton interactions start to cause trouble. But before claiming this, we should make more precise the behavior in the $\bar{\phi}_1 \gg M_P/\sqrt{\xi}$ limit, and check what happens if we consider the Higgs to be a complex doublet, as it is in the Standard Model. This is what the next two sections are devoted to.

2 Extension to general background

Let us now extend the previous calculation to a general background $\bar{\phi}_1$, that is, without making the assumption $\bar{\phi}_1^2 \gg M_P^2/\xi$. The only requirement is that this background varies

slowly in time and space, for it to be considered constant in the region and timescale of the interactions between ϕ_1' and ϕ_2. Setting:

$$\bar{\Omega}^2 = 1 + \frac{\xi\bar{\phi}_1^2}{M_P^2} \quad \text{and} \quad x^2 = \frac{\xi\bar{\phi}_1^2}{M_P^2\bar{\Omega}^2} \tag{2.1}$$

and proceed to the computation just as before, first in the Palatini and then in the metric formalism.

2.1 Palatini formalism

In the Palatini formalism, after developing (1.3), we extract the relevant interactions of the canonically normalized fields for the process $\chi_1'\chi_2 \to \chi_1'\chi_2$:

$$\mathcal{L}_{\chi_1'^3} = \frac{x\sqrt{\xi}}{M_P}\chi_1'(\partial\chi_1')^2, \qquad\qquad \mathcal{L}_{\chi_1'\chi_2^2} = \frac{x\sqrt{\xi}}{M_P}\chi_1'(\partial\chi_2)^2,$$

$$\mathcal{L}_{\chi_1'^2\chi_2^2} = \frac{(1-4x^2)\xi}{2M_P^2}\chi_1'^2(\partial\chi_2)^2 + \frac{\xi}{2M_P^2}\chi_2^2(\partial\chi_1)^2 \tag{2.2}$$

when $\bar{\phi}_1^2 \to M_P^2/\xi$ we have $x \to 1$ and of course we recover (1.4). Here as before, the $\chi_i = \phi_i/\bar{\Omega}$ are the canonically normalized counterparts of the ϕ_i. From these interactions, we deduce the corresponding vertices; and using the same notations as in equation (1.7), we compute the following subamplitudes:

$$\mathcal{M}^4 = \frac{2(1-2x^2)\xi t}{M_P^2}, \quad \mathcal{M}_{\chi_1'}^t = \frac{\xi x^2 t}{M_P^2}, \quad \mathcal{M}_{\chi_2}^s = -\frac{\xi x^2 s}{M_P^2}, \quad \mathcal{M}_{\chi_2}^u = -\frac{\xi x^2 u}{M_P^2} \tag{2.3}$$

adding them together yields:

$$\mathcal{M}(\chi_1'\chi_2 \to \chi_1'\chi_2) = \frac{2(1-x^2)\xi t}{M_P^2} \tag{2.4}$$

which is vanishing when $x \to 1$. More precisely, we can develop $x^2 \sim 1 - M_P^2/(\xi\bar{\phi}_1^2)$ when $\bar{\phi}_1^2 \gg M_P^2/\xi$, so that:

$$\mathcal{M}(\chi_1'\chi_2 \to \chi_1'\chi_2) \simeq \frac{2t}{\bar{\phi}_1^2} \sim \frac{E^2}{\bar{\phi}_1^2} \quad \text{when} \quad \bar{\phi}_1^2 \gg M_P^2/\xi \tag{2.5}$$

it then turns out that in this limit, the energy at which $\mathcal{M} \sim 1$, identified with the cutoff of the effective theory, is $\Lambda \sim \bar{\phi}_1$ in the Einstein frame. This is above the usual $M_P/\sqrt{\xi}$. We will discuss it more later.

2.2 Metric formalism

In the metric formalism, after developing (1.3), the relevant interactions for $\chi_1'\chi_2 \to \chi_1'\chi_2$ are:

$$\mathcal{L}_{\chi_1'^3} = \frac{(1-6\xi+12\xi x^2)x\sqrt{\xi}}{(1+6\xi x^2)^{3/2}M_P}\chi_1'(\partial\chi_1')^2 \tag{2.6}$$

$$\mathcal{L}_{\chi_1'\chi_2^2} = \frac{x\sqrt{\xi}}{(1+6\xi x^2)^{1/2}M_P}\chi_1'(\partial\chi_2)^2 - \frac{6\xi^{3/2}x}{(1+6\xi x^2)^{1/2}M_P}\chi_2(\partial\chi_1' \cdot \partial\chi_2) \tag{2.7}$$

and:

$$\mathcal{L}_{\chi_1'^2\chi_2^2} = \frac{(1-4x^2)\xi}{2(1+6\xi x^2)M_P^2}\chi_1'^2(\partial\chi_2)^2 + \frac{(1+12\xi x^2)\xi}{2(1+6\xi x^2)M_P^2}\chi_2^2(\partial\chi_1')^2 - \frac{6(1-4x^2)\xi^2}{(1+6\xi x^2)M_P^2}\chi_1'\chi_2(\partial\chi_1'\cdot\partial\chi_2)$$

(2.8)

where $\chi_1' = (1+6\xi x^2)\phi_1'/\bar{\Omega}$ and $\chi_2 = \phi_2/\bar{\Omega}$ are the canonically normalized counterparts of ϕ_i' and ϕ_2, respectively.

From these interaction terms, using the same notations as in equation (1.7), we compute the following subamplitudes:

$$\mathcal{M}^4 = \frac{2(1-2x^2)(1+3\xi)}{1+6\xi x^2}\frac{\xi t}{M_P^2}, \qquad \mathcal{M}_{\chi_1'}^t = \frac{(1+6\xi)(1-6\xi+12\xi x^2)}{(1+6\xi x^2)^2}\frac{\xi x^2 t}{M_P^2} \qquad (2.9)$$

$$\mathcal{M}_{\chi_2}^s = -\frac{\xi x^2 s}{(1+6\xi x^2)M_P^2}, \qquad\qquad \mathcal{M}_{\chi_2}^u = -\frac{\xi x^2 u}{(1+6\xi x^2)M_P^2} \qquad (2.10)$$

summing them yields:

$$\mathcal{M}(\chi_1'\chi_2 \to \chi_1'\chi_2) = \frac{2((1-x^2)+3\xi(1-x^4))\xi t}{(1+6\xi x^2)^2 M_P^2} \qquad (2.11)$$

A first observation here is that the terms proportional to ξ^3 cancel in the numerator, meaning that for fixed x, in the limit where $\xi \gg 1$, we have $\mathcal{M} \sim E^2/M_P^2$, giving a cutoff $\Lambda \sim M_P$, above $M_P/\sqrt{\xi}$. Then, $\mathcal{M}$ vanishes when $x \to 1$. More precisely, developing $x^2 \sim 1 - M_P^2/(\xi\bar{\phi}_1^2)$ when $\bar{\phi}_1^2 \gg M_P^2/\xi$, we obtain:

$$\mathcal{M}(\chi_1'\chi_2 \to \chi_1'\chi_2) \simeq \frac{2t}{(1+6\xi)\bar{\phi}_1^2} \sim \frac{E^2}{\xi\bar{\phi}_1^2} \quad \text{when} \quad \bar{\phi}_1^2 \gg M_P^2/\xi \quad \text{and} \quad \xi \gg 1 \quad (2.12)$$

Then it turns out that in this limit the Einstein frame cutoff of the effective theory is $\Lambda \sim \bar{\phi}_1\sqrt{\xi}$.

Thus, in both the Palatini and metric formalisms, cancellations in the amplitude of $\chi_1'\chi_2 \to \chi_1'\chi_2$ scattering, between the canonically normalized physical Higgs and Goldstone boson suggest a cutoff scale $\Lambda \sim \bar{\phi}_1$ or $\lambda \sim \bar{\phi}_1\sqrt{\xi}$ that can be much higher than the usual $M_P/\sqrt{\xi}$ in a large background. We will now check whether this result holds if we consider the scalar to be an SU(2) doublet, as is the Higgs in the Standard Model.

3   Extension to complex Higgs doublet

When H is a complex SU(2) doublet, we can parameterize it as:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \qquad (3.1)$$

then, we may, without loss of generality, introduce a background in the direction of ϕ_1 as before. In this case, the Goldstone bosons ϕ_2, ϕ_3, ϕ_4 play exactly the same role and are interchangeable, so the amplitudes of e.g. $\chi_1'\chi_3 \to \chi_1'\chi_3$ are the same as in equations (2.5) and (2.12). Note however that now there are new possible scattering processes between different Goldstone bosons. Without loss of generality we shall consider $\chi_2\chi_3 \to \chi_2\chi_3$.

3.1 Palatini formalism

In this case, the relevant interactions we obtain after developing the action (1.3) are $\mathcal{L}_{\chi_1'\chi_3^2}$, which is the same as $\mathcal{L}_{\chi_1'\chi_2^2}$ in (2.2), and:

$$\mathcal{L}_{\chi_2^2\chi_3^2} = \frac{\xi}{2}\chi_2^2(\partial\chi_3)^2 + \frac{\xi}{2}\chi_3^2(\partial\chi_2)^2 \tag{3.2}$$

The corresponding vertices follow as previously. Ignoring graviton exchange as before, there are now two graphs corresponding to the $\chi_2\chi_3 \to \chi_2\chi_3$ amplitude at tree level: one for the quartic vertex and one for χ_1' exchange in the t channel. We denote the corresponding subamplitudes $\mathcal{M}^4$ and $\mathcal{M}_{\chi_1'}^t$ respectively.

We get:

$$\mathcal{M}^4 = \frac{2\xi t}{M_P^2} \quad \text{and} \quad \mathcal{M}_{\chi_1'}^t = -\frac{\xi x^2 t}{M_P^2} \quad \text{so that} \quad \mathcal{M}(\chi_2\chi_3 \to \chi_2\chi_3) = \frac{(2-x^2)\xi t}{M_P^2} \tag{3.3}$$

this time, the amplitude does not vanish when $x \to 1$, so $\mathcal{M} \sim \xi E^2/M_P^2$, giving an Einstein frame cutoff at $\Lambda \sim M_P/\sqrt{\xi}$.

3.2 Metric formalism

In this case, the relevant interactions we obtain after developing the action (1.3) are $\mathcal{L}_{\chi_1'\chi_3^2}$, which is the same as $\mathcal{L}_{\chi_1'\chi_2^2}$ in (2.7), and:

$$\mathcal{L}_{\chi_2^2\chi_3^2} = \frac{\xi}{2}\chi_2^2(\partial\chi_3)^2 + \frac{\xi}{2}\chi_3^2(\partial\chi_2)^2 - 6\xi^2\chi_2\chi_3(\partial\chi_2 \cdot \partial\chi_3) \tag{3.4}$$

using the same notations as in (3.3), we compute the following subamplitudes:

$$\mathcal{M}^4 = \frac{2(1+3\xi)\xi t}{M_P^2} \quad \text{and} \quad \mathcal{M}_{\chi_1'}^t = -\frac{(1+6\xi)^2\xi x^2 t}{(1+6\xi x^2)M_P^2} \tag{3.5}$$

summing them yields:

$$\mathcal{M}(\chi_2\chi_3 \to \chi_2\chi_3) = \frac{2 - x^2 + 6\xi}{1 + 6\xi x^2}\frac{\xi t}{M_P^2} \tag{3.6}$$

again, this does not vanish in the large background limit $x \to 1$, but goes to $\mathcal{M} \sim \xi t/M_P^2$. Therefore, in this limit or in the limit where $\xi \gg 1$, we have $\mathcal{M} \sim \xi E^2/M_P^2$, giving the usual cutoff at $\Lambda \sim M_P/\sqrt{\xi}$.

4 Discussion and conclusion

Let us collect the results (2.5), (2.12), (3.3), (3.6) at leading order when $\xi \gg 1$ and $\bar{\phi}_1^2 \gg M_P^2/\xi$:

	amplitude $\mathcal{M}$		cutoff Λ (Einstein)		cutoff Λ (Jordan)	
	Palatini	metric	Palatini	metric	Palatini	metric
$\chi_1'\chi_2 \to \chi_1'\chi_2$	$E^2/\bar{\phi}_1^2$	$E^2/(\xi\bar{\phi}_1^2)$	$\bar{\phi}_1$	$\bar{\phi}_1\sqrt{\xi}$	$\bar{\phi}_1^2\sqrt{\xi}/M_P$	$\xi\bar{\phi}_1/M_P$
$\chi_2\chi_3 \to \chi_2\chi_3$	$\xi E^2/M_P^2$	$\xi E^2/M_P^2$	$M_P/\sqrt{\xi}$	$M_P/\sqrt{\xi}$	$\bar{\phi}_1$	$\bar{\phi}_1$

here, in addition to the amplitudes and associated cutoff in the Einstein frame, in which we worked here, we reported cutoff in the Jordan frame. The two are simply linked by the conformal transformation (1.2):

$$\Lambda^{(J)} = \bar{\Omega}\Lambda^{(E)} \quad \text{and} \quad \bar{\Omega} \simeq \frac{\bar{\phi}_1\sqrt{\xi}}{M_P} \quad \text{when} \quad \bar{\phi}_1^2 \gg \frac{M_P^2}{\xi} \tag{4.1}$$

this can help to compare with [1], where as it has been pointed out the background $\bar{\phi}_1$ is the effective Jordan frame cutoff in all cases for both the Palatini and metric formalisms. However, here, we stick to the Einstein frame.

In [1], we investigated a simplified model where the Higgs was a singlet. In this case, only the first line of the table is relevant. We find that the cutoff of this simplified model can be much higher, at $\bar{\phi}_1$ (Palatini) or $\bar{\phi}_1\sqrt{\xi}$ (metric). When the Higgs is, rightfully, considered to be a doublet, there are new, nonvanishing Goldstone-Goldstone amplitudes (second line of the table), for which no cancellation occur yielding the usual cutoff at $M_P/\sqrt{\xi}$ as in [2]. Therefore, the singlet model has a softer behaviour than the more realistic doublet model at high energies.

Acknowledgments

References

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