Reduction of Couplings and its application in a Finite Unified Theory and the MSSM

S. Heinemeyer[1,2,3*], J. Kalinowski[4†], W. Kotlarski[5‡], M. Mondragón[6§], G. Patellis[7¶], N. Tracas[7‖] and G. Zoupanos[7,8,9**]

[1]*Instituto de Física Teórica (UAM/CSIC), Universidad Autónoma de Madrid Cantoblanco, 28049 Madrid, Spain*
[2]*Campus of International Excellence UAM+CSIC, Cantoblanco, 28049 Madrid, Spain*
[3]*Instituto de Física de Cantabria (CSIC-UC), E-39005 Santander, Spain*
[4]*University of Warsaw - Faculty of Physics, ul. Pasteura 5, 02-093 Warsaw, Poland*
[5]*Technische Universität Dresden - Institut für Kern- und Teilchenphysik (IKTP) 01069 Dresden, Germany*
[6]*Instituto de Física, Universidad Nacional Autónoma de México, A.P. 20-364, CDMX 01000 México*
[7] *Physics Department, Nat. Technical University, 157 80 Zografou, Athens, Greece*
[8] *Max-Planck Institut für Physik, Föhringer Ring 6, D-80805 München, Germany*
[9] *Theoretical Physics Department, CERN, Geneva, Switzerland*

Abstract

The search for relations among parameters that are renormalization group invariant to all orders in perturbation theory is the basis of the reduction of couplings idea. This method has been applied to some $N = 1$ supersymmetric Grand Unified Theories, few of which can become all-loop finite. We review the basic idea and the tools developed, as well as two resulting theories in which reduction of couplings has been achieved: (i) an all-loop finite $N = 1$ $SU(5)$ model and (ii) a reduced version of the Minimal Supersymmetric Standard Model. We present three benchmark scenarios for the finite $SU(5)$ model and investigate their observability at existing and future hadron colliders. The model's heavy supersymmetric spectrum lies beyond the reach of the 14 TeV HL-LHC. Concerning the 100 TeV FCC-hh, it is found that large parts of the predicted spectrum can be tested, but the higher mass regions are beyond the reach even of the FCC-hh. It is also found that the reduced version of the MSSM is ruled out by the LHC searches for heavy neutral MSSM Higgs bosons for the allowed parameter space.

[*]email: Sven.Heinemeyer@cern.ch
[†]email: kalino@fuw.edu.pl
[‡]email: wojciech.kotlarski@tu-dresden.de
[§]email: myriam@fisica.unam.mx
[¶]email: patellis@central.ntua.gr
[‖]email: ntrac@central.ntua.gr
[**]email: George.Zoupanos@cern.ch

1 Introduction

The reduction of couplings idea [1–4] (see also [5–7]) is a promising technique which relates seemingly independent parameters of a renormalizable theory to a single coupling. The method requires resulting relation among the parameters to be valid at all energy scales, i.e. Renormalization Group Invariant (RGI).

After the introduction of a novel symmetry through a Grand Unified Theory (GUT) [8–13]), in order to achieve reduction of the number of free parameters of the Standard Model (SM), the next step is the unification of the gauge and Yukawa sectors (Gauge Yukawa Unification, GYU). This was the main characteristic of the *reduction of couplings* early-stage application in $N = 1$ GUTs [14–27], where RGI relations are set between the unification scale and the Planck scale. Moreover, RGI relations which guarantee all-loop finiteness can be found. The method predicted the top quark mass in the finite $N = 1$ supersymmetric $SU(5)$ model [14, 15], as well as in the Reduced Minimal $N = 1$ supersymmetric $SU(5)$ one [16] before its experimental discovery [28].

Since *reduction of couplings* can only be applied in models with Supersymmetry (SUSY), a supersymmetry breaking sector (SSB) has to be included, which involves couplings with non-zero mass dimension. The supergraph method and the spurion superfield technique played an important role for the progress in that sector, leading to complete all-loop finite models, i.e. including the SSB sector. The all-loop finite $N = 1$ supersymmetric $SU(5)$ model [29, 30] has given a prediction for the light Higgs boson mass in agreement with the experimental results [31–33] and a heavy supersymmetric mass spectrum, consistent with the experimental non-observation of these particles. In the past two decades the reduction of couplings technique has been applied to many cases, including a reduced version of the minimal $N = 1$ supersymmetric $SU(5)$ [16] and a reduced version of the $N = 1$ supersymmetric $SU(3) \times SU(3) \times SU(3)$ model [34–36]. The full analyses of the most successful models, that includes predictions in agreement with the experimental measurements of the top and bottom quark masses for each model, can be found in [37].

In the present article we review the examination of two of these models, namely the all-loop finite $N = 1$ $SU(5)$ model and the Reduced Minimal Supersymmetric Standard Model (Reduced MSSM). Specifically for the finite model we address the question to what extent the reduction of couplings idea can be experimentally tested at HL-LHC and future FCC hadron collider. To this end we propose three benchmark points. We present the SUSY breaking parameters used as input in each benchmark to calculate the corresponding Higgs boson and supersymmetric particles masses. Then we compute the expected production cross sections at the 14 TeV (HL-)LHC and the 100 TeV FCC-hh and investigate which production channels can be observed. The complete analyses for both models (and two more) are included in our recent work [38].

The present work is structured as follows. In Sections 2 and 3 we review the basic idea of the reduction of couplings and finiteness. In Section 4 we list the phenomenological constraints used in our analyses, while in Section 5 we explain the computational setup. In Sections 6 and 7 we review the two above-mentioned models. We briefly review some earlier results of our phenomenological analysis. In this context the new version of the `FeynHiggs` [39–42] code plays a crucial role, which was used to calculate the Higgs-boson predictions, in particular the mass of the lightest CP-even Higgs boson. The improved predictions of `FeynHiggs` are compared with the LHC measurements and the Beyond Standard Model (BSM) Higgs boson searches. Furthermore, in the case of the finite $SU(5)$ model, we examine the discovery potential of the Higgs and SUSY spectrum at approved future and hypothetical future hadron colliders. Finally, Section 8 containts some brief conclusive remarks.

2 Reduction of Couplings Basics

2.1 Reduction of Dimensionless Parameters

We start by reviewing the basic *reduction of couplings* idea. The aim is to express the parameters of a theory that are considered free in terms of one independent parameter, which we call primary. The basic idea is to search for RGI relations among couplings and use them to reduce the number of seemingly independent parameters. Any RGI relation among parameters $g_1, \cdots, g_A$ of a given renormalizable theory can be expressed implicitly as $\Phi(g_1, \cdots, g_A) = \text{const}$. This expression must satisfy the partial differential equation (PDE)

$$\mu \frac{d\Phi}{d\mu} = \vec{\nabla}\Phi \cdot \vec{\beta} = \sum_{a=1}^{A} \beta_a \frac{\partial \Phi}{\partial g_a} = 0 , \tag{1}$$

with β_a the β-functions of g_a. Solving this PDE is equivalent to solving a set of ordinary differential equations (ODEs), the reduction equations (REs) [2–4],

$$\beta_g \frac{dg_a}{dg} = \beta_a , \; a = 1, \cdots, A-1 , \tag{2}$$

Here, g and β_g are the primary coupling and its β-function, respectively. The Φ_a's can impose up to $(A-1)$ independent RGI constraints in the A-dimensional parameter space. As a result, all couplings can be (in principle) expressed in terms of the primary coupling g.

This is not enough, as the number of integration constants of the general solutions of Eq. (2) matches the number of these equations, meaning that we just traded an integration constant for each ordinary renormalized coupling, and therefore these cannot be considered as reduced solutions.

The crucial requirement is the demand that the REs admit power series solutions,

$$g_a = \sum_n \rho_a^{(n)} \, g^{2n+1} , \tag{3}$$

that preserve perturbative renormalizability. This way, the integration constant corresponding to each RE is fixed and the RE is picked up as a special solution out of the set of the general ones. It is worth noting that a one-loop level examination is enough to decide for the uniqueness of these solutions [2–4]. As an illustration on the above, we assume β-functions of the form

$$\begin{aligned} \beta_a &= \frac{1}{16\pi^2}\left[\sum_{b,c,d\neq g} \beta_a^{(1)\,bcd} g_b g_c g_d + \sum_{b\neq g} \beta_a^{(1)\,b} g_b g^2\right] + \cdots , \\ \beta_g &= \frac{1}{16\pi^2}\beta_g^{(1)} g^3 + \cdots , \end{aligned} \tag{4}$$

Here $\cdots$ stands for higher order terms and $\beta_a^{(1)\,bcd}$'s are symmetric in b, c and d. We assumethat $\rho_a^{(n)}$ with $n \leq r$ are already determined uniquely. In order to obtain $\rho_a^{(r+1)}$, the power series (3) are inserted into the REs (2) and we collect terms of $\mathcal{O}(g^{2r+3})$. Thus, we find

$$\sum_{d\neq g} M(r)_a^d \, \rho_d^{(r+1)} = \text{lower order quantities} \ ,$$

where the right-hand side is known by assumption and

$$M(r)_a^d = 3 \sum_{b,c\neq g} \beta_a^{(1)\,bcd} \, \rho_b^{(1)} \, \rho_c^{(1)} + \beta_a^{(1)\,d} - (2r+1)\, \beta_g^{(1)} \, \delta_a^d \ , \tag{5}$$

$$0 = \sum_{b,c,d\neq g} \beta_a^{(1)\,bcd} \, \rho_b^{(1)} \, \rho_c^{(1)} \, \rho_d^{(1)} + \sum_{d\neq g} \beta_a^{(1)\,d} \, \rho_d^{(1)} - \beta_g^{(1)} \, \rho_a^{(1)} \ . \tag{6}$$

Therefore, the $\rho_a^{(n)}$ for all $n > 1$ for a given set of $\rho_a^{(1)}$ can be uniquely determined if $\det M(n)_a^d \neq 0$ for all $n \geq 0$. This is checked in all models that reductions of couplings is applied.

The search for power series solutions to the REs like (3) is more than justified in SUSY theories, where parameters often behave asymptotically in a similar way. This "completely reduced" theory features only one independent parameter, rendering this unification very attractive. It is often unrealistic, however, and, usually, fewer RGI constraints are imposed, leading to a partial reduction [43, 44].

All the above give rise to hints towards an underlying connection among the requirement of reduction of couplings and SUSY.

As an example, we consider a $SU(N)$ gauge theory with $\phi^i(\mathbf{N})$ and $\hat{\phi}_i(\overline{\mathbf{N}})$ complex scalars, $\psi^i(\mathbf{N})$ and $\hat{\psi}_i(\overline{\mathbf{N}})$ left-handed Weyl spinors and $\lambda^a (a = 1, \ldots, N^2-1)$ right-handed Weyl spinors in the adjoint representation of $SU(N)$.

The Lagrangian (kinetic terms are omitted) includes

$$\mathcal{L} \supset i\sqrt{2}\{\ g_Y \overline{\psi} \lambda^a T^a \phi - \hat{g}_Y \overline{\hat{\psi}} \lambda^a T^a \hat{\phi} + \text{h.c.}\ \} - V(\phi, \overline{\phi}), \tag{7}$$

where

$$V(\phi,\overline{\phi}) = \frac{1}{4}\lambda_1 (\phi^i \phi_i^*)^2 + \frac{1}{4}\lambda_2 (\hat{\phi}_i \hat{\phi}^{*\ i})^2 + \lambda_3 (\phi^i \phi_i^*)(\hat{\phi}_j \hat{\phi}^{*\ j}) + \lambda_4 (\phi^i \phi_j^*)(\hat{\phi}_i \hat{\phi}^{*\ j}), \tag{8}$$

This is the most general renormalizable form in four dimensions. In search of a solution of the form of Eq. (3) for the REs, among other solutions, one finds in lowest order:

$$\begin{aligned} g_Y &= \hat{g}_Y = g \ , \\ \lambda_1 &= \lambda_2 = \frac{N-1}{N} g^2 \ , \\ \lambda_3 &= \frac{1}{2N} g^2 \ , \ \lambda_4 = -\frac{1}{2} g^2 \ , \end{aligned} \tag{9}$$

which corresponds to a $N = 1$ SUSY gauge theory. While these remarks do not provide an answer about the relation of reduction of couplings and SUSY, they certainly point to further study in that direction.

2.2 Reduction in N = 1 Supersymmetric Gauge Theories - Partial Reduction

Consider a chiral, anomaly free, $N = 1$ globally supersymmetric gauge theory that is based on a group G and has gauge coupling g. The superpotential of the theory is:

$$W = \frac{1}{2} m_{ij} \, \phi_i \, \phi_j + \frac{1}{6} C_{ijk} \, \phi_i \, \phi_j \, \phi_k \ , \tag{10}$$

m_{ij} and C_{ijk} are gauge invariant tensors and the chiral superfield ϕ_i belongs to the irreducible representation R_i of the gauge group. The renormalization constants associated with the superpotential, for preserved SUSY, are:

$$\phi_i^0 = \left(Z_i^j\right)^{(1/2)} \phi_j \ , \tag{11}$$

$$m_{ij}^0 = Z_{ij}^{i'j'} m_{i'j'} \ , \tag{12}$$

$$C_{ijk}^0 = Z_{ijk}^{i'j'k'} C_{i'j'k'} \ . \tag{13}$$

By virtue of the $N = 1$ non-renormalization theorem [45–48] there are no mass and cubic-interaction-term infinities. Therefore:

$$\begin{aligned} Z_{ij}^{i'j'} \left(Z_{i'}^{i''}\right)^{(1/2)} \left(Z_{j'}^{j''}\right)^{(1/2)} &= \delta_{(i}^{i''} \delta_{j)}^{j''} \ , \\ Z_{ijk}^{i'j'k'} \left(Z_{i'}^{i''}\right)^{(1/2)} \left(Z_{j'}^{j''}\right)^{(1/2)} \left(Z_{k'}^{k''}\right)^{(1/2)} &= \delta_{(i}^{i''} \delta_{j}^{j''} \delta_{k)}^{k''} \ . \end{aligned} \tag{14}$$

The only surviving infinities are the wave function renormalization constants Z_i^j, so just one infinity per field. The β-function of the gauge coupling g at the one-loop level is given by [49–53]

$$\beta_g^{(1)} = \frac{dg}{dt} = \frac{g^3}{16\pi^2}\left[\sum_i T(R_i) - 3\, C_2(G)\right] \ , \tag{15}$$

where $C_2(G)$ is the quadratic Casimir operator of the adjoint representation of the gauge group G and $\mathrm{Tr}[T^a T^b] = T(R)\delta^{ab}$, where T^a are the group generators in the appropriate representation. The β-functions of C_{ijk} are related to the anomalous dimension matrices γ_{ij} of the matter fields as:

$$\beta_{ijk} = \frac{dC_{ijk}}{dt} \ = \ C_{ijl}\,\gamma_k^l + C_{ikl}\,\gamma_j^l + C_{jkl}\,\gamma_i^l \ . \tag{16}$$

The one-loop γ_j^i is given by [49]:

$$\gamma^{(1)}{}_j^i = \frac{1}{32\pi^2}\,[\,C^{ikl}\,C_{jkl} - 2\,g^2\,C_2(R_i)\delta_j^i\,], \tag{17}$$

where $C^{ijk} = C_{ijk}^*$. We take C_{ijk} to be real so that C_{ijk}^2 are always positive. The squares of the couplings are convenient to work with, and the C_{ijk} can be covered by a single index i $(i = 1, \cdots, n)$:

$$\alpha = \frac{g^2}{4\pi} \ , \ \alpha_i \ = \ \frac{g_i^2}{4\pi} \ . \tag{18}$$

Then the evolution of α's in perturbation theory will take the form

$$\begin{aligned} \frac{d\alpha}{dt} = \beta \ &= \ -\,\beta^{(1)}\alpha^2 + \cdots \ , \\ \frac{d\alpha_i}{dt} = \beta_i \ &= \ -\,\beta_i^{(1)}\,\alpha_i\,\alpha + \sum_{j,k} \beta_{i,jk}^{(1)}\,\alpha_j\,\alpha_k + \cdots \ , \end{aligned} \tag{19}$$

Here, $\cdots$ denotes higher-order contributions and $\beta_{i,jk}^{(1)} = \beta_{i,kj}^{(1)}$. For the evolution equations (19) we investigate the asymptotic properties. First, we define [2,4,6,54,55]

$$\tilde{\alpha}_i \equiv \frac{\alpha_i}{\alpha} \ , \ i = 1, \cdots, n \ , \tag{20}$$

and derive from Eq. (19)

$$\alpha \frac{d\tilde{\alpha}_i}{d\alpha} = -\tilde{\alpha}_i + \frac{\beta_i}{\beta} = \left(-1 + \frac{\beta_i^{(1)}}{\beta^{(1)}}\right) \tilde{\alpha}_i - \sum_{j,k} \frac{\beta_{i,jk}^{(1)}}{\beta^{(1)}} \tilde{\alpha}_j \, \tilde{\alpha}_k + \sum_{r=2} \left(\frac{\alpha}{\pi}\right)^{r-1} \tilde{\beta}_i^{(r)}(\tilde{\alpha}) \ , \tag{21}$$

where $\tilde{\beta}_i^{(r)}(\tilde{\alpha})$ $(r = 2, \cdots)$ are power series of $\tilde{\alpha}$'s and can be computed from the r^{th}-loop β-functions. We then search for fixed points ρ_i of Eq. (20) at $\alpha = 0$. We have to solve the equation

$$\left(-1 + \frac{\beta_i^{(1)}}{\beta^{(1)}}\right) \rho_i - \sum_{j,k} \frac{\beta_{i,jk}^{(1)}}{\beta^{(1)}} \rho_j \, \rho_k = 0 \ , \tag{22}$$

assuming fixed points of the form

$$\rho_i = 0 \text{ for } i = 1, \cdots, n' \ ; \ \rho_i \ > 0 \text{ for } i = n'+1, \cdots, n \ . \tag{23}$$

Next, we treat $\tilde{\alpha}_i$ with $i \leq n'$ as small perturbations to the undisturbed system (defined by setting $\tilde{\alpha}_i$ with $i \leq n'$ equal to zero). It is possible to verify the existence of the unique power series solution of the reduction equations (21) to all orders already at one-loop level [2–4, 54]:

$$\tilde{\alpha}_i = \rho_i + \sum_{r=2} \rho_i^{(r)} \, \alpha^{r-1} \ , \ i = n'+1, \cdots, n \ . \tag{24}$$

These are RGI relations among parameters, and preserve formally perturbative renormalizability. So, in the undisturbed system there is only one independent parameter, the primary coupling α.

The nonvanishing $\tilde{\alpha}_i$ with $i \leq n'$ cause small perturbations that enter in a way that the reduced couplings ($\tilde{\alpha}_i$ with $i > n'$) become functions both of α and $\tilde{\alpha}_i$ with $i \leq n'$. Investigating such systems with partial reduction is very convenient to work with the following PDEs:

$$\left\{ \tilde{\beta} \frac{\partial}{\partial \alpha} + \sum_{a=1}^{n'} \tilde{\beta}_a \frac{\partial}{\partial \tilde{\alpha}_a} \right\} \tilde{\alpha}_i(\alpha, \tilde{\alpha}) = \tilde{\beta}_i(\alpha, \tilde{\alpha}) \ , \qquad \tilde{\beta}_{i(a)} = \frac{\beta_{i(a)}}{\alpha^2} - \frac{\beta}{\alpha^2} \tilde{\alpha}_{i(a)}, \qquad \tilde{\beta} \equiv \frac{\beta}{\alpha} \ . \tag{25}$$

These equations are equivalent to the REs (21), where, in order to avoid any confusion, we let a, b run from 1 to n' and i, j from $n'+1$ to n. Then, we search for solutions of the form

$$\tilde{\alpha}_i = \rho_i + \sum_{r=2} \left(\frac{\alpha}{\pi}\right)^{r-1} f_i^{(r)}(\tilde{\alpha}_a) \ , \ i = n'+1, \cdots, n \ , \tag{26}$$

where $f_i^{(r)}(\tilde{\alpha}_a)$ are power series of $\tilde{\alpha}_a$. The requirement that in the limit of vanishing perturbations we obtain the undisturbed solutions (24) [44, 56] suggests this type of solutions. Once more, one can obtain the conditions for uniqueness of $f_i^{(r)}$ in terms of the lowest order coefficients.

2.3 Reduction of Parameters of Dimension-1 and -2

The extension of reduction of couplings to massive parameters is not straightforward, since the technique was originally aimed at massless theories on the basis of the Callan-Symanzik equation [2, 3]. Many requirements have to be met, such as the normalization conditions imposed on irreducible Green's functions [57], etc. Significant progress has been made towards this goal, starting from [58], where, as an assumption, a mass-independent renormalization scheme renders all RG functions only trivially dependent on dimensional parameters. Mass parameters can then be introduced similarly to couplings.

This was justified later [59,60], where it was demonstrated that, apart from dimensionless parameters, pole masses and gauge couplings, the model can also include couplings carrying a dimension and masses. To simplify the analysis, we follow Ref. [58] and use a mass-independent renormalization scheme as well.

Consider a renormalizable theory that contains $(N+1)$ dimension-0 couplings, $(\hat{g}_0, \hat{g}_1, ..., \hat{g}_N)$, L parameters with mass dimension-1, $\left(\hat{h}_1, ..., \hat{h}_L\right)$, and M parameters with mass dimension-2, $\left(\hat{m}_1^2, ..., \hat{m}_M^2\right)$. The renormalized irreducible vertex function Γ satisfies the RG equation

$$\mathcal{D}\Gamma\left[\Phi' s; \hat{g}_0, \hat{g}_1, ..., \hat{g}_N; \hat{h}_1, ..., \hat{h}_L; \hat{m}_1^2, ..., \hat{m}_M^2; \mu\right] = 0 \ , \tag{27}$$

with

$$\mathcal{D} = \mu\frac{\partial}{\partial\mu} + \sum_{i=0}^{N}\beta_i\frac{\partial}{\partial\hat{g}_i} + \sum_{a=1}^{L}\gamma_a^h\frac{\partial}{\partial\hat{h}_a} + \sum_{\alpha=1}^{M}\gamma_\alpha^{m^2}\frac{\partial}{\partial\hat{m}_\alpha^2} + \sum_J \Phi_I\gamma^{\phi I}{}_J\frac{\delta}{\delta\Phi_J} \ , \tag{28}$$

where β_i are the β-functions of the dimensionless couplings g_i and Φ_I are the matter fields. The mass, trilinear coupling and wave function anomalous dimensions, respectively, are denoted by $\gamma_\alpha^{m^2}$, γ_a^h and $\gamma^{\phi I}{}_J$ and μ denotes the energy scale. For a mass-independent renormalization scheme, the γ's are given by

$$\begin{aligned}
\gamma_a^h &= \sum_{b=1}^{L}\gamma_a^{h,b}(g_0, g_1, ..., g_N)\hat{h}_b, \\
\gamma_\alpha^{m^2} &= \sum_{\beta=1}^{M}\gamma_\alpha^{m^2,\beta}(g_0, g_1, ..., g_N)\hat{m}_\beta^2 + \sum_{a,b=1}^{L}\gamma_\alpha^{m^2,ab}(g_0, g_1, ..., g_N)\hat{h}_a\hat{h}_b \ .
\end{aligned} \tag{29}$$

The $\gamma_a^{h,b}$, $\gamma_\alpha^{m^2,\beta}$ and $\gamma_\alpha^{m^2,ab}$ are power series of the (dimensionless) g's.

We search for a reduced theory where

$$g \equiv g_0, \qquad h_a \equiv \hat{h}_a \quad \text{for } 1 \le a \le P, \qquad m_\alpha^2 \equiv \hat{m}_\alpha^2 \quad \text{for } 1 \le \alpha \le Q$$

are independent parameters. The reduction of the rest of the parameters, namely

$$\begin{aligned}
\hat{g}_i &= \hat{g}_i(g), \qquad (i = 1, ..., N), \\
\hat{h}_a &= \sum_{b=1}^{P} f_a^b(g)h_b, \qquad (a = P+1, ..., L), \\
\hat{m}_\alpha^2 &= \sum_{\beta=1}^{Q} e_\alpha^\beta(g)m_\beta^2 + \sum_{a,b=1}^{P} k_\alpha^{ab}(g)h_a h_b, \qquad (\alpha = Q+1, ..., M)
\end{aligned} \tag{30}$$

is consistent with the RGEs (27,28). The following relations should be satisfied

$$
\begin{aligned}
\beta_g \frac{\partial \hat{g}_i}{\partial g} &= \beta_i, \qquad (i = 1, ..., N), \\
\beta_g \frac{\partial \hat{h}_a}{\partial g} + \sum_{b=1}^{P} \gamma_b^h \frac{\partial \hat{h}_a}{\partial h_b} &= \gamma_a^h, \qquad (a = P+1, ..., L), \\
\beta_g \frac{\partial \hat{m}_\alpha^2}{\partial g} + \sum_{a=1}^{P} \gamma_a^h \frac{\partial \hat{m}_\alpha^2}{\partial h_a} + \sum_{\beta=1}^{Q} \gamma_\beta^{m^2} \frac{\partial \hat{m}_\alpha^2}{\partial m_\beta^2} &= \gamma_\alpha^{m^2}, \qquad (\alpha = Q+1, ..., M).
\end{aligned}
\tag{31}
$$

Using Eqs. (29) and (30), they reduce to

$$
\begin{aligned}
&\beta_g \frac{df_a^b}{dg} + \sum_{c=1}^{P} f_a^c \left[\gamma_c^{h,b} + \sum_{d=P+1}^{L} \gamma_c^{h,d} f_d^b \right] - \gamma_a^{h,b} - \sum_{d=P+1}^{L} \gamma_a^{h,d} f_d^b = 0, \\
&\qquad\qquad\qquad\qquad (a = P+1, ..., L;\ b = 1, ..., P), \\
&\beta_g \frac{de_\alpha^\beta}{dg} + \sum_{\gamma=1}^{Q} e_\alpha^\gamma \left[\gamma_\gamma^{m^2,\beta} + \sum_{\delta=Q+1}^{M} \gamma_\gamma^{m^2,\delta} e_\delta^\beta \right] - \gamma_\alpha^{m^2,\beta} - \sum_{\delta=Q+1}^{M} \gamma_\alpha^{m^2,d} e_\delta^\beta = 0, \\
&\qquad\qquad\qquad\qquad (\alpha = Q+1, ..., M;\ \beta = 1, ..., Q), \\
&\beta_g \frac{dk_\alpha^{ab}}{dg} + 2 \sum_{c=1}^{P} \left(\gamma_c^{h,a} + \sum_{d=P+1}^{L} \gamma_c^{h,d} f_d^a \right) k_\alpha^{cb} + \sum_{\beta=1}^{Q} e_\alpha^\beta \left[\gamma_\beta^{m^2,ab} + \sum_{c,d=P+1}^{L} \gamma_\beta^{m^2,cd} f_c^a f_d^b \right. \\
&\left. +2 \sum_{c=P+1}^{L} \gamma_\beta^{m^2,cb} f_c^a + \sum_{\delta=Q+1}^{M} \gamma_\beta^{m^2,d} k_\delta^{ab} \right] - \left[\gamma_\alpha^{m^2,ab} + \sum_{c,d=P+1}^{L} \gamma_\alpha^{m^2,cd} f_c^a f_d^b \right. \\
&\left. +2 \sum_{c=P+1}^{L} \gamma_\alpha^{m^2,cb} f_c^a + \sum_{\delta=Q+1}^{M} \gamma_\alpha^{m^2,\delta} k_\delta^{ab} \right] = 0, \\
&\qquad\qquad\qquad\qquad (\alpha = Q+1, ..., M;\ a, b = 1, ..., P)\ .
\end{aligned}
\tag{32}
$$

The above relations ensure that the irreducible vertex function of the reduced theory

$$
\begin{aligned}
&\Gamma_R \left[\Phi\text{'s}; g; h_1, ..., h_P; m_1^2, ..., m_Q^2; \mu \right] \equiv \\
&\quad \Gamma \left[\Phi\text{'s}; g, \hat{g}_1(g) ..., \hat{g}_N(g); h_1, ..., h_P, \hat{h}_{P+1}(g,h), ..., \hat{h}_L(g,h); \right. \\
&\qquad\qquad \left. m_1^2, ..., m_Q^2, \hat{m}_{Q+1}^2(g,h,m^2), ..., \hat{m}_M^2(g,h,m^2); \mu \right]
\end{aligned}
\tag{33}
$$

has the same renormalization group flow as the original one.

Assuming a perturbatively renormalizable reduced theory, the functions $\hat{g}_i$, f_a^b, e_α^β and k_α^{ab} are expressed as power series in the primary coupling:

$$
\begin{aligned}
\hat{g}_i &= g \sum_{n=0}^{\infty} \rho_i^{(n)} g^n, \qquad & f_a^b &= g \sum_{n=0}^{\infty} \eta_a^{b(n)} g^n, \\
e_\alpha^\beta &= \sum_{n=0}^{\infty} \xi_\alpha^{\beta(n)} g^n, \qquad & k_\alpha^{ab} &= \sum_{n=0}^{\infty} \chi_\alpha^{ab(n)} g^n.
\end{aligned}
\tag{34}
$$

These expansion coefficients are found by inserting the above power series into Eqs. (31), (32) and requiring the equations to be satisfied at each order of g. It is not trivial to have a unique power series solution; it depends both on the theory and the choice of independent couplings.

If there are no independent dimension-1 parameters ($\hat{h}$), their reduction becomes

$$\hat{h}_a = \sum_{b=1}^{L} f_a^b(g) M,$$

where M is a dimension-1 parameter (i.e. a gaugino mass, corresponding to the independent gauge coupling). If there are no independent dimension-2 parameters ($\hat{m}^2$), their reduction takes the form

$$\hat{m}_a^2 = \sum_{b=1}^{M} e_a^b(g) M^2.$$

2.4 Reduction of Soft Breaking Terms in $N = 1$ Supersymmetric Theories

The reduction of dimensionless couplings was extended [58, 61] to the SSB dimensionful parameters of $N = 1$ supersymmetric theories. It was also found [25, 62] that soft scalar masses satisfy a universal sum rule.
We consider the superpotential (10)

$$W = \frac{1}{2}\,\mu^{ij}\,\Phi_i\,\Phi_j + \frac{1}{6}\,C^{ijk}\,\Phi_i\,\Phi_j\,\Phi_k \ , \tag{35}$$

and the SSB Lagrangian

$$-\mathcal{L}_{\text{SSB}} = \frac{1}{6}\,h^{ijk}\,\phi_i\phi_j\phi_k + \frac{1}{2}\,b^{ij}\,\phi_i\phi_j + \frac{1}{2}\,(m^2)^j_i\,\phi^{*\,i}\phi_j + \frac{1}{2}\,M\,\lambda_i\lambda_i + \text{h.c.} \tag{36}$$

The ϕ_i's are the scalar parts of chiral superfields Φ_i, λ are gauginos and M the unified gaugino mass.

The one-loop gauge β-function (15) is given by [49–53]

$$\beta_g^{(1)} = \frac{dg}{dt} = \frac{g^3}{16\pi^2}\left[\sum_i T(R_i) - 3\,C_2(G)\right] \ , \tag{37}$$

whereas the one-loop C_{ijk}'s β-function (16) is given by

$$\beta_C^{ijk} = \frac{dC_{ijk}}{dt} \ = \ C_{ijl}\,\gamma^l_k + C_{ikl}\,\gamma^l_j + C_{jkl}\,\gamma^l_i \ , \tag{38}$$

and the (one-loop) anomalous dimension $\gamma^{(1)}{}^i_j$ of a chiral superfield (17) is

$$\gamma^{(1)}{}^i_j = \frac{1}{32\pi^2}\left[C^{ikl}\,C_{jkl} - 2\,g^2\,C_2(R_i)\delta^i_j\right] \ . \tag{39}$$

Then the $N = 1$ non-renormalization theorem [45, 46, 48] guarantees that the β-functions of C_{ijk} are expressed in terms of the anomalous dimensions.
We make the assumption that the REs admit power series solutions:

$$C^{ijk} = g\sum_{n=0}\rho^{ijk}_{(n)}g^{2n} \ . \tag{40}$$

Since we want to obtain higher-loop results instead of knowledge of explicit β-functions, we require relations among β-functions. The spurion technique [48, 63–66] gives all-loop relations among SSB β-functions [67–74]:

$$\beta_M = 2\mathcal{O}\left(\frac{\beta_g}{g}\right) , \tag{41}$$

$$\begin{aligned}\beta_h^{ijk} &= \gamma^i_l h^{ljk} + \gamma^j_l h^{ilk} + \gamma^k_l h^{ijl} \\ &\quad - 2\,(\gamma_1)^i_l\, C^{ljk} - 2\,(\gamma_1)^j_l\, C^{ilk} - 2\,(\gamma_1)^k_l\, C^{ijl} ,\end{aligned} \tag{42}$$

$$(\beta_{m^2})^i_j = \left[\Delta + X\frac{\partial}{\partial g}\right]\gamma^i_j , \tag{43}$$

where

$$\mathcal{O} = \left(Mg^2\frac{\partial}{\partial g^2} - h^{lmn}\frac{\partial}{\partial C^{lmn}}\right) , \tag{44}$$

$$\Delta = 2\mathcal{O}\mathcal{O}^* + 2|M|^2 g^2\frac{\partial}{\partial g^2} + \tilde{C}_{lmn}\frac{\partial}{\partial C_{lmn}} + \tilde{C}^{lmn}\frac{\partial}{\partial C^{lmn}} , \tag{45}$$

$$(\gamma_1)^i_j = \mathcal{O}\gamma^i_j, \tag{46}$$

$$\tilde{C}^{ijk} = (m^2)^i_l C^{ljk} + (m^2)^j_l C^{ilk} + (m^2)^k_l C^{ijl} . \tag{47}$$

Assuming (following [71]) that the relation among couplings

$$h^{ijk} = -M(C^{ijk})' \equiv -M\frac{dC^{ijk}(g)}{d\ln g} , \tag{48}$$

is RGI and the use of the all-loop gauge β-function of [75–77]

$$\beta_g^{\rm NSVZ} = \frac{g^3}{16\pi^2}\left[\frac{\sum_l T(R_l)(1-\gamma_l/2) - 3C_2(G)}{1 - g^2C_2(G)/8\pi^2}\right] , \tag{49}$$

we are led to an all-loop RGI sum rule [78] (assuming $(m^2)^i_j = m^2_j\delta^i_j$),

$$\begin{aligned} m^2_i + m^2_j + m^2_k = |M|^2 &\left\{ \frac{1}{1 - g^2C_2(G)/(8\pi^2)}\frac{d\ln C^{ijk}}{d\ln g} + \frac{1}{2}\frac{d^2\ln C^{ijk}}{d(\ln g)^2} \right\} \\ &+ \sum_l \frac{m^2_l T(R_l)}{C_2(G) - 8\pi^2/g^2}\frac{d\ln C^{ijk}}{d\ln g} .\end{aligned} \tag{50}$$

It is worth noting that the all-loop result of Eq. (50) coincides with the superstring result for the finite case in a certain class of orbifold models [25, 79, 80] if $\frac{d\ln C^{ijk}}{d\ln g} = 1$ [15].

As mentioned above, the all-loop results on the SSB β-functions, Eqs.(41)-(47), lead to all-loop RGI relations. We assume:
(a) the existence of an RGI surface on which $C = C(g)$, or equivalently that the expression

$$\frac{dC^{ijk}}{dg} = \frac{\beta_C^{ijk}}{\beta_g} \tag{51}$$

holds (i.e. reduction of couplings is possible)
(b) the existence of a RGI surface on which

$$h^{ijk} = -M\frac{dC(g)^{ijk}}{d\ln g} \tag{52}$$

holds to all orders.
Then it can be proven [81,82] that the relations that follow are all-loop RGI (note that in both assumptions we do not rely on specific solutions of these equations)

$$M = M_0 \, \frac{\beta_g}{g}, \tag{53}$$

$$h^{ijk} = -M_0 \, \beta_C^{ijk}, \tag{54}$$

$$b^{ij} = -M_0 \, \beta_\mu^{ij}, \tag{55}$$

$$(m^2)^i_j = \frac{1}{2} \, |M_0|^2 \, \mu \frac{d\gamma^i{}_j}{d\mu}, \tag{56}$$

where M_0 is an arbitrary reference mass scale to be specified shortly. Assuming

$$C_a \frac{\partial}{\partial C_a} = C_a^* \frac{\partial}{\partial C_a^*} \tag{57}$$

for an RGI surface $F(g, C^{ijk}, C^{*ijk})$ we are led to

$$\frac{d}{dg} = \left(\frac{\partial}{\partial g} + 2 \frac{\partial}{\partial C} \frac{dC}{dg} \right) = \left(\frac{\partial}{\partial g} + 2 \frac{\beta_C}{\beta_g} \frac{\partial}{\partial C} \right) , \tag{58}$$

where Eq. (51) was used. Let us now consider the partial differential operator $\mathcal{O}$ in Eq. (44) which (assuming Eq. (48)), becomes

$$\mathcal{O} = \frac{1}{2} M \frac{d}{d \ln g} \tag{59}$$

and β_M, given in Eq. (41), becomes

$$\beta_M = M \frac{d}{d \ln g} (\frac{\beta_g}{g}) \ , \tag{60}$$

which by integration provides us [74, 81] with the generalized, i.e. including Yukawa couplings, all-loop RGI Hisano - Shifman relation [70]

$$M = \frac{\beta_g}{g} M_0 \ .$$

M_0 is the integration constant and can be associated to the unified gaugino mass M (of an assumed covering GUT), or to the gravitino mass $m_{3/2}$ in a supergravity framework. Therefore, Eq. (53) becomes the all-loop RGI Eq. (53). β_M, using Eqs.(60) and (53) can be written as follows:

$$\beta_M = M_0 \frac{d}{dt} (\beta_g / g) \ . \tag{61}$$

Similarly

$$(\gamma_1)^i_j = \mathcal{O} \gamma^i_j = \frac{1}{2} \, M_0 \, \frac{d\gamma^i_j}{dt} \ . \tag{62}$$

Next, from Eq.(48) and Eq.(53) we get

$$h^{ijk} = -M_0 \, \beta_C^{ijk} \ , \tag{63}$$

while β_h^{ijk}, using Eq.(62), becomes [81]

$$\beta_h^{ijk} = -M_0 \, \frac{d}{dt} \beta_C^{ijk}, \tag{64}$$

which shows that Eq. (63) is RGI to all loops. Eq. (55) can similarly be shown to be all-loop RGI as well.

Finally, it is important to note that, under the assumptions (a) and (b), the sum rule of Eq. (50) has been proven [78] to be RGI to all loops, which (using Eq. (53)) generalizes Eq. (56) for application in cases with non-universal soft scalar masses, a necessary ingredient in the models that will be examined in the next Sections. Another important point to note is the use of Eq. (53), which, in the case of product gauge groups (as in the MSSM), takes the form

$$M_i = \frac{\beta_{g_i}}{g_i} M_0 \ , \tag{65}$$

where $i = 1, 2, 3$ denotes each gauge group, and will be used in the Reduced MSSM case.

3 Finiteness in N=1 Supersymmetric Gauge Theories

We start by considering a chiral, anomaly free, $N = 1$ globally supersymmetric gauge theory with gauge group G and g the theory's coupling constant. Again, the theory's superpotential is given by Eq. (10). Because of the $N = 1$ non-renormalization theorem, the one-loop β-function is given by Eq. (15), the β-function of C_{ijk} by Eq. (16) and the one-loop anomalous dimensions of the chiral superfields by Eq. (17).
It is obvious from Eqs. (15) and (17) that all one-loop β-functions of the theory vanish if $\beta_g^{(1)}$ and $\gamma^{(1)i}_{j}$ vanish:

$$\sum_i T(R_i) = 3C_2(G) \,, \tag{66}$$

$$C^{ikl} C_{jkl} = 2\delta^i_j g^2 C_2(R_i) \ . \tag{67}$$

In [83] one can find the finiteness conditions for $N = 1$ theories with $SU(N)$ gauge symmetry, while [84] discusses the requirements of anomaly-free and no-charge renormalization. Remarkably, the conditions (66,67) are necessary and sufficient for finiteness at the two-loop level as well [49–53].

In the case of soft SUSY breaking, requiring finiteness in the one-loop SSB sector imposes additional constraints among soft terms [85]. Again, the one-loop SSB finiteness conditions are enough to render the soft sector two-loop finite [86].

The above finiteness conditions impose considerable restrictions on the choice of irreducible representations (irreps) R_i for a given group G as well as the Yukawa couplings. These conditions cannot be applied to the MSSM, because the $U(1)$ gauge group is not compatible with condition (66), since $C_2[U(1)] = 0$. This points to the grand unified level, with the MSSM just being the low-energy theory.

Additionally, one(two)-loop finiteness causes SUSY to break only softly. Since gauge singlets are not acceptable, due to the condition given in Eq. (67) ($C_2(1) = 0$, i.e. singlets do not couple to the theory), F-type spontaneous symmetry breaking [87] terms are incompatible with finiteness, as well as D-type [88] spontaneous breaking which requires the existence of a $U(1)$ gauge group.

One can see that conditions (66,67) impose relations between the gauge and Yukawa sector. Imposing such relations, that make the parameters mutually dependent at a given renormalization point, is trivial. What is not trivial is to guarantee that relations leading to a reduction of the couplings hold at any renormalization point. As explained (see Eq. (51)), the necessary and sufficient condition is to require that such relations are solutions to the REs

$$\beta_g \frac{dC_{ijk}}{dg} = \beta_{ijk} \tag{68}$$

and hold at all orders. It is reminded that the existence of all-order power series solutions to (68) can be decided at one-loop level.

Concerning higher loop orders, a theorem [89, 90] exists that states the necessary and sufficient conditions to achieve all-loop finiteness for an $N = 1$ SUSY theory. It relies on the structure of the supercurrent in an $N = 1$ SUSY theory [91–93], and on the non-renormalization properties of $N = 1$ chiral anomalies [89, 90, 94–96]. Details and further discussion can be found in [89, 90, 94–98] Following [98] we briefly discuss the proof.

Consider an $N = 1$ SUSY gauge theory, with simple Lie group G. The content of this theory is given at the classical level by the matter supermultiplets S_i, which contain a scalar field ϕ_i and a Weyl spinor ψ_{ia}, and the vector supermultiplet V_a, which contains a gauge vector field A^a_μ and a gaugino Weyl spinor λ^a_α.

Let us first recall certain facts about the theory:

(1) A massless $N = 1$ SUSY theory is invariant under a $U(1)$ chiral transformation R under which the various fields transform as follows

$$\begin{aligned} A'_\mu = A_\mu, \quad \lambda'_\alpha = \exp(-i\theta)\lambda_\alpha \\ \phi' = \exp(-i\frac{2}{3}\theta)\phi, \quad {}'_\alpha = \exp(-i\frac{1}{3}\theta)\psi_\alpha, \ \cdots \end{aligned} \tag{69}$$

The corresponding axial Noether current $J^\mu_R(x)$ is

$$J^\mu_R(x) = \bar{\lambda}\gamma^\mu\gamma^5\lambda + \cdots \tag{70}$$

is conserved classically, while in the quantum case is violated by the axial anomaly

$$\partial_\mu J^\mu_R = r\left(\epsilon^{\mu\nu\sigma\rho}F_{\mu\nu}F_{\sigma\rho} + \cdots\right). \tag{71}$$

From its known topological origin in ordinary gauge theories [99–101], one would expect the axial vector current J^μ_R to satisfy the Adler-Bardeen theorem and receive corrections only at the one-loop level. Indeed it has been shown that the same non-renormalization theorem holds also in SUSY theories [94–96]. Therefore

$$r = \hbar\beta^{(1)}_g. \tag{72}$$

(2) The massless theory we consider is scale invariant at the classical level and, in general, there is a scale anomaly due to radiative corrections. The scale anomaly appears in the trace of the energy momentum tensor $T_{\mu\nu}$, which is traceless classically. It has the form

$$T^\mu_\mu = \beta_g F^{\mu\nu}F_{\mu\nu} + \cdots \tag{73}$$

(3) Massless, $N = 1$ SUSY gauge theories are classically invariant under the supersymmetric extension of the conformal group – the superconformal group. Examining the superconformal algebra, it can be seen that the subset of superconformal transformations consisting of translations, SUSY transformations, and axial R transformations is closed under SUSY, i.e. these transformations form a representation of SUSY. It follows that the conserved currents corresponding to these transformations make up a supermultiplet represented by an axial vector superfield called the supercurrent J,

$$J \equiv \left\{ J'^{\mu}_{R},\ Q^{\mu}_{\alpha},\ T^{\mu}_{\nu}, ... \right\}, \tag{74}$$

where J'^{μ}_{R} is the current associated to R-invariance, Q^{μ}_{α} is the one associated to SUSY invariance, and T^{μ}_{ν} the one associated to translational invariance (energy-momentum tensor).

The anomalies of the R-current J'^{μ}_{R}, the trace anomalies of the SUSY current, and the energy-momentum tensor, form also a second supermultiplet, called the supertrace anomaly

$$S = \{Re\ S,\ Im\ S,\ S_{\alpha}\} = \left\{ T^{\mu}_{\mu},\ \partial_{\mu} J'^{\mu}_{R},\ \sigma^{\mu}_{\alpha\dot{\beta}} \bar{Q}^{\dot{\beta}}_{\mu}\ +\ \cdots \right\}$$

where T^{μ}_{μ} is given in Eq. (73) and

$$\partial_{\mu} J'^{\mu}_{R} \ = \ \beta_g \epsilon^{\mu\nu\sigma\rho} F_{\mu\nu} F_{\sigma\rho} + \cdots \tag{75}$$

$$\sigma^{\mu}_{\alpha\dot{\beta}} \bar{Q}^{\dot{\beta}}_{\mu} \ = \ \beta_g \lambda^{\beta} \sigma^{\mu\nu}_{\alpha\beta} F_{\mu\nu} + \cdots \tag{76}$$

(4) It is important to note that the Noether current defined in (70) is not the same as the current associated to R-invariance that appears in the supercurrent J in (74), but they coincide in the tree approximation. So starting from a unique classical Noether current $J^{\mu}_{R(class)}$, the Noether current J^{μ}_{R} is defined as the quantum extension of $J^{\mu}_{R(class)}$ which allows for the validity of the non-renormalization theorem. On the other hand, J'^{μ}_{R}, is defined to belong to the supercurrent J, together with the energy-momentum tensor. The two requirements cannot be fulfilled by a singl current operator at the same time.

Although the Noether current J^{μ}_{R} which obeys (71) and the current J'^{μ}_{R} belonging to the supercurrent multiplet J are not the same, there is a relation [89, 90] between quantities associated with them

$$r = \beta_g(1 + x_g) + \beta_{ijk} x^{ijk} - \gamma_A r^A\ , \tag{77}$$

where r is given in Eq. (72). The r^A are the non-renormalized coefficients of the anomalies of the Noether currents associated to the chiral invariances of the superpotential, and –like r– are strictly one-loop quantities. The γ_A's are linear combinations of the anomalous dimensions of the matter fields, and x_g, and x^{ijk} are radiative correction quantities. The structure of Eq. (77) is independent of the renormalization scheme.

One-loop finiteness, i.e. vanishing of the β-functions at one loop, implies that the Yukawa couplings λ_{ijk} must be functions of the gauge coupling g. To find a similar condition to all orders it is necessary and sufficient for the Yukawa couplings to be a formal power series in g, which is solution of the REs (68).

We can now state the theorem for all-order vanishing β-functions [90].

Theorem:

Consider an $N=1$ SUSY Yang-Mills theory, with simple gauge group. If the following conditions are satisfied

1. There is no gauge anomaly.
2. The gauge β-function vanishes at one loop
$$\beta_g^{(1)} = 0 = \sum_i T(R_i) - 3\,C_2(G). \tag{78}$$
3. There exist solutions of the form
$$C_{ijk} = \rho_{ijk} g, \qquad \rho_{ijk} \in \mathbb{C} \tag{79}$$
to the conditions of vanishing one-loop matter fields anomalous dimensions
$$\gamma^{(1)}{}^i_j \;=\; 0 = \frac{1}{32\pi^2}\,[\; C^{ikl}\,C_{jkl} - 2\;g^2\;C_2(R)\delta^i_j]. \tag{80}$$
4. These solutions are isolated and non-degenerate when considered as solutions of vanishing one-loop Yukawa β-functions:
$$\beta_{ijk} = 0. \tag{81}$$

Then, each of the solutions (79) can be uniquely extended to a formal power series in g, and the associated super Yang-Mills models depend on the single coupling constant g with a β-function which vanishes at all orders.

Important note: The requirement of isolated and non-degenerate solutions guarantees the existence of a unique formal power series solution to the reduction equations. The vanishing of the gauge β-function at one loop, $\beta_g^{(1)}$, is equivalent to the vanishing of the R-current anomaly (71). The vanishing of the anomalous dimensions at one loop implies the vanishing of the Yukawa couplings β-functions at that order. It also implies the vanishing of the chiral anomaly coefficients r^A. This last property is a necessary condition for having β-functions vanishing at all orders.[a]

Proof:

Insert β_{ijk} as given by the REs into the relationship (77). Since these chiral anomalies vanish, we get for β_g an homogeneous equation of the form
$$0 = \beta_g(1 + O(\hbar)). \tag{82}$$
The solution of this equation in the sense of a formal power series in $\hbar$ is $\beta_g = 0$, order by order. Therefore, due to the REs (68), $\beta_{ijk} = 0$ too.

Thus we see that finiteness and reduction of couplings are intimately related. Since an equation like Eq. (77) is absent in non-SUSY theories, one cannot extend the validity of a similar theorem in such theories.

A very interesting development was done in [68]. Based on the all-loop relations among the β-functions of the soft SUSY breaking terms and those of the rigid supersymmetric

[a]There is an alternative way to find finite theories [102–104, 106].

theory with the help of the differential operators, discussed in Sect. 2.4, it was shown that certain RGI surfaces can be chosen, so as to reach all-loop finiteness of the full theory. More specifically, it was shown that on certain RGI surfaces the partial differential operators appearing in Eq. (41),(42) acting on the β- and γ-functions of the rigid theory can be transformed to total derivatives. Then the all-loop finiteness of the β and γ-functions of the rigid theory can be transferred to the β-functions of the SSB terms. Therefore, a totally all-loop finite $N = 1$ SUSY gauge theory can be constructed, including the soft SUSY breaking terms.

4 Experimental Constraints

In this section we review the phenomenological constraints that were applied in the phenomenological analysis. The fact that the used values do not correspond to the latest experimental results has a negligible impact on our analysis.

In each of our models we evaluate the pole mass of the top quark, while the bottom quark mass is evaluated at the M_Z scale, in order to avoid uncertainties to its pole mass. The experimental values [107] are:

$$m_t^{\rm exp} = 173.1 \pm 0.9 \text{ GeV} \quad , \quad m_b(M_Z) = 2.83 \pm 0.10 \text{ GeV} . \tag{83}$$

The Higgs-like particle discovered in July 2012 by ATLAS and CMS [31,32] is interpreted as the light CP-even Higgs boson of the MSSM [108–110]. The Higgs boson experimental average mass is [107] [b]

$$M_h^{\rm exp} = 125.10 \pm 0.14 \text{ GeV} . \tag{84}$$

The theoretical uncertainty [39, 40], however, for the prediction of M_h in the MSSM is much larger than the experimental one and thus dominates the total uncertainty. In the following sections we shall use the updated `FeynHiggs` code [39–41] (Version 2.16.0) to predict the light Higgs mass.[c] `FeynHiggs` evaluates all Higgs masses based on a combination of fixed order diagrammatic calculations and resummation of the (sub)leading logarithmic contributions at all orders. This gives a reliable M_h even for large SUSY scales. This version gives a downward shift on the Higgs mass M_h of $\mathcal{O}(2 \text{ GeV})$ for large supersymmetric masses. In particular, it gives a reliable point-by-point calculation of the Higgs boson mass uncertainty [42]. The theoretical uncertainty calculated is added linearly to the experimental error in Eq. (84).

Furthermore, recent ATLAS results [112] set limits to the pseudoscalar Higgs mass, M_A, compared to $\tan\beta$. For $\tan\beta \sim 45 - 55$ the lowest limit for the physical M_A is

$$M_A \gtrsim 1900 \text{ GeV}. \tag{85}$$

Finally, we consider four flavor observables where SUSY has non-negligible impact. For the branching ratio $\text{BR}(b \to s\gamma)$ we use a value from [113,114], while for $\text{BR}(B_s \to \mu^+\mu^-)$ we take a combination of [115–119]:

$$\frac{\text{BR}(b \to s\gamma)^{\rm exp}}{\text{BR}(b \to s\gamma)^{\rm SM}} = 1.089 \pm 0.27 \quad , \quad \text{BR}(B_s \to \mu^+\mu^-) = (2.9 \pm 1.4) \times 10^{-9} . \tag{86}$$

[b] This is the latest available LHC combination. More recent measurements confirm this value.
[c] For a discussion of the impact of the improved M_h calculation in several SUSY models see [111].

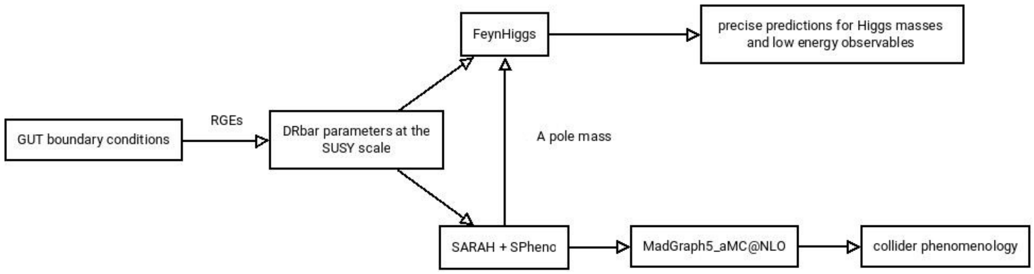

Figure 1: Flow of information between computer codes.

For the B_u decay to $\tau\nu$ [114, 120, 121] is used and for ΔM_{B_s} we take [122, 123]:

$$\frac{\mathrm{BR}(B_u \to \tau\nu)^{\mathrm{exp}}}{\mathrm{BR}(B_u \to \tau\nu)^{\mathrm{SM}}} = 1.39 \pm 0.69 \qquad , \qquad \frac{\Delta M_{B_s}^{\mathrm{exp}}}{\Delta M_{B_s}^{\mathrm{SM}}} = 0.97 \pm 0.2 \; . \tag{87}$$

5 Computational setup for phenomenological analysis

Here we briefly explain the setup that was used for our analysis. We start from a set of MSSM boundary conditions at the unification scale, then parameters are run down to the SUSY scale using a private code. Two-loop RGEs are used throughout, with the exception of the one-loop soft sector. The running parameters are then used as inputs for `FeynHiggs` [39–42] and a `SARAH` [124] generated, custom MSSM module for `SPheno` [125,126]. Since `FeynHiggs` requires the $m_b(m_b)$ scale, the *physical* top quark mass m_t as well as *the physical* pseudoscalar boson mass M_A as input, the first two values are calculated by the private code, while M_A is calculated only in $\overline{\mathrm{DR}}$ scheme. This single value is obtained from the `SPheno` output where it is calculated at two loops in the gaugeless limit [127, 128]. Fig. 1 summarizes the flow of information between codes.

Although at this point both codes contain a consistent set of all required parameters, the SM-like Higgs boson mass and the B-physics observables are evaluated using `FeynHiggs`, while to obtain collider predictions we use `SARAH` to generate `UFO` [129, 130] model for `MadGraph` event generator. Based on SLHA spectrum files generated by `SPheno`, we use `MadGraph5_aMC@NLO` [131] to calculate cross sections for Higgs boson and SUSY particle production at the HL-LHC and a 100 TeV FCC-hh. All processes are generated at the leading order, using `NNPDF31_lo_as_0130` [132] structure functions interfaced through `LHAPDF6` [133]. The respective cross sections are computed using dynamic scale choice, where the scale is considered equal to the transverse mass of an event, in 4 or 5-flavor scheme depending on the presence or not of b-quarks in the final state.

6 The Finite $N = 1$ Supersymmetric $SU(5)$ Model

We start with the finite to all-orders $SU(5)$ theory, where we restrict the application of the reduction of couplings method to the third generation. An older analysis of this Finite Unified Theory (FUT) was in agreement with the experimental constraints at the time [29] and predicted the light Higgs mass in the correct range almost five years before its discovery. As reviewed below, improved Higgs calculations predict a somewhat different interval that is still within current experimental limits.

The particle content of the model consists of three ($\overline{\mathbf{5}}+\mathbf{10}$) supermultiplets for the three generations of leptons and quarks, while the Higgs sector is accommodated in four supermultiplets ($\overline{\mathbf{5}}+\mathbf{5}$) and one $\mathbf{24}$. The finite $SU(5)$ group is broken to the MSSM, which is no longer a finite theory, as expected [14–17, 21, 24].

The following characteristics are essential in order for this all-loop finite $SU(5)$ model to achieve Gauge Yukawa Unification (GYU):
(i) The one-loop anomalous dimensions must be diagonal i.e., $\gamma_i^{(1)\,j} \propto \delta_i^j$.
(ii) The fermions of $\overline{\mathbf{5}}_i$ and $\mathbf{10}_i$ ($i = 1, 2, 3$) do not couple to $\mathbf{24}$.
(iii) The MSSM Higgs doublets are mostly composed from the 5 and $\bar{5}$ that couple to the third generation.
The superpotential (with an enhanced symmetry due to the reduction of couplings) is given by [25, 27]:

$$W = \sum_{i=1}^{3} \left[\, \frac{1}{2} g_i^u \,\mathbf{10}_i \mathbf{10}_i H_i + g_i^d \,\mathbf{10}_i \overline{\mathbf{5}}_i \,\overline{H}_i \,\right] + g_{23}^u \,\mathbf{10}_2 \mathbf{10}_3 H_4 \tag{88}$$
$$+ g_{23}^d \,\mathbf{10}_2 \overline{\mathbf{5}}_3 \,\overline{H}_4 + g_{32}^d \,\mathbf{10}_3 \overline{\mathbf{5}}_2 \,\overline{H}_4 + g_2^f \,H_2 \,\mathbf{24}\, \overline{H}_2 + g_3^f \,H_3 \,\mathbf{24}\, \overline{H}_3 + \frac{g^\lambda}{3} \,(\mathbf{24})^3 \ .$$

The model is discussed in more detail in [14–16]. The *non-degenerate* and *isolated* solutions to the vanishing of $\gamma_i^{(1)}$ are:

$$\begin{aligned}
&(g_1^u)^2 = \frac{8}{5}\, g^2 \ ,\ (g_1^d)^2 = \frac{6}{5}\, g^2 \ ,\ (g_2^u)^2 = (g_3^u)^2 = \frac{4}{5}\, g^2 \ ,\\
&(g_2^d)^2 = (g_3^d)^2 = \frac{3}{5}\, g^2 \ ,\ (g_{23}^u)^2 = \frac{4}{5}\, g^2 \ ,\ (g_{23}^d)^2 = (g_{32}^d)^2 = \frac{3}{5}\, g^2 \ ,\\
&(g^\lambda)^2 = \frac{15}{7} g^2 \ ,\ (g_2^f)^2 = (g_3^f)^2 = \frac{1}{2}\, g^2 \ ,\ (g_1^f)^2 = 0 \ ,\ (g_4^f)^2 = 0 \ .
\end{aligned} \tag{89}$$

Regarding the parameters of non-zero dimension, we have the relation $h = -MC$, while the sum rules lead to:

$$m_{H_u}^2 + 2m_{\mathbf{10}}^2 = M^2 \ ,\ m_{H_d}^2 - 2m_{\mathbf{10}}^2 = -\frac{M^2}{3} \ ,\ m_{\overline{\mathbf{5}}}^2 + 3m_{\mathbf{10}}^2 = \frac{4M^2}{3} \ . \tag{90}$$

We therefore result in just two free dimensionful parameters, $m_{\mathbf{10}}$ and M.

When the GUT breaks to the MSSM, a suitable rotation in the Higgs sector [14, 15, 134–137], allows only one two Higgs doublets (coupled mostly to the third family) to remain light and acquire vevs. Fast proton decay is avoided with the usual doublet-triplet splitting.

Below the GUT scale we get the MSSM, where the third generation is given by the finiteness conditions (the first two remain unrestricted). However, these conditions do not restrict the low-energy renormalization properties, so the above relations between gauge, Yukawa and the various dimensionful parameters serve as blundary conditions at M_{GUT}. The third generation quark masses $m_b(M_Z)$ and m_t are predicted within 2σ and 3σ uncertainties, respectively, of their experimental values (see the complete analysis in [37]). $\mu < 0$ is the only phenomenologically viable option, as shown in [37, 138–145].

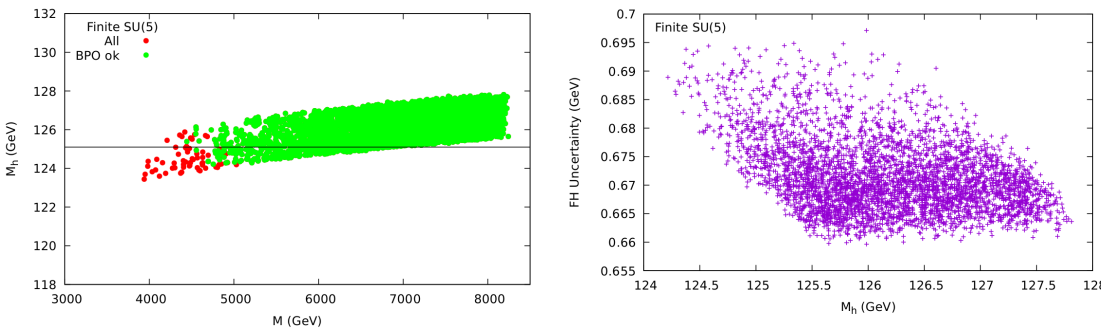

Figure 2: Plot for the Finite $SU(5)$ model. Left: M_h as a function of M. The green points satisfy all four B-physics constraints. Right: The lightest Higgs boson mass theoretical uncertainty (calculated with `FeynHiggs` 2.16.0 [42]).

The plot of the light Higgs mass is given in Fig. 2 (left) and its theoretical uncertainty [42] is given in Fig. 2 (right). It should be noted that this point-by-point uncertainty drops significantly (w.r.t. the previous analysis) to $0.65 - 0.70$ GeV.

The improved evaluation of M_h and its uncertainty prefer a heavier (Higgs) spectrum (compared to previous analyses [37, 138–144, 146–150]), and thus allows only a heavy supersymmetric spectrum, which is in agreement with all existing experimental data. Very heavy colored supersymmetric particles are favored, in agreement with the non-observation of such particles at the LHC [151].

	M_1	M_2	M_3	$\|\mu\|$	b	A_u	A_d	A_e	$\tan\beta$	$m^2_{Q_{1,2}}$
FUTSU5-1	2124	3815	8804	4825	854^2	7282	7710	2961	49.9	8112^2
FUTSU5-2	2501	4473	10198	5508	1048^2	8493	9023	3536	50.1	9387^2
FUTSU5-3	3000	5340	11996	6673	2361^2	10086	10562	4243	49.9	11030^2
	$m^2_{Q_3}$	$m^2_{\tilde{L}_{1,2}}$	$m^2_{\tilde{L}_3}$	$m^2_{\tilde{\bar{u}}_{1,2}}$	$m^2_{\tilde{\bar{u}}_3}$	$m^2_{\tilde{\bar{d}}_{1,2}}$	$m^2_{\tilde{\bar{d}}_3}$	$m^2_{\tilde{\bar{e}}_{1,2}}$	$m^2_{\tilde{\bar{e}}_3}$	
FUTSU5-1	6634^2	3869^2	3120^2	7684^2	5053^2	7635^2	4177^2	3084^2	2241^2	
FUTSU5-2	7669^2	4521^2	3747^2	8887^2	6865^2	8826^2	6893^2	3602^2	2551^2	
FUTSU5-3	9116^2	5355^2	3745^2	10419^2	8170^2	10362^2	7708^2	4329^2	3403^2	

Table 1: Finite $N = 1$ $SU(5)$ predictions that are used as input to `SPheno` (see [38]).

As explained in more detail in [38], the three benchmarks chosen feature the LSP above 2100 GeV, 2400 GeV and 2900 GeV, respectively. The input of `SPheno` 4.0.4 [125, 126] can be found in Table 1, where M_i are gaugino masses and the rest are squared sfermion masses which are diagonal ($\mathbf{m^2} = \mathrm{diag}(\mathrm{m}_1^2, \mathrm{m}_2^2, \mathrm{m}_3^2)$) and soft trilinear couplings, which are also diagonal $\mathbf{A_i} = \mathbb{1}_{3\times3} A_i$).

The resulting masses that are relevant to our analysis are listed in Table 2. The three first values are the heavy Higgs masses. The gluino mass is $M_{\tilde{g}}$, the neutralinos and the charginos are denoted as $M_{\tilde{\chi}_i^0}$ and $M_{\tilde{\chi}_i^\pm}$, while the slepton and sneutrino masses for all three generations are given as $M_{\tilde{e}_{1,2,3}}$, $M_{\tilde{\nu}_{1,2,3}}$. Similarly, the squarks are denoted as $M_{\tilde{d}_{1,2}}$ and $M_{\tilde{u}_{1,2}}$ for the first two generations. The third generation masses are given by $M_{\tilde{t}_{1,2}}$ for stops and $M_{\tilde{b}_{1,2}}$ for sbottoms.

	M_H	M_A	$M_{H^\pm}$	$M_{\tilde{g}}$	$M_{\tilde{\chi}_1^0}$	$M_{\tilde{\chi}_2^0}$	$M_{\tilde{\chi}_3^0}$	$M_{\tilde{\chi}_4^0}$	$M_{\tilde{\chi}_1^\pm}$	$M_{\tilde{\chi}_2^\pm}$
FUTSU5-1	5.688	5.688	5.688	8.966	2.103	3.917	4.829	4.832	3.917	4.833
FUTSU5-2	7.039	7.039	7.086	10.380	2.476	4.592	5.515	5.518	4.592	5.519
FUTSU5-3	16.382	16.382	16.401	12.210	2.972	5.484	6.688	6.691	5.484	6.691
	$M_{\tilde{e}_{1,2}}$	$M_{\tilde{\nu}_{1,2}}$	$M_{\tilde{\tau}}$	$M_{\tilde{\nu}_\tau}$	$M_{\tilde{d}_{1,2}}$	$M_{\tilde{u}_{1,2}}$	$M_{\tilde{b}_1}$	$M_{\tilde{b}_2}$	$M_{\tilde{t}_1}$	$M_{\tilde{t}_2}$
FUTSU5-1	3.102	3.907	2.205	3.137	7.839	7.888	6.102	6.817	6.099	6.821
FUTSU5-2	3.623	4.566	2.517	3.768	9.059	9.119	7.113	7.877	7.032	7.881
FUTSU5-3	4.334	5.418	3.426	3.834	10.635	10.699	8.000	9.387	8.401	9.390

Table 2: Masses for each of the three benchmarks of the Finite $N = 1$ $SU(5)$ (in TeV) [38].

Table 3 lists all the expected production cross sections for the various final states at the 100 TeV future FCC-hh collider (for the full analysis see [38]). At 14 TeV HL-LHC none of the Finite $SU(5)$ scenarios listed above has a SUSY production cross section above 0.01 fb, and thus will most probably remain unobservable, since all superpartners are too heavy for pair production and the heavy Higgs bosons are far outside the reach of the collider [152]. For this reason we do not show any $\sqrt{s} = 14$ TeV cross sections.

The discovery prospects for the heavy Higgs-boson spectrum is significantly better at the FCC-hh [153]. Theoretical analyses [154, 155] have shown that for large $\tan\beta$ heavy Higgs mass scales up to ~ 8 TeV could be accessible. The relevant decay channels are $H/A \to \tau^+\tau^-$ and $H^\pm \to \tau\nu_\tau, tb$. Since in this model we have $\tan\beta \sim 50$, the first two benchmark points are well within the reach of the FCC-hh. The third point, however, where $M_A \sim 16$ TeV, will be far outside the reach of the collider.

The energy of 100 TeV is big enough to produce SUSY particles in pairs. However, prospects for detecting production of squark pairs and squark-gluino pairs are very dim since their production cross section is also at the level of a few fb. This is as a result of a heavy spectrum in this class of models.

The SUSY discovery reach at the FCC-hh with $3\,\text{ab}^{-1}$ was evaluated in [156] for a certain set of simplified models. In the following we will compare these simplified model limits with our benchmark points to get an idea, which part of the spectrum can be covered at the FCC-hh. A more detailed evaluation with the future limits implemented into proper recasting tools would be necessary to obtain a firmer statement. However, such a detailed analysis goes beyond the scope of our work and we restrict ourselves to the simpler direct comparison of the simplified model limits with our benchmark predictions.

The lighter stop might be accessible in FUTSU5-1. For the squarks of the first two generations have somewhat better prospects of testing the model. All benchmarks could possibly be excluded at the $2\,\sigma$ level, but no discovery at the $5\,\sigma$ can be expected and the same holds for the gluino. The heavy LSP will keep charginos and neutralinos unobservable. We have to conclude that again large parts of the possible mass spectra will not be observable at the FCC-hh.

scenarios $\sqrt{s}$	FUTSU5-1 100 TeV	FUTSU5-2 100 TeV	FUTSU5-3 100 TeV	scenarios $\sqrt{s}$	FUTSU5-1 100 TeV	FUTSU5-2 100 TeV	FUTSU5-3 100 TeV
$\tilde{\chi}_2^0\tilde{\chi}_3^0$	0.01	0.01		$\tilde{\nu}_i\tilde{\nu}_j^*$	0.02	0.01	0.01
$\tilde{\chi}_3^0\tilde{\chi}_4^0$	0.03	0.01		$\tilde{u}_i\tilde{\chi}_1^-, \tilde{d}_i\tilde{\chi}_1^+ + h.c.$	0.15	0.06	0.02
$\tilde{\chi}_2^0\tilde{\chi}_1^+$	0.17	0.08	0.03	$\tilde{q}_i\tilde{\chi}_1^0, \tilde{q}_i^*\tilde{\chi}_1^0$	0.08	0.03	0.01
$\tilde{\chi}_3^0\tilde{\chi}_2^+$	0.05	0.03	0.01	$\tilde{q}_i\tilde{\chi}_2^0, \tilde{q}_i^*\tilde{\chi}_2^0$	0.08	0.03	0.01
$\tilde{\chi}_4^0\tilde{\chi}_2^+$	0.05	0.03	0.01	$\tilde{\nu}_i\tilde{e}_j^*, \tilde{\nu}_i^*\tilde{e}_j$	0.09	0.04	0.01
$\tilde{g}\tilde{g}$	0.20	0.05	0.01	$Hb\bar{b}$	2.76	0.85	
$\tilde{g}\tilde{\chi}_1^0$	0.03	0.01		$Ab\bar{b}$	2.73	0.84	
$\tilde{g}\tilde{\chi}_2^0$	0.03	0.01		$H^+b\bar{t} + h.c.$	1.32	0.42	
$\tilde{g}\tilde{\chi}_1^+$	0.07	0.03	0.01	H^+W^-	0.38	0.12	
$\tilde{q}_i\tilde{q}_j, \tilde{q}_i\tilde{q}_j^*$	3.70	1.51	0.53	HZ	0.09	0.03	
$\tilde{\chi}_1^+\tilde{\chi}_1^-$	0.10	0.05	0.02	AZ	0.09	0.03	
$\tilde{\chi}_2^+\tilde{\chi}_2^-$	0.03	0.02	0.01				
$\tilde{e}_i\tilde{e}_j^*$	0.23	0.13	0.05				
$\tilde{q}_i\tilde{g}, \tilde{q}_i^*\tilde{g}$	2.26	0.75	0.20				

Table 3: Expected production cross sections (in fb) for supersymmetric particles (for the original analysis see [38]).

7 The Reduced MSSM

Our second phenomenological analysis concerns the application of the reduction of couplings method to a version of the MSSM, where we assume a covering GUT. The original work can be found in refs. [157, 158]. Considering the reduction only on the third fermionic generation, the superpotential reads:

$$W = Y_t H_2 Q t^c + Y_b H_1 Q b^c + Y_\tau H_1 L \tau^c + \mu H_1 H_2 \,, \tag{91}$$

where $Y_{t,b,\tau}$ are third family parameters. The SSB Lagrangian is given by (again only for the third family sector)

$$\begin{aligned} -\mathcal{L}_{\rm SSB} = \sum_\phi m_\phi^2 \hat{\phi}^* \hat{\phi} + \left[m_3^2 \hat{H}_1 \hat{H}_2 + \sum_{i=1}^3 \frac{1}{2} M_i \lambda_i \lambda_i + \text{h.c} \right] \\ + \left[h_t \hat{H}_2 \hat{Q} \hat{t^c} + h_b \hat{H}_1 \hat{Q} \hat{b^c} + h_\tau \hat{H}_1 \hat{L} \hat{\tau^c} + \text{h.c.} \right]. \end{aligned} \tag{92}$$

Starting with the dimensionless sector, we consider initially the top and bottom Yuakwa couplings and the strong gauge coupling, whereas the rest of the couplings are treated as corrections. For $Y^2_{(t,b)}/(4\pi) \equiv \alpha_{(t,b)}$, the REs and the Yukawa RGEs give

$$\alpha_i = G_i^2 \alpha_3, \quad \text{where} \quad G_i^2 = \frac{1}{3}, \qquad i = t, b.$$

If we include the tau Yukawa in the reduction, the corresponding G^2 coefficient for tau turns negative [159]. This is the reason that this coupling cannot be reduced and is also treated as a correction.

We assume that the ratios of the quark Yukawa couplings to the strong coupling are constant at the unification scale (they have negligible scale dependence),

$$\frac{d}{dg_3}\left(\frac{Y_{t,b}^2}{g_3^2}\right) = 0.$$

Then, if we include the corrections from the $SU(2)$, $U(1)$ and tau couplings at the GUT scale, we get the full coefficients $G_{t,b}^2$:

$$G_t^2 = \frac{1}{3} + \frac{71}{525}\rho_1 + \frac{3}{7}\rho_2 + \frac{1}{35}\rho_\tau, \qquad G_b^2 = \frac{1}{3} + \frac{29}{525}\rho_1 + \frac{3}{7}\rho_2 - \frac{6}{35}\rho_\tau \ , \tag{93}$$

where

$$\rho_{1,2} = \frac{g_{1,2}^2}{g_3^2} = \frac{\alpha_{1,2}}{\alpha_3}, \qquad \rho_\tau = \frac{g_\tau^2}{g_3^2} = \frac{\frac{Y_\tau^2}{4\pi}}{\alpha_3} \ . \tag{94}$$

We shall treat Eqs.(93) as boundary conditions at the unification scale.

Going to two loops, we assume the form of the corrections to be:

$$\alpha_i = G_i^2\alpha_3 + J_i^2\alpha_3^2, \qquad i = t, b \ .$$

Then, the two-loop coefficients, J_i, with the inclusion of the above-mentioned corrections, are:

$$J_t^2 = \frac{1}{4\pi}\frac{N_t}{D}, \quad J_b^2 = \frac{1}{4\pi}\frac{N_b}{5D},$$

where D, N_t and N_b are known quantities which can be found in ref. [160].

We may now proceed to the the SSB Lagrangian, Eq. (92), and the dimension-one parameters, i.e the trilinear couplings $h_{t,b,\tau}$. We first reduce $h_{t,b}$:

$$h_i = c_i Y_i M_3 = c_i G_i M_3 g_3, \quad \text{where} \quad c_i = -1 \qquad i = t, b,$$

where M_3 is the gluino mass. Adding the gauge and the tau corrections we have

$$c_t = -\frac{A_A A_{bb} + A_{tb}B_B}{A_{bt}A_{tb} - A_{bb}A_{tt}}, \qquad c_b = -\frac{A_A A_{bt} + A_{tt}B_B}{A_{bt}A_{tb} - A_{bb}A_{tt}}.$$

where A_{tt}, A_{bb} and A_{tb} can be found in ref. [160].

Finally, we treat the soft scalar masses m_ϕ^2 of the SSB Lagrangian. Assuming the relations $m_i^2 = c_i M_3^2$ ($i = Q, u, d, H_u, H_d$), and adding the gauge, the tau couplings and h_τ corrections, we get

$$c_Q = -\frac{c_{Q\text{Num}}}{D_m}, \quad c_u = -\frac{1}{3}\frac{c_{u\text{Num}}}{D_m}, \quad c_d = -\frac{c_{d\text{Num}}}{D_m}, \quad c_{H_u} = -\frac{2}{3}\frac{c_{H_u\text{Num}}}{D_m}, \quad c_{H_d} = -\frac{c_{H_d\text{Num}}}{D_m}, \tag{95}$$

where D_m, $c_{Q\text{Num}}$, $c_{u\text{Num}}$, $c_{d\text{Num}}$, $c_{H_u\text{Num}}$, $c_{H_d\text{Num}}$ and the complete analysis are given in ref. [160]. The values of Eq. (95) do not obey any soft scalar mass sum rule.

If only the reduced system was used (with no corrections), i.e. the strong, top and bottom Yukawa couplings as well as the h_t and h_b, the coefficients would be

$$c_Q = c_u = c_d = \frac{2}{3}, \quad c_{H_u} = c_{H_d} = -1/3,$$

which obey the sum rules

$$\frac{m_Q^2 + m_u^2 + m_{H_u}^2}{M_3^2} = c_Q + c_u + c_{H_u} = 1, \qquad \frac{m_Q^2 + m_d^2 + m_{H_d}^2}{M_3^2} = c_Q + c_d + c_{H_d} = 1. \tag{96}$$

We should also mention an essential point for the gaugino masses here. The application of the Hisano-Shifman relation (Eq. (53)) is made for each gaugino mass as a boundary condition at $M_{\rm GUT}$. Then, at one-loop level, the gaugino mass depends on the one-loop coefficient of the corresponding β-function and an arbitrary mass $M_0, \quad M_i = b_i M_0$. This fact permits, with a suitable choice of M_0, to have the gluino mass equal to the unified gaugino mass, while the gauginos masses of the other two gauge groups are given by the gluino mass multiplied by the ratio of the appropriate one-loop β coefficient.

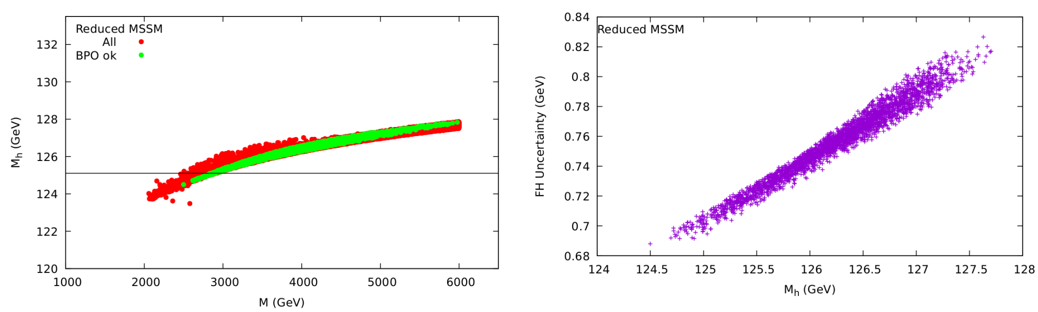

Figure 3: Left: The lightest Higgs mass, M_h in the Reduced MSSM. Right: the Higgs mass theoretical uncertainty [42].

We choose the GUT scale to apply the corrections to all these RGI relations in our analysis. A detailed discussion on the free parameters selection of the model can be found in [37]. In total, we vary ρ_τ, ρ_{h_τ}, M and μ. The predictions for the bottom and the top quark masses are within 2σ of Eq. (83). The light Higgs mass M_h is shown in Fig. 3 (left) and is predicted within experimental measured range, while its theoretical uncertainty shown in Fig. 3 (right) now drops below 1 GeV.

As demonstrated in [38], M_h sets a limit on the low-energy supersymmetric masses, which we briefly discuss. The three benchmarks selected correspond to $\overline{\rm DR}$ pseudoscalar Higgs masses above 1900 GeV, 1950 GeV and 2000 GeV respectively. The values used as input to SPheno 4.0.4 [125, 126] are listed in Table 4 (notation as in Sect. 6).

The resulting masses of Higgs bosons and some of the lightest supersymmetric particles are given in Table 5. In particular, we find $M_A \lesssim 1.5$ TeV This means that in this model, because of the large $\tan\beta \sim 45$, the physical mass of the pseudoscalar

	M_1	M_2	M_3	$\|\mu\|$	b	A_u	A_d	A_e	$\tan\beta$	$m^2_{Q_{1,2}}$
RMSSM-1	3711	1014	7109	4897	284^2	5274	5750	20	44.9	5985^2
RMSSM-2	3792	1035	7249	4983	294^2	5381	5871	557	44.6	6103^2
RMSSM-3	3829	1045	7313	5012	298^2	5427	5942	420	45.3	6161^2
	$m^2_{Q_3}$	$m^2_{L_{1,2}}$	$m^2_{L_3}$	$m^2_{\overline{u}_{1,2}}$	$m^2_{\overline{u}_3}$	$m^2_{\overline{d}_{1,2}}$	$m^2_{\overline{d}_3}$	$m^2_{\overline{e}_{1,2}}$	$m^2_{\overline{e}_3}$	
RMSSM-1	5545^2	2106^2	2069^2	6277^2	5386^2	5989^2	5114^2	3051^2	4491^2	
RMSSM-2	5656^2	2122^2	2290^2	6385^2	5476^2	6110^2	5219^2	3153^2	4181^2	
RMSSM-3	5708^2	2106^2	2279^2	6427^2	5506^2	6172^2	5269^2	3229^2	3504^2	

Table 4: Reduced MSSM predictions used as input to `SPheno` (see [38]).

Higgs boson, M_A, is excluded by the searches $H/A \to \tau\tau$ at ATLAS with 139/fb [112] for all three benchmarks, and, as it was shown in [38], this holds for the entire allowed parameter space. If we considered a heavier sectrum instead (in which we would have $M_A \gtrsim 1900$ GeV) the light Higgs boson mass would be above its acceptable region. Thus, this version of the model is ruled out experimentally. Consequently, we do not present any cross sections.

	M_H	M_A	$M_{H^\pm}$	$M_{\tilde{g}}$	$M_{\tilde{\chi}^0_1}$	$M_{\tilde{\chi}^0_2}$	$M_{\tilde{\chi}^0_3}$	$M_{\tilde{\chi}^0_4}$	$M_{\tilde{\chi}^\pm_1}$	$M_{\tilde{\chi}^\pm_2}$
RMSSM-1	1.393	1.393	1.387	7.253	1.075	3.662	4.889	4.891	1.075	4.890
RMSSM-2	1.417	1.417	1.414	7.394	1.098	3.741	4.975	4.976	1.098	4.976
RMSSM-3	1.491	1.491	1.492	7.459	1.109	3.776	5.003	5.004	1.108	5.004
	$M_{\tilde{e}_{1,2}}$	$M_{\tilde{\nu}_{1,2}}$	$M_{\tilde{\tau}}$	$M_{\tilde{\nu}_\tau}$	$M_{\tilde{d}_{1,2}}$	$M_{\tilde{u}_{1,2}}$	$M_{\tilde{b}_1}$	$M_{\tilde{b}_2}$	$M_{\tilde{t}_1}$	$M_{\tilde{t}_2}$
RMSSM-1	2.124	2.123	2.078	2.079	6.189	6.202	5.307	5.715	5.509	5.731
RMSSM-2	2.297	2.139	2.140	2.139	6.314	6.324	5.414	5.828	5.602	5.842
RMSSM-3	2.280	2.123	2.125	2.123	6.376	6.382	5.465	5.881	5.635	5.894

Table 5: Masses for each of the three benchmarks of the Reduced MSSM (in TeV). Original analysis in [38].

8 Conclusions

The basis of the reduction of couplings scheme is the search for RGE relations among parameters that hold to all orders in perturbation theory. It is realized in certain $N = 1$ theories, rendering them more predictive. In the present work, after a review of the ideas concerning the reduction of couplings of renormalizable theories and the theoretical methods which have been developed to confront the problem, we turn to the question of testing experimentally the idea of reduction of couplings. Two specific models, namely the all-loop Finite $N = 1$ $SU(5)$ and the Reduced MSSM, have been considered and new results have been obtained for both, using the updated Higgs-boson mass calculation of `FeynHiggs`. In each case low-mass region benchmark points have been chosen, for which the `SPheno` code was used to calculate the spectrum of supersymmetric particles and their respective decay modes. Finally, the `MadGraph` event generator has been used (in the case of the Finite $SU(5)$) for the computation of the production cross sections of relevant final states at the 14 TeV (HL-)LHC and 100 TeV FCC-hh colliders.

The finite model was found to be in agreement with LHC measurements. Both models predict relatively heavy spectra, which evade largely the detection in the HL-LHC. However, the Reduced MSSM features a relatively light heavy Higgs spectrum. Combined with its relatively high $\tan\beta$, this spectrum is excluded by current searches at ATLAS for in the $pp \to H/A \to \tau^+\tau^-$ mode. Concerning the finite model, we examined the accessibility of the SUSY and heavy Higgs spectrum at the FCC-hh with $\sqrt{s} = 100$ TeV. The lower parts of the parameter space will be testable at the $2\,\sigma$ level, with only an even smaller part discoverable at the $5\,\sigma$ level. However, the heavier parts of the possible SUSY spectra will remain elusive even at the FCC-hh.

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