

DarkSUSY 6.3 – Freeze-in, out-of-equilibrium freeze-out, cosmic-ray upscattering and further new features

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DarkSUSY is a versatile tool for precision calculations of a large variety of dark matter-related signals, ranging from predictions for the dark matter relic density to dark matter self-interactions and rates relevant for direct and indirect detection experiments. In all of these areas significant new code additions have been made in recent years, since the release of DarkSUSY 6 in 2018, which we summarize in this overview. In particular, DarkSUSY now allows users to compute the relic density for feebly interacting massive particles via the freeze-in mechanism, but also offers new routines for freeze-out calculations in the presence of secluded dark sectors as well as for models where kinetic equilibrium is not fully established during the freeze-out process. On the direct detection side, the effect of cosmic-ray upscattering of dark matter has been fully implemented, leading to a subdominant relativistic component in the expected dark matter flux at Earth. Finally, updated yields relevant for indirect searches with gamma rays, neutrinos or charged cosmic rays have been added; the new default spectra are based on a large number of Pythia 8 runs, but users can also easily switch between various alternative spectra. Further code details, including a manual and various concrete example applications, are provided at www.darksusy.org.

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Contents

1	Introduction	2
2	Relic density: Boltzmann equations	3
2.1	Dark sector freeze-out	4
2.2	Freeze-out beyond kinetic equilibrium	5
2.3	Freeze-in	7
3	Direct detection: cosmic-ray upscattering of dark matter	8
4	Indirect detection: particle yields	10
5	Summary	11
A	Technical updates: installation and make system	11

1. Introduction

The identity of dark matter (DM) remains one of the central questions of fundamental physics, even though its present abundance of $\Omega_\chi h^2 = 0.12$ has been precisely measured over an impressive range of cosmological distance scales [1]. The minimal requirement on any theory beyond the standard model (BSM) that includes a potential DM candidate is therefore a DM production mechanism in the early universe explaining the observed value of $\Omega_\chi h^2$. Further experimental observables, based on non-gravitational interactions introduced in such BSM theories, are generally needed in order to actually close in on the nature of DM. Consequently, there is a huge demand on precision calculations both for the relic density and rates associated to other potentially observable DM signals. DarkSUSY [2, 3] is one of the major public and generic codes to perform such calculations (complemented by MicrOMEGAs [4], MadDM [5] and SuperIso Relic [6], which each have a somewhat different focus [7]).

The upgrade to DarkSUSY 6 [3] represented a major overhaul and restructuring of the code. The main novelty introduced in that release, besides many new physics features, is a highly modular structure that allows users to numerically compute DM properties beyond supersymmetric models (and more generally beyond weakly interacting massive particles as DM candidates). Since then, DarkSUSY has been widely used in the community. Recent major applications include sensitivity studies for a DM signal from the Galactic center by the Cherenkov Telescope Array (CTA) collaboration [8], searches for DM-related gamma-rays from the sun by the HAWC [9] and Fermi [10] collaborations, as well as searches for neutrino signals with Super-Kamionkande [11] and Ice-Cube [12]; furthermore, DarkSUSY is one of the main backends that DarkBit [13] relies on, for

its rate and relic density calculations, and as such decisive for the global fits performed by the GAMBIT collaboration [14, 15]. At the same time, the code has seen further active development and new features added, warranting an updated description beyond what is regularly reported on the homepage.¹

In these proceedings we summarize the most important updates since version 6.1. [3] of the code, both in terms of physics and actual implementation.² The text is organized along the three main directions where significant code updates have been implemented, namely relic density calculations (section 2), direct detection (section 3) and indirect detection routines (section 4). After a brief summary, in section 5, we also include a more technical Appendix A where we describe recent updates to the installation and make system that address in particular commonly encountered problems when building contributed code like HEALPix.

2. Relic density: Boltzmann equations

The evolution of the DM phase-space density $f_\chi(t, p)$ in the early universe is governed by the Boltzmann equation

$$E (\partial_t - Hp\partial_p) f_\chi = C_{\text{ann}}[f_\chi] + C_{\text{el}}[f_\chi], \quad (1)$$

where

$$C_{\text{ann}} = \frac{1}{2g_\chi} \int \frac{d^3\tilde{p}}{(2\pi)^3 2\tilde{E}} \int \frac{d^3k}{(2\pi)^3 2\omega} \int \frac{d^3\tilde{k}}{(2\pi)^3 2\tilde{\omega}} (2\pi)^4 \delta^{(4)}(\tilde{p} + p - \tilde{k} - k) |\mathcal{M}|_{\chi\chi \leftrightarrow \psi\psi}^2 \\ \times [f_\psi(\omega) f_\psi(\tilde{\omega}) \bar{f}_\chi(E) \bar{f}_\chi(\tilde{E}) - f_\chi(E) f_\chi(\tilde{E}) \bar{f}_\psi(\omega) \bar{f}_\psi(\tilde{\omega})], \quad (2)$$

$$C_{\text{el}} = \frac{1}{2g_\chi} \int \frac{d^3\tilde{p}}{(2\pi)^3 2\tilde{E}} \int \frac{d^3k}{(2\pi)^3 2\omega} \int \frac{d^3\tilde{k}}{(2\pi)^3 2\tilde{\omega}} (2\pi)^4 \delta^{(4)}(\tilde{p} + p - \tilde{k} - k) |\mathcal{M}|_{\chi\psi \leftrightarrow \chi\psi}^2 \\ \times [f_\chi(E) f_\psi(\omega) \bar{f}_\chi(\tilde{E}) \bar{f}_\psi(\tilde{\omega}) - f_\chi(\tilde{E}) f_\psi(\tilde{\omega}) \bar{f}_\chi(E) \bar{f}_\psi(\omega)] \quad (3)$$

describe the effect of two-body annihilations and elastic scattering processes with non-DM particles ψ , respectively. Here $H = \dot{a}/a$ is the Hubble parameter, with a the Friedman-Robertson-Walker scale factor, $|\mathcal{M}|^2$ is the squared amplitude summed over *both* initial and final state internal or spin degrees of freedom, g_i , and for ease of notation we suppressed the implicit sum over all contributing particle species ψ . We also introduced $\bar{f}_i \equiv 1 - \varepsilon_i f_i$ to capture the effect of final-state Pauli blocking for fermions ($\varepsilon_{\chi,\psi} = +1$) and Bose enhancement for Bosons ($\varepsilon_{\chi,\psi} = -1$).

For standard freeze-out calculations, it is assumed that all relevant particle species ψ are in full thermal equilibrium with the standard model, such that $f_\psi(\omega) = 1/[\exp(\omega/T) \pm 1]$ at a photon temperature T . Assuming furthermore that the DM particles are non-relativistic and stay in kinetic equilibrium with ψ during the entire freeze-out process, Eq. (1) can then be integrated [16] to an evolution equation for the DM number density n_χ that takes the familiar form

$$\dot{n}_\chi + 3Hn_\chi = -\langle\sigma v\rangle (n_\chi^2 - n_{\chi,\text{MB}}^2). \quad (4)$$

¹<http://www.darksusy.org>

²These proceedings do however *not* replace Ref. [3] as the correct way of referring to the most recent version of DarkSUSY 6. If you use DarkSUSY, please also consider citing Ref. [2] for code prior to version 4.2 that is still contained in the current release. Finally, most routines in DarkSUSY have been implemented in the context of original research work. Therefore, when using those routines, please also give proper credit to the relevant articles indicated in section 5 of the manual (as well as in the respective sections in these proceedings).

If the total entropy s is conserved, this can equivalently be stated as an equation for the DM abundance $Y \equiv n_\chi/s$:

$$\frac{dY}{dx} = \frac{\langle\sigma v\rangle}{x s \tilde{H}} \left(n_\chi^2 - n_{\chi,\text{MB}}^2 \right). \quad (5)$$

In the above equations, $n_\chi^{\text{MB}} \equiv g_\chi (2\pi)^{-3} \int d^3p f_\chi^{\text{MB}} = g_\chi m_\chi^2 T K_2(x) / (2\pi^2)$ denotes the number density of DM particles following a Maxwell-Boltzmann distribution of temperature T , with $x \equiv m_\chi/T$ and K_2 being a modified Bessel function of the second kind, and

$$\langle\sigma v\rangle = \int_1^\infty d\tilde{s} \frac{4x\sqrt{\tilde{s}}(\tilde{s}-1) K_1(2\sqrt{\tilde{s}}x)}{K_2^2(x)} \sigma_{\chi\chi \rightarrow \psi\psi} \equiv \int_1^\infty d\tilde{s} \frac{x\sqrt{\tilde{s}-1} K_1(2\sqrt{\tilde{s}}x)}{2m_\chi^2 K_2^2(x)} W_{\text{eff}}(s) \quad (6)$$

is the thermally averaged annihilation cross section w.r.t. such a distribution. Further, $\tilde{H} \equiv H/[1 + (1/3)d(\log g_*)/d(\log T)]$, where the number of entropy degrees of freedom g_* is defined through the relation $s \equiv (2\pi^2/45)g_*T^3$. W_{eff} is referred to as the (effective) invariant rate.

Eqs. (4–6) are widely used in relic density calculations, not the least because they take the same form even when including co-annihilations [17]. These equations have been implemented to a high numerical precision since the early versions of DarkSUSY, and we refer to Refs. [2, 3] for a detailed description. Notable additions since DarkSUSY 6.1 include updated degrees-of-freedom tables, based on lattice simulations as well as perturbative computations up to the 3-loop level [18, 19], and a major revision of how $\sigma_{\chi\chi \rightarrow \psi\psi}$ in Eq. (6) is tabulated while solving Eq. (5). In detail this method uses an adaptive integrator to on the fly tabulate W_{eff} where needed; if nearby points have already been tabulated, an interpolation is used instead of the numerically expensive calculation of W_{eff} . On top of this, before this tabulation starts, possible resonances in W_{eff} are inspected to see if they can be accurately fit with a Breit-Wigner form. If this is the case, the analytic Breit-Wigner form will be used instead of the actual expensive calculation of W_{eff} . Compared to earlier tabulation methods in DarkSUSY, this new method is typically both faster and more accurate.

A further addition in DarkSUSY 6.3 is that the Hubble expansion rate now is a replaceable function, `dsrdHubble`, allowing e.g. for relic density calculations in cosmologies with non-standard expansion histories.

In the following we describe in some more detail recent code updates that allow relic density calculations also in situations where one or more of the assumptions leading to Eqs. (4–6) are not satisfied. This has highly relevant applications for example when the DM particles are part of a secluded dark sector (section 2.1), the relic density is dominantly set by annihilation through a narrow resonance (section 2.2) or for freeze-in production of feebly interacting DM particles (section 2.3).

2.1 Dark sector freeze-out

Secluded dark sectors [20–22] constitute a prominent class of models where the DM relic density is not as usual set by freeze-out from the SM heat bath. The underlying idea of such scenarios is that DM might be interacting very weakly with the SM, but still sufficiently strongly to equilibrate with itself or other new particles. In general one therefore has to distinguish the photon temperature, T , from that of the DM (and other new ‘dark’) particles, T_χ . For example, the visible and the dark sector could have been in thermal contact at high temperatures, with $T_\chi = T$, and

only later decoupled at some temperature T_{dec} . In that case, since entropy is typically conserved separately in the two sectors, the temperature ratio will evolve as

$$\xi(T) \equiv \frac{T_\chi(T)}{T} = \frac{[g_*^{\text{SM}}(T)/g_*^{\text{SM}}(T_{\text{dec}})]^{\frac{1}{3}}}{[g_*^{\text{DS}}(T)/g_*^{\text{DS}}(T_{\text{dec}})]^{\frac{1}{3}}}, \quad (7)$$

where $g_*^{\text{SM,DS}}$ refers to the effective number of relativistic entropy d.o.f. in the visible and dark sector, respectively.³

In order to accurately describe the freeze-out of DM particles from such a secluded dark sector, the standard Boltzmann equation (4) must be adapted at three places:

1. the equilibrium density $n_{\chi,\text{MB}}$ must be evaluated at T_χ rather than the SM temperature T ;
2. the same replacement, $x \rightarrow x/\xi$, must also be made in the expression for $\langle\sigma v\rangle$ in Eq. (6);
3. the additional energy content of the dark sector must be reflected in an increased Hubble rate. During radiation domination, in particular, this implies $H^2 = (8\pi^3/90)g_{\text{tot}}M_{\text{pl}}^{-2}T^4$, with $g_{\text{tot}} = g_{\text{SM}} + \xi^4 g_{\text{DS}}$ and $g_{\text{SM,DS}}$ denoting the number of relativistic *energy* d.o.f. in the visible and dark sector, respectively.

The option to perform such relic density calculations has been implemented in DarkSUSY in the context of a more general endeavour [23] to update precision calculations of the ‘thermal’ annihilation cross section, i.e. the size of $\langle\sigma v\rangle$ that is needed – close to freeze-out – in order to match the observed DM relic abundance, and how the presence of dark sectors affects the numerical value of this quantity. Concretely, two new functions `dsrddofDS` and `dsrdxi` have been introduced and are now consistently used in all freeze-out calculations, returning $g_{\text{DS}}(T_\chi)$ and $\xi(T)$, respectively. If any of these functions is declared in a particle module, or when linking a main program, this automatically replaces their trivial implementation in the `main` library that corresponds to standard freeze-out (i.e. $g_{\text{DS}} \equiv 0, \xi \equiv 1$). In order to facilitate the implementation of such a model-specific version of `dsrddofDS`, DarkSUSY furthermore provides an auxiliary function `dsrdsingledof` that returns the temperature-dependent effective number of relativistic degrees of freedom for a single massive particle (resulting in exactly 1 and 7/8 for bosons and fermions, respectively, in the massless limit). An example of `dsrdxi` implementing Eq. (7) can be found in the `vdSIDM` module.

2.2 Freeze-out beyond kinetic equilibrium

The standard treatment of the freeze-out process consists in numerically solving Eq. (4) – which rests, just like the extensions described in section 2.1, on the assumption that DM remains in *kinetic* equilibrium during freeze-out.⁴ There is however an important subset of DM models that feature a strongly velocity-dependent annihilation cross section and where this assumption is not met. For example, using the conventional Eq. (4) may result in a relic density that does not even

³It is worth noting that this relation tacitly assumes that at least one of the additional particles ψ that the DM particles interact with has vanishing chemical potential. If all other dark sector particles are massive, too, this is in general no longer the case. See Ref. [23] for how to treat such situations.

⁴Technically, what enters in the derivation [16] of Eqs. (4–6) is that the DM phase-space distribution is of the form $f_\chi(p, T) = A(T) \exp(-E/T)$ both before and during the entire decoupling process.

have the correct order of magnitude in models with narrow resonances (like in particular for the standard model Higgs boson), Sommerfeld-enhanced annihilation, or for DM particles degenerate in mass with the annihilation products [24–28]. In such situations, a convenient alternative to the numerically challenging integration of the full Boltzmann equation at the phase-space level, Eq. (1), is to consider a set of Boltzmann equations that couple the evolution of the DM number density and velocity dispersion, respectively (as first introduced in Ref. [24], and later refined in Ref. [26]).

Introducing $T_\chi \equiv g_\chi/(3n_\chi) \int d^3p (2\pi)^{-3} (p^2/E) f_\chi$ and $y(x) \equiv m_\chi T_\chi s^{-2/3}$, in analogy to n_χ and $Y = n_\chi/s$, these equations generalize Eq. (5) to

$$\frac{x}{Y} \frac{dY}{dx} = \frac{sY}{\tilde{H}} \left[\frac{Y_{\text{eq}}^2}{Y^2} \langle \sigma v \rangle_T - \langle \sigma v \rangle_{T_\chi} \right], \quad (8)$$

$$\frac{x}{y} \frac{dy}{dx} = \frac{\gamma w}{\tilde{H}} \left[\frac{y_{\text{eq}}}{y} - 1 \right] + \frac{sY}{\tilde{H}} \left[\langle \sigma v \rangle_{T_\chi} - \langle \sigma v \rangle_{2,T_\chi} \right] + \frac{sY}{\tilde{H}} \frac{Y_{\text{eq}}^2}{Y^2} \left[\frac{y_{\text{eq}}}{y} \langle \sigma v \rangle_{2,T} - \langle \sigma v \rangle_T \right] + 2(1-w) \frac{H}{\tilde{H}}. \quad (9)$$

Here, a subscript T or T_χ indicates the temperature at which to take thermal averages, and $\langle \sigma v \rangle_2$ is a variant of Eq. (6) that is explicitly stated in Ref. [26]; the parameter $w(T_\chi) \equiv 1 - \langle p^4/E^3 \rangle_{T_\chi}/(6T_\chi)$ indicates deviations from DM being highly non-relativistic (where $w = 1$). The momentum transfer rate $\gamma(T)$, finally, is given by

$$\gamma = \frac{1}{3g_\chi m_\chi T} \int \frac{d^3k}{(2\pi)^3} g^\pm(\omega) [1 \mp g^\pm(\omega)] \int_{-4k_{\text{cm}}^2}^0 dt (-t) \frac{d\sigma}{dt} v, \quad (10)$$

where $k_{\text{cm}}^2 \equiv m_\chi^2 k^2/(m_\chi^2 + 2\omega m_\chi + m_f^2)$ and $|\mathcal{M}|^2$ in $(d\sigma/dt)v \equiv |\mathcal{M}|_{\chi f \leftrightarrow \chi f}^2/(64\pi k\omega m_\chi^2)$ is evaluated at $s \simeq m_\chi^2 + 2\omega m_\chi + m_f^2$. By construction, these equations accurately describe the evolution of $Y(x)$ as long as efficient DM self-interactions force $f_\chi(t, p)$ into a thermal shape with $T_\chi \neq T$ (or, rather, as long as the resulting thermal averages are not significantly affected by deviations from such a thermal shape); even more generally, in fact, relic density calculations based on these equations often provide a good estimate of the relic density that results from directly integrating Eq. (1). For a more detailed discussion see Ref. [28].

The numerical solution of the coupled system of Eqs. (8,9) has been implemented in DarkSUSY in the context of Ref. [28]. Concretely, a routine `dsrdomega_cBE` has been added that returns the final DM relic density based on these coupled Boltzmann equations – just as the conventional `dsrdomega` returns the relic density based on a solution of Eq. (4). The usage of both routines is illustrated in detail in a number of example programs located at `examples/aux/oh2_*.f`. A call to `dsrdomega_cBE` also initializes the function `dsrdthav_select` which for convenience returns the various thermal averages appearing in Eqs. (8,9), including the momentum exchange rate γ . In order to enable a main program to use `dsrdomega_cBE`, finally, a particle module must provide the typical interface functions required by *both* relic density and kinetic decoupling routines, i.e. `dsrdparticles`, `dsanwx`, `dskdparticles` and `dskdm2` (see the manual or Ref. [3] for more details).

2.3 Freeze-in

Another important exception to the validity of Eqs. (4) are DM particles with interactions so weak that they never thermalized with the heat bath. Such feebly interacting massive particles (FIMPs) could still obtain the correct relic density through continuous production from the thermal bath of standard model particles, a mechanism known as *freeze-in production* of DM [29, 30]. Starting from Eq. (1), the main technical difference to the freeze-out case is that a much larger range of temperatures are relevant for determining the final relic abundance, implying in particular that the DM particles can no longer be assumed to be non-relativistic and that the effect of quantum statistics ($f_i \neq 1$), but also other thermal effects, potentially become relevant. Still, it is possible to capture all these effects with a description that closely resembles that of the freeze-out case [31].

As long as the FIMP abundance stays far below the equilibrium abundance, in particular, it increases as

$$\frac{dY_\chi}{dx} = \frac{n_{\chi, \text{MB}}^2}{xs\tilde{H}} \langle \sigma v \rangle. \quad (11)$$

While this is *formally* the same as Eq. (5), without the ‘backreaction’ term that would describe the annihilation of FIMPs into standard model particles, there are some important differences:

1. The term n_{χ}^{MB} in Eq. (11) refers to the number density of a *would-be* Maxwell-Boltzmann distribution of DM particles. Unlike for WIMPs, cf. footnote 4, the above formulation does not assume in any way that the *actual* DM distribution is related to a thermal one.
2. Just as in the case of WIMPs, Eq. (11) is formulated in terms of the DM *annihilation* cross section σ . The thermal effects appearing due to the presence of relativistic DM particles, however, require a generalization of the thermal average given in Eq. (6).

Conveniently, the quantity $\langle \sigma v \rangle$ can still be expressed in terms of the same model-independent thermal kernel as in Eq. (6). The model-dependent invariant rate W_{eff} , on the other hand, now is also temperature-dependent and in general given by

$$W_{\text{eff}}(s, T) \equiv 16m_\chi^2 \frac{x\tilde{s}\sqrt{\tilde{s}-1}}{K_1(2\sqrt{\tilde{s}}x)} \int_1^\infty d\gamma \sqrt{\gamma^2-1} e^{-2\sqrt{\tilde{s}}x\gamma} \sum_{\psi_1\psi_2} \sigma_{\chi\chi \rightarrow \psi_1\psi_2}(s, \gamma). \quad (12)$$

Here the integration is over Lorentz boosts γ from the center-of-mass to the cosmic rest frame, and the in-medium cross section can be written as

$$\sigma_{\chi\chi \rightarrow \psi_1\psi_2}(s, \gamma) = \frac{N_\psi^{-1}}{8\pi s} \frac{|\mathbf{k}_{\text{CM}}|}{\sqrt{s-4m_\chi^2}} \int_{-1}^1 \frac{d\cos\theta}{2} |\mathcal{M}|_{\chi\chi \rightarrow \psi_1\psi_2}^2(s, \cos\theta) G_{\psi_1\psi_2}(\gamma, s, \cos\theta), \quad (13)$$

with $N_\psi = 2$ for identical SM particles ($\psi_1 = \psi_2$) and $N_\psi = 1$ otherwise. The quantity $G_{\psi_1\psi_2}(\gamma, s, \cos\theta)$ is explicitly stated in Ref. [31] and encodes the effect of quantum statistics in the final state, leading to an enhancement ($G_{\psi_1\psi_2} > 1$) or decrease ($G_{\psi_1\psi_2} < 1$) of the corresponding cross section in vacuum; for $G_{\psi_1\psi_2} = 1$, in particular, the definition of $W_{\text{eff}}(s, T)$ in Eq. (12) becomes identical to that of the conventional invariant rate given in Eq. (6). It is worth noting that a T -dependent W_{eff} also allows to include temperature-dependent effects other than

those due to quantum statistics, such as thermal masses and phase transitions, which can affect both interaction rates and the spectrum of relevant SM states.

The capability of DarkSUSY to perform freeze-in calculations has been added in the context of Ref. [31], which discusses in detail freeze-in production of Scalar Singlet DM [32]. In particular, `dsfi2to2oh2` numerically solves Eq. (11) and returns the resulting DM relic density, $\Omega_\chi h^2$, as a function of the reheating temperature (defined as the starting point of the integration). During the numerical integration, special care is taken to model the effects of QCD and EW phase transitions to sufficient accuracy. The thermally averaged cross section is provided by the function `dsfithav` (rather than `dsrdthav` as in the WIMP case). There are two interface functions that a particle physics module must provide for the freeze-in routines in the DarkSUSY core library to work: a subroutine `dsrdparticles`, providing kinematic information about (potentially T -dependent) thresholds and resonances, and a function `dsanwx_finiteT` returning the temperature-dependent effective invariant rate as defined in Eq. (12). Two example programs, `examples/aux/FreezeIn_ScalarSinglet` and `examples/aux/FreezeIn_generic_fimp`, illustrate the usage of the freeze-in routines for the Scalar Singlet model and a ‘generic’ FIMP model, respectively (where the latter implements a simplified contact-like interaction with $|\mathcal{M}|^2 \equiv c (s/\Lambda^2)^n$; for further details we refer to the manual and Ref. [31]). As of version 6.3, DarkSUSY also provides temperature-dependent SM masses (`dsmass_finiteT`) and Higgs vev (`dshvev_finiteT`), as well as an improved treatment of the partial Higgs decay width including in particular hadronic final states (`dssmgammahpartial.f`) – all of which must be modelled accurately e.g. for freeze-in calculations involving Higgs portal models [31].

3. Direct detection: cosmic-ray upscattering of dark matter

Conventional direct detection experiments were long thought to be insensitive to sub-GeV DM particles [33], because the typical kinetic energy of Galactic DM is too small to trigger nuclear recoil energies above the necessary threshold. For large elastic scattering cross sections with nuclei, however, there inevitably exists an irreducible flux of relativistic DM particles that are up-scattered by high-energy cosmic rays [34, 35]. This sub-dominant component of the expected DM flux at Earth allows to constrain both very light DM particles and DM particles in the GeV range that would otherwise be stopped in the overburden before reaching the detector [36–41].

The local interstellar flux of such cosmic-ray upscattered DM (CRDM) particles is given by

$$\frac{d\Phi_\chi}{dT_\chi} = D_{\text{eff}} \frac{\rho_\chi^{\text{local}}}{m_\chi} \sum_N \int_{T_N^{\text{min}}}^{\infty} dT_N \frac{d\sigma_{\chi N}}{dT_\chi} \frac{d\Phi_N^{\text{LIS}}}{dT_N}, \quad (14)$$

where $d\sigma_{\chi N}/dT_\chi$ is the differential elastic scattering cross section to accelerate DM to a kinetic recoil energy of T_χ , for an incident cosmic-ray (CR) nucleus N with energy T_N , and $d\Phi_N^{\text{LIS}}/dT_N$ is the *local* interstellar CR flux; ρ_χ is the local DM density and $D_{\text{eff}} \sim 10$ kpc is an effective distance out to which the above expression for the production of the CRDM component holds (for further details, see Refs. [40, 41]). The scattering rate of relativistic DM particles in underground detectors is formally given by the same expression as for the standard non-relativistic contribution, i.e.

$$\frac{d\Gamma_N}{dT_N} = \int_{T_\chi^{\text{min}}}^{\infty} dT_\chi \frac{d\sigma_{\chi N}}{dT_N} \frac{d\Phi_\chi}{dT_\chi}, \quad (15)$$

with T_χ^{\min} being the minimal DM energy needed in order to induce a nuclear recoil T_N . Here, the nuclear scattering cross section is in general a function of both the center-of-mass energy and the (spatial) momentum transfer, $Q^2 = 2m_N T_N$. If the dominant dependence on Q^2 factorizes – like in particular for form factors – the above rate has an identical Q^2 -dependence for relativistic and non-relativistic DM. It is then straight-forward to re-interpret published conventional direct detection results into limits on the CRDM component (the same also applies to neutrino detectors sensitive to nuclear recoils, after converting the nuclear recoil to the detected apparent electron energy T_e) [34].

For large scattering cross sections the complication arises that the CRDM flux in Eq. (14) is not necessarily the one that is relevant for underground laboratories, because the original CRDM flux is attenuated by scattering with nuclei in the overburden of the experimental location. In other words, the kinetic energy T_χ^z of a DM particle at depth z of the detector may be significantly less than its initial energy T_χ at the top of the atmosphere ($z = 0$). The expression for the rate in Eq. (15) then continues to apply, after a change of variables from T_χ^z to $T_\chi(T_\chi^z)$, but the scattering cross section $d\sigma_{\chi N}/dT_N$ must still be evaluated at the actual DM energy T_χ^z at the detector location. In order to find the *average* kinetic energy at the detector location, one needs to solve the energy loss equation

$$\frac{dT_\chi^z}{dz} = - \sum_N n_N \int_0^{T_N^{\max}} dT_N \frac{d\sigma_{\chi N}}{dT_N} T_N, \quad (16)$$

where the sum runs over the most dominant nuclei in the overburden, i.e. no longer over the CR species as in Eq. (14). When expressed in terms of an integration over Q^2 instead of T_N , the above expression can also be used to include the attenuation due to *inelastic* scattering, which dominates at high momentum transfers; to a good approximation, this can be modelled by adding a model-independent function $I^2(Q^2)$ to the usual nuclear form factors [41].

The necessary routines to compute DM limits resulting from the irreducible CRDM flux have been implemented and released with version 6.2 of DarkSUSY in the context of Ref. [34], with significant additions (full Q^2 and s -dependent elastic scattering cross sections, inelastic scattering, increased number of nuclei contributing to scattering processes) added subsequently [41, 42]. The interstellar CRDM flux, Eq. (14), is returned by the function `dsddMCRflux`, based on the dominant species in the CR flux $d\Phi_N^{\text{LIS}}/dT_N$ [43, 44] (provided by `dsCrISRflux`). The full relativistic scattering cross sections $d\sigma_{\chi N}/dT_N$ and $d\sigma_{\chi N}/dT_\chi$ that appear in the above expressions are implemented as the conventional cross sections in the highly non-relativistic limit – including state-of-the-art nuclear form factors – and then multiplied by a relativistic correction factor `dsddsigmarel` to take into account the model-dependent dependence on s and Q^2 ; in order to use the CDMR routines for arbitrary scattering cross sections, one thus only has to replace the function `dsddsigmarel`. While there is a separate function to calculate the rate in Eq. (15), namely `dsddMCRdgammaadt` directly, it is in practice most convenient to call the ‘driver’ routine `dsddMCRcountrate` which only takes the name of a given – direct detection or neutrino – experiment as input and directly returns the experimental count rate from the CRDM component, divided by the rate corresponding to the published limit of that experiment; this ensures that all experiment-specific settings are initialized correctly (for example the depth of the detector location, and the composition of the material in the overburden). The usage of `dsddMCRcountrate` is demonstrated in the example program

`examples/aux/DDCR_limits.f` (while `examples/aux/DDCR_flux.f` provides an illustration of how to compute and tabulate the interstellar CRDM flux).

4. Indirect detection: particle yields

The particle yields from DM annihilation or decay in the halo and in the Sun/Earth have traditionally, in DarkSUSY, been generated with Pythia 6 [45]. However, to allow for more flexibility and improved yield calculations, the possibility to use tables from Pythia 8 [46] runs is being added in DarkSUSY 6.3, based on Pythia 8.306 with default settings. For annihilations in the Sun/Earth, WimpSim [47] is used to handle hadron interactions and neutrino oscillations, and has correspondingly been updated to be based on Pythia 8 as an event generator as well.

For the Pythia 8 simulations we include for a range of (annihilating) DM masses between 5 GeV and 20 TeV. We use a total of 30 different masses and simulate annihilations into 15 different final states (all quark-antiquark final states, glue-gluon, W^+W^- , Z^0Z^0 , $\tau^+\tau^-$, prompt neutrino final states, hh and Z^0h , with h being the standard model Higgs boson). For each mass and annihilation channel, 10^7 annihilation events have been simulated. For neutrino oscillations, we use the NuFit 5.1 normal ordering best fit values [48, 49]. For interactions at a detector on Earth, simulations were performed for IceCube during the austral winter, but the results are to within a few percent applicable also for other detectors. The final yields are tabulated in both energy and angle, and then interpolated via the routines in DarkSUSY. Compared to the Pythia 6 runs, the Pythia 8 runs currently have lower statistics and hence the Pythia 6 runs will initially still be kept as the default. Eventually, with higher statistics runs becoming available, the default will change to yield tables based on Pythia 8. Independently of the default settings, the user can easily change which yield tables to use by calling `dsseyield_set`.

Similarly, for indirect DM detection rates related to annihilations or decays in the halo, we also rely on event generators to calculate the yield of positrons, antiprotons, gamma rays, neutrinos and anti-deuterons for a range of different DM masses and annihilation/decay channels. The traditional high-statistics tables based on Pythia 6 runs remain the default here, but we are in the process of adding Pythia 8 tables also for these yields, where we use the same range of masses and set of annihilation channels as described above for annihilation in the Sun/Earth (except that we here also include annihilation into $\mu^+\mu^-$).

On top of this, DarkSUSY now allows to use alternative yield calculations in its indirect detection routines; currently, this includes yield tables provided by Refs. [50–53]. The main difference between the various implemented yield tables concerns statistics, masses and channels that are simulated, but also assumptions regarding the underlying physics. Concretely, the (current) default tables based on Pythia 6 runs with very high statistics are particularly well tested and also include anti-deuteron yields; corresponding tables based on Pythia 8 runs, with default settings, will soon be available – but initially ship with somewhat reduced statistics compared to the present DarkSUSY implementation, and include all final states except anti-deuterons in the initial release. Externally provided yield tables are best described in the respective references. Roughly speaking, the tables by Bauer *et al.* [52] focus specifically on accurately modelling the yield in the multi-TeV regime and beyond, where Pythia ceases to be reliable, while those by Plehn *et al.* [51] focus on the sub-GeV regime and the impact of hadronic resonances that are not (fully) included in Pythia,

either. The yield tables by Jueid *et al.* [53] (as well as an earlier version by Amoroso *et al.* [50]), on the other hand, are also based on Pythia 8 and cover a similar DM mass range as in the DarkSUSY default implementation – but are based on a number of different treatments of the underlying QCD uncertainties related to the fragmentation of light quarks; the present DarkSUSY implementation returns their central prediction for the spectra.

Accessing the yield routines is straight-forward, as demonstrated in the example program `examples/aux/wimpyields.f`. In particular, a simple initial call to `dsanyield_set` allows to switch between the implemented yield tables, as well as to choose between various options (e.g. SM-like or $B-L$ -like models for the tables from Plehn *et al.* [51]). Such a call to `dsanyield_set` does not only affect the output of `dsanyield_sim`, as demonstrated in this program, but automatically adjusts the output of all routines returning indirect detection rates (e.g. gamma-ray or positron fluxes) as well.

5. Summary

DarkSUSY is a versatile numerical tool to compute potential DM observables that is widely used in the community. Here we have presented the most important updates since version 6.1 of the code, ranging from relic density computations beyond the standard thermal equilibrium assumption (section 2) to direct detection routines including the effect of cosmic-ray upscattering (section 3) and various versions of updated yield routines relevant for indirect detection (section 4). DarkSUSY continues to be in a state of active development, and the range of possible applications is expected to further increase in the near future. We thus recommend to regularly check `www.darksusy.org` for new releases, updated documentation, as well as new concrete example applications to get started. Using the code is simple and straightforward (see also Appendix A), but we happily encourage users to get in touch with the developers in case of problems – as well as for suggestions concerning further code additions or improvements.

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A. Technical updates: installation and make system

The installation of DarkSUSY is in principle as straightforward as downloading the most recent version from `www.darksusy.org/download.html`, unpacking the `darksusy-6.x.x.tgz` file and running

```
./configure  
make
```

in the newly created directory `darksusy-6.x.x/`. While typically not necessary, special options can as usual be specified in the `configure` step, for example to choose a specific compiler and compiler-specific performance flags (see, e.g., the script `./conf.gfortran` for an example optimized for the use with `gfortran`). Assuming that several cores are available, furthermore, the `make` command can typically be made significantly faster by adding the `-j` flag (which, however, makes the output harder to follow).

As indicated by the terminal output, the `make` command first builds *contributed* code, i.e. external numerical packages that a subset of DarkSUSY’s routines relies on,⁵ before proceeding to compile genuine DarkSUSY code (resulting in the main library, `lib/libds_core.a`, as well as a separate library for each of the shipped particle modules). The compilation of those external libraries is included for convenience, not the least to ensure a seamless interface with the rest of the code, but sometimes more demanding in terms of system requirements than that of the proper DarkSUSY libraries. The problem with this setup used to be that failing to compile contributed code would also stop the installation process for the rest of DarkSUSY. The updated `make` system takes care of this by just displaying a warning in such a situation and then, if possible with minimal damage, automatically removing any dependence of DarkSUSY on the respective contributed code package. If HEALPix fails to compile, e.g., the DarkSUSY main and module libraries will still be built without any problems, the only compromise being that a call to the integration routine `dshealpixint` will result in a corresponding warning message (for an example of HEALPIX-based line-of-sight integrations of the astrophysical J - or D -factors, another newly added feature, see the short demonstration program `examples/aux/DMhalo_los`). Likewise, the only noticeable impact of failing to compile HiggsBounds or HiggsSignals is that a call to `dshiggsbounds` will not as usual compute the p -value based on Higgs observables, in the `mssm` module, but instead return a warning that support for these libraries is disabled.

For compilation problems beyond the cases that can be handled automatically, furthermore, a new simplified target is available. The sequence of calls

```
make distclean
./configure
make darksusy_light
```

restores the pristine version of the code (i.e. the one after downloading and unpacking), and then installs a ‘*light*’ version of DarkSUSY that does not depend on any contributed code at all. While this will disable particle modules that heavily rely on contributed code, in particular the `mssm` module, most of the functionality of the core library as well as the majority of the particle modules will not be affected and build as usual.

Finally it is worth mentioning that the result of the standard installation process, as described above, consists of *static* libraries located in `lib/`. Interfacing DarkSUSY with other codes, however, sometimes requires *shared* libraries instead. We therefore now also provide corresponding targets, e.g. `make ds_mssm_shared`, which are heavily used for example in GAMBIT [14, 15].

⁵Presently, these include HEALPix [54], FeynHiggs [55, 56], HiggsBounds [57], HiggsSignals [58], ISAJET [59] and SuperIso [60].

References

- [1] **Planck Collaboration**, N. Aghanim et al., *Planck 2018 results. VI. Cosmological parameters*, *Astron. Astrophys.* **641** (2020) A6, [1807.06209]. [Erratum: *Astron. Astrophys.* 652, C4 (2021)].
- [2] P. Gondolo, J. Edsjo, P. Ullio, L. Bergstrom, M. Schelke, et al., *DarkSUSY: Computing supersymmetric dark matter properties numerically*, *JCAP* **07** (2004) 008, [astro-ph/0406204].
- [3] T. Bringmann, J. Edsjö, P. Gondolo, P. Ullio, and L. Bergström, *DarkSUSY 6 : An Advanced Tool to Compute Dark Matter Properties Numerically*, *JCAP* **07** (2018) 033, [1802.03399].
- [4] G. Bélanger, F. Boudjema, A. Goudelis, A. Pukhov, and B. Zaldivar, *micrOMEGAs5.0 : Freeze-in*, *Comput. Phys. Commun.* **231** (2018) 173–186, [1801.03509].
- [5] F. Ambrogio, C. Arina, M. Backovic, J. Heisig, F. Maltoni, et al., *MadDM v.3.0: a Comprehensive Tool for Dark Matter Studies*, *Phys. Dark Univ.* **24** (2019) 100249, [1804.00044].
- [6] A. Arbey and F. Mahmoudi, *SuperIso Relic: A Program for calculating relic density and flavor physics observables in Supersymmetry*, *Comput. Phys. Commun.* **181** (2010) 1277–1292, [0906.0369].
- [7] C. Arina, *Review on Dark Matter Tools*, *PoS TOOLS2020* (2021) 001, [2012.09462].
- [8] **CTA Collaboration**, A. Acharyya et al., *Sensitivity of the Cherenkov Telescope Array to a dark matter signal from the Galactic centre*, *JCAP* **01** (2021) 057, [2007.16129].
- [9] **HAWC Collaboration**, A. Albert et al., *Constraints on Spin-Dependent Dark Matter Scattering with Long-Lived Mediators from TeV Observations of the Sun with HAWC*, *Phys. Rev. D* **98** (2018) 123012, [1808.05624].
- [10] **Fermi-LAT Collaboration**, D. Serini, F. Loparco, and M. N. Mazziotta, *Constraints on dark matter scattering with long-lived mediators using gamma-rays from the Sun*, *PoS ICRC2019* (2020) 544.
- [11] **Super-Kamiokande Collaboration**, K. Abe et al., *Indirect search for dark matter from the Galactic Center and halo with the Super-Kamiokande detector*, *Phys. Rev. D* **102** (2020), no. 7 072002, [2005.05109].
- [12] **IceCube Collaboration**, C. Tönnes, *Search for secluded dark matter with 6 years of IceCube data*, in *37th International Cosmic Ray Conference*, 7, 2021. 2107.10778.
- [13] **GAMBIT Dark Matter Workgroup Collaboration**, T. Bringmann et al., *DarkBit: A GAMBIT module for computing dark matter observables and likelihoods*, *Eur. Phys. J. C* **77** (2017), no. 12 831, [1705.07920].

- [14] **GAMBIT Collaboration**, P. Athron et al., *GAMBIT: The Global and Modular Beyond-the-Standard-Model Inference Tool*, *Eur. Phys. J. C* **77** (2017), no. 11 784, [1705.07908]. [Addendum: *Eur.Phys.J.C* 78, 98 (2018)].
- [15] S. Bloor, T. E. Gonzalo, P. Scott, C. Chang, A. Raklev, et al., *The GAMBIT Universal Model Machine: from Lagrangians to likelihoods*, *Eur. Phys. J. C* **81** (2021), no. 12 1103, [2107.00030].
- [16] P. Gondolo and G. Gelmini, *Cosmic abundances of stable particles: Improved analysis*, *Nucl. Phys. B* **360** (1991) 145–179.
- [17] J. Edsjo and P. Gondolo, *Neutralino relic density including coannihilations*, *Phys. Rev. D* **56** (1997) 1879–1894, [hep-ph/9704361].
- [18] M. Laine and Y. Schroder, *Quark mass thresholds in QCD thermodynamics*, *Phys. Rev. D* **73** (2006) 085009, [hep-ph/0603048].
- [19] M. Laine and M. Meyer, *Standard Model thermodynamics across the electroweak crossover*, *JCAP* **07** (2015) 035, [1503.04935].
- [20] M. Pospelov, A. Ritz, and M. B. Voloshin, *Secluded WIMP Dark Matter*, *Phys. Lett. B* **662** (2008) 53–61, [0711.4866].
- [21] M. Pospelov, *Secluded $U(1)$ below the weak scale*, *Phys. Rev. D* **80** (2009) 095002, [0811.1030].
- [22] J. L. Feng, H. Tu, and H.-B. Yu, *Thermal Relics in Hidden Sectors*, *JCAP* **10** (2008) 043, [0808.2318].
- [23] T. Bringmann, P. F. Depta, M. Hufnagel, and K. Schmidt-Hoberg, *Precise dark matter relic abundance in decoupled sectors*, *Phys. Lett. B* **817** (2021) 136341, [2007.03696].
- [24] L. G. van den Aarssen, T. Bringmann, and Y. C. Goedecke, *Thermal decoupling and the smallest subhalo mass in dark matter models with Sommerfeld-enhanced annihilation rates*, *Phys. Rev. D* **85** (2012) 123512, [1202.5456].
- [25] M. Duch and B. Grzadkowski, *Resonance enhancement of dark matter interactions: the case for early kinetic decoupling and velocity dependent resonance width*, *JHEP* **09** (2017) 159, [1705.10777].
- [26] T. Binder, T. Bringmann, M. Gustafsson, and A. Hryczuk, *Early kinetic decoupling of dark matter: when the standard way of calculating the thermal relic density fails*, *Phys. Rev. D* **96** (2017), no. 11 115010, [1706.07433]. [Erratum: *Phys.Rev.D* 101, 099901 (2020)].
- [27] T. Binder, M. Gustafsson, A. Kamada, S. M. R. Sandner, and M. Wiesner, *Reannihilation of self-interacting dark matter*, *Phys. Rev. D* **97** (2018), no. 12 123004, [1712.01246].
- [28] T. Binder, T. Bringmann, M. Gustafsson, and A. Hryczuk, *DRAKE: Dark matter Relic Abundance beyond Kinetic Equilibrium*, *Eur. Phys. J. C* **81** (2021) 577, [2103.01944].

- [29] L. J. Hall, K. Jedamzik, J. March-Russell, and S. M. West, *Freeze-In Production of FIMP Dark Matter*, *JHEP* **03** (2010) 080, [0911.1120].
- [30] X. Chu, T. Hambye, and M. H. G. Tytgat, *The Four Basic Ways of Creating Dark Matter Through a Portal*, *JCAP* **05** (2012) 034, [1112.0493].
- [31] T. Bringmann, S. Heeba, F. Kahlhoefer, and K. Vangsnes, *Freezing-in a hot bath: resonances, medium effects and phase transitions*, 2111.14871.
- [32] V. Silveira and A. Zee, *SCALAR PHANTOMS*, *Phys. Lett. B* **161** (1985) 136–140.
- [33] **XENON Collaboration**, E. Aprile et al., *Dark Matter Search Results from a One Ton-Year Exposure of XENON1T*, *Phys. Rev. Lett.* **121** (2018), no. 11 111302, [1805.12562].
- [34] T. Bringmann and M. Pospelov, *Novel direct detection constraints on light dark matter*, *Phys. Rev. Lett.* **122** (2019), no. 17 171801, [1810.10543].
- [35] C. V. Cappiello, K. C. Y. Ng, and J. F. Beacom, *Reverse Direct Detection: Cosmic Ray Scattering With Light Dark Matter*, *Phys. Rev. D* **99** (2019), no. 6 063004, [1810.07705].
- [36] Y. Ema, F. Sala, and R. Sato, *Light Dark Matter at Neutrino Experiments*, *Phys. Rev. Lett.* **122** (2019), no. 18 181802, [1811.00520].
- [37] C. V. Cappiello and J. F. Beacom, *Strong New Limits on Light Dark Matter from Neutrino Experiments*, *Phys. Rev. D* **100** (2019), no. 10 103011, [1906.11283]. [Erratum: *Phys.Rev.D* 104, 069901 (2021)].
- [38] J. B. Dent, B. Dutta, J. L. Newstead, and I. M. Shoemaker, *Bounds on Cosmic Ray-Boosted Dark Matter in Simplified Models and its Corresponding Neutrino-Floor*, *Phys. Rev. D* **101** (2020), no. 11 116007, [1907.03782].
- [39] C. Xia, Y.-H. Xu, and Y.-F. Zhou, *Constraining light dark matter upscattered by ultrahigh-energy cosmic rays*, *Nucl. Phys. B* **969** (2021) 115470, [2009.00353].
- [40] C. Xia, Y.-H. Xu, and Y.-F. Zhou, *Production and attenuation of cosmic-ray boosted dark matter*, 2111.05559.
- [41] T. B. J. Alvey and K. Kolesova, *No room to hide: cosmic-ray upscattering of GeV-scale dark matter*, in prep. (2022).
- [42] K. Bondarenko, A. Boyarsky, T. Bringmann, M. Hufnagel, K. Schmidt-Hoberg, et al., *Direct detection and complementary constraints for sub-GeV dark matter*, *JHEP* **03** (2020) 118, [1909.08632].
- [43] M. J. Boschini et al., *Deciphering the local Interstellar spectra of primary cosmic ray species with HelMod*, *Astrophys. J.* **858** (2018), no. 1 61, [1804.06956].
- [44] M. J. Boschini et al., *Inference of the Local Interstellar Spectra of Cosmic-Ray Nuclei $Z \leq 28$ with the GalProp–HelMod Framework*, *Astrophys. J. Suppl.* **250** (2020), no. 2 27, [2006.01337].

- [45] T. Sjöstrand, S. Mrenna, and P. Z. Skands, *PYTHIA 6.4 Physics and Manual*, *JHEP* **05** (2006) 026, [[hep-ph/0603175](#)].
- [46] T. Sjöstrand, S. Ask, J. R. Christiansen, R. Corke, N. Desai, et al., *An introduction to PYTHIA 8.2*, *Comput. Phys. Commun.* **191** (2015) 159–177, [[1410.3012](#)].
- [47] J. Edsjö, “NuSigma Neutrino Interaction Monte Carlo.” <http://www.fysik.su.se/~edsjo/wimpsim/>, 2007.
- [48] I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, T. Schwetz, and A. Zhou, *The fate of hints: updated global analysis of three-flavor neutrino oscillations*, *JHEP* **09** (2020) 178, [[2007.14792](#)].
- [49] I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, I. Martinez-Soler, and T. Schwetz, “NuFIT 5.0.” <http://www.nu-fit.org/>, 2021.
- [50] S. Amoroso, S. Caron, A. Jueid, R. Ruiz de Austri, and P. Skands, *Estimating QCD uncertainties in Monte Carlo event generators for gamma-ray dark matter searches*, *JCAP* **05** (2019) 007, [[1812.07424](#)].
- [51] T. Plehn, P. Reimitz, and P. Richardson, *Hadronic Footprint of GeV-Mass Dark Matter*, *SciPost Phys.* **8** (2020) 092, [[1911.11147](#)].
- [52] C. W. Bauer, N. L. Rodd, and B. R. Webber, *Dark matter spectra from the electroweak to the Planck scale*, *JHEP* **06** (2021) 121, [[2007.15001](#)].
- [53] A. Jueid, J. Kip, R. R. de Austri, and P. Skands, *Impact of QCD uncertainties on antiproton spectra from dark-matter annihilation*, [2202.11546](#).
- [54] K. M. Górski, E. Hivon, A. J. Banday, B. D. Wandelt, F. K. Hansen, et al., *HEALPix - A Framework for high resolution discretization, and fast analysis of data distributed on the sphere*, *Astrophys. J.* **622** (2005) 759–771, [[astro-ph/0409513](#)].
- [55] S. Heinemeyer, W. Hollik, and G. Weiglein, *FeynHiggs: A Program for the calculation of the masses of the neutral CP even Higgs bosons in the MSSM*, *Comput. Phys. Commun.* **124** (2000) 76–89, [[hep-ph/9812320](#)].
- [56] T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak, and G. Weiglein, *FeynHiggs: A program for the calculation of MSSM Higgs-boson observables - Version 2.6.5*, *Comput. Phys. Commun.* **180** (2009) 1426–1427.
- [57] P. Bechtle, O. Brein, S. Heinemeyer, O. Stål, T. Stefaniak, et al., *HiggsBounds – 4: Improved Tests of Extended Higgs Sectors against Exclusion Bounds from LEP, the Tevatron and the LHC*, *Eur. Phys. J. C* **74** (2014), no. 3 2693, [[1311.0055](#)].
- [58] P. Bechtle, S. Heinemeyer, O. Stål, T. Stefaniak, and G. Weiglein, *HiggsSignals: Confronting arbitrary Higgs sectors with measurements at the Tevatron and the LHC*, *Eur. Phys. J. C* **74** (2014), no. 2 2711, [[1305.1933](#)].

- [59] F. E. Paige, S. D. Protopopescu, H. Baer, and X. Tata, *ISAJET 7.69: A Monte Carlo event generator for pp , anti- $p p$, and $e+e-$ reactions*, [hep-ph/0312045](#).
- [60] F. Mahmoudi, *SuperIso v2.3: A Program for calculating flavor physics observables in Supersymmetry*, *Comput. Phys. Commun.* **180** (2009) 1579–1613, [[0808.3144](#)].