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Wilson coefficients and natural zeros from the on-shell viewpoint

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Abstract: We show how to simplify the calculation of the finite contributions from heavy particles to EFT Wilson coefficients by using on-shell methods. We apply the technique to the one-loop calculation of $g-2$ and $H\gamma\gamma$, showing how finite contributions can be obtained from the product of tree-level amplitudes. In certain cases, due to a parity symmetry of these amplitudes, the total contribution adds up to zero, as previously found in the literature. Our method allows to search for new natural zeros, as well as to obtain non-zero contributions in a straightforward way.

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Contents

1 Introduction

The Effective Field Theory (EFT) approach to describe low-energy experiments is based on the idea of integrating out heavy particles from the theory, keeping only the relevant light degrees of freedom. The effects of these heavy particles are then captured in the (Wilson) coefficients of the EFT which become the relevant parameters to be measured in low-energy experiments, as for example the magnetic dipole moment of the SM fermions, $g-2$.

When performing these calculations using Feynman rules, it has been found that in certain models the different contributions to the Wilson coefficients add up to zero with no apparent explanation. An example for the $g-2$ case has been extensively studied in [1], and other examples can be found e.g. in [2] for the Higgs coupling to photons, $H\gamma\gamma$. In these cases, there is no obvious symmetry which would explain this cancellation, jeopardizing the idea of naturalness which states that *contributions not forbidden by symmetries are compulsory*. In [1] instead the vanishing of the $g-2$ Wilson coefficient was found to arise since the total contribution is a total derivative. In the following, we will show that the cancellations can also be understood from exchange symmetries which act at the amplitude level.

We will analyze the cancellations by calculating the finite contributions to the Wilson coefficients using on-shell methods. These methods have already been useful to understand certain cancellations in the anomalous dimensions which looked mysterious from the

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Feynman approach [3]. We will show that the finite contributions to $g-2$ arising from integrating out heavy fermions can also be obtained by a product of tree-level amplitudes. In certain models, these amplitudes are odd under the interchange of the heavy fermions, while the total contribution must be even under this interchange. This provides an explanation in terms of symmetries acting at the amplitude level of the vanishing Wilson coefficients in the models in [1, 2]. It will also allow us to find new cases where the contributions add up to zero. For models without this parity, the contributions will not cancel, and our method will explicitly provide the Wilson coefficients as a simple product of tree-level amplitudes.

We will extend the analysis also to the Wilson coefficient of the $H\gamma\gamma$ coupling (for tree-level calculations using on-shell methods, see [4]). We will see that the same argument as for the $g-2$ case can lead to an explanation for the absence of the total contribution to this Wilson coefficient in certain models of heavy fermions.

The calculation of Wilson coefficients using on-shell amplitudes has been previously studied in the literature — see for example [5] and references therein. Nevertheless, in these cases first the full amplitude is calculated, and later the heavy mass limit is taken to match with the EFT. This makes the method too long and probably not so competitive with the Feynman approach. The main purpose here will be to understand what cuts in the one-loop amplitudes are needed in order to simply extract the finite contributions to the Wilson coefficients, specially for cases of phenomenological interest. This will also allow us to more clearly understand the origin of the rational number appearing in the Wilson coefficients.

While this work was being written, the article [6] appeared where also a symmetry argument was presented as an explanation of the zeros found in [1]. Although the symmetry is also an interchange parity, the approach in [6] is different from the one followed here [7].

2 Finite contributions to Wilson coefficients via on-shell methods

Amplitudes at the one-loop level can have a Passarino-Veltman decomposition given by

$$\mathcal{A}_{\text{loop}} = \sum_a C_1^{(a)} I_1^{(a)} + \sum_b C_2^{(b)} I_2^{(b)} + \sum_c C_3^{(c)} I_3^{(c)} + \sum_d C_4^{(d)} I_4^{(d)} + R\,, \tag{2.1}$$

where I_n are master scalar integrals with n propagators ($n=1,2,3,4$) and C_n are mass- and kinematic-dependent coefficients. The master integrals are given by

$$I_n = (-1)^n \mu^{4-D} \int \frac{d^D\ell}{i(2\pi)^D} \frac{1}{(\ell^2 - M_0^2)\,((\ell - P_1)^2 - M_1^2)\,((\ell - P_1 - P_2)^2 - M_2^2)\cdots}\,, \tag{2.2}$$

where $P_1, P_2, \ldots, P_{n-1}$ are sums of external particle momenta p_i. The first four contributions to eq. (2.1) are called respectively tadpoles, bubbles, triangles and boxes, according to the topology of the scalar integral. Terms collected under R are rational functions of kinematic invariants. We will be using dimensional regularization, $D = 4 - 2\epsilon$.

In general $\mathcal{A}_{\text{loop}}$ can be divergent. Nevertheless, here we are only interested in one-loop effects from renormalizable theories contributing to processes, such as the magnetic dipole moment, which must go to zero as the heavy masses go to infinity, and are therefore UV convergent. Even in these cases, it is still possible to have IR divergencies ($\propto 1/M_i^2 \ln M_i/\mu$)

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which would signal the presence of nonzero anomalous dimensions. The processes that we will consider here will however also be IR convergent.[1]

To match with the EFT, we must take the limit in which the masses of the heavy particles in $\mathcal{A}_{\text{loop}}$ are larger than all the external momenta p_i. For simplicity in this section we take these masses to be equal to M. Note that I_n can involve both massless, $M_i = 0$, as well as massive states, $M_j = M$. The external states will be assumed to be massless, $p_i^2 = 0$. By performing an expansion for $P_i \ll M$ in eq. (2.1), the amplitude $\mathcal{A}_{\text{loop}}$ should match with the amplitude associated with a higher-dimensional operator $\mathcal{A}_{\mathcal{O}_i} = \langle 12\ldots|\mathcal{O}_i|0\rangle$. Assuming that the leading operator in this expansion has dimension six, we have

$$\mathcal{A}_{\text{loop}} \to \frac{C_i}{M^2}\mathcal{A}_{\mathcal{O}_i} + \cdots\,, \tag{2.3}$$

where C_i is a rational number times some couplings divided by $16\pi^2$, and is often referred to as the Wilson coefficient of the corresponding operator. In appendix A we will prove that no contribution at order $1/M^2$ can arise from R in eq. (2.1). Therefore we have that the Wilson coefficients are given by

$$C_i = \frac{1}{\mathcal{A}_{\mathcal{O}_i}} \lim_{P_i/M\to 0} M^2 \left(\sum_a C_1^{(a)} I_1^{(a)} + \sum_b C_2^{(b)} I_2^{(b)} + \sum_c C_3^{(c)} I_3^{(c)} + \sum_d C_4^{(d)} I_4^{(d)}\right). \tag{2.4}$$

From eq. (2.4) we see that in order to determine the Wilson coefficients we need to know the coefficients $C_n^{(a)}$. These coefficients can however be easily obtained using on-shell methods. In particular generalized unitarity methods, extensively developed in the literature in recent years [8], allow one to calculate $C_n^{(a)}$ without the need to perform loop calculations. Instead one uses products of tree-level amplitudes (integrated over some phase space), making the determination of the Wilson coefficients C_i clearer. The idea is to obtain the coefficients $C_n^{(a)}$ from performing n-cuts in the loop. Although this can look like a lengthy procedure, we will see that in many cases, and specially those we are interested in, the situation is quite simple and only one or two 2-cuts are needed.

3 Dipole moment Wilson coefficient

We start by calculating the Wilson coefficient of the magnetic dipole moment induced by different models with heavy fermions. For the SM leptons this operator is defined as

$$\frac{C_\gamma}{2M^2}\frac{q_e}{\sqrt{2}}\,\bar{\ell}_L\sigma_{\mu\nu}e_R H F^{\mu\nu} = \frac{C_\gamma}{M^2}\,q_e\,\ell_\alpha e_\beta H F^{\alpha\beta}\,, \tag{3.1}$$

where we have introduced the 2-component Weyl spinors ℓ_α and e_α of helicity $h = -1/2$, and F refers to the field strength of the photon. We follow the usual definition of the gauge coupling used when calculating amplitudes with spinors ($\Delta\mathcal{L} = q_f A_\mu \bar{f}\gamma^\mu f/\sqrt{2}$) which avoids the proliferation of factors of $\sqrt{2}$ in the calculations. In particular, eq. (3.1) leads to the amplitude

$$\frac{C_\gamma}{M^2}\mathcal{A}_D(1_\ell, 2_e, 3_{\gamma_-}, 4_{H^0}) = \frac{C_\gamma}{M^2}\,q_e\,\langle 13\rangle\langle 23\rangle\,, \tag{3.2}$$

[1] For obtaining anomalous dimensions via on-shell methods, see for example [9–16].

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where H^0 is the neutral Higgs component, and with an abuse of notation we have denoted by ℓ also the charged-lepton component of the $\mathrm{SU}(2)_L$ doublet. Subindices $\pm$ denote helicities $h = \pm 1$ and all particles are taken to be incoming. We use spinor-helicity notation [8] using properties and conventions which are summarized in appendix B. A key point for our calculation is to set the Higgs momentum to zero, $p_{H^0} = 0$, as this enormously simplifies the loop amplitude. This is possible because the amplitude (3.2) does not explicitly depend on the Higgs momentum.

3.1 Massive vector-like singlet S and doublet L

The first model we consider is the one studied in ref. [1] which consists of two massive vector-like fermions, a singlet (S) and an $\mathrm{SU}(2)_L$ doublet (L). The Lagrangian in Weyl notation is given by (omitting Lorentz indices)

$$\mathcal{L} = -Y_L \ell S \tilde{H} - Y_R L e H - Y_V \tilde{H}^\dagger L^c S^c - Y'_V L S \tilde{H} - M_S S S^c - M_L L L^c + \text{h.c.}\,, \tag{3.3}$$

where $\tilde{H} = i\sigma_2 H^*$. For simplicity we choose the couplings to be real. As we will see, it will be useful for our calculation to take $M_S \neq M_L$ with both being larger than the P_i. As in ref. [1], we set the SM Yukawa coupling for the muon to zero, $Y_\ell = 0$ (at tree-level).

$Y'_V \neq 0$ case. Let us start by considering the case with $Y_V = 0$, $Y'_V \neq 0$. The Feynman diagram which contributes to the SM lepton dipole moment is given in figure 1. We will follow eq. (2.4) for the calculation. The coefficients C_n which enter this relation can be obtained by applying n-cuts on both sides of eq. (2.1), after analytically continuing to complex momenta. In particular, performing 4-cuts, the coefficients C_4 are given by products of four amplitudes. Similarly, C_3 and C_2 can be obtained from 3-cuts and 2-cuts, respectively, provided that the contributions from boxes and triangles are subtracted.

In this example and the others that will follow, however, all the coefficients C_4 and C_3 vanish, as all possible 4-cuts and 3-cuts of figure 1 give zero. The reason is the following. Since we are taking $p_{H^0} = 0$, we have $p_S = p_L$ and then the condition to have S and L simultaneously on-shell cannot be fulfilled as both have different masses. This implies that the 4-cut gives zero and there are thus no boxes. The only potential nonzero 3-cut must then arise from cutting two massless states and one massive state. The coefficient of the corresponding triangle is however also zero. Indeed, one can follow the arguments of ref. [11] to prove that in the absence of IR divergencies (as it is our case), IR-divergent triangles cannot be present when there are no boxes. We are thus left only with bubbles and can obtain their coefficients C_2 directly from applying 2-cuts to both sides of eq. (2.1). The bubble coefficients are then given by a phase-space integral over the product of two amplitudes. Note that there could also be momentum-independent bubbles $I_2(p^2 = 0, M^2, 0)$ and tadpoles $I_1(M^2) = M^2 I_2(0, M^2, 0)$. However, these bubbles and the tadpoles, added to the momentum-independent terms of the other bubbles (see e.g. eq. (3.8) below), must sum to zero since the one-loop amplitude cannot have divergent terms. We can therefore ignore them in the following.

According to eq. (2.4), we must perform an expansion in p/M. For that purpose, the momentum controlling the expansion can be freely chosen among those of all the external

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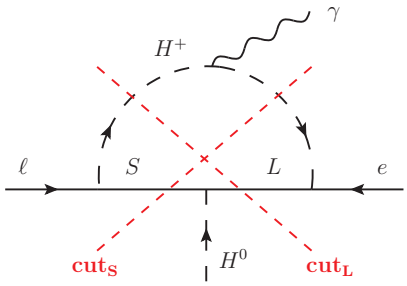

Figure 1. One-loop contribution to the $g-2$ of the SM leptons from the model eq. (3.3) with $Y_V = 0$ and $Y'_V \neq 0$, with the relevant 2-cuts.

lines. Notice that since $p_{H^0} = 0$, we have $p_1 + p_2 + p_3 = 0$. Therefore if we work in the limit in which $s_{13} = (p_1 + p_3)^2$ is small but nonzero, we will have to take also $p_2^2 = s_{13} \neq 0$. This means that the fermion e is either slightly off-shell or has a small mass equal to s_{13}.[2] Alternatively, we can take the limit $s_{23} = (p_2 + p_3)^2 \to 0$ and then $p_1^2 = s_{23} \neq 0$ (i.e. the fermion ℓ slightly off-shell or massive). Let us choose the first option and consider the 2-cuts where S becomes on-shell. There are in principle two possible 2-cuts of this type. However, the one leaving ℓ alone as an external leg is proportional to $I_2(p_1^2 = 0, M_S^2, 0)$ and cannot give any contribution of $O(s_{13}/M^2)$. The only relevant 2-cut is then the one depicted by $\mathbf{cut_S}$ in figure 1. We have

$$C_2^{(13)} = \int d\text{LIPS}\,(-1)^F \mathcal{A}(1_\ell, 3_{\gamma_-}, 1'_S, 3'_{H^+}) \times \mathcal{A}(3'_{\bar{H}^+}, 1'_{\bar{S}}, 2_e, 4_{H^0})\,, \tag{3.4}$$

where the integral is over the Lorentz-Invariant Phase Space (LIPS) associated with the momenta of the two cut states, $p_{1'}$ and $p_{3'}$, normalized as $\int d\text{LIPS} = 1$. With a bar over a state we denote that the signs of the momentum, helicity and all other quantum numbers of the state have been reversed, and F is the number of internal fermions ($F = 1$ in this case) [11].

The tree-level amplitudes in eq. (3.4) can be easily calculated from the model eq. (3.3). We use the spinor-helicity formalism for massive particles from ref. [17], using properties and conventions which are summarized in appendix B. This gives (recall that $p_{H^0} = 0$)

$$\mathcal{A}(1_\ell, 3_{\gamma_-}, 1'_S, 3'_{H^+}) = q_e Y_L M_S \frac{[3'\mathbf{1}']}{[3'3][13]}\,, \qquad \mathcal{A}(3'_{\bar{H}^+}, 1'_{\bar{S}}, 2_e, 4_{H^0}) = Y_R Y'_V \frac{[-\mathbf{1}'|p_{1'}|2\rangle}{M_S^2 - M_L^2}\,. \tag{3.5}$$

Writing the SU(2) little-group indices of the bold spinor-helicity variables explicitly, the integrand in eq. (3.4) is then given by

$$\begin{aligned}\mathcal{A}(1_\ell, 3_{\gamma_-}, 1'^I_S, 3'_{H^+})\,\epsilon_{IJ}\,\mathcal{A}(3'_{\bar{H}^+}, 1'^J_{\bar{S}}, 2_e, 4_{H^0}) &= -q_e Y_L Y_R Y'_V \frac{M_S^2}{M_S^2 - M_L^2} \frac{[3'|(p_3 + p_1)|2\rangle}{[3'3][13]} \\ &= -q_e Y_L Y_R Y'_V \frac{M_S^2}{M_S^2 - M_L^2} \left(\frac{\langle 32\rangle}{[13]} + \frac{[3'1]\langle 12\rangle}{[3'3][13]} \right), \end{aligned} \tag{3.6}$$

[2] Note that below we work with amplitudes where all external particles are on-shell and massless. However, the extension to massive external particles is straightforward by bolding the corresponding spinor-helicity variables. After taking the massless limit, the result is the same as working with massless particles from the start.

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where we have used that $|1']^I[-1'|_I = M_S$ (see appendix B). Only the first term can give a contribution to the dipole since in the second term the two external spinors are contracted among themselves, $\langle 12\rangle$ (cf. eq. (3.2)).[3] The first term of eq. (3.6) does not depend on the internal spinors so the dLIPS integration in eq. (3.4) is trivial leading to

$$C_2^{(13)} = Y_L Y_R Y'_V \frac{M_S^2}{M_S^2 - M_L^2} \frac{1}{s_{13}} \mathcal{A}_D(1_l, 2_e, 3_{\gamma_-}, 4_{H^0}) \,. \tag{3.7}$$

Plugging this into eq. (2.4), and expanding the bubble integral to $O(s_{13}/M_S^2)$,

$$I_2^{(13)}(s_{13}, M_S^2, 0) \simeq \frac{1}{16\pi^2} \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{M_S^2} + 1 + \frac{s_{13}}{2M_S^2} + \cdots \right) , \tag{3.8}$$

we obtain a finite term[4] corresponding to the contribution from this 2-cut to the Wilson coefficient:

$$\frac{\Delta C_\gamma}{M^2} = \frac{Y_L Y_R Y'_V}{32\pi^2} \frac{1}{M_S^2 - M_L^2} \,. \tag{3.9}$$

There are also 2-cuts where L instead of S is put on-shell, in particular $\mathbf{cut_L}$ of figure 1. It is clear however that this 2-cut is identical to $\mathbf{cut_S}$ by the exchange[5]

$$S \leftrightarrow L \,, \qquad \ell \leftrightarrow e \,. \tag{3.10}$$

Therefore the contribution must be the same as eq. (3.9) with the replacement $M_S \leftrightarrow M_L$. Since eq. (3.19) is odd under this transformation, the total contribution to C_γ adds up to zero.

It is easy to understand this cancellation without the need to go through all the details of the calculation. The first important thing to know is how M_L enters into $\mathbf{cut_S}$, since the dependence on M_S can then be fixed by dimensional analysis. Now, by inspection of the second amplitude of eq. (3.5) we see that, due to the L propagator, M_L can only appear as $\Delta C_\gamma \propto 1/(M_S^2 - M_L^2)$, being then odd under eq. (3.10). Since the total contribution from $\mathbf{cut_S}$ and $\mathbf{cut_L}$ must be symmetric under eq. (3.10), this must be zero.

[3] The vanishing of the second term can also be explicitly seen by integrating over the phase space as in eq. (3.4) which can be easily done by relating the internal spinor $|3']$ with the external ones $|1]$ and $|3]$. This relation is given by $|3'] = \sqrt{1 - M_S^2/s_{12}} \left(c_{\theta/2}|1] + s_{\theta/2} e^{-i\phi}|3] \right)$ which fulfills the kinematic constraint $p_1 + p_3 = p_{3'} + p_{1'}$ with $p_1^2 = p_3^2 = p_{3'}^2 = 0$ and $p_{1'}^2 = M_S^2$. The integral measure is $\int \text{dLIPS} = \int_0^{2\pi} d\phi \int_0^\pi d\theta \, s_\theta/(4\pi)$.

[4] Again, we are neglecting the divergent and the constant term in eq. (3.8) which must cancel other divergent and $1/s_{13}$ contributions arising from bubbles proportional to $I_2(0, M_S^2, 0)$ which, as explained, we also neglect.

[5] If we keep $s_{13} \neq 0$ the contribution from this 2-cut will not contain terms of $O(s_{ij}/M_L^2)$ and can be neglected. However, there is then another 2-cut which isolates the fermion e with $p_2^2 \neq 0$ and which does not vanish. Since the contribution from cutting the L state should not depend on our choice of whether we take s_{12} or s_{13} nonzero, we can just choose $s_{12} \neq 0$ for this calculation in which case $\mathbf{cut_L}$ gives the only relevant contribution.

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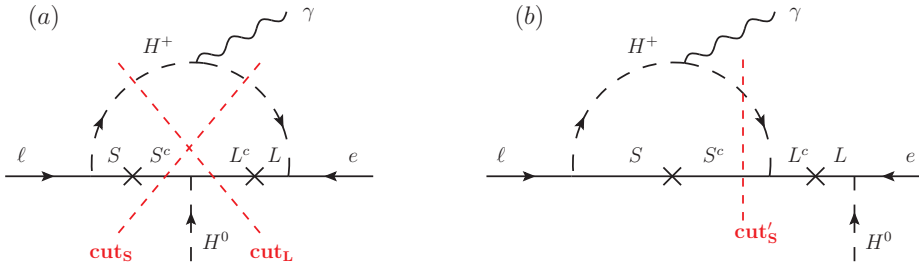

Figure 2. One-loop contributions to the $g-2$ of the SM leptons from the model eq. (3.3) with $Y'_V = 0$ and $Y_V \neq 0$, with the relevant 2-cuts.

$Y_V \neq 0$ case. Let us next move to the case $Y'_V = 0$, $Y_V \neq 0$. We now have two Feynman diagrams, see figure 2. Let us first consider diagram (a). The contribution from $\mathbf{cut_S}$ is again given by eq. (3.4) with the only difference with respect to the previous case that now

$$\mathcal{A}(3'_{\bar{H}^+}, 1'_{\bar{S}}, 2_e, 4_{H^0}) = Y_R Y_V \langle -\mathbf{1}'2\rangle \frac{M_L}{M_S^2 - M_L^2} \,. \tag{3.11}$$

Plugging eq. (3.11) into eq. (3.4) it is easy to see that we get the same as in eq. (3.9) with the only differences being that, due to the L mass insertion in eq. (3.11), we have an extra factor of M_L/M_S and that it depends on Y_V instead of Y'_V:

$$\frac{\Delta C_\gamma}{M^2} = \frac{Y_L Y_R Y_V}{32\pi^2} \frac{M_L/M_S}{M_S^2 - M_L^2} \,. \tag{3.12}$$

This contribution is not odd under the symmetry eq. (3.10). Therefore, when adding $\mathbf{cut_L}$ of figure 2, obtained by performing $M_S \leftrightarrow M_L$ in eq. (3.12), we get a nonzero result:

$$\frac{\Delta C_\gamma}{M^2} = \frac{Y_L Y_R Y_V}{32\pi^2} \frac{(M_L/M_S - M_S/M_L)}{M_S^2 - M_L^2} = -\frac{Y_L Y_R Y_V}{32\pi^2} \frac{1}{M_S M_L} \,. \tag{3.13}$$

There is, however, another contribution coming from the 2-cut of diagram (b) in figure 2. This contribution can be considered as arising from an extra term to the amplitude eq. (3.11) given by

$$\mathcal{A}(3'_{\bar{H}^+}, 1'_{\bar{S}}, 2_e, 4_{H^0}) = -Y_R Y_V \langle -\mathbf{1}'2\rangle \frac{M_L}{p_2^2 - M_L^2} \simeq Y_R Y_V \langle -\mathbf{1}'2\rangle \frac{1}{M_L} \,, \tag{3.14}$$

which exactly cancels the leading term of $O(1/M_L)$ of this amplitude in the limit $M_L \gg M_S$. Notice that this leading term was crucial in obtaining the nonzero result at $O(1/M_L M_S)$ in eq. (3.13). We therefore have that by adding the contribution from eq. (3.14) in eq. (3.4) we again find a vanishing Wilson coefficient.

The origin of this cancellation can again be understood from symmetries acting at the amplitude level. For $M_L \gg M_S$, the leading term of the total amplitude $\mathcal{A}(3'_{\bar{H}^+}, 1'_{\bar{S}}, 2_e, 4_{H^0})$ is captured by the dimension-5 operator $\tilde{H}^\dagger H S^c e/M_L$. However, this operator is zero since $\tilde{H}^\dagger H = \epsilon_{ab} H^a H^b = 0$ (a, b being $\mathrm{SU}(2)_L$ indices). Were this property absent, as we will find in the next model, the Wilson coefficient would have been generated.

We conclude then that the *a priori* non-trivial result that the contributions to the Wilson coefficient of the dipole moment add up to zero in this model boils down, by inspection with on-shell methods, to a clash of *even* $\times$ *odd* under a given parity.

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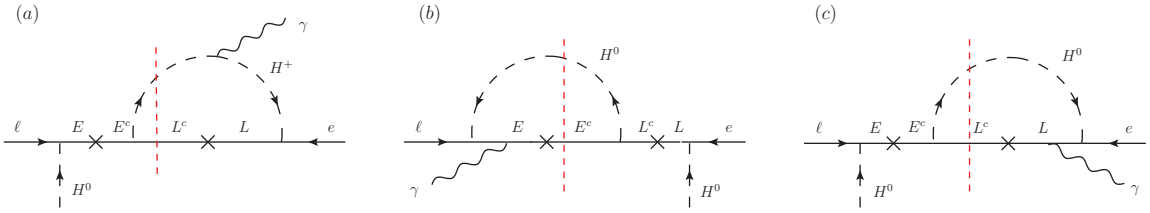

Figure 3. One-loop contributions to the $g-2$ of the SM leptons from the model eq. (3.15), with the relevant 2-cuts.

3.2 Massive vector-like charged E and doublet L

Let us next consider a model where the extra vector-like fermions, E and L, have the same quantum numbers as the leptons of the SM. The Lagrangian is now given by

$$\mathcal{L} = -Y_L \ell E H - Y_R L e H - Y_V H^\dagger L^c E^c - Y_V' L E H - M_E\, E E^c - M_L\, L L^c + \text{h.c.} \tag{3.15}$$

For the case $Y_V' \neq 0$, $Y_V = 0$, there is no possible Feynman diagram. Therefore we only have to study the opposite case $Y_V \neq 0$, $Y_V' = 0$. The Feynman diagrams are given in figure 3. Let us first consider the contribution coming from the charged Higgs, diagram (a). The calculation is identical to the one of the 2-cut of diagram (b) in figure 2 which we already found to give

$$\frac{\Delta C_\gamma}{M^2} = \frac{Y_L Y_R Y_V}{32\pi^2} \frac{1}{M_E M_L}\,. \tag{3.16}$$

Next we study the contributions from the neutral Higgs. For the 2-cut of diagram (b) in figure 3, the involved amplitudes read

$$\mathcal{A}(1_\ell, 3_{\gamma_-}, 1'_E, 3'_{H^0}) = q_e Y_L \frac{M_E}{2p_3 p_{1'}} \frac{\langle 33'\rangle [3'\mathbf{1}']}{[31]}\,, \qquad \mathcal{A}(3'_{\bar{H}^0}, 1'_{\bar{E}}, 2_e, 4_{H^0}) = Y_R Y_V \frac{M_L \langle -\mathbf{1}'2\rangle}{p_2^2 - M_L^2}\,. \tag{3.17}$$

This leads to

$$\mathcal{A}(1_\ell, 3_{\gamma_-}, 1'^I_E, 3'_{H^0})\, \epsilon_{IJ}\, \mathcal{A}(3'_{\bar{H}^0}, 1'^J_{\bar{E}}, 2_e, 4_{H^0}) \simeq -q_e Y_L Y_R Y_V \frac{M_E}{M_L} \frac{\langle 32\rangle}{[31]} + \cdots\,, \tag{3.18}$$

where we have used that $\langle 33'\rangle[3'1'^I]\langle -1'_I 2\rangle = \langle 3|p_{3'}p_{1'}|2\rangle = -\langle 3|p_{3'}(p_3+p_1)|2\rangle = -2p_3 p_{3'}\langle 32\rangle + \cdots \simeq 2p_3 p_{1'}\langle 32\rangle + \cdots$ with the dots corresponding to terms $\propto \langle 12\rangle$ which do not contribute to the dipole and to terms which are subdominant for $s_{13}/M^2_{L,E} \ll 1$. Since eq. (3.18) does not depend on the internal spinors, we can trivially integrate over the phase space and from this get

$$\frac{\Delta C_\gamma}{M^2} = -\frac{Y_L Y_R Y_V}{32\pi^2} \frac{1}{M_E M_L}\,. \tag{3.19}$$

As opposed to eq. (3.9) this contribution is symmetric under $E \leftrightarrow L, \ell \leftrightarrow e$. Therefore we get a factor 2 when adding the 2-cut of diagram (c) in figure 3. Summing the three contributions, we find a result in agreement with ref. [18].

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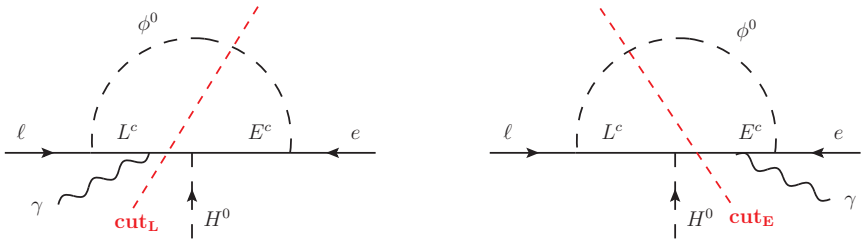

Figure 4. One-loop contributions to the $g-2$ of the SM leptons from the model eq. (3.20), with the relevant 2-cuts.

3.2.1 A natural zero for models with an extra (massless) scalar singlet

We have seen that in the model eq. (3.15) we do not find a vanishing contribution from the diagrams (b)+(c) since each contribution is even under $E \leftrightarrow L$, $\ell \leftrightarrow e$. To have a contribution which is odd under this interchange, we need to have the same type of diagram as the one in figure 1 with no mass insertions in the heavy fermion lines. Unfortunately, diagrams of this type are identically zero in the model eq. (3.15) as the Higgs line cannot be closed if we do not insert fermion masses. Nevertheless, diagrams of this type can be generated if we add an extra massless scalar singlet ϕ^0 to the model with the following couplings:

$$\Delta\mathcal{L} = Y_L^{\phi}\,\phi^0 \ell L^c + Y_R^{\phi}\,\phi^0 E^c e + \text{h.c.} \tag{3.20}$$

The Feynman diagrams involving this scalar are given in figure 4. Now, we can follow the same reasoning as in section 3.1 to show that this contribution to the dipole moment is zero. Indeed, we can get the dependence on M_L of $\mathbf{cut_E}$ (where E is put on-shell) by noticing that it only enters in the L propagator, so it must appear as $1/(M_E^2 - M_L^2)$. Dimensional analysis tells us then that $\Delta C_\gamma \propto 1/(M_E^2 - M_L^2)$. The dependence on the masses for $\mathbf{cut_L}$ is determined by a permutation similar to eq. (3.10) with S replaced by E which gives $\Delta C_\gamma \propto 1/(M_L^2 - M_E^2)$. Adding both contributions we get zero. It is clear that the cancellations have nothing to do with where the photon is attached, either to the Higgs line as in figure 1 or to the fermion line as in figure 4.

4 $|H|^2F^2$ Wilson coefficient

Let us now move to the calculation of the Wilson coefficient of the operator contributing to the decay of a Higgs to two photons. The operator reads

$$\frac{C_{\gamma\gamma}}{M^2}\frac{q_e^2}{2}\,|H|^2 F_{\mu\nu}^2\,, \tag{4.1}$$

and the resulting amplitude is

$$\frac{C_{\gamma\gamma}}{M^2}\mathcal{A}_{H^2F^2}(1_{\gamma_-}, 2_{\gamma_-}, 3_{H^0}, 4_{H^0}) = -\frac{C_{\gamma\gamma}}{M^2}\,q_e^2\,\langle 12\rangle^2\,. \tag{4.2}$$

We consider the same model as eq. (3.15), containing two vector-like fermions, L and E, with the same quantum numbers as the SM leptons. Here we assume vanishing

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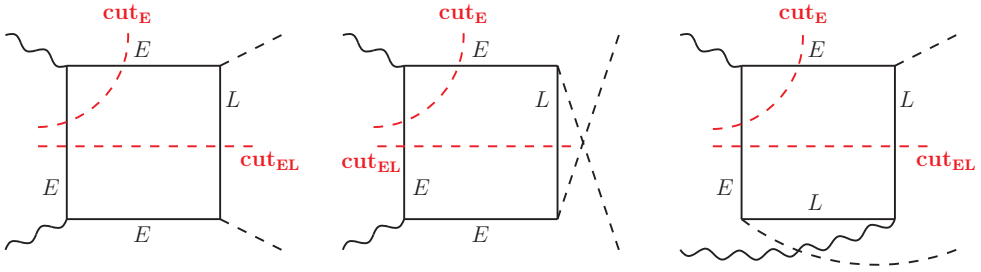

Figure 5. One-loop contributions to $H\gamma\gamma$ from the model eq. (3.15), with the relevant 2-cuts. There is a similar 2-cut isolating the other photon that we do not show. Fermion lines can be clockwise and counterclockwise.

Yukawa couplings between the new fermions and the SM leptons though, $Y_{L,R} = 0$. In the following, we will focus on the case $Y_V = 0, Y'_V \neq 0$. The discussion for the opposite case $Y_V \neq 0, Y'_V = 0$ is identical.

Since the amplitude eq. (4.2) does not depend on the Higgs momenta, we can take them to be zero, $p_3 = p_4 = 0$. In this case we can take the limit $p_i/M \to 0$ by giving to the photons a small nonzero mass $p^2 \equiv p_1^2 = p_2^2 = -p_1 p_2$. An alternative is to set only one Higgs momentum to zero, say $p_3 = 0$, but in this case we have nonzero 3-cuts as we elaborate in appendix C.

There are three different diagrams which can contribute to the Wilson coefficient, shown in figure 5. Additional contributions arise from the same diagrams with $E \leftrightarrow L$. So the total contribution must be symmetric under $E \leftrightarrow L$. As we will see, this will clash with the fact that the contributions from figure 5 are odd under $E \leftrightarrow L$. Although to show that the total contribution is zero is quite easy, we will proceed here with the details of the calculation which can be useful for cases where they do not add up to zero.

As in the $g-2$ case, there are no possible 3-cuts or 4-cuts since the on-shell conditions cannot be simultaneously fulfilled for vanishing Higgs momenta. Furthermore, one can show that the tadpoles do not contribute to the dimension-6 operator since their coefficients cannot have the required dependence on the masses. This leaves the 2-cuts shown in figure 5. The 2-cut denoted as $\mathbf{cut_{EL}}$ isolates two amplitudes involving a photon coupled to two different fermions which are zero by gauge invariance. This can explicitly seen by calculating these amplitudes,

$$\mathcal{A}(1_\gamma, 2_E, 3_L, 4_{H^0}) = \frac{1}{s - M_E^2}\left(\langle \mathbf{1}|p_2|\mathbf{1}]\,\frac{\langle \mathbf{23}\rangle}{p} + \langle \mathbf{12}\rangle\langle \mathbf{13}\rangle\right) + \frac{1}{u - M_L^2}\left(\langle \mathbf{1}|p_3|\mathbf{1}]\,\frac{\langle \mathbf{32}\rangle}{p} + \langle \mathbf{12}\rangle\langle \mathbf{13}\rangle\right), \tag{4.3}$$

where $s = (p_3 + p_4)^2$, $u = (p_2 + p_4)^2$ and $p = \sqrt{p^2}$. In the limit $p_4 \to 0$, we have $s \to M_L^2$ and $u \to M_E^2$, and the amplitude vanishes after symmetrizing over the SU(2) indices of the photon.

The only remaining 2-cut is $\mathbf{cut_E}$ of figure 5. This involves a coupling of a massive photon to massive fermions given by [19]

$$\mathcal{A}(1_\gamma, \ell_E, \ell'_E) = \frac{q_e}{p}\left(\langle \mathbf{1}\boldsymbol{\ell}\rangle\,[\mathbf{1}\boldsymbol{\ell'}] + [\mathbf{1}\boldsymbol{\ell}]\langle \mathbf{1}\boldsymbol{\ell'}\rangle\right). \tag{4.4}$$

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On the other side of the cut, we have the same type of amplitude $\mathcal{A}(1_\gamma, \ell_E, \ell'_E)$ but with the external fermion line corrected by the insertion of two Higgs. This can be considered as a correction to the E propagator which can be absorbed into a renormalization of the wavefunction δZ_E and of the mass,

$$M_E \to \hat{M}_E = M_E + \frac{1}{2}|Y'_V H^0|^2 \frac{M_E}{M_E^2 - M_L^2}\,, \tag{4.5}$$

where the Higgs has been considered a constant configuration. The correction from δZ_E is expected to be exactly cancelled by a correction to the photon vertex, as dictated by gauge invariance. This vertex correction is given by the third diagram in figure 5 where a Higgs is inserted on each of the two fermion lines. This leaves us with eq. (4.5) as the only effect of the Higgs. Notice that this is odd under $E \leftrightarrow L$.

The $\mathbf{cut_E}$ of figure 5 can therefore be obtained by the dLIPS integral of the product of two photon couplings to the fermion E of mass given by eq. (4.5),

$$\mathcal{A}(1_\gamma, \ell_E^I, \ell_E'^K)\,\epsilon_{IJ}\epsilon_{KL}\,\mathcal{A}(\ell_{\bar{E}}'^L, \ell_{\bar{E}}^J, 2_\gamma) = \frac{q_e^2}{p^2}\left[2\,\hat{M}_E^2\langle\mathbf{12}\rangle[\mathbf{12}] + (\langle\mathbf{1}|\ell|\mathbf{2}]\langle\mathbf{2}|\ell'|\mathbf{1}] + (\ell \leftrightarrow \ell'))\right], \tag{4.6}$$

where we have used eq. (4.4). In order to simplify the integration of eq. (4.6) over dLIPS, we can go to the rest frame of the photon where $p_1^\mu = -p_2^\mu = (p, 0, 0, 0)$. In this frame, we furthermore have $\ell^\mu = (-p/2, \vec{\ell})$, where $\vec{\ell} = \ell(s_\theta c_\phi, s_\theta s_\phi, c_\theta)$ with $\ell = (p^2/4 - M_E^2)^{1/2}$, and $\ell'^\mu = -\ell^\mu - p_1^\mu$. We next have the freedom to choose a particular basis for the photon polarizations, for which we take (equivalent to setting $|s| = \sin(\theta/2) = 0$ in appendix C of ref. [17]):

$$|1^{I=1}\rangle = \sqrt{p}\begin{pmatrix}1\\0\end{pmatrix}\,, \qquad |1^{I=2}\rangle = \sqrt{p}\begin{pmatrix}0\\1\end{pmatrix}\,, \tag{4.7}$$

and $|1^{I=1,2}] = |1^{I=1,2}\rangle$. Since $p_1 = -p_2$, we can set $|\mathbf{2}\rangle = -|\mathbf{1}\rangle$ and $|\mathbf{2}] = |\mathbf{1}]$. Furthermore, we will only calculate the result for one of the three polarizations of the massive photon, as the others must give the same result. We take the longitudinal one, which leads to

$$\langle\mathbf{12}\rangle[\mathbf{12}] \to \langle 1^{I=1}2^{I=2}\rangle[1^{I=1}2^{I=2}] = p^2, \tag{4.8}$$

and similarly for the other terms in eq. (4.6). This gives

$$\begin{aligned}\int d\text{LIPS}\,\mathcal{A}(1_\gamma, \ell_E, \ell'_E) \times \mathcal{A}(\ell'_{\bar{E}}, \ell_{\bar{E}}, 2_\gamma) \to\; & q_e^2 \int_0^{2\pi}\frac{d\phi}{4\pi}\int_0^\pi d\theta\, s_\theta\left[2\,\hat{M}_E^2(1-c_\theta^2) + \frac{1}{2}p^2(1+c_\theta^2)\right] \\ &= \frac{2q_e^2}{3}\left(2\hat{M}_E^2 + p^2\right).\end{aligned} \tag{4.9}$$

Furthermore, from the bubble integral we have expanding in p^2/M^2 and $|H^0|^2/M^2$ that

$$I_2(p^2, \hat{M}_E^2, \hat{M}_E^2) = \frac{1}{16\pi^2}\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{\hat{M}_E^2} + \dots\right) = \frac{1}{16\pi^2}\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{M_E^2} - \frac{|Y'_V H^0|^2}{M_E^2 - M_L^2} + \cdots\right). \tag{4.10}$$

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In order to match the product of eq. (4.9) and eq. (4.10) with the amplitude eq. (4.2), we must take the photons in the latter to be massive too, $\langle 12\rangle^2 \to \langle \mathbf{12}\rangle^2$, and project the photons into their longitudinal components, $\langle \mathbf{12}\rangle^2 \to p^2$. The Wilson coefficient then arises from the $p^2|H^0|^2$-dependent part of this product which gives[6]

$$\frac{\Delta C_{\gamma\gamma}}{M^2} = \frac{1}{16\pi^2}\frac{2}{3}\frac{|Y_V'|^2}{M_E^2 - M_L^2}\,. \tag{4.11}$$

This is again odd under the interchange $E \leftrightarrow L$, so when adding the contribution from the same diagrams as in figure 5 but with $E \leftrightarrow L$, we see that the total contribution to the Wilson coefficient vanishes.

5 Conclusions

In this paper we have shown how to efficiently calculate finite contributions to Wilson coefficients using amplitude methods which are known to significantly simplify loop calculations compared to the Feynman approach. The Wilson coefficients can be extracted from one-loop amplitudes by expanding them in powers of the masses of the heavy particles. Using a Passarino-Veltman decomposition, one-loop amplitudes can in turn be expressed in terms of basic scalar integrals called bubbles, triangles and boxes. By applying generalized unitarity cuts to this relation, the coefficients of these integrals are obtained from products of tree-level amplitudes integrated over some phase-space. In general, the one-loop amplitudes receive additional contributions from rational terms whose calculation is more involved. We have shown, however, that these rational terms cannot contribute to the Wilson coefficients. Combining everything, the Wilson coefficients can then be calculated from products of, in general, two, three and four on-shell amplitudes.

We have applied this method to calculate finite contributions to the dipole-moment operator and the operator coupling the Higgs to photons, $|H|^2F^2$. This was done for several theories containing heavy vector-like fermions. We have shown that the calculation simplifies significantly by taking the momenta of the (one respectively two) Higgs fields in these operators to zero. The reason is that in this limit, triangles and boxes in the Passarino-Veltman decomposition necessarily vanish due to kinematical constraints. The calculation of the Wilson coefficients then boils down to products of two on-shell amplitudes, integrated over the phase space of the two intermediate particles. In many cases, this phase-space integral is trivial, further simplifying the calculation.

Our method has allowed to shine light on the mysterious cancellations in the contributions to these Wilson coefficients, recently discussed in detail in [1] for the dipole-moment operator, but also noted (e.g. in [2]) for $|H|^2F^2$. This has been shown to happen in certain models with heavy fermions, even though the contributions do not seem to be forbidden by any symmetry. We have seen that the Wilson coefficients can be expressed as the sum of two different products of amplitudes, corresponding to two possible ways of applying

[6] The leading terms (proportional to $\hat{M}_E^2$ in eq. (4.9)) must vanish when adding other bubbles which do not involve p^2 terms. These have been omitted here such as the one related to the vertical 2-cut of the diagrams of figure 5 which isolates a two-photon amplitude.

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2-cuts to the one-loop amplitude. We have found that these contributions to the Wilson coefficients are odd under the exchange of the heavy fermions, while the total contribution has to be even under this exchange. Therefore, upon inspection with amplitude methods, the cancellation boils down to a clash of *even* $\times$ *odd* under the exchange parity. This understanding has allowed us to find other models where this cancellation also occurs.

Acknowledgments

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A The absence of rational terms

Here we will show that the rational terms R of eq. (2.1) do not contribute to the Wilson coefficients arising from integrating out heavy particles.

Rational terms are related to UV divergences and, in $D = 4 - 2\epsilon$ dimensions, within the Passarino-Veltman reduction, they appear from the product ϵI_1 or ϵI_2, where the scalar integrals I_1 and I_2 carry $1/\epsilon$ UV-divergent terms. Having no imaginary parts, they can in principle not be obtained by performing cuts of the loop diagrams. Nevertheless, it was shown in [20] that by extending eq. (2.1) to a D-dimensional Passarino-Veltman decomposition, generalized unitarity methods can also be used to obtain these rational terms.

Let us here review the argument of [20]. This is based on the observation that, by promoting the loop integration momentum l to $D = 4-2\epsilon$ dimensions, any rational term can only appear from the -2ϵ component of l^2, namely $l^2 = l^2_{(4)} + l^2_{(-2\epsilon)} \equiv l^2_{(4)} - \mu^2$. The usual basis of the Passarino-Veltman decomposition of eq. (2.1) is then enlarged with new master integrals whose integrands contain powers of μ^2.[7] Within this framework, the rational terms can be obtained by exploiting generalized unitarity methods and can be written as [20]

$$R = -\frac{1}{6}\sum_{i,j}\tilde{C}_2^{(ij)}(s_{ij} - 3(M_i^2 + M_j^2)) - \frac{1}{2}\sum_b \tilde{C}_3^{(b)} - \frac{1}{6}\sum_c \tilde{C}_4^{(c)}\,. \tag{A.1}$$

Without going into the details of the proof of the above formula (we refer the interested reader to ref. [20]), it will be enough to highlight how the coefficients $\tilde{C}_n$ can be extracted. Firstly, since the extra component μ^2 in the integration momentum l^2 can be effectively seen as a mass, $l^2_{(4)} = M_i^2 + \mu^2$, all internal masses in the one-loop amplitude must be shifted by the same mass parameter μ^2, i.e. $M_i^2 \to M_i^2 + \mu^2$. Secondly, we must perform the

[7] In [20], the new master integrals are denoted as $I_n^{4-2\epsilon}[\mu^{2k}]$ where μ^{2k} is understood as being integrated over. The explicit computation of these integrals gives rise to the coefficients in eq. (A.1).

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corresponding n-cuts of the amplitude, and take the limit of large μ^2; $\tilde{C}_2$ and $\tilde{C}_3$ are given from the coefficient of the μ^2-term of this expansion, while $\tilde{C}_4$ is obtained from the μ^4-term.

Equipped with these observations, we are ready to show the absence of rational terms in the Wilson coefficients considered in this work. We are interested in one-loop contributions arising from renormalizable theories in the limit $M \gg p_i$, where we match to the EFT at order $1/M^2$. Therefore the loop integrals must converge to zero for large M. Nevertheless, since the rational terms are obtained in the large-μ^2 limit ($\mu^2 \gg M^2, s_{ij}$), M^2 is always subleading with respect to μ^2. Indeed, the internal propagators will be given by $1/(s_{ij} - M^2 - \mu^2)$ and M^2 can only appear as powers of M^2/μ^2 in the $\mu^2 \to \infty$ limit, and therefore cannot contribute at $O(1/M^2)$.[8]

B Massless and massive spinor-helicity variables

We begin with specifying our conventions for massless and massive spinor-helicity variables. We choose the metric $\eta_{\mu\nu} = \text{diag}(+,-,-,-)$. For a massless particle, the momentum can be written as

$$p_{\alpha\dot{\alpha}} = |p\rangle_\alpha [p|_{\dot{\alpha}}\,, \tag{B.1}$$

where $|p\rangle_\alpha$ and $[p|_{\dot{\alpha}}$ are two-component spinors which transform under the little group with helicity $h = \mp 1/2$, respectively. The Lorentz indices α and $\dot{\alpha}$ are raised and lowered with the two-component Levi-Civita symbol, which we choose such that $\epsilon^{12} = -\epsilon_{12} = 1$. In particular, we have

$$\langle p|^\beta = \epsilon^{\beta\alpha}|p\rangle_\alpha\,, \qquad |p]^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}[p|_{\dot{\beta}}\,. \tag{B.2}$$

The Lorentz indices of two angle or square brackets are contracted as

$$\langle pq\rangle = \langle p|^\alpha\, |p\rangle_\alpha\,, \qquad [pq] = [p|_{\dot{\alpha}}\, |p]^{\dot{\alpha}}\,. \tag{B.3}$$

For a massive particle, on the other hand, we need twice as many spinors which combine into two vectors transforming under the little group SU(2), $|p\rangle^I_\alpha$ and $|p]^{\dot{\alpha}\,I}$ [17]. The SU(2) little-group indices I are again raised and lowered with the two-component Levi-Civita symbol. The momentum is then given by

$$p_{\alpha\dot{\alpha}} = \epsilon_{IJ}|p\rangle^I_\alpha [p|^J_{\dot{\alpha}} = |p\rangle^I_\alpha [p|_{\dot{\alpha}\,I}\,. \tag{B.4}$$

The massive spinor-helicity variables fulfil the identities

$$\begin{aligned} \langle p^I p^J\rangle &= -M\epsilon^{IJ}, & [p_I p_J] &= -M\epsilon_{IJ}\,, \\ |p\rangle^I_\alpha\, \langle p|^\beta_I &= -M\delta^\beta_\alpha\,, & |p]^{\dot{\alpha}\,I}\, [p|_{\dot{\beta}\,I} &= M\delta^{\dot{\alpha}}_{\dot{\beta}}\,, \end{aligned} \tag{B.5}$$

where M is the mass of the particle. From this, we in particular obtain the Dirac equation

$$p|p]^I = M|p\rangle^I, \qquad p|p\rangle^I = M|p]^I\,. \tag{B.6}$$

[8] Rational terms can appear at $O(1/s_{ij})$. However, these singular contributions are cancelled by contributions from $C_n I_n$ to make the one-loop amplitude non-singular in the limit of small momenta.

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We will often not write the SU(2) indices explicitly and will then use bold letters for the massive spinor-helicity variables, $|p\rangle^I = |\boldsymbol{p}\rangle$ and $|p]^I = |\boldsymbol{p}]$, to distinguish them from the massless ones. In amplitudes involving massive spinors, the SU(2) indices $I, J, \ldots$ of a given state must be symmetrized [17].

When contracting amplitudes to obtain the coefficients of the Pasarino-Veltman decomposition, we need to flip the momenta of the particles on one side of the contraction from incoming to outgoing (amplitudes are defined with all momenta being incoming). The spinor-helicity variables for the flipped momenta then satisfy different contraction rules from the ones given above [19]:

$$\begin{aligned} |p\rangle^I_\alpha [-p|_{\dot\alpha\, I} &= p_{\alpha\dot\alpha}\,, & |p]^{\dot\alpha\, I} \langle -p|^\alpha_I &= p^{\dot\alpha\alpha} \\ |p\rangle^I_\alpha \langle -p|^\beta_I &= M\delta^\beta_\alpha\,, & |p]^{\dot\alpha I} [-p|_{\dot\beta I} &= M\delta^{\dot\alpha}_{\dot\beta}\,. \end{aligned} \tag{B.7}$$

C An alternative way to calculate the $|H|^2F^2$ Wilson coefficient

In the following, we will discuss an alternative way to calculate the Wilson coefficient for the operator $|H|^2F^2$, corresponding to the amplitude in eq. (4.2). Here we keep the photons on-shell and take the momentum of only one Higgs to be identically zero, $p_3 = 0$. Since then $p_4^2 = 2p_1p_2$, the other Higgs must be slightly off-shell. The diagrams which contribute to the Wilson coefficient are those shown in figure 5 plus the same diagrams with $E \leftrightarrow L$. We will see, however, that in the chosen kinematical configuration the nonvanishing cuts are different from those depicted in figure 5. Let us first consider 3-cuts and 4-cuts. As before, cutting both fermion lines attached to the Higgs with vanishing momentum gives zero since the on-shell conditions cannot be simultaneously fulfilled. This eliminates all 4-cuts and several of the 3-cuts. Furthermore, we have shown in section 4 that the amplitude in eq. (4.3) for a massive photon vanishes identically for zero Higgs momentum. Since this amplitude contains the corresponding amplitudes for a massless photon (taking the high-energy limit), the latter are zero too. Any 3-cut which isolates this amplitude therefore gives no contribution. We are then left with only one 3-cut, the one that puts the three fermions E in the first two diagrams in figure 5 on-shell (plus the corresponding 3-cut in the diagrams with $E \leftrightarrow L$).

Let us now calculate this 3-cut. We denote the momentum of the fermion line connecting the two photons as ℓ. One solution for this momentum after restricting three of the propagators in the loop to be on-shell can be parametrized as [21]

$$\ell = \tau\, |1\rangle[2| - \tau^{-1}\frac{M_E^2}{s_{12}}\, |2\rangle[1|\,, \tag{C.1}$$

where τ is the remaining integration variable. The other two cut momenta are $\widetilde{\ell} = -\ell - p_1$ and $\widehat{\ell} = \ell - p_2$, altogether satisfying $\ell^2 = \widetilde{\ell}^2 = \widehat{\ell}^2 = M_E^2$ as required. The other solution ℓ^* is given by eq. (C.1) with $1 \leftrightarrow 2$. The triangle coefficient then is[9]

$$C_3 = \sum_{\ell,\ell^*} \int d\tau J_\tau\, \mathcal{A}(1_{\gamma_-}, \widetilde{\ell}_E, \ell_E) \times \mathcal{A}(2_{\gamma_-}, \ell_{\bar E}, \widehat{\ell}_{\bar E}) \times \mathcal{A}(\widetilde{\ell}_{\bar E}, \widehat{\ell}_E, 3_{H^0}, 4_{H^0})\,, \tag{C.2}$$

[9] In general, one has to expand the integrand for large τ. This is not necessary in our case.

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where J_τ is a Jacobian arising from the transformation to the integration variable τ. We have $\int d\tau J_\tau \tau^n = \delta_{0n}$ which leads to

$$C_3 = \sum_{\ell,\ell^*} \left[\mathcal{A}(1_{\gamma_-}, \widetilde{\ell}_E, \ell_E) \times \mathcal{A}(2_{\gamma_-}, \ell_{\bar{E}}, \widehat{\ell}_{\bar{E}}) \times \mathcal{A}(\widetilde{\ell}_{\bar{E}}, \widehat{\ell}_E, 3_{H^0}, 4_{H^0})\right]_{\tau^0} . \tag{C.3}$$

The 4-point amplitude which is isolated in the 3-cut gets contributions from both the first and second diagram in figure 5 and reads

$$\begin{aligned}\mathcal{A}(\widetilde{\ell}_{\bar{E}}, \widehat{\ell}_E, 3_{H^0}, 4_{H^0}) &= |Y_V'|^2 \left(\frac{[-\widetilde{\boldsymbol{\ell}}|(\widehat{\ell}+p_3)|\widehat{\boldsymbol{\ell}}\rangle}{(\widehat{\ell}+p_3)^2 - M_L^2} - \frac{[\widehat{\boldsymbol{\ell}}|(-\widetilde{\ell}+p_3)|-\widetilde{\boldsymbol{\ell}}\rangle}{(-\widetilde{\ell}+p_3)^2 - M_L^2} \right) \\ &\underset{p_3\to 0}{=} 2|Y_V'|^2 \frac{M_E\,[-\widetilde{\boldsymbol{\ell}}\,\widehat{\boldsymbol{\ell}}]}{M_E^2 - M_L^2} .\end{aligned} \tag{C.4}$$

In the second step we have taken the Higgs momentum p_3 to zero. Furthermore, the 3-point amplitudes are given by

$$\mathcal{A}(1_{\gamma_-}, \widetilde{\ell}_E, \ell_E) = q_e \frac{\langle 1|\ell|\xi]}{M_E[1\xi]}\,[\widetilde{\boldsymbol{\ell}}\boldsymbol{\ell}] , \tag{C.5}$$

with ξ being a reference spinor, and similarly for $\mathcal{A}(2_{\gamma_-}, \ell_{\bar{E}}, \widehat{\ell}_{\bar{E}})$. Using eq. (C.1), the reference spinors drop out. Combining the three amplitudes and summing over the SU(2) indices of the internal fermions, eq. (C.3) then yields

$$C_3 = \frac{16\,|Y_V'|^2}{M_E^2 - M_L^2} \frac{M_E^4}{s_{12}} q_e^2 \langle 12\rangle^2 . \tag{C.6}$$

Here we have included a factor 2 due to the fact that the fermion lines in the loop can be clockwise or counterclockwise (or, alternatively, that the photons can be interchanged, $1 \leftrightarrow 2$). From eq. (2.1), the contribution to the one-loop amplitude $\mathcal{A}_{H^2F^2}$ is obtained by multiplying eq. (C.6) with the triangle function corresponding to this 3-cut,

$$I_3(s_{12}, 0, 0; M_E, M_E, M_E) = \frac{1}{16\pi^2}\left(\frac{1}{2M_E^2} + \frac{s_{12}}{24 M_E^4} + \dots\right) , \tag{C.7}$$

with $s_{12} = (p_1 + p_2)^2$ and which we have expanded to $O(s_{12}/M_E^2)$. The leading term in this expansion gives rise to an unphysical pole term when multiplied with eq. (C.6). This is cancelled by a rational term. Using eq. (2.4), the next term in the expansion then yields the contribution of the 3-cut to the Wilson coefficient

$$\frac{\Delta C_{\gamma\gamma}}{M^2} = \frac{1}{16\pi^2}\frac{2}{3}\frac{|Y_V'|^2}{M_E^2 - M_L^2} , \tag{C.8}$$

in agreement with eq. (4.11). This is again odd under the exchange $E \leftrightarrow L$ and is therefore exactly cancelled by the contribution which arises from the diagrams in figure 5 with $E \leftrightarrow L$. We thus conclude that neither 4-cuts nor 3-cuts contribute to the Wilson coefficient $C_{\gamma\gamma}$.

We still need to discuss the 2-cuts. Since the 3-cut already accounts for the result in eq. (4.11), we expect that the 2-cuts give no additional contributions to the Wilson coefficient. As before, the 2-cut which leaves the zero-momentum Higgs alone vanishes since

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the on-shell conditions cannot be fulfilled. Furthermore, any 2-cut which isolates an on-shell photon (with $p^2 = 0$) does not contribute to the Wilson coefficient since the corresponding bubble integrals do not depend on any momentum (cf. the discussion in section 3.1). Other 2-cuts are zero since they again cut out an amplitude of the type $\mathcal{A}(1_{\gamma_-}, 3_{H^0}, \ell_E, \ell'_L)$ which vanishes for $p_3 = 0$. Finally, also the 2-cut which leaves the off-shell Higgs alone does not contribute. Indeed, the amplitude on the other side of this cut is $\mathcal{A}(1_{\gamma_-}, 2_{\gamma_-}, 3_{H^0}, \ell_E, \ell'_L)$ whose only kinematically allowed factorization channels once again involve amplitudes of the type $\mathcal{A}(1_{\gamma_-}, 3_{H^0}, \ell_E, \ell'_L)$ with $p_3 = 0$. The amplitude $\mathcal{A}(1_{\gamma_-}, 2_{\gamma_-}, 3_{H^0}, \ell_E, \ell'_L)$ and the corresponding 2-cut therefore vanish too. This leaves only the 2-cut which puts the upper and lower E line in the first two diagrams in figure 5 on-shell. Notice that the same two lines are also cut in the non-vanishing 3-cut. The 2-cut therefore obtains a contribution from the corresponding triangle and is guaranteed to be non-zero. It can be shown that the 2-cut exactly matches the latter and the Wilson coefficient therefore does not get any additional contribution from this 2-cut either.

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