ANNUAL REVIEWS

Annual Review of Nuclear and Particle Science

Rare Kaon Decays

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Annu. Rev. Nucl. Part. Sci. 2021. 71:113–37

First published as a Review in Advance on
May 4, 2021

The *Annual Review of Nuclear and Particle Science*
is online at nucl.annualreviews.org

https://doi.org/10.1146/annurev-nucl-102419-054905

Keywords

CKM, rare processes, *CP* violation

Abstract

Historically important in the development of the Standard Model (SM) of particle physics, rare kaon decays are still a privileged tool for looking beyond it. The main reasons to continue the study of rare kaon decays are to test the CKM quark-mixing and *CP*-violation paradigm, to make quantitative comparisons with the B sector, and to search for explicit violations of the SM. Current research on rare kaon decays focuses mostly on $K \to \pi\nu\bar{\nu}$ decays, which are predicted with good accuracy within the SM and beyond. Experimentally, these decays, especially that of the charged kaon, have a long history. Their theoretical importance is matched only by their experimental difficulty. This article reviews the progress of the past 10 years, describes the state of the art, and looks toward future perspectives.

Contents

1. INTRODUCTION

1.1. Kaons and Particle Physics

The Standard Model (SM) of particle physics is a collection of theories that can explain the electromagnetic, weak, and strong interactions in terms of a few guiding principles (local gauge symmetries) and at least 26 fundamental parameters to be determined experimentally. Among these parameters, there are (assuming three generations of fermions and universality of the weak interaction) six quark masses, three quark-mixing parameters, and one *CP*-violating phase. To test the validity of the SM, it is essential to focus on those few genuine short distance (SD)-dominated processes (1) in which hadronic uncertainties do not shadow the sensitivity of the predictions. Among these processes, $K \to \pi\nu\bar{\nu}$ decays are notable for their pristine theoretical precision and the experimental challenges they raise. We are still lacking a theory to explain why fundamental fermions appear in three families, and many outstanding questions, such as the baryon asymmetry of the Universe (BAU) and the puzzle of dark matter (DM), remain unanswered. The interplay of *CP* symmetry violation and the mixing of quarks remains one of the most active areas of experimental research in high-energy physics. In addition to the general-purpose LHC experiments, there are strong dedicated flavor programs both at colliders (LHCb at the LHC and Belle II at KEK) and at fixed-target experiments (KOTO at J-PARC and NA62 at CERN). With the LHC energy regime all but fully explored and without a collider with larger center-of-mass energy for at least two more decades, the question is how to probe what lies beyond the SM. Rare kaon decays offer a well-defined strategy to address very high-energy scales. In this review, we focus particularly on the SD information that can be extracted from processes forbidden at tree level and strongly suppressed even at higher orders. Bridging the gap between the precise theoretical predictions and the available experimental information will be paramount to the completion of the rare kaon decay program. Rare kaon decays offer the opportunity to measure higher-order electroweak interactions in which deviations from the SM predictions may appear. In this respect, they are complementary to searches for New Physics performed at colliders and should be pursued vigorously.

1.2. Historical Foreword

We often lament that the SM passes every experimental test, but we tend to forget that the SM was not always the same; it has grown and incorporated, step-by-step, the discoveries made along the way. The aim of particle physics is to continue to build the SM rather than to break it. Several of the discoveries that informed the SM originated from kaon physics, such as flavor (2), Cabibbo theory (3), *CP* violation (4), and the GIM mechanism (5); all of these pillars of the SM trace their origins to strange particles. The study of rare or forbidden kaon decays is linked to an understanding of weak interactions that dates back to the 1950s. A striking feature is the absence of flavor-changing neutral currents (FCNCs). The first decisive tests looked for decays of kaons into pairs of charged leptons. One might note that searches in muon decays (e.g., $\mu \to 3e$) are not a good test in the search for weak neutral currents because such currents might be absent as a result of strict lepton number conservation in muon decays (6). In the past, there was strong theoretical prejudice against weak neutral currents. For more than 30 years after the publication of the Fermi weak interaction paper (7), it was assumed that these interactions were charged: The lepton retained its electric charge, whereas the nucleon (nucleus) did not. Many different processes could be explained by the adaptation of the Fermi theory. The study of rare kaon decays appeared to be a good way to find the neutral currents if they existed. Some of the earliest results are shown in **Table 1**. An especially low limit on the absence of the $K^+ \to \pi^+ \nu\bar{\nu}$ decay seemed to confirm a clear message—namely, the lack of FCNCs in nature. The growing necessity of describing the weak interactions in terms of quantum field theories (in analogy with electrodynamics) led to the introduction of massive intermediate vector bosons (11). These bosons were generally understood to be electrically charged. The Weinberg–Salam theory (12, 13) with spontaneous symmetry breaking requires one of the three massive bosons to be neutral, but this theory was not a major contender until it was proven to be renormalizable (14). Neutrino scattering experiments focused mostly on the discovery of the charged intermediate vector bosons and the parton hypothesis. Given the compelling limit on the $K^+ \to \pi^+ \nu\bar{\nu}$ process, there was no reason to believe in the existence of neutral currents. The GIM mechanism (5) explained the smallness of the neutral kaon mass splitting and the absence of $K_L^0 \to \mu^+\mu^-$ and $K^+ \to \pi^+\ell^+\ell^-$ decays by introducing a fourth quark. The discovery of neutral currents in neutrino scattering (15) and increasingly stringent limits on the absence of FCNCs led to the consolidation of the SM with rare kaon decays as a tool to test it in a significant way.

Even before the discovery of *P* violation, Lee et al. (16) looked at the possibilities of *C* and *T* violation under *CPT* conservation and found that the decay of the neutral kaon was of particular

Table 1 Earliest rare kaon decay results

Decay	Upper limit (90% confidence level)	Year	Reference
$K^+ \to \pi^+e^+e^-$	2.45×10^{-6}	1964	6
$K^+ \to \pi^+\mu^+\mu^-$	3×10^{-6}	1965	8
$K_L^0 \to \mu^+\mu^-$	1.6×10^{-6}	1967	9
$K^+ \to \pi^+\nu\bar{\nu}$	1×10^{-4}	1969	10

interest. *CP* violation was discovered in 1964 in the relatively abundant $\pi^+\pi^-$ decays of the long-lived neutral kaon (4). The phenomenology of $K \to \pi\pi$ decays was developed by Wu & Yang (17), and this work led to a parameterization distinguishing between indirect *CP* violation in the mass mixing, ε, and direct *CP* violation in the decay, ε'. These two parameters are related by

$$\eta_{\pm} = \varepsilon + \varepsilon',$$

$$\eta_{00} = \varepsilon - 2\varepsilon',$$

where $\eta_{\pm} = \frac{A(K_L^0 \to \pi^+\pi^-)}{A(K_S^0 \to \pi^+\pi^-)}$ and $\eta_{00} = \frac{A(K_L^0 \to \pi^0\pi^0)}{A(K_S^0 \to \pi^0\pi^0)}$. The existence of *CP* violation was unexpected, and it was suggested to be the unique manifestation of a new superweak (18) interaction that only the smallness of the mass difference between the two neutral kaons could reveal. Only with the six-quark model of Kobayashi & Maskawa (19) could *CP* violation be accommodated within the known weak interaction framework.

Following the breakthrough of GIM, which explained that FCNCs are strongly suppressed at loop level, it was realized that rare kaon decays such as $K_L^0 \to \mu^+\mu^-$, $K \to \pi\nu\bar{\nu}$, $K \to \gamma\gamma$, and others that exhibit induced $|\Delta S| = 1$ transitions are theoretically interesting because they allow the direct observation of higher-order electromagnetic and weak interactions (20). Extending the calculations to six quarks, following the Kobayashi–Maskawa prescription, Ellis et al. (21) found that it was possible to differentiate between models because of the different sensitivity of the rare kaon decay to *CP* violation in the decay and in the mixing. In 1981, Inami & Lim (22) found that the value of the branching ratio for rare decays such as $K \to \pi\nu\bar{\nu}$ could be significantly increased by the exchange of heavy quarks or leptons where the inverse power of the fermion masses could be compensated for by large Yukawa couplings. With the mass of the top being excluded at higher and higher values, the implications of this work became more and more interesting.

A larger-than-usual prediction (23) for ε'/ε motivated the experimental effort. The quantity to measure direct *CP* violation in two-pion decays of the neutral kaons is

$$R = \frac{\Gamma(K_L^0 \to \pi^0\pi^0)/\Gamma(K_S^0 \to \pi^0\pi^0)}{\Gamma(K_L^0 \to \pi^+\pi^-)/\Gamma(K_S^0 \to \pi^+\pi^-)} \simeq 1 - 6\,\varepsilon'/\varepsilon.$$

The first evidence for a nonzero ε'/ε was given by the NA31 (24) experiment at CERN. The evidence was not confirmed by the E731 experiment at Fermilab (25), and a new round of experiments—KTeV (26) at Fermilab and NA48 (27) at CERN—was necessary to establish direct *CP* violation. The measurement of a nonzero ε'/ε (28),

$$\varepsilon'/\varepsilon_{(\mathrm{PDG\,avg})} = (1.68 \pm 0.20) \times 10^{-3},$$

ruled out superweak models and was a strong endorsement for the CKM explanation of *CP* violation, which was then confirmed by the discovery of *CP* violation in the *B* system.

It was emphasized (29) that a very clean way to study *CP* violation in the frame of the CKM matrix was to study the decay $K_L^0 \to \pi^0\nu\bar{\nu}$. Several remarkable phenomenological advantages are at play in this decay:

- absence of long-distance (LD) processes from two-photon contributions, which can plague the final states with pairs of charged leptons;
- highly suppressed *CP* violation from mixing, which again is a consequence of the above because the $K_S^0 \to \pi^0\nu\bar{\nu}$ contribution is heavily suppressed; and
- absence of backgrounds from rare kaon radiative decays, which represent an essentially irreducible limitation (30) for $K_L^0 \to \pi^0\ell^+\ell^-$.

The $K_L^0 \rightarrow \pi^0 \nu\bar{\nu}$ decay is unique in that the measurement of the branching fraction translates directly into the measurement of the *CP*-violation invariant J (31) in the SM practically without hadronic and parametric uncertainties. The challenge is purely experimental. To make quantitative predictions, the theoretical calculations need to be placed on solid grounds taking into account QCD and electroweak corrections. A detailed review of the theoretical status, including a record of the primary literature and results obtained by the mid-1990s, can be found in Reference 32. More recently, NNLO QCD (33), NNLO electroweak (34), and (small) light quark (35) corrections to $K^+ \rightarrow \pi^+ \nu\bar{\nu}$ have also been computed.

One might be tempted to conclude that with the advent of specialized B physics experiments such as the B factories and LHCb at the LHC collider, the role of kaons is exhausted. This is not the case; we still need to overconstrain the CKM matrix with precise determinations of its parameters extracted independently using different quark systems. A discrepancy between the determinations of the parameters obtained from kaon and B mesons would have profound consequences. What makes rare kaon decays special is the unique combination of precise theoretical predictions and the strong suppression of the SM contributions. It should be emphasized that to explore the flavor structure, it is not enough to check the unitarity of the CKM matrix, which occurs almost by construction: One has to study genuine (1) electroweak FCNC processes. Among these, the $K \rightarrow \pi\nu\bar{\nu}$ ones described in the next sections offer a particularly interesting combination of theoretical cleanliness and experimental challenge.

2. KAONS, FLAVOR MIXING, AND *CP* VIOLATION

2.1. General Framework and Notation

In the modern description of the SM, the masses and the mixing of the quarks originate from the Yukawa interactions with the Higgs condensate. The Higgs field acquisition of a nonzero vacuum expectation value by spontaneous symmetry breaking yields the quark masses. As a result, the left-handed quarks u_{Lj} and d_{Lk} are coupled to the charged-current weak interaction via the relations

$$\frac{-g}{\sqrt{2}}(\overline{u_L}, \overline{c_L}, \overline{t_L})\gamma^\mu W_\mu^+ V_{\mathrm{CKM}} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + \mathrm{h.c.},$$

where

$$V_{\mathrm{CKM}} \equiv V_L^u V_L^{d\dagger} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

is the CKM matrix (3, 19), a 3 × 3 unitary matrix. It can be parameterized by three mixing angles and a *CP*-violating phase (19). Of the many possible conventions, the following has become a standard choice (36):

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \qquad 1.$$

where $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$, and δ is the phase responsible for all *CP*-violating phenomena in flavor-changing processes in the SM. The angles θ_{ij} can be chosen to lie in the first quadrant, so $s_{ij}, c_{ij} \geq 0$. Experimentally, we know that $s_{13} \ll s_{23} \ll s_{12} \ll 1$, and it is convenient to show this hierarchy using the Wolfenstein parameterization. Defining (18, 37, 38)

$$s_{12} = \lambda = \frac{|V_{us}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}}, \qquad s_{23} = A\lambda^2 = \lambda\left|\frac{V_{cb}}{V_{us}}\right|,$$

$$s_{13}e^{i\delta} = V_{ub}^* = A\lambda^3(\rho + i\eta) = \frac{A\lambda^3(\bar\rho + i\bar\eta)\sqrt{1 - A^2\lambda^4}}{\sqrt{1-\lambda^2}\,[1 - A^2\lambda^4(\bar\rho + i\bar\eta)]}, \qquad 2.$$

these relations ensure that $\bar\rho + i\bar\eta = -(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*)$ is independent of the phase convention and that the CKM matrix written in terms of λ, A, $\bar\rho$, and $\bar\eta$ is unitary to all orders in λ. Because of unitarity, the CKM matrix satisfies the relations

$$V_{\mathrm{CKM}}V_{\mathrm{CKM}}^\dagger = V_{\mathrm{CKM}}^\dagger V_{\mathrm{CKM}} = \mathbf{1},$$

which, in terms of matrix elements, can be written as

$$\sum_i V_{ij}V_{ik}^* = \delta_{jk}$$

and

$$\sum_j V_{ij}V_{kj}^* = \delta_{ik},$$

where $\delta_{jk} = 1$ if $j = k$ and $\delta_{jk} = 0$ otherwise. The nonvanishing relations are important because they provide a test of weak universality. In the absence of other generations of quarks, the sum of the square of the strengths of the charged current for each up quark into down quarks (and vice versa) must equal one to conserve probability.

To obtain information about the *CP*-violating phase, one can use the six independent relations for which $j \neq k$ and $i \neq k$. These six relations can be displayed in the form of six different triangles on the complex plane. While these triangles differ greatly in shape (because of the very different lengths of the sides due to the hierarchical form of the matrix), what makes them equal in the SM is their area, which measures one half of the unique parameter describing *CP* violation in the SM: the Jarlskog invariant, J (31). Among the six triangles, the most used is the one obtained from the following relation:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0.$$

The angles of this triangle are defined as follows:

$$\beta = \phi_1 = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right),$$

$$\alpha = \phi_2 = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right),$$

$$\gamma = \phi_3 = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right). \qquad 3.$$

To avoid increasing the uncertainties by translating kaon quantities into *B* triangles, a kaon triangle can be used:

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0,$$

which can be written more concisely as

$$\lambda_u + \lambda_c + \lambda_t = 0,$$

with $\lambda_i = V_{id} V_{is}^*$.

This notation is particularly interesting as a way to express $J = \lambda_u \times \Im\lambda_t$, which in turn can be expressed directly in terms of the Cabibbo angle and just one rare kaon decay branching ratio without introducing uncertainties due to large powers of V_{cb} and to other parameters measured from B physics. For instance, following Marciano (39), one can write

$$J = 5.60 \times \sqrt{B(K_L \to \pi^0 \nu \bar{\nu})}.$$

Interpreting the CP violation in the two-pion decays of neutral kaons in the SM leads to expressions that are functions of the CKM elements—for instance (40),

$$\varepsilon = C_\varepsilon \hat{B}_K \Im\lambda_t \left\{ \Re\lambda_c \left[\eta_1 S_0(x_c) - \eta_3 S_0(x_c, x_t) \right] - \Re\lambda_t \eta_2 S_0(x_t) \right\} \exp(i\pi/4),$$

where C_ε is a numerical factor, $S_0(x_i)$ are Inami–Lim loop functions with $x_i = m_i^2/M_W^2$, $\hat{B}_K$ is the bag parameter computed in lattice QCD, and $\eta_{1,2,3}$ are NLO QCD factors. As can be seen from the formula, the SD contributions have good sensitivity to the CKM parameters. Recent lattice QCD results have led to small errors on the nonperturbative quantity $\hat{B}$. Experimentally, ε is very well determined, and its small theoretical uncertainty allows one to place a significant constraint on the CKM parameters. In the SM, ε'/ε can be expressed, quoting Andrzej Buras (40), in a crude approximation as follows:

$$\frac{\varepsilon'}{\varepsilon} \simeq 13\, \Im\lambda_t \left[\frac{110\,\mathrm{MeV}}{m_s(2\,\mathrm{GeV})} \right]^2 \left(\frac{\Lambda^{(4)}_{\overline{MS}}}{340\,\mathrm{MeV}} \right) \left[B^{(1/2)}(1 - \Omega_{\eta+\eta'}) - 0.4 B_8^{(3/2)} \left(\frac{m_t}{165\,\mathrm{GeV}} \right)^{2.5} \right],$$

where B_i are hadronic parameters, and $\Omega_{\eta+\eta'}$ parameterizes isospin breaking. The ε'/ε formula shows cancellations between hadronic matrix elements such that while a nonzero value is an unambiguous signal of direct CP violation, cancellations of the B_6 and B_8 matrix elements require precise QCD calculations to exploit the experimental precision and make a precise determination of $\Im\lambda_t$. Notwithstanding the hadronic uncertainties, ε'/ε places strong bounds on extra CP-violating contributions in $|\Delta S| = 1$ transitions.

To summarize the modern view of flavor physics, we can describe its overall status as follows:

1. The Higgs sector is the source of flavor violation. It is when the Higgs acquires a vacuum expectation value that the quarks get a physical mass and the charged-current $W^\pm$ interactions couple to the physical quarks with couplings proportional to the CKM matrix elements.
2. So far, all manifestations of CP violation and quark mixing are compatible with the single complex phase of the CKM matrix.
3. Tree-level FCNCs are forbidden by weak universality (CKM unitarity).
4. FCNCs at loop level are suppressed by the GIM mechanism so that in the SM these processes are reduced, rendering contributions from beyond the SM relatively important.

Experimental investment in the field of flavor physics is thus justified by the high-energy scales addressed by these studies; in a few cases, they exceed the direct reach achievable at colliders.

2.2. $K^+ \to \pi^+ \nu\bar{\nu}$: Gold-Plated Decay to Study Higher-Order Weak Interactions

As mentioned in Section 2.1, the two decays $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L^0 \to \pi^0 \nu\bar{\nu}$ can be used to quantitatively check the description of quark mixing and CP violation independently from information extracted from the B system. Box and Penguin diagrams of the processes mediating these decays

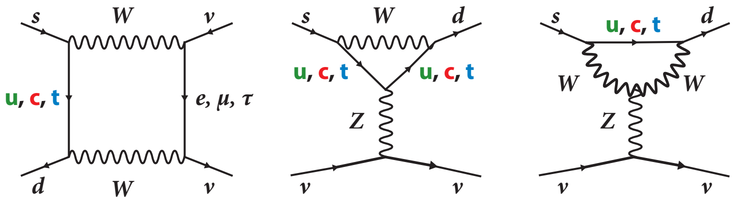

Figure 1

Boxes and Penguin diagrams of the processes contributing to $K \to \pi\nu\bar{\nu}$ decays. Diagrams provided by Joachim Brod.

in the SM are shown in **Figure 1**. Expressing the branching ratio in terms of contributions from the different loop functions following Reference 41, the SM predictions can be written as

$$B(K^+ \to \pi^+\nu\bar{\nu}) = \kappa_+(1+\Delta_{\mathrm{EM}})\left[\left(\frac{\Im\lambda_t}{\lambda^5}X(x_t)\right)^2 + \left(\frac{\Re\lambda_c}{\lambda}P_c(X) + \frac{\Re\lambda_t}{\lambda^5}X(x_t)\right)^2\right],$$

where $\Delta_{\mathrm{EM}} = -0.003$ is the electromagnetic radiative correction; $x_t = m_t^2/M_W^2$, $\lambda = |V_{us}|$, and $\lambda_i = V_{is}^* V_{id}$ are the relevant combinations of CKM matrix elements; X and $P_c(X)$ are the loop functions for the top and charm quarks, respectively; and

$$\kappa_+ = (5.173 \pm 0.025) \times 10^{-11}\left[\frac{\lambda}{0.225}\right]^8$$

is the parameter encoding the relevant hadronic matrix elements extracted from a suitable combination of semileptonic rates. As the formula shows, $B(K^+ \to \pi^+\nu\bar{\nu})$ depends on the sum of the square of the imaginary part of the top loop (*CP* violating) and the square of the sum of the charm contribution and the real part of the top loop. Inserting the numerical factors and making the dependence of the CKM explicit, one obtains

$$B(K^+ \to \pi^+\nu\bar{\nu}) = (8.39 \pm 0.30) \times 10^{-11}\left[\frac{|V_{cb}|}{40.7 \times 10^{-3}}\right]^{2.8}\left[\frac{\gamma}{73.2^\circ}\right]^{0.74}.$$

In the above formula, the explicit numerical uncertainty is the theoretical one originating from QCD and electroweak uncertainties, which amounts to 3.6%. Taking the latest values (28) for $|V_{cb}|_{\mathrm{avg}} = (41.0 \pm 1.4) \times 10^{-3}$ and $\gamma = (72.1^{+4.1}_{-4.5})^\circ$, one finds the following:

$$B(K^+ \to \pi^+\nu\bar{\nu})_{\mathrm{SM}} = (8.5 \pm 1.0) \times 10^{-11}.$$

The predictions are currently dominated by the parametric uncertainty that will plausibly be reduced by new measurements of $|V_{cb}|$ and γ by LHCb and Belle II.

With the discovery of the Higgs boson, the particle content of the SM is complete; nevertheless, we know that the SM itself cannot be the full story. There are many extensions of the SM in which one could expect sizable contributions to $K^+ \to \pi^+\nu\bar{\nu}$, including warped extra dimensions (42), minimal supersymmetric SM analyses (43–45), simplified Z and Z' models (46), littlest Higgs with T parity (47), lepton flavor universality violation models (48), and leptoquarks (49). As a generic example of complementarity between $K \to \pi\nu\bar{\nu}$ and B meson physics, one can mention the case of $B_s^0 \to \mu^+\mu^-$: The B decay is sensitive to possible new pseudoscalar (e.g., charged Higgs) interactions, whereas the rare kaon decays are sensitive to possible new vector ones, such as a Z'.

The SM theoretical precision is waiting to be matched experimentally. The experiments are difficult because the three-body final state lacks a clear, unambiguous signature and the $\nu\bar{\nu}$ pair cannot be detected. A long series of decay-at-rest searches for $K^+ \to \pi^+\nu\bar{\nu}$ have culminated with the final results of the BNL E787/E949 experiments, which found the following (50):

$$B(K^+ \to \pi^+\nu\bar{\nu})_{\mathrm{E787/E949}} = (17.3^{+11.5}_{-10.5}) \times 10^{-11}.$$

Although the mean value is about twice the SM prediction, the result is consistent with the SM because of the large statistical uncertainty. The E787/E949 decay-at-rest technique is described in detail in Reference 51. In addition to the purity of the separated kaon beam, the advantages of the stopped kaon technique include good kinematic constraint to kill backgrounds from two-body kaon decays and the possibility of enforcing good charged particle identification following the full $K^+ \to \pi^+ \to \mu^+ \to e^+$ decay chain. Limitations of the technique are the small acceptance left once adequate levels of muon and photon rejection are achieved and the presence of the material of the stopping target, which leads to π^+ scattering and loss of energy. Prospects to continue measurements of $K^+ \to \pi^+\nu\bar{\nu}$ with kaon decays at rest were presented in the framework of the Project X study (52).

The CERN NA62 experiment at the Super Proton Synchrotron (SPS) was built to make a precise study of $K^+ \to \pi^+\nu\bar{\nu}$ with kaon decays in flight (53). With respect to the earlier CKM design (54), NA62 uses a much higher kaon momentum to enable the detection of the $K^+ \to \pi^+\nu\bar{\nu}$ decay when the π^+ goes backward in the center of mass. In this way, the most dangerous background—the one originating from $K^+ \to \pi^+\pi^0$ decays when the π^0 is lost and the two-body kinematics is not well reconstructed—can be vetoed effectively because a lot of electromagnetic energy from the π^0 decay is deposited in the forward region, which is hermetically instrumented with high-quality electromagnetic calorimeters. The $K^+ \to \pi^+\pi^0$ decays, which split the squared missing mass signal region into two parts, have to be suppressed by 11 orders of magnitude. The experiment has completed the first period of data taking in 2018 and is expected to resume operations in July 2021. A drawback of the high-momentum kaon beam is that the pions and protons cannot be separated from the kaons. Only about 6% of the beam particles are kaons, and of those, only about 10% are usefully decaying in the fiducial volume. For each useful kaon decay, NA62 has to track almost 200 beam particles. To be able to accumulate a sufficient number of decays, the beam rate is very high ($\simeq$750 MHz). The SPS beam is delivered by a slow extraction to minimize the instantaneous intensity. Contrary to colliders, the beam is not bunched because, if it were, the kaon decays from the same bunch might veto each other. Nevertheless, even when employing a slow extraction and an overall duty cycle of about 20%, the instantaneous beam intensity is so high that accidental activity in the beam tracker can lead to the wrong association between a beam track (assumed to be the kaon) and the decay product. To minimize the mistag and to correctly associate the kaon track with the pion one, NA62 has developed a novel Si pixel tracking detector (55) called the Gigatracker (GTK). The time resolution achieved with three GTK stations is as good as 65 ps. Other state-of-the-art detectors of NA62 include a large straw tracker, housed in the vacuum tank, to track the π^+; a liquid krypton electromagnetic calorimeter to veto photons; and a long (17-m focal length) ring-imaging Cherenkov counter for π-μ separation (53). The experiment has already published the results based on the data collected in 2016 (56) and 2017 (57). From these analyses, the best upper limit, at 90% confidence level (CL), has been obtained:

$$B(K^+ \to \pi^+\nu\bar{\nu})_{\mathrm{NA62(2016\text{–}2017)}} \leq 17.8 \times 10^{-11}.$$

The 2016–2017 data also allow one to set a 68% CL mean value for the branching ratio:

$$B(K^+ \to \pi^+\nu\bar{\nu})_{\mathrm{NA62(2016\text{–}2017)}} = (4.8^{+7.2}_{-4.8}) \times 10^{-11}.$$

A detailed description of the NA62 analysis technique can be found in Reference 57. Preliminary results from the analysis of the NA62 2018 data set were presented recently (58). The inspection

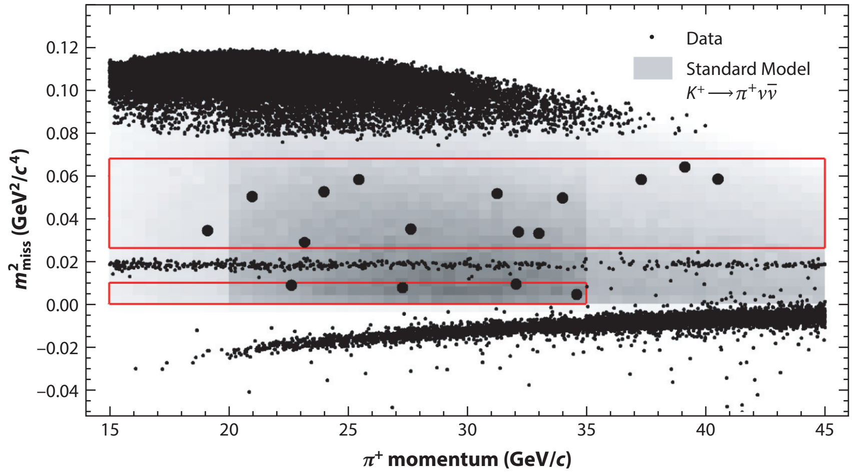

Figure 2

NA62 2018 data: distribution of the squared missing mass (m^2_{miss}), computed under the hypothesis that the charged track is a π^+, versus track momentum. The intensity of the gray tone indicates the signal acceptance computed with Monte Carlo simulation. The two signal boxes show 17 candidate events. Residual $K^+ \rightarrow \mu^+\nu$ events are found at negative m^2_{miss}. $K^+ \rightarrow \pi^+\pi^0$ events lie in between the two signal boxes, whereas $K^+ \rightarrow \pi^+\pi^+\pi^-$ events are kinematically confined at $m^2_{\text{miss}} > 4m^2_{\pi^+}$. Figure adapted with permission from Reference 58; copyright 2020 CERN for the benefit of the NA62 Collaboration.

of the blind signal box for the 2018 sample revealed 17 candidates, as shown in **Figure 2**. To visualize the signal and the components of the expected backgrounds, the candidates are integrated in momentum bins in **Figure 3**. Based on 20 candidate events (17 events in 2018 data, 2 events in 2017 data, and 1 event in 2016 data) and with an expected background of approximately 7 events,

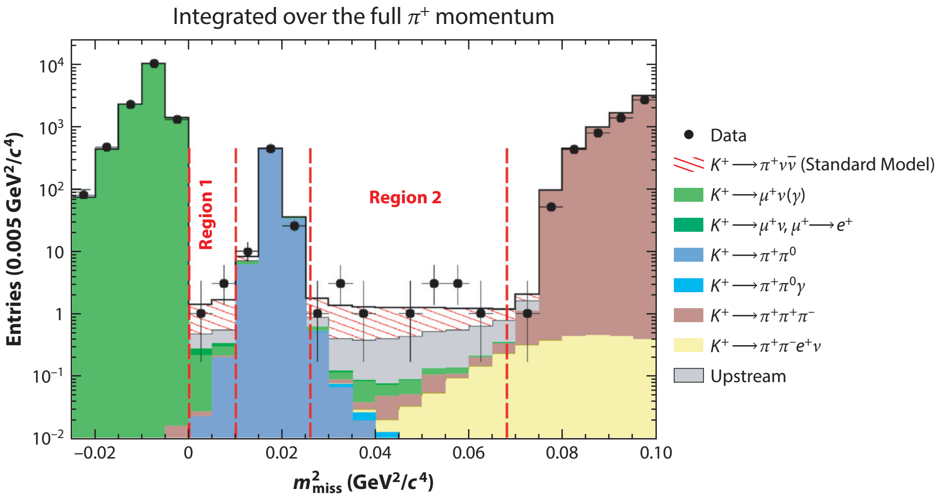

Figure 3

Comparison of the squared missing mass distribution (m^2_{miss}) for signal and backgrounds. The result is based on the NA62 2018 data sample. Figure adapted with permission from Reference 58; copyright 2020 CERN for the benefit of the NA62 Collaboration.

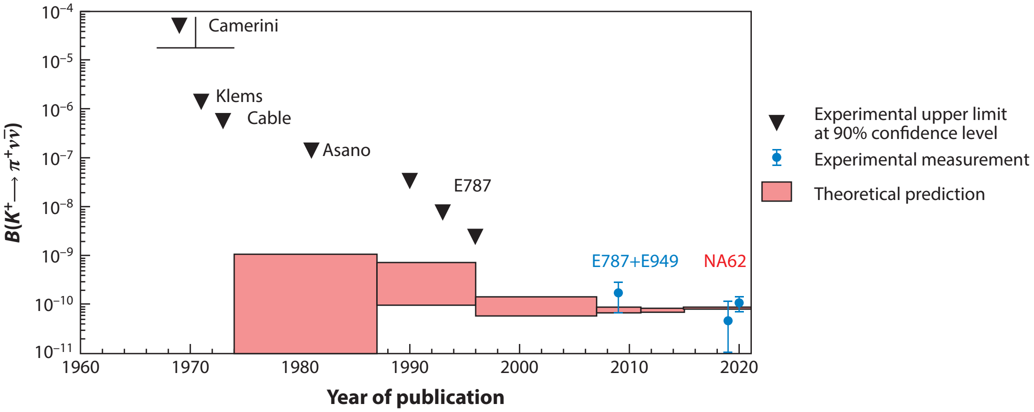

Figure 4

Timeline of theoretical predictions and experimental results for $K^+ \to \pi^+ \nu\bar{\nu}$ (10, 51, 57–64). Figure adapted with permission from Reference 58; copyright 2020 CERN for the benefit of the NA62 Collaboration.

the NA62 Collaboration reported the following:

$$B(K^+ \to \pi^+ \nu\bar{\nu})_{\text{NA62(2016–2018)}} = (11.0^{+4.0}_{-3.5}{}_{\text{stat}} \pm 0.3_{\text{syst}}) \times 10^{-11},$$

in agreement with the SM. This result is the strongest evidence of the $K^+ \to \pi^+ \nu\bar{\nu}$ decay obtained so far and demonstrates the potential of the new in-flight technique. It is interesting to plot the theoretical and experimental results concerning this elusive decay as a function of time, as can be appreciated in **Figure 4**. A large effort is underway in NA62 to increase the acceptance and reconstruction efficiencies and to further reduce backgrounds; for instance, an anticounter to veto upstream decays and a fourth GTK station to reduce the mistags are being installed for the next data taking. To achieve the goal of 10% precision on $B(K^+ \to \pi^+ \nu\bar{\nu})$, NA62 is scheduled to resume data taking after the CERN Long Shutdown 2 (LS2). The possibility to further improve the measurement toward 5% accuracy for an SM signal is being studied (65). Assuming that the infrastructure can be updated to take four times the proton intensity, the NA62 detector would need to be upgraded to accomplish the following objectives:

1. Improve the time resolution of the beam tracker to less than 50 ps per station. The experiment is limited not by the number of protons deliverable by the SPS but, rather, by the time resolution required to resolve the tracks in the beam tracker and to correctly match the kaon track to the pion reconstructed in the downstream tracker. As stated above, the NA62 GTK has a time resolution of 120 ps per station, or 65 ps for the three stations combined. The success of the NA62 GTK has inspired several new R&D efforts to develop Si pixel detectors with even better timing in the view, for example, of Upgrade 2 of LHCb (66). A significant improvement of the time resolution would enable NA62 to operate efficiently with increased beam intensity.
2. Reduce the material budget of the trackers. While any amount of material in the beam leads to unwanted scattering and interactions, for this experiment it is also important to minimize the thickness of the main pion tracker because large scatters couple to several sources of backgrounds. The NA62 straw tracker has a thickness of about 1.7% of a radiation length. It is made of straws 9.8 mm in diameter. It is operated in the decay tank under vacuum to minimize scattering. The main source of material is the plastic wall of the straw, which amounts to 36 μm. Reducing the diameter of each straw to approximately 5 mm opens the

possibility of employing much thinner walls and reducing the material budget by up to a factor of two. In addition, a shorter drift time will improve the high rate capability of the detector.

3. Reduce random veto. One of the essential aspects of NA62 is the capability to detect photons with high efficiency. Considerably increasing the beam intensity will require faster veto counters to avoid large signal losses due to random veto and/or optimized readout and reconstruction algorithms.

2.3. $K_L^0 \to \pi^0 \nu\bar{\nu}$: Champion of *CP* Violation

We turn now to the description of the $K_L^0 \to \pi^0 \nu\bar{\nu}$ decay, which is in principle the best way to measure *CP* violation in the SM, though in practice it is essentially impossible to measure. The SM formulas for K_L^0 are (41)

$$B(K_L^0 \to \pi^0 \nu\bar{\nu}) = \kappa_L \left(\frac{\Im \lambda_t}{\lambda^5} X(x_t) \right)^2,$$

with

$$\kappa_L = (2.231 \pm 0.013) \times 10^{-10} \left[\frac{\lambda}{0.225} \right]^8.$$

The $B(K_L^0 \to \pi^0 \nu\bar{\nu})$ depends only on the square of the imaginary part of the top loop that is *CP* violating. The charm contributions drop out because K_L^0 is mostly an odd linear combination of K^0 and $\bar{K}^0$. This makes the theoretical prediction for the K_L^0 rate even cleaner than for the K^+ (the theoretical error is only about 1.5%). A measurement of the $K_L^0 \to \pi^0 \nu\bar{\nu}$ mode would amount to the measurement of the square of the *CP*-violating parameter η in the Wolfenstein parameterization (18), which is directly related to the Jarlskog invariant, J (31)—the unique measure of *CP* violation in the SM. Parametrically, one can find

$$B(K_L^0 \to \pi^0 \nu\bar{\nu}) = (3.36 \pm 0.05) \times 10^{-11} \left[\frac{|V_{ub}|}{3.88 \times 10^{-3}} \right]^2 \left[\frac{|V_{cb}|}{40.7 \times 10^{-3}} \right]^2 \left[\frac{\sin\gamma}{\sin 73.2^\circ} \right]^2,$$

which, taking the latest values (28) for $|V_{cb}|_{\text{avg}} = (41.0 \pm 1.4) \times 10^{-3}$, $|V_{ub}|_{\text{avg}} = (3.82 \pm 0.24) \times 10^{-3}$, and $\gamma = (72.1^{+4.1}_{-4.5})^\circ$, leads to the following numerical prediction:

$$B(K_L^0 \to \pi^0 \nu\bar{\nu}) = (3.2 \pm 0.6) \times 10^{-11}.$$

While the experimental situation for $K^+ \to \pi^+ \nu\bar{\nu}$ shows that we have two independent experimental techniques that can reach SM sensitivities, with the NA62 experiment on the way to making a precise measurement, the situation for the neutral mode is more complex. Progress has been hampered by the lack of a clean experimental signature because no redundancy is available once the π^0 mass is used as a constraint to reconstruct the decay vertex. The KOTO experiment at J-PARC builds on the experience of the predecessor experiment E391a (67), which was performed at KEK. It is based on the technique of letting a well-collimated "pencil" beam enter the decay region surrounded by high-performance photon vetoes. By vetoing extra photons and applying a transverse momentum cut (150 MeV/c) to eliminate residual $\Lambda \to n\pi^0$ decays, KOTO is expected to reach SM sensitivities by the mid-2020s. The KOTO experiment has published the best upper limit (68):

$$B(K_L^0 \to \pi^0 \nu\bar{\nu})_{\text{KOTO}} < 3.0 \times 10^{-9} \text{ (90\% CL)}.$$

The analysis is based on the definition of the signal region (**Figure 5**) in terms of π^0 transverse momentum and kaon decay vertex; the latter is determined assuming that the two photons originate from a π^0 (the π^0 mass is used as a constraint). A preliminary analysis of the KOTO sample

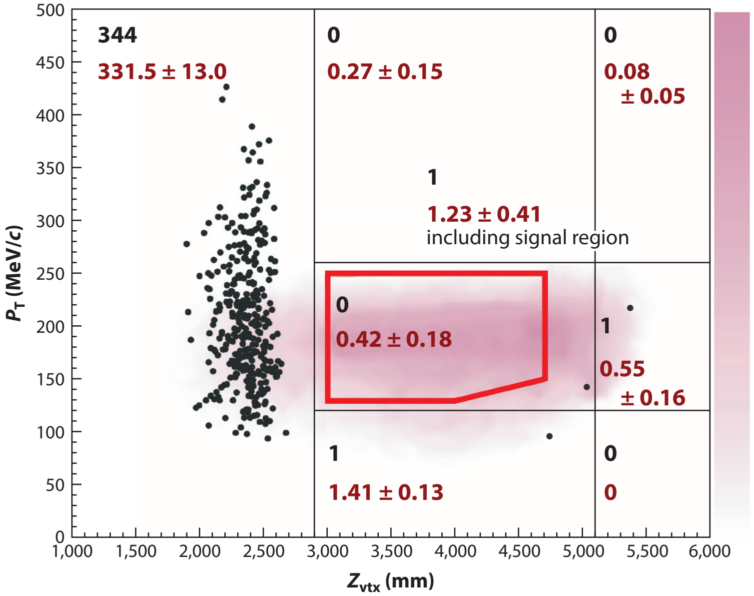

Figure 5

Signal (surrounded by *red lines*), control, and background regions for the KOTO $K_L \to \pi^0\nu\bar{\nu}$ analysis based on the 2015 data sample. The plot shows the reconstructed π^0 transverse momentum (P_T) as a function of the π^0 decay vertex position (Z_vtx). The black and red numbers indicate the numbers of observed and expected background events, respectively. Figure adapted from Reference 68 (CC BY 4.0).

collected in 2018 is presented in Reference 69; the results are consistent with the above-mentioned upper limit, and more detailed studies of residual events are underway (70). A historical timeline indicating the progress made on the upper limits of $B(K_L \to \pi^0\nu\bar{\nu})$ is shown in **Figure 6**. For KOTO, the next step will be to improve the model-independent Grossman–Nir limit (79), which has been updated recently to incorporate the published NA62 upper limit:

$$B(K_L^0 \to \pi^0\nu\bar{\nu})_{\text{Grossman–Nir}} < 8.14 \times 10^{-10}\ (90\%\ \text{CL}).$$

The relation between this upper limit and the corresponding K^+ decay measurement is displayed in **Figure 7**.

The KOTO Collaboration is planning a new phase to significantly improve sensitivity to the $K_L^0 \to \pi^0\nu\bar{\nu}$ decay (80). This project requires the planned extension of the hadron hall of J-PARC. The possibility of exploring the neutral decay mode at CERN once NA62 is completed has been considered in the framework of the European Strategy for Particle Physics upgrade. The experiment under study, KLEVER (81), is based on the same technique employed by KOTO but at much higher kaon energies. Higher kaon energies are expected, as shown by KTeV and KAMI (82), to simplify the task of rejecting the photons from $K_L^0 \to \pi^0\pi^0$, which is the dominant source of background from kaon decays. One should note that with respect to the charged mode, the two-pion branching ratio is *CP* violating and is therefore suppressed by a factor of about 200. It is this suppression that makes the pencil beam approach to study $K_L^0 \to \pi^0\nu\bar{\nu}$ at all thinkable. The KOTO and KLEVER Collaborations are holding joint meetings to address issues of mutual interest. For a different, more constrained approach to measuring $K_L^0 \to \pi^0\nu\bar{\nu}$ (à la KOPIO), readers may consult Reference 52 and the references therein.

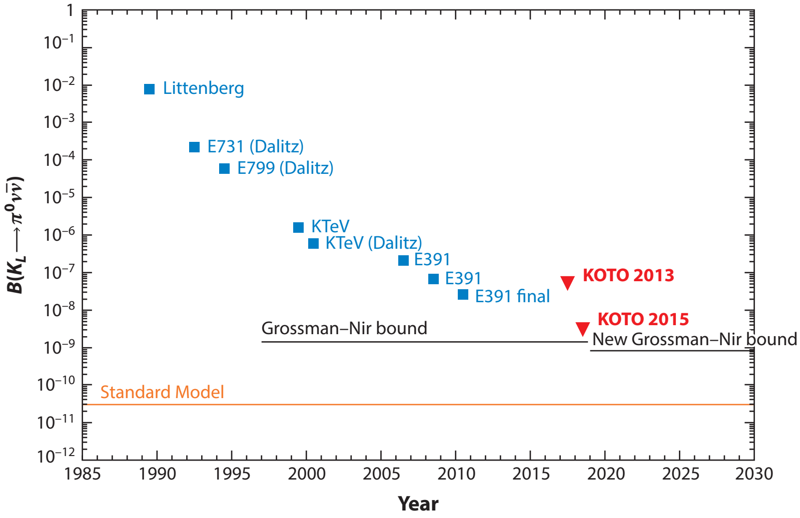

Figure 6

Upper limits for $B(K_L \to \pi^0\nu\bar{\nu})$ as a function of time. Data from References 29, 67, 68, 71–77. Figure adapted with permission from Reference 78.

In conclusion, what makes the case to continue the study of these rare decays compelling is that sensitivity beyond the SM is there for most of the proposed extensions. Together with the study of rare muon decays and searches for electric dipole moments of elementary particles, rare kaon decays like $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L^0 \to \pi^0\nu\bar{\nu}$ offer a genuine window of sensitivity to access high-energy scales thanks to the absence of tree level (CKM unitarity), the absence of LD contributions ($\nu\bar{\nu}$ pair in the final state), and the hard GIM suppression at loop level in the SM.

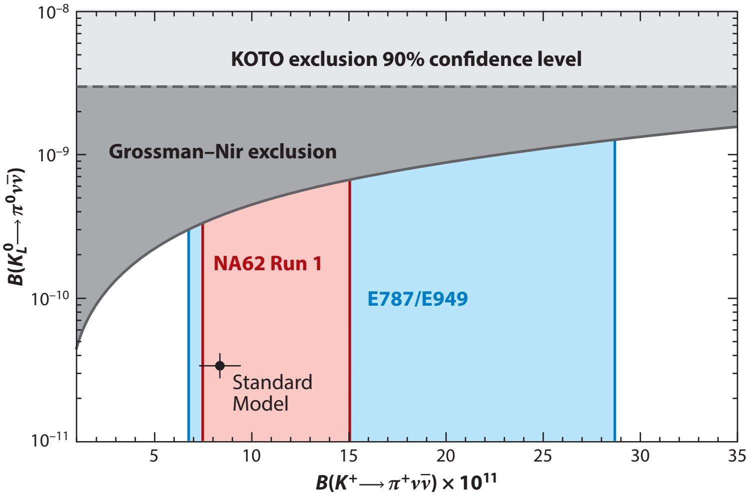

Figure 7

Direct and indirect limits on $K_L^0 \to \pi^0\nu\bar{\nu}$. Figure adapted with permission from Reference 58; copyright 2020 CERN for the benefit of the NA62 Collaboration.

2.4. $K^0_L \to \mu^+\mu^-$

The unambiguous and clean experimental signature of the decay $K^0_L \to \mu^+\mu^-$ established very early the remarkable FCNC suppression, which played a crucial role in the modern formulation of the weak interactions (GIM). The current best measurement of K^0_L is based on more than 6,200 candidates from the BNL E871 Collaboration (83):

$$B(K^0_L \to \mu^+\mu^-) = (6.84 \pm 0.11) \times 10^{-9}.$$

Phenomenologically, the process is characterized by an absorptive and a dispersive amplitude. The absorptive part of the LD $\gamma\gamma$ contribution is calculable (84) and practically saturates the experimental rate:

$$B(K^0_L \to \mu\mu)_{\mathrm{abs}} = \frac{2\alpha^2_{em} r_\mu \beta_\mu}{\pi^2} \left[\frac{\pi}{2\beta_\mu} \ln \frac{1-\beta_\mu}{1+\beta_\mu}\right]^2 B(K^0_L \to \gamma\gamma) = (6.59 \pm 0.05) \times 10^{-9},$$

where $r_\mu = m^2_\mu / m^2_K$ and $\beta_\mu = \sqrt{1 - 4r_\mu}$. The SD contribution has been computed to NNLO QCD (85):

$$B(K^0_L \to \mu^+\mu^-)_{\mathrm{SD}} = (0.79 \pm 0.12) \times 10^{-9},$$

where the error is dominated by the parametric uncertainty of the CKM elements. Although the smallness of the total dispersive amplitude is well established (86), the comparison of the SD contribution extracted from the measured rate with the SM prediction is complicated by the unknown phase of the LD dispersive contribution (87). Incidentally, the corresponding electronic mode is the rarest kaon decay measured so far (88): $B(K^0_L \to e^+e^-) = (8.7^{+5.7}_{-4.1}) \times 10^{-12}$, in agreement with the theoretical prediction (89).

2.5. $K^0_S \to \mu^+\mu^-$

The short-lived kaon has a lifetime that is around 600 times shorter than the K^0_L one, and therefore the branching ratios related to SD processes are correspondingly strongly suppressed. Nevertheless, some of the rare K^0_S decays are interesting in their own right. This is the case for $K^0_S \to \mu^+\mu^-$, where the LD two-photon contribution is calculable (90) as follows:

$$B(K^0_S \to \mu^+\mu^-)_{\mathrm{LD}} = 5.1 \times 10^{-12}.$$

The SD component can be expressed in terms of CKM parameters (86) as follows:

$$B(K^0_S \to \mu^+\mu^-)_{\mathrm{SD}} = 1.5 \times |\Im V^*_{ts} V_{td}|^2 = 1.4 \times 10^{-12} \left|\frac{V_{cb}}{0.041}\right|^4 \times \left|\frac{\lambda}{0.223}\right|^2 \times \bar{\eta}^2 \approx 1.8 \times 10^{-13}.$$

An interesting possibility is to study the $K^0_S K^0_L$ interference in the $\mu\mu$ channel (87), where the interference contribution is due to direct *CP* violation and can modify the effective branching ratio up to 60%.

The old K^0_S experimental limit dating from the 1970s (91) has been improved by upper limits at 90% CL by the LHCb experiment (92, 93):

$$B(K^0_S \to \mu^+\mu^-) < 2.1 \times 10^{-10}.$$

The current limit already places constraints on leptoquark models (94, 95), supersymmetry (96, 97), and other extensions of the SM (87). In the Phase II upgrade during the High-Luminosity LHC era, LHCb is expected to collect around 300 fb^{-1}. If the trigger is fully efficient, LHCb will be able to explore branching ratios below 10^{-11} (98). The best upper limit for the corresponding electronic mode belongs to KLOE (99): $B(K^0_S \to e^+e^-) < 9.0 \times 10^{-9}$ (90% CL).

2.6. $K_S^0 \to \pi^0\ell^+\ell^-$

The measurement of $K_S^0 \to \pi^0 e^+e^-$ determines the indirect (mixing) *CP*-violating contribution present in the corresponding K_L^0 mode. This is important because the direct and indirect components can interfere, amplifying (depending on the relative sign of the amplitudes) the $K_L^0 \to \pi^0 e^+e^-$ signal. The relative sign of the amplitudes cannot be measured directly, so one relies on lattice QCD calculations (100). The precision of the NA48/1 measurement (101),

$$B(K_S^0 \to \pi^0 e^+e^-) = (5.8^{+2.8}_{-2.3} \pm 0.8) \times 10^{-9},$$

is good enough to justify a new K_L^0 campaign as long as realistic plans for dealing with the backgrounds originating from radiative decays (30) are taken into account. The $K_S^0 \to \pi^0\mu^+\mu^-$ NA48/1 measurement (102),

$$B(K_S^0 \to \pi^0\mu^+\mu^-) = (2.9^{+1.5}_{-1.2} \pm 0.2) \times 10^{-9},$$

might be improved by LHCb (98). This would be useful for reducing the error on the indirect *CP*-violating amplitude in the corresponding K_L^0 decay. The $\mu^+\mu^-$ channel has the advantage that no dilepton invariant mass cut at the π^0 and below is needed to reject the background originating from π^0 Dalitz decays, so the extrapolation to determine the form factor parameter is smaller than for the e^+e^- case. In the SM, the process $K_S^0 \to \pi^0\ell^+\ell^-$ is *CP* conserving and dominated by a single photon exchange (103). In chiral perturbation theory (ChPT) expansion beyond leading order, the K_S decays can be expressed as

$$B(K_S^0 \to \pi^0 e^+e^-) = (0.01 - 0.76a_S - 0.21b_S + 46.5a_S^2 + 12.9a_Sb_S + 1.44b_S^2) \times 10^{-10},$$
$$B(K_S^0 \to \pi^0\mu^+\mu^-) = (0.07 - 4.52a_S - 1.50b_S + 98.7a_S^2 + 57.7a_Sb_S + 8.95b_S^2) \times 10^{-11},$$

where a_S and b_S are free, real ChPT parameters of the polynomial expansion of the electromagnetic form factor in terms of the dilepton invariant mass q^2 (with b_S being the coefficient of the linear term in q^2) (104).

2.7. $K_L^0 \to \pi^0\ell^+\ell^-$

Because of SD sensitivity and the excellent experimental signature, the K_L^0 modes with $\ell = e, \mu$ were among the most promising decays through which to explore direct *CP* violation in rare kaon decays. Unfortunately, it is difficult to extract the SD information for several reasons:

- Radiative backgrounds: It was realized by Greenlee (30) that very little acceptance remains once the tight cuts to reject the radiative decays $K_L^0 \to e^+e^-\gamma\gamma$ are made. To extract a significant signal would require an enormous number of kaon decays.
- Indirect *CP* violation from $\varepsilon A(K_S^0 \to \pi^0\ell^+\ell^-)$: SD sensitivity is enhanced in case of positive interference of the K_S^0 and K_L^0 amplitudes, but to determine the sign of $A(K_S^0 \to \pi^0\ell^+\ell)$, lattice calculations are required.
- *CP*-conserving contributions from $A(K_L^0 \to \pi^0\gamma\gamma)$: This component seems to be small with respect to the other two because it is driven by the $m_{\gamma\gamma}$ component of $K_L^0 \to \pi^0\gamma\gamma$, which is measured to be small.

For the K_L^0 process, *CP*-violating contributions can originate from $K^0\bar{K}^0$ mixing via a decay of the *CP*-even component of the K_L^0, and direct *CP*-violating contributions from SD physics via loops sensitive to $Im(\lambda_t) = \Im(V_{td}V_{ts}^*)$. The indirect and direct *CP*-violating contributions can interfere, and the expression for the total *CP*-violating branching ratio can be written as follows (104, 105):

$$B(K_L^0 \to \pi^0 \ell^+ \ell^-)_{\text{CPV}} = \left[C_{\text{MIX}} \pm C_{\text{INT}} \frac{\Im \lambda_t}{10^{-4}} + C_{\text{DIR}} \left(\frac{\Im \lambda_t}{10^{-4}} \right)^2 \right] \times 10^{-12},$$

where C_{INT} is due to the interference between the direct (C_{DIR}) and indirect (C_{MIX}) *CP*-violating (CPV) components. Here, C_{MIX} and C_{INT} depend on $B(K_S^0 \to \pi^0 \ell^+ \ell^-)$ and $\sqrt{B(K_S^0 \to \pi^0 \ell^+ \ell^-)}$, respectively. The SM predictions for the branching ratios of $K_L \to \pi^0 \ell^+ \ell^-$ are (106)

$$B(K_L^0 \to \pi^0 e^+ e^-)_{\text{SM}} = (3.5 \pm 0.9, 1.6 \pm 0.6) \times 10^{-11},$$
$$B(K_L^0 \to \pi^0 \mu^+ \mu^-)_{\text{SM}} = (1.4 \pm 0.3, 0.9 \pm 0.2) \times 10^{-11}.$$

In each equation, the first set of values is for constructive interference and the second is for destructive interference. Better measurements of the $K_S \to \pi^0 \ell^+ \ell^-$ decay rate would allow for improvement of the quoted errors, which are currently dominated by the uncertainty due to the ChPT parameters a_S and b_S. A joint study of the Dalitz plot variables and the components of the μ^+ polarization could directly allow the separation of the indirect, direct *CP*-violating, and direct *CP*-conserving contributions (107). The 90% CL limits for the K_L^0 modes come from KTeV (108, 109):

$$B(K_L^0 \to \pi^0 e^+ e^-) < 2.8 \times 10^{-10},$$
$$B(K_L^0 \to \pi^0 \mu^+ \mu^-) < 3.8 \times 10^{-10}.$$

The K_L^0 channel is sensitive to beyond-the-SM models. For instance, searches of extra dimensions with enhancements of the branching ratio of both K_L^0 modes by a factor of about five are possible without violating any constraints (106). Specific flavor structures can correlate effects in $K_S^0, K_L^0 \to \pi^0 \ell^+ \ell^-$ with *B* physics anomalies—for example, leptoquark models with rank-one flavor violation (95, 110). As we await future plans to further study the $K_L^0 \to \pi^0 \ell^+ \ell^-$ decays, it should be pointed out that if the interference sign is positive, then the KTeV upper limit for the electronic channel may be just above the SM prediction for the total *CP*-violating component. Under this assumption, if a sufficiently large number of K_L^0 decays becomes available in the future, the extraction of some SD information could be possible.

2.8. $K_S^0 \to 3\pi^0$

Since the kaon is spinless and the $3\pi^0$ final state is a pure $CP = -1$ eigenstate, the decay $K_S \to 3\pi^0$ is *CP* violating. The *CP*-violating amplitude is $\eta_{000} = A(K_S \to 3\pi^0)/A(K_L^0 \to 3\pi^0) = \varepsilon + i\Im A_1/\Re A_1$, where A_1 is the isospin $I = 1$ amplitude. Neglecting direct *CP* violation, $\eta_{000} = \varepsilon$, and the prediction of the branching ratio is simply

$$B(K_S^0 \to 3\pi^0)_{\text{SM}} = |\varepsilon|^2 \frac{\tau_S}{\tau_L} \times B(K_L^0 \to 3\pi^0) = 1.7 \times 10^{-9}.$$

The upper limit for the above prediction sets limits on direct *CP* violation. The best limit has been set with KLOE (111) data:

$$B(K_S^0 \to 3\pi^0) < 2.6 \times 10^{-8} \text{ (90\% CL)}.$$

The decay is also interesting in testing *CPT* symmetry. For many years, until the NA48/1 measurement, it limited the test of *CPT* conservation based on the Bell–Steinberger relation (112).

Table 2 Recent determinations of R_K

Experiment	Value (10^{-5})	Year	Reference
KLOE	$2.493 \pm 0.025 \pm 0.019$	2009	116
NA62	$2.488 \pm 0.007 \pm 0.007$	2013	117
PDG	2.488 ± 0.009	2020	28

3. LEPTON UNIVERSALITY, FLAVOR, AND NUMBER VIOLATION

In this section, three different concepts are reviewed. Lepton universality violation (LUV) refers to differences in the strength of the weak interaction in processes distinguished only by the nature of the lepton in the final state. This concept has recently received additional attention because of hints of LUV in B decays. Lepton flavor violation (LFV) is automatically forbidden in the SM apart from the unmeasurably small contributions due to neutrino mixing. Only the existence of doublets of right-handed heavy neutral leptons (113) or a significant modification of the SM could lead to measurable effects. LFV has been extensively tested in rare muon decays, where the recent limit (114) $B(\mu \to e\gamma) < 4.2 \times 10^{-13}$ (90% CL) stands out. Limits on LFV from kaon decays cannot compete with those from rare muon decays but are important because they test possible new interactions that connect quarks and leptons. Lepton number violation (LNV) can occur if the neutrino masses are of Majorana nature as opposed to Dirac.

3.1. $K^+ \to e^+\nu$: Lepton Universality

The decays of pseudoscalar mesons into leptons are helicity suppressed because of the V-A structure of the charged weak current. For kaons, the SM partial width can be written as

$$\Gamma^{\mathrm{SM}}(K^+ \to \ell^+\nu) = \frac{G_{\mathrm{F}}^2 M_K M_\ell^2}{8\pi}\left(1 - \frac{M_\ell^2}{M_K^2}\right)^2 f_K^2 |V_{us}|^2,$$

where G_{F} is the Fermi constant, M_K and M_ℓ are the kaon and lepton masses, and f_K is the kaon decay constant. The ratio of partial widths $R_K = \Gamma(K^+ \to e^+\nu)/\Gamma(K^+ \to \mu^+\nu)$ for electrons and muons can be used to make a precise test of lepton universality in kaon decays. In the SM it is precisely predicted (115), including inner bremsstrahlung, to be

$$R_K^{\mathrm{SM}} = \left(\frac{M_e}{M_\mu}\right)^2 \left(\frac{M_K^2 - M_e^2}{M_K^2 - M_\mu^2}\right)^2 (1 + \delta R_{\mathrm{QED}}) = (2.477 \pm 0.001) \times 10^{-5},$$

where $\delta R_{\mathrm{QED}} = (-3.79 \pm 0.04)\%$. Precise experimental determinations were published by the KLOE and NA62 Collaborations. They are reported in **Table 2** together with their PDG average.

3.2. Lepton Flavor and Lepton Number Violation

Limits from kaon physics cannot compete with those from dedicated rare muon decays (such as $\mu^+ \to e^+\gamma$) or the coherent muon–electron conversion in nuclei ($\mu^- N \to e^- N$), but the presence of two different generations of quarks and leptons in the kaons makes these searches interesting. For example, models that involve leptoquarks or the exchange of multi-TeV particles can be well tested in rare kaon decays. Possible anomalies in the B system may lead to visible effects in rare kaon decays (118) with branching ratios expected in the range of 10^{-12} to 10^{-13}. Other models advocate heavy Majorana neutrinos as a source of LNV (119, 120). Limits (at 90% CL) on the branching ratios of the most studied kaon channels are reported in **Table 3**. The NA62 experiment has the prospect of pushing the limits for the K^+ modes to the range of 10^{-12} to 10^{-11}, considering the data collection that is foreseen after LS2. These modes also can be pursued at LHCb in the

Table 3 **Limits (90% confidence level) on several lepton-flavor- and lepton-number-violating decays**

Decay	Upper limit	Experiment	Reference
$K_L^0 \to e^\pm \mu^\mp$	$<4.7 \times 10^{-12}$	BNL 871	121
$K_L^0 \to \pi^0 e^\pm \mu^\mp$	$<7.6 \times 10^{-11}$	FNAL KTeV	122
$K^+ \to \pi^+ e^- \mu^+$	$<1.3 \times 10^{-11}$	PDG	28
$K^+ \to \pi^+ e^+ \mu^-$	$<5.2 \times 10^{-10}$	BNL 865	123
$K^+ \to \pi^- e^+ \mu^+$	$<5 \times 10^{-10}$	BNL 865	124
$K^+ \to \pi^- \mu^+ \mu^+$	$<4.2 \times 10^{-11}$	CERN NA62	125
$K^+ \to \pi^- e^+ e^+$	$<2.2 \times 10^{-10}$	CERN NA62	125

upgrade phase, benefiting from the large strange-production cross-section and from the improved efficiency for kaon decays; LHCb may be able to extend the existing limits and probe a sizable part of the parameter space suggested by the discrepancies in B physics (126).

4. KAON STRUCTURE AND CHIRAL PERTURBATION THEORY

Rare decays of kaons are a laboratory for the study of the strong interaction at low energy. Chiral perturbation theory (ChPT) is an effective field theory formulated (127) in terms of meson masses and powers the external particle momenta p^2 where, at each order, a finite number of constants (counterterms) is determined experimentally. This is a very broad subject since the corresponding physics processes are not as rare as those presented in the sections above (for a broader kaon physics review, see, for instance, 128).

4.1. $K^+ \to \pi^+ \ell^+ \ell^-$

The decays $K^+ \to \pi^+ \ell^+ \ell^-$ are dominated by LD contributions involving one photon exchange ($K^+ \to \pi^+ \gamma \to \pi^+ \ell^+ \ell^-$), and their branching fraction can be derived within the framework of ChPT in terms of a vector-interaction form factor, which describes the single-photon exchange and characterizes the dilepton invariant-mass spectrum. The form factor includes a small contribution from the two-pion-loop intermediate state and is dominated by a term phenomenologically described as a first-order polynomial $(a_+ + b_+ z)$, where $z = (m_{\ell\ell}/M_K)^2$ and a_+ and b_+ are free parameters used to describe the nonperturbative QCD effects in the chiral expansion (104, 129). To obtain the parameters and the corresponding branching ratios, the differential decay rate spectrum must be reconstructed from experimental data.

Similar to $B \to K\ell^+\ell^-$, these decays can be described by an effective Lagrangian with nonzero Wilson coefficients for the semileptonic operators Q_{7V} and Q_{7A} (103), where possible New Physics processes can be interpreted as deviations from the SM Wilson coefficients C_{7V} and C_{7A}. In particular, the Wilson coefficient C_{7A} can be related to a_+, making the form factor measurement a test of beyond-the-SM effects (118). Lepton universality implies that the free parameters are the same for both the electron and muon channels and that any deviation is a sign of SD New Physics dynamics (130).

The current best experimental measurements of the $K^+ \to \pi^+ \ell^+ \ell^-$ branching ratios are from the NA48/2 Collaboration (131, 132):

$$B(K^+ \to \pi^+ e^+ e^-) = (3.11 \pm 0.04_{\mathrm{stat}} \pm 0.12_{\mathrm{sys}}) \times 10^{-7},$$

$$B(K^+ \to \pi^+ \mu^+ \mu^-) = (9.62 \pm 0.21_{\mathrm{stat}} \pm 0.13_{\mathrm{sys}}) \times 10^{-7}.$$

Table 4 **Measurements of K_L^0 decays into four leptons**

Decay	Branching ratio	Reference(s)
$K_L^0 \to e^+e^-e^+e^-$	$3.56 \pm 0.21 \times 10^{-8}$	136, 137
$K_L^0 \to e^+e^-\mu^+\mu^-$	$2.69 \pm 0.27 \times 10^{-9}$	138

Both NA48/2 and E865 (133, 134) have extracted the free parameters a_+ and b_+ for muon and electron channels, placing limits on LUV. However, such a test is at present limited by the uncertainties of the measurements, especially in the muon channel. At NA62, both larger and significantly cleaner samples of both channels are expected to be collected over the lifetime of the experiment because of vast increases in instantaneous rate and improved tracking resolution compared with NA48/2. The LHCb mass resolution is sufficient to separate the muon decay from the kinematically similar three-pion decay; the experiment can collect of order 10^4 decays in the muon channel per year of upgraded LHCb data taking. Similar considerations apply to the electron channel, where a reduced reconstruction efficiency is somehow compensated for by the larger branching fraction (98).

4.2. $K_{L,S}^0 \to \ell^+\ell^-\ell^+\ell^-$

The decays of the neutral kaons into four leptons are of interest not only to measure the structure of the vertex with two virtual photons but also to fix the sign of the amplitude of $K_L^0 \to \gamma\gamma$. This sign, in turn, is important in determining the SD contributions to $K_L^0 \to \mu^+\mu^-$. To do so, one would need to measure the sign of the K_L^0-K_S^0 interference component (135). No K_S^0 has been measured yet; the K_L^0 modes already measured are shown in **Table 4**.

Recently, LHCb has shown good prospects for the K_S^0 modes (139). Future high-intensity neutral kaon experiments with intensities around five times those of the most recent ones would be required to address all the modes with two or four electrons.

4.3. $K_S^0 \to \gamma\gamma$, $K_L^0 \to \pi^0\gamma\gamma$, and $K^+ \to \pi^+\pi^0e^+e^-$

$K_S^0 \to \gamma\gamma$ is predicted unambiguously in ChPT at $O(p^4)$ to be 2.1×10^{-4} (140). Corrections of the order $O(p^6) \approx M_K^2/(4\pi F_\pi)^2 \approx 0.2$ are possible. The NA48 measurement (141) indicates that such corrections exist, while the KLOE (142) measurement is in line with the $O(p^4)$ prediction. The two experimental techniques are quite complementary. The channel is also interesting because the amplitude is the input for the determination of the $K_S^0 \to \mu^+\mu^-$ amplitude discussed above. Unfortunately, no new measurements are currently foreseen to clarify the issue.

Concerning $K_L^0 \to \pi^0\gamma\gamma$, in ChPT this decay is important in extracting the a_V parameter, which is used to estimate the *CP*-conserving component of $K_L^0 \to \pi^0\ell^+\ell^-$. Two determinations are available [NA48 (143) and KTeV (144)], and another determination from the KOTO experiment is expected in the future. The two existing measurements are in agreement and indicate that the *CP*-conserving contribution in $K_L^0 \to \pi^0e^+e^-$ is negligible.

The first observation of the decay $K^+ \to \pi^+\pi^0e^+e^-$ was made by the NA48/2 Collaboration (145). Both the branching ratio and the fraction of magnetic component with respect to the inner bremsstrahlung, extracted from a Dalitz plot analysis, were found to be in agreement with the predicted value (146).

5. HEAVY NEUTRAL LEPTONS AND EXOTICS

The long lifetime of kaons opens the interesting possibility of investigating, with good sensitivity, decays of kaons in exotic final states, including heavy neutral leptons and exotics such as

Table 5 Determinations of the Jarlskog invariant, *J*

Mode	$J\ (\times 10^5)$	Notes
$K_L^0 \to \pi^0 \nu\bar{\nu}$	≤ 30	KOTO 90% CL (68)
$K_{S,L}^0 \to \pi^0 e^+ e^-$	≤ 9	$\lvert\Im\lambda_t\rvert \leq 1.3 \times 10^{-3}$ (105)
$K^+ \to \pi^+ \nu\bar{\nu}$	≤ 5	GN limit (79), NA62 90% CL (57)
ε'/ε	3.60 ± 1.29	Taking into account typical theoretical estimates (153–155)
SM	3.18 ± 0.15	Global fit (PDG) (28)

Abbreviations: CL, confidence level; GN, Grossman–Nir; SM, Standard Model.

$K^+ \to \pi^+ X$, where X is a long-lived boson. A generic possibility of k new sterile neutrino mass eigenstates is

$$\nu_\alpha = \sum_{i=1}^{3+k} U_{\alpha i} \nu_i \qquad (\alpha = e, \mu, \tau).$$

On general grounds, the extension of the neutrino sector is motivated by its relation to the neutrino mass generation mechanism. The νMSM (147, 148) is the most economical theory that accounts for neutrino masses and oscillations, baryogenesis, and DM. Three heavy neutral leptons are posited to provide a DM candidate ($m_1 \approx 10$ keV/c^2) while two more massive neutrinos could exist with $m_{2,3} \approx 1$ GeV/c^2.

While pion decays allow exploration of the mass region between 60 and 135 MeV/c^2 (149), kaon decays enable us to extend a very sensitive search up to about 450 MeV/c^2. Limits at 90% CL on the square of the mixing angle extend down to about 10^{-8} for $K^+ \to \mu^+ N$ (150) and close to 10^{-9} for $K^+ \to e^+ N$ (151). One by-product of the $K \to \pi\nu\bar{\nu}$ analyses is the ability to search for new stable neutral bosons in two-body decays of the type $K^+ \to \pi^+ X$ (152) and $K_L^0 \to \pi^0 X$ (68).

6. PROSPECTS AND CONCLUSIONS

The current picture is that each manifestation of *CP* violation is compatible with the idea that the complex CKM phase is the sole source of *CP* violation. This contribution is generally not enough to generate the observed BAU, for which the presence of *C* and *CP* violation is one of the three necessary Sakharov conditions together with baryon number violation and lack of thermal equilibrium. It is clear that the search for new sources of *CP* violation is a compelling effort and that the study of as many systems as possible should be strongly pursued. This includes not only the study of kaons and *B* and *D* mesons but also the searches for *CP* violation in neutrino mixing and for *T*-violating electric dipole moments of elementary particles. Disagreeing determinations of fundamental SM parameters (like the CKM matrix elements) measured from different systems (e.g., V_{td} obtained from kaon and *B* systems) may well point to physics beyond the SM.

By 2025, the experimental precision on $K^+ \to \pi^+ \nu\bar{\nu}$ should match the current parametric uncertainty. NA62 might further improve the setup and increase the beam intensity to ultimately make it possible to measure the branching ratio with 5% precision. This would be an important test for higher-order weak interactions involving all the main ingredients of the SM: weak decays, GIM suppression, and electroweak and QCD higher-order corrections. The $K^+ \to \pi^+ \nu\bar{\nu}$ amplitude is sensitive to both *CP*-violating and *CP*-conserving contributions, which, in the SM, are dominant. Concerning the investigations of purely *CP*-violating contributions, the following question may arise: What is the interest in measuring $K_L^0 \to \pi^0 \nu\bar{\nu}$ with 20% precision when extra *CP*-violating contributions are already strongly bound by ε'/ε? To put things in perspective, in **Table 5** the current bounds on *J* from different kaon channels are compared with the result from the SM fit. One can see that only ε'/ε has a sensitivity approaching the SM fit, although the theoretical uncertainties are still large. Theoretical work, especially including lattice gauge field theories, will hopefully reduce the uncertainty and make it possible to exploit the good experimental precision (154). The situation is the opposite for $K_L^0 \to \pi^0 \nu\bar{\nu}$, where the experiments are orders of magnitude away from the interesting level of sensitivity. In a certain sense, theoretical improvement on the prediction of ε'/ε and experimental progress on $K_L^0 \to \pi^0 \nu\bar{\nu}$ may healthily compete to provide another decisive comparison between the kaon and the B system.

DISCLOSURE STATEMENT

The author was spokesperson for the NA62 experiment from its inception until the end of 2018. The author is not aware of any other affiliations, memberships, funding, or financial holdings that might be perceived as affecting the objectivity of this review.

ACKNOWLEDGMENTS

The author thanks the NA62 Collaboration for providing the preliminary data presented in this paper. Many people have made rare kaon decays a lively and stimulating field by contributing theoretical and experimental ideas. The risk of forgetting somebody is too high to be taken; thanks go to all.

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Annual Review of
Nuclear and
Particle Science

Volume 71, 2021

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