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3HDM with $\Delta(27)$ symmetry and its phenomenological consequences

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Abstract: We perform a comprehensive analysis of a version of the 3-Higgs doublet model whose scalar potential is invariant under a global $\Delta(27)$ discrete symmetry and where the three scalar doublets are chosen to transform as a triplet under this discrete group. For each of the known tree-level minima we study the mass spectra and use the oblique parameters STU as well as perturbative unitarity to constrain the parameter space of the model. We then discuss phenomenological consequences of some leading order flavour mixing quark Yukawa couplings by considering the flavour violation process $b \to s\gamma$.

We show that perturbative unitarity significantly constrains parameters of the model while, conversely, the beyond the Standard Model contributions to the $b \to s\gamma$ decay are automatically tamed by the symmetry.

Keywords: Multi-Higgs Models, Specific BSM Phenomenology, Higgs Properties

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Contents

1 Introduction

The discovery of the Higgs boson and the (so far) non-observation of new particles at the LHC have confirmed the Standard Model as our paradigm. Only the observation of neutrino mixing requires its extension in the leptonic sector. However, a number of theoretical arguments as well as experimental observations still motivate searches for a more fundamental theory. Extensions of the scalar sector of the Standard Model (SM) are particularly well motivated. One of the simplest ones is the well-known two-Higgs doublet model (2HDM) which, in its different incarnations, has been already extensively studied (see for example [1, 2]). Models with more Higgs doublets transforming non-trivially under non-Abelian discrete symmetries have also gained interest, in particular in the context of predicting the observed mixing patterns of leptons or quarks. That is because, in its most general form, already a three-Higgs doublet model (3HDM) has a large number of free parameters in the scalar sector alone. Therefore it is particularly appealing from the point of view of model building to impose additional symmetries in the scalar sector. Symmetries play an important role in reducing the number of free parameters and, as a result, they increase the predictive power of the model. Continuous symmetries, when spontaneously broken, lead to undesired massless Goldstone bosons. There is however a small list of finite symmetries a 3HDM can possess without having also a continuous symmetry [3] (see also [4, 5]). Among the more predictive potentials are those where the 3 Higgs fields transform as a triplet of a non-Abelian discrete symmetry. The highlights of these cases are the A_4 and $\Delta(27)$ symmetries.

The phenomenology of the A_4 invariant potential has been analysed in detail already several years ago [6, 7]. Meanwhile, while the $\Delta(27)$ symmetry has been often used in particle physics, it was usually done so in a slightly different context (e.g. [8–33]). Namely, an interesting feature of a model with such a symmetry is that it exhibits Geometrical CP Violation [8, 11–13, 15–17, 19, 22], which means that there are minima of the potential that violate CP independently of the parameters of the potential, their form being fixed by the symmetry.

In this paper we therefore aim to fill this gap, by performing a thorough phenomenological analysis of the $\Delta(27)$ invariant potential (with a $\Delta(27)$ triplet of $\mathrm{SU}(2)_L$ doublets), which in fact coincides with the $\Delta(54)$ invariant potential for the same field content. We refer throughout this work to it as the $\Delta(27)$ potential as we assign fermions also to $\Delta(27)$ representations to study leading order in terms of flavour mixing Yukawa structures and associated flavour violating processes.

We perform an analysis analogous to the study done for the A_4 potential in [6, 7]. Namely, we investigate how the parameter space of this potential becomes constrained by STU parameters, perturbative unitarity and, when considering Yukawa structures, the $b \to s\gamma$ process.

In section 2 we describe the model in detail, covering regions of parameter space, possible minima and relations between the physical parameters (masses etc.) and the parameters of the potential. We also discuss possible leading order extensions of the model to the fermion sector for each minimum. As mentioned before, we extend the model to the fermionic sector by choosing how the fermions transform under the $\Delta(27)$ symmetry, thus determining the structure of the Yukawa couplings. Although realistic fermion mixing requires further symmetry breaking, nevertheless even in this simple implementation we can already discuss some important features such as the leading aspects of the Higgs mediated flavour changing neutral currents.

In section 3 we present the constraints from STU and unitarity in terms of physical masses of the charged and neutral Higgs bosons. Finally, we conclude in section 4.

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Appendix A contains the relevant details on the discrete group $\Delta(27)$, while appendix B gives some technical details on `SPheno` setup and calculation of unitarity constraints. We also give an example of non-SM Higgs decays pattern in appendix C.

2 The $\Delta(27)$ symmetric 3HDM

In this section we discuss the scalar potential, its tree-level minima, Higgs mass matrices as well as the fermion sector of the model.

We consider the most general CP-conserving 3HDM scalar potential, where the Higgs field content is a triplet of $\Delta(27)$ in which each of the three components of the triplet is an $\mathrm{SU}(2)_L$ doublet. For phenomenological analysis the model is encoded into `SARAH` [34–37] version 4.14.5 which, among others, automatically derives aforementioned mass matrices and tadpole equations (minima conditions), as well as interaction vertices.[1]

2.1 Scalar potential

Using the notation as in [38, 39], the scalar potential has a contribution that is common to all $\Delta(3n^2)$ $(n \geq 3)$ potentials

$$\begin{aligned} V_0(\Phi) = & -\mu^2 \sum_{i,\alpha} \Phi_{i\alpha}\Phi^{*i\alpha} + s \sum_{i,\alpha,\beta} (\Phi_{i\alpha}\Phi^{*i\alpha})(\Phi_{i\beta}\Phi^{*i\beta}) \\ & + \sum_{i,j,\alpha,\beta} \left[r_1 (\Phi_{i\alpha}\Phi^{*i\alpha})(\Phi_{j\beta}\Phi^{*j\beta}) + r_2 (\Phi_{i\alpha}\Phi^{*i\beta})(\Phi_{j\beta}\Phi^{*j\alpha}) \right], \end{aligned} \tag{2.1}$$

where the indices $i, j = 1, 2, 3$ refer to the doublets and $\alpha, \beta = 1, 2$ to the $\mathrm{SU}(2)_L$ components of the doublet, either up or down. $\Delta(3n^2)$ $(n \geq 3)$ is a discrete subgroup of the continuous SU(3) group (not to be confused with the $\mathrm{SU}(3)_C$ gauge symmetry of the SM), and V_0 would be invariant under the continuous SU(3) if the coefficient s was set to zero. Note that, (omitting here the SU(2) indices and contractions), the other terms depend on $(|\Phi_1|^2 + |\Phi_2|^2 + |\Phi_3|^2)$ and are SU(3) invariant - they depend only on the overall magnitude - as opposed to $s\,(|\Phi_1|^4 + |\Phi_2|^4 + |\Phi_3|^4)$, which as we discuss later, depends also on the direction. The potential that is invariant under $\Delta(27)$ ($\Delta(3n^2)$ with $n = 3$) contains V_0 and in addition the following term

$$V_{\Delta(27)}(\Phi) = V_{\Delta(54)}(\Phi) = V_0(\Phi) \; + \; \sum_{\alpha,\beta} \left[d \left(\Phi_{1\alpha}\Phi_{1\beta}\Phi^{*2\alpha}\Phi^{*3\beta} + \text{cycl.} \right) + \text{h.c.} \right], \tag{2.2}$$

which is not invariant under A_4 or $\Delta(3n^2)$ for $n > 3$ but still coincides with the $\Delta(54)$ potential. Only when we extend the model to the fermionic sector and choose how the fermions transform under the symmetry, thus determining the structure of the Yukawa couplings, will it be possible to distinguish the two types of models. The parameter d in eq. (2.2) is the only parameter of the potential that can be complex. Choosing d to be real (as we do in this work) leads to a explicit CP-conserving potential and it allows for a simple CP transformation under which each doublet transforms trivially, i.e. each doublet transforms into its complex conjugate. We note that CP can also be conserved with a complex d

[1] The model files are attached to the `arXiv` version of this work.

provided that its phase is of the form $\pm i\frac{2\pi}{3}$. In the latter case the CP transformation is no longer the trivial one and will relate different doublets and their complex conjugates. The potential (2.2) has the interesting feature of having minima with spontaneous geometrical CP violation [8, 11] for real d, i.e., for a large region of parameter space there is a CP violating minimum where the phases of the vacuum expectation value (VEV) are fixed to a specific value (in this case, an integer multiple of $2\pi/3$), with this value not depending on the parameters of the potential (however, for sufficiently large variation of the parameters, one gets into a separate region of parameter space, where the minima belong to a different class).

2.2 The minima of the potential

After spontaneous gauge symmetry breakdown the Higgs doublets can be decomposed as

$$\Phi_j = e^{i\alpha_j}\begin{pmatrix} \phi_j^+ \\ \frac{1}{\sqrt{2}}(v_j + ia_j + \rho_j + i\eta_j) \end{pmatrix}, \qquad j = 1,2,3 \tag{2.3}$$

with real scalar fields ρ_j, η_j and $v_j + ia_j$ the (complex) vacuum expectation values (where v_i and a_i are real numbers).

The minima of the potential (2.2) have been classified previously [39, 40]. As discussed above, the real parameter s governs a term that is not SU(3) invariant and that distinguishes directions of VEVs. For $s < 0$, the global minimum favoured is in the $(v, 0, 0)$ direction, while for $s > 0$ in the $(v, v, v)/\sqrt{3}$ direction. The d parameter (complex in general) governs the phase-dependent term that makes the potential invariant under $\Delta(27)$ (or $\Delta(54)$ to be more precise). If the magnitude of this coefficient is large it disfavours $(v, 0, 0)$ being the global minimum, as for this VEV d does not contribute to the potential — a direction like $(v, v, v)/\sqrt{3}$ or $(\omega v, v, v)/\sqrt{3}$ or similar becomes the global minimum when d dominates ($\omega \equiv \exp(2\pi i/3)$). Then, if a CP symmetry is imposed making d a real parameter, the sign of d determines the class of the minima, for positive d the minimum is of the class $(\omega v, v, v)/\sqrt{3}$, with spontaneous geometrical CP violation.

2.2.1 General VEVs $v_i + ia_i$

Here we list the extrema conditions for completely general complex VEVs

$$\begin{aligned}\frac{\partial V}{\partial v_1} =& -\mu^2 v_1 + s v_1(v_1^2+a_1^2) + (r_1+r_2)v_1(v_1^2+a_1^2+v_2^2+a_2^2+v_3^2+a_3^2) \\ &+ d\left[v_1(v_2v_3 - a_2a_3) + v_2a_3(a_1+a_2) + a_2v_3(a_1+a_3) + \frac{1}{2}v_2(v_3^2-a_3^2) + \frac{1}{2}v_3(v_2^2-a_2^2)\right],\end{aligned} \tag{2.4}$$

$$\begin{aligned}\frac{\partial V}{\partial a_1} =& -\mu^2 a_1 + s a_1(v_1^2+a_1^2) + (r_1+r_2)a_1(v_1^2+a_1^2+v_2^2+a_2^2+v_3^2+a_3^2) \\ &+ d\left[a_1(a_2a_3 - v_2v_3) + v_2a_3(v_1+v_3) + a_2v_3(v_1+v_2) + \frac{1}{2}a_2(a_3^2-v_3^2) + \frac{1}{2}a_3(a_2^2-v_2^2)\right]\end{aligned} \tag{2.5}$$

and the other 4 eqs. obtained by a cyclic permutation of (123).

The above equations are simplified when considering real VEVs. Then we have as equations

$$-\mu^2 v_1 + s v_1^3 + (r_1+r_2)v_1(v_1^2+v_2^2+v_3^2) + d v_1 v_2 v_3 + \frac{1}{2}v_2v_3(v_2+v_3) = 0 \tag{2.6}$$

plus two cyclic permutations. We found several solutions to these equations, which we separate depending on whether their direction does or does not depend directly on the parameters of the potential. Within the solutions we find the (known) minima: $(v, 0, 0)$ and (v, v, v), as well as solutions of the type $(v, -v, 0)$ which we verified analytically can not be minima for any region of parameter space. There are also solutions, whose entries are complicated functions of the parameters, which we checked numerically are not minima for any of the points in parameter space we sampled. This is in agreement with [39, 40]. For complex VEVs we relied on the known solutions $(\omega v, v, v)$ and $(\omega^2 v, v, v)$ (and cyclic permutations) [39, 40]. As we are considering cases with CP symmetry of the potential, $(\omega^2 v, v, v)$ is related by the CP symmetry to $(\omega v, v, v)$, so we do not need to consider it separately.

2.2.2 The $(v, 0, 0)$ case

Solving extrema conditions for μ^2, v_2 and v_3 and renaming v_1 to v gives

$$\mu^2 = (r_1 + r_2 + s)v^2, \tag{2.7}$$

$$v_2 = v_3 = 0, \tag{2.8}$$

as one of the solutions. In this case the Φ_1 plays the role of the SM Higgs doublet, while the other two doublets remain VEV-less and couple to gauge bosons only through quartic couplings even after the Electroweak Symmetry Breaking (EWSB). Therefore we identify v with the SM VEV $v_{\rm SM}$, $v \approx v_{\rm SM} \approx 247\,\text{GeV}$. The CP-even Higgs mass matrix takes the form

$$m_H^2 = \begin{pmatrix} 2(r_1 + r_2 + s) & 0 & 0 \\ 0 & -s & \frac{1}{2}d \\ 0 & \frac{1}{2}d & -s \end{pmatrix} v^2 \tag{2.9}$$

with eigenvalues $m_h^2 = 2(r_1 + r_2 + s)v^2$, $m_{H_1}^2 = \frac{1}{2}(d - 2s)v^2$, $m_{H_2}^2 = -\frac{1}{2}(d + 2s)v^2$, where m_{H_1} and m_{H_2} are not mass ordered, and h is to be identified with the SM-like Higgs.

The CP-odd Higgs mass matrix is given by

$$m_A^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -s & -\frac{1}{2}d \\ 0 & -\frac{1}{2}d & -s \end{pmatrix} v^2 + \text{gauge dependent terms} \tag{2.10}$$

with eigenvalues $m_{A_1}^2 = \frac{1}{2}(d - 2s)v^2$, $m_{A_2}^2 = -\frac{1}{2}(d + 2s)v^2$ and a Goldstone boson G^0. The $m_{H^\pm}^2$ matrix is diagonal, with $m_{H_1^\pm}^2 = m_{H_2^\pm}^2 = -(r_2 + s)v^2$ and a Goldstone boson $G^\pm$.

The Lagrangian parameters can be expressed in terms of physical masses of Higgses as

$$s = -\frac{1}{2v^2}(m_{H_1}^2 + m_{H_2}^2), \tag{2.11}$$

$$r_1 = \frac{1}{2v^2}(m_h^2 + 2m_{H^\pm}^2), \tag{2.12}$$

$$r_2 = \frac{1}{2v^2}(m_{H_1}^2 + m_{H_2}^2 - 2m_{H^\pm}^2), \tag{2.13}$$

$$d = \frac{1}{v^2}(m_{H_1}^2 - m_{H_2}^2). \tag{2.14}$$

Here $H^\pm$ is a simplified notation for either $H_1^\pm$ or $H_2^\pm$, which are degenerate in mass. Likewise A_i are pairwise mass degenerate with H_i.

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2.2.3 The (v, v, v) case

Solving extrema conditions for μ^2, v_2 and v_3 and renaming v_1 to v gives also the case

$$\mu^2 = (2d + 3r_1 + 3r_2 + s)v^2, \tag{2.15}$$

$$v_2 = v, \tag{2.16}$$

$$v_3 = v, \tag{2.17}$$

where $v \approx v_{\mathrm{SM}}/\sqrt{3} \approx 143\,\mathrm{GeV}$ in order to provide correct gauge boson masses (as in this case all 3 doublets contribute to those masses, with equal weights).

The CP-even Higgs mass matrix takes the form ($r_{12} \equiv r_1 + r_2$)

$$m_H^2 = \frac{1}{2}\begin{pmatrix} 4(r_{12}+s)-2d & 5d+4r_{12} & 5d+4r_{12} \\ 5d+4r_{12} & 4(r_{12}+s)-2d & 5d+4r_{12} \\ 5d+4r_{12} & 5d+4r_{12} & 4(r_{12}+s)-2d \end{pmatrix} v^2 \tag{2.18}$$

with eigenvalues $m_h^2 = 2(2d + 3r_{12} + s)v^2$ (to be identified with the SM-like Higgs boson mass) and $m_H^2 = 1/2(-7d + 4s)v^2$ (the mass of the pair of degenerate Higgs bosons H_1 and H_2).

The CP-odd mass matrix is given by

$$m_A^2 = \frac{3}{2}\begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix} dv^2 + \text{gauge dependent terms.} \tag{2.19}$$

The two physical CP-odd Higgs bosons A_1 and A_2 are mass degenerate with a common mass $m_A^2 = -9dv^2/2$. Similarly the charged Higgs bosons mass matrix is given by

$$m_{H^\pm}^2 = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix} (r_2 + d)v^2 + \text{gauge dependent terms,} \tag{2.20}$$

where $H_1^\pm$ and $H_2^\pm$ are also mass degenerate, with a common mass $m_{H^\pm}^2 = -3(d + r_2)v^2$.

The Lagrangian parameters can be expressed in terms of physical masses of Higgses as

$$s = \frac{1}{18v^2}(9m_H^2 - 7m_A^2), \tag{2.21}$$

$$r_1 = \frac{1}{18v^2}(m_A^2 + 6m_{H^\pm}^2 + 3m_h^2 - 3m_H^2), \tag{2.22}$$

$$r_2 = \frac{1}{9v^2}(2m_A^2 - 3m_{H^\pm}^2), \tag{2.23}$$

$$d = -\frac{2}{9v^2}m_A^2. \tag{2.24}$$

2.2.4 The $(\omega v, v, v)$ case

Solving extrema conditions for μ^2, v_1 and v_3, renaming v_2 to v and rearranging the order of VEVs gives finally the case[2]

$$\mu^2 = (-d + 3r_{12} + s)v^2, \tag{2.25}$$
$$v_1 = \omega v, \tag{2.26}$$
$$v_3 = v, \tag{2.27}$$

where $v \approx 143\,\text{GeV}$ (following the same argument as in the (v, v, v) case).

Because in this vacuum CP is spontaneously broken, this time all Higgs bosons mix and the mass matrix for the neutral ones takes the form

$$m_H^2 = \begin{pmatrix} m_{HH}^2 & m_{HA}^2 \\ (m_{HA}^2)^T & m_{AA}^2 \end{pmatrix} v^2 + \text{gauge dependent terms}, \tag{2.28}$$

where the 3x3 submatrices are as follows

$$m_{HH}^2 = \begin{pmatrix} \frac{1}{2}(4d + r_{12} + s) & d - r_{12} & d - r_{12} \\ d - r_{12} & \frac{1}{2}(d + 4(r_{12} + s)) & \frac{1}{4}(8r_{12} - 5d) \\ d - r_{12} & \frac{1}{4}(8r_{12} - 5d) & \frac{1}{2}(d + 4(r_{12} + s)) \end{pmatrix}, \tag{2.29}$$

$$m_{HA}^2 = \frac{\sqrt{3}}{2} \begin{pmatrix} -(r_{12} + s) & -d + 2r_{12} & -d + 2r_{12} \\ d & d & -d/2 \\ d & -d/2 & d \end{pmatrix}, \tag{2.30}$$

$$m_{AA}^2 = \frac{3}{2} \begin{pmatrix} r_{12} + s & 0 & 0 \\ 0 & d & -d/2 \\ 0 & -d/2 & d \end{pmatrix} \tag{2.31}$$

with the eigenvalues

$$m_h^2 = 2v^2(-d + 3r_1 + 3r_2 + s), \tag{2.32}$$
$$m_{H_1}^2 = m_{H_2}^2 = \frac{1}{2}\left(4d + 2s - \sqrt{7d^2 - 2ds + 4s^2}\right) v^2, \tag{2.33}$$
$$m_{H_3}^2 = m_{H_4}^2 = \frac{1}{2}\left(4d + 2s + \sqrt{7d^2 - 2ds + 4s^2}\right) v^2. \tag{2.34}$$

We assume that the single mass-nondegenerate state h is identified with the SM-like Higgs.

The mass matrix for the charged Higgs bosons reads

$$m_{H^-H^+}^2 = (d - 2r_2) \begin{pmatrix} 1 & -\frac{\omega}{2} & -\frac{\omega}{2} \\ -\frac{\omega^*}{2} & 1 & -\frac{1}{2} \\ -\frac{\omega^*}{2} & -\frac{1}{2} & 1 \end{pmatrix} v^2 + \text{gauge dependent terms}. \tag{2.35}$$

Physical charged states are mass degenerate, with mass $m_{H_1^\pm}^2 = m_{H_2^\pm}^2 = \frac{3}{2}(d - 2r_2)v^2$.

[2]Because we are taking d to be real, the potential is invariant under the trivial CP symmetry (as discussed above), so the $(\omega^2 v, v, v)$ case (which is in general distinct from this case) collapses into the same orbit as $(\omega v, v, v)$ and we do not discuss it here separately.

Because of the quadratic nature of expressions for masses $m^2_{H_i}$ there are two combinations of potential parameters that give the same 4 masses m_h, m_{H_1}, m_{H_3} and $m_{H^\pm}$, namely

$$s = \frac{1}{18v^2}\left(3m^2_{H_1} + 3m^2_{H_3} \mp 2\Omega\right), \tag{2.36}$$

$$r_1 = \frac{1}{18v^2}\left(6m^2_h - 3m^2_{H_1} - 3m^2_{H_3} + 12m^2_{H^\pm} \pm \Omega\right), \tag{2.37}$$

$$r_2 = \frac{1}{18v^2}\left(3m^2_{H_1} + 3m^2_{H_3} - 12m^2_{H^\pm} \pm \Omega\right), \tag{2.38}$$

$$d = \frac{1}{18v^2}\left(3m^2_{H_1} + 3m^2_{H_3} \pm \Omega\right), \tag{2.39}$$

where $m^2_{H_3} > 3m^2_{H_1}$ in order for $\Omega \equiv \sqrt{9m^4_{H_1} - 30m^2_{H_1}m^2_{H_3} + 9m^4_{H_3}}$ to be real.

2.3 Yukawa couplings

Having discussed the scalar potential and its vacua, we now proceed with description of the Yukawa interactions.

The general form of the quark Yukawa couplings in models with three Higgs doublets is

$$\begin{aligned} L_Y = &-\overline{Q_L}\ \Gamma_1\Phi_1 d_R - \overline{Q_L}\ \Gamma_2\Phi_2 d_R - \overline{Q_L}\ \Gamma_3\Phi_3 d_R \\ &- \overline{Q_L}\ \Delta_1\tilde{\Phi}_1 u_R - \overline{Q_L}\ \Delta_2\tilde{\Phi}_2 u_R\ - \overline{Q_L}\ \Delta_3\tilde{\Phi}_3 u_R\ + \text{h.c.}, \end{aligned} \tag{2.40}$$

where Γ_i and Δ_i denote the Yukawa couplings of the left-handed (LH) quark doublets Q_L to the Higgs doublets Φ_j and, respectively, right-handed (RH) quarks d_R or u_R. After spontaneous symmetry breaking quark mass matrices are generated with the form

$$\begin{aligned} M_d &= \frac{1}{\sqrt{2}}(v_1 e^{i\alpha_1}\Gamma_1 + v_2 e^{i\alpha_2}\Gamma_2 + v_3 e^{i\alpha_3}\Gamma_3), \\ M_u &= \frac{1}{\sqrt{2}}(v_1 e^{-i\alpha_1}\Delta_1 + v_2 e^{-i\alpha_2}\Delta_2 + v_3 e^{-i\alpha_3}\Delta_3). \end{aligned} \tag{2.41}$$

The matrices M_d, M_u are then diagonalised by the usual bi-unitary transformations

$$U^\dagger_{dL} M_d U_{dR} = D_d \equiv \text{diag}\ (m_d, m_s, m_b), \tag{2.42}$$

$$U^\dagger_{uL} M_u U_{uR} = D_u \equiv \text{diag}\ (m_u, m_c, m_t). \tag{2.43}$$

Before we can continue with the phenomenological analysis we need to assign the fermions to $\Delta(27)$ representations to obtain semi-realistic Yukawa couplings. The RH quarks we choose to be $\Delta(27)$ triplets because, as we present in more detail for each of the VEVs, this choice allows us to make semi-realistic $\Delta(27)$ invariants for both up and down quark sectors. The 3 LH quark doublets will be distinct $\Delta(27)$ singlets but the choice of representations depends on the VEV, in order to have Yukawa structures that are realistic at leading order - we are able to obtain distinct masses for each generation but with a CKM matrix that is the unit matrix, which is a consequence of breaking $\Delta(27)$ with just one triplet scalar that leaves too much residual unbroken flavour symmetry. While such mixing angles are clearly not viable, we consider this to be a good leading-order approximation. We consider these Yukawa structures as toy models to obtain constraints on the scalar sector. More realistic mixing angles can be obtained by adding further sources of $\Delta(27)$ breaking. This is demonstrated in previous works such as [13, 17, 19], where there is one additional scalar SU(2) singlet that transforms non-trivially under $\Delta(27)$, which we refer to here generically as θ for the sake of our discussion (this notation matches that of [17]). Through its VEV $\langle\theta\rangle$, the Yukawa couplings receive additional contributions and a good fit to all quark mixing angles is obtained. Some further comments about this are in order - the additional source of $\Delta(27)$ breaking is necessary for realistic quark mixing, and comes at the cost of enlarging the scalar content of the model. In this paper we are mostly concerned with the study of bounds on parameters of the scalar potential coming from e.g. unitarity, as in the similar study of the A_4 potential [6]. We argue that the results obtained for the potential with just the three SU(2) doublets Φ_i are relevant, neglecting the contributions of eventual SU(2) singlet θ, whose VEV is expected to be above the electroweak scale. A relevant concern is whether it is valid to consider in this situation the potential of Φ to be $\Delta(27)$ invariant, as terms coupling Φ and θ in the scalar potential will in general give rise to apparently non-invariant terms for Φ proportional to $\langle\theta\rangle$. That this needs not occur, and does not occur e.g. in [17], is a consequence of additional symmetry which allows only terms with $(\theta\theta^\dagger)$ to couple with Φ in the scalar potential. In some sense, the $\Delta(27)$ breaking of $\langle\theta\rangle$ can manifest itself in the Yukawa couplings (where it is needed for realistic models) without spoiling $V_{\Delta(27)}(\Phi)$.

In the following subsections we present Yukawa couplings as Y_u, Y_d and Y_e matrices, and later show the respective Γ, Δ matrices that couple to each of the Φ_i fields.

2.3.1 The $(v, 0, 0)$ case

For the simplest VEV, the choice of singlets is one trivial (1_{00}) and two non-trivial 1_{0i} ($i = 1, 2$). This choice corresponds to having the three singlets that do not transform under the generator c of $\Delta(27)$ (see appendix A), such that we have the desired composition rules from the product of the (anti-)triplet RH quark (either d_R or u_R) and the (anti-)triplet Higgs (either Φ_j for the down sector or its conjugate for the up sector).

In particular, with $(d_R)_i$ ($i = 1 \ldots 3$, the generation index) transforming as an anti-triplet $\bar{3}$, the $\Delta(27)$ invariant terms are then of the type $\bar{Q}_i(\Phi d_R)_{0j}$, whereas $(u_R)_i$ should transform as a triplet 3 to construct similarly the invariants with $\tilde{\Phi}$ which transforms as an anti-triplet $\bar{3}$, i.e. $\bar{Q}_i(u_R\tilde{\Phi})_{0j}$.

For $\bar{Q}_1, \bar{Q}_2, \bar{Q}_3$ belonging to, respectively, $1_{00}, 1_{02}$ and 1_{01} the corresponding Yukawa terms are

$$\begin{aligned} L_Y^d = \bar{Q}Y_d d_R \equiv y_1^d \bar{Q}_1 \left(\Phi_1(d_R)_1 + \Phi_2(d_R)_2 + \Phi_3(d_R)_3\right)_{00} \\ + y_2^d \bar{Q}_2 \left(\Phi_3(d_R)_1 + \Phi_1(d_R)_2 + \Phi_2(d_R)_3\right)_{01} \\ + y_3^d \bar{Q}_3 \left(\Phi_2(d_R)_1 + \Phi_3(d_R)_2 + \Phi_1(d_R)_3\right)_{02}\,. \end{aligned} \tag{2.44}$$

These expressions follow from the specific choice of non-trivial singlets made for the LH quarks and the composition rules of $\Delta(27)$ listed in appendix A.

The Yukawa terms above correspond to Yukawa matrices of the form (in LR convention)

$$Y_d = \begin{pmatrix} y_1^d\Phi_1 & y_1^d\Phi_2 & y_1^d\Phi_3 \\ y_2^d\Phi_3 & y_2^d\Phi_1 & y_2^d\Phi_2 \\ y_3^d\Phi_2 & y_3^d\Phi_3 & y_3^d\Phi_1 \end{pmatrix}. \tag{2.45}$$

The mass matrices are now easy to read of when Φ acquires its VEV, breaking $SU(2)_L$ and $\Delta(27)$. In this subsection we are dealing with the $(v, 0, 0)$ VEV, which means that we are already in the Higgs basis. The down quark mass matrix then looks as follows

$$M_d = \frac{1}{\sqrt{2}} \begin{pmatrix} y_1^d v & 0 & 0 \\ 0 & y_2^d v & 0 \\ 0 & 0 & y_3^d v \end{pmatrix}. \tag{2.46}$$

Because the LH quarks are $\Delta(27)$ singlets, it is easy to adapt the above assignments to the up quark sector giving

$$\begin{aligned} L_Y^u = {} & y_1^u \bar{Q}_1 \left(\tilde{\Phi}_1 (u_R)_1 + \tilde{\Phi}_2 (u_R)_2 + \tilde{\Phi}_3 (u_R)_3\right)_{00} \\ & + y_2^u \bar{Q}_2 \left(\tilde{\Phi}_2 (u_R)_1 + \tilde{\Phi}_3 (u_R)_2 + \tilde{\Phi}_1 (u_R)_3\right)_{01} \\ & + y_3^u \bar{Q}_3 \left(\tilde{\Phi}_3 (u_R)_1 + \tilde{\Phi}_1 (u_R)_2 + \tilde{\Phi}_2 (u_R)_3\right)_{02}. \end{aligned} \tag{2.47}$$

Note the change compared to the down sector. We have to construct the same singlets of $\Delta(27)$ from $3 \times \bar{3}$ (see appendix A), but in the down sector the scalar Φ is in the 3 and the RH fermion is in the $\bar{3}$, whereas in the up sector the scalar $\tilde{\Phi}$ is in the $\bar{3}$ and it is the RH fermion that transforms as 3. Thus

$$Y_u = \begin{pmatrix} y_1^u\tilde{\Phi}_1 & y_1^u\tilde{\Phi}_2 & y_1^u\tilde{\Phi}_3 \\ y_2^u\tilde{\Phi}_2 & y_2^u\tilde{\Phi}_3 & y_2^u\tilde{\Phi}_1 \\ y_3^u\tilde{\Phi}_3 & y_3^u\tilde{\Phi}_1 & y_3^u\tilde{\Phi}_2 \end{pmatrix}. \tag{2.48}$$

For the $(v, 0, 0)$ VEV we therefore have

$$M_u = \frac{1}{\sqrt{2}} \begin{pmatrix} y_1^u v & 0 & 0 \\ 0 & 0 & y_2^u v \\ 0 & y_3^u v & 0 \end{pmatrix}, \tag{2.49}$$

which gives

$$M_u M_u^\dagger = \frac{1}{2} \begin{pmatrix} |y_1^u v|^2 & 0 & 0 \\ 0 & |y_2^u v|^2 & 0 \\ 0 & 0 & |y_3^u v|^2 \end{pmatrix}. \tag{2.50}$$

Given that both $M_u M_u^\dagger$ and $M_d M_d^\dagger$ are diagonal, the CKM matrix in this limit is the identity matrix. For this phenomenological study we consider this as a reasonable leading order approximation.

The extension to the charged lepton sector is likewise simple, where we choose to take the $\mathrm{SU}(2)_L$ doublet L as a triplet so that the RH charged leptons are singlets. The charged lepton masses can be easily obtained in this basis, e.g. with e_1^c, e_2^c, e_3^c respectively transforming as $\Delta(27)$ singlets $1_{00}, 1_{02}, 1_{01}$

$$\begin{aligned} L_Y^e = y_1^e &\left(\bar{L}_1\Phi_1 + \bar{L}_2\Phi_2 + \bar{L}_3\Phi_3\right)_{00} e_1^c \\ &+ y_2^e \left(\bar{L}_1\Phi_3 + \bar{L}_2\Phi_1 + \bar{L}_3\Phi_2\right)_{01} e_2^c \\ &+ y_3^e \left(\bar{L}_1\Phi_2 + \bar{L}_2\Phi_3 + \bar{L}_3\Phi_1\right)_{02} e_3^c, \end{aligned} \tag{2.51}$$

corresponding to Yukawa matrices of the form (LR convention)

$$Y_e = \begin{pmatrix} y_1^e\Phi_1 & y_2^e\Phi_3 & y_3^e\Phi_2 \\ y_1^e\Phi_2 & y_2^e\Phi_1 & y_3^e\Phi_3 \\ y_1^e\Phi_3 & y_2^e\Phi_2 & y_3^e\Phi_1 \end{pmatrix}, \tag{2.52}$$

leading again to a diagonal matrix for the $(v, 0, 0)$ VEV

$$M_e = \frac{1}{\sqrt{2}} \begin{pmatrix} y_1^e v & 0 & 0 \\ 0 & y_2^e v & 0 \\ 0 & 0 & y_3^e v \end{pmatrix}. \tag{2.53}$$

This alternative choice of $\bar{L} \sim 3_{02}$ allows for interesting possibilities for obtaining large leptonic mixing, see e.g. [19], but as this depends on the mechanism that gives neutrinos their masses (the type of seesaw for example), it is beyond the scope of this paper.

2.3.2 The (v, v, v) case

For this VEV we use singlets transforming only under the generator of $\Delta(27)$ which we refer to as the c generator (see appendix A).

With d_R transforming as a $\bar{3}$, the $\Delta(27)$ invariant terms are then of the type $\bar{Q}_i(H(d_R))_{j0}$, whereas u_R should transform as a 3 to construct similarly the invariants with $\tilde{\Phi}$ which transforms as a $\bar{3}$, i.e. $\bar{Q}_i((u_R)\tilde{\Phi})_{j0}$.

For $\bar{Q}_1, \bar{Q}_2, \bar{Q}_3$ chosen as, respectively, $1_{00}, 1_{20}, 1_{10}$ (note these are not the same choices as above), the corresponding Yukawa terms expanded are

$$\begin{aligned} L_Y^d = y_1^d\bar{Q}_1 &\left(\Phi_1(d_R)_1 + \Phi_2(d_R)_2 + \Phi_3(d_R)_3\right)_{00} \\ &+ y_2^d\bar{Q}_2 \left(\Phi_1(d_R)_1 + \omega^2\Phi_2(d_R)_2 + \omega\Phi_3(d_R)_3\right)_{10} \\ &+ y_3^d\bar{Q}_3 \left(\Phi_1(d_R)_1 + \omega\Phi_2(d_R)_2 + \omega^2\Phi_3(d_R)_3\right)_{20} \end{aligned} \tag{2.54}$$

corresponding to Yukawa matrices of the form (LR convention)

$$Y_d = \begin{pmatrix} y_1^d\Phi_1 & y_1^d\Phi_2 & y_1^d\Phi_3 \\ y_2^d\Phi_1 & \omega^2 y_2^d\Phi_2 & \omega y_2^d\Phi_3 \\ y_3^d\Phi_1 & \omega y_3^d\Phi_2 & \omega^2 y_3^d\Phi_3 \end{pmatrix}. \tag{2.55}$$

The mass matrices are now easy to construct when Φ acquires its VEV, breaking SU(2) and $\Delta(27)$. With the (v, v, v) VEV, we are not in the Higgs basis. The down mass matrix looks like

$$M_d = \frac{1}{\sqrt{2}} \begin{pmatrix} y_1^d v & y_1^d v & y_1^d v \\ y_2^d v & \omega^2 y_2^d v & \omega y_2^d v \\ y_3^d v & \omega y_3^d v & \omega^2 y_3^d v, \end{pmatrix}, \tag{2.56}$$

which also in this case is not diagonal, but gives

$$M_d M_d^\dagger = \frac{1}{2} \begin{pmatrix} |y_1^d v|^2 & 0 & 0 \\ 0 & |y_2^d v|^2 & 0 \\ 0 & 0 & |y_3^d v|^2 \end{pmatrix}. \tag{2.57}$$

Because the LH quarks are singlets, it is easy to adapt the above assignments to the up-quark sector. In this case the change of roles of triplet and anti-triplets does not change the type of invariant, and we have

$$\begin{aligned} L_Y^u = y_1^u \bar{Q}_1 & \left(\tilde{\Phi}_1 (u_R)_1 + \tilde{\Phi}_2 (u_R)_2 + \tilde{\Phi}_3 (u_R)_3 \right)_{00} \\ & + y_2^u \bar{Q}_2 \left(\tilde{\Phi}_1 (u_R)_1 + \omega^2 \tilde{\Phi}_2 (u_R)_2 + \omega \tilde{\Phi}_3 (u_R)_3 \right)_{10} \\ & + y_3^u \bar{Q}_3 \left(\tilde{\Phi}_1 (u_R)_1 + \omega \tilde{\Phi}_2 (u_R)_2 + \omega^2 \tilde{\Phi}_3 (u_R)_3 \right)_{20} \end{aligned} \tag{2.58}$$

and

$$Y_u = \begin{pmatrix} y_1^u \tilde{\Phi}_1 & y_1^u \tilde{\Phi}_2 & y_1^u \tilde{\Phi}_3 \\ y_2^u \tilde{\Phi}_1 & \omega^2 y_2^u \tilde{\Phi}_2 & \omega y_2^u \tilde{\Phi}_3 \\ y_3^u \tilde{\Phi}_1 & \omega y_3^u \tilde{\Phi}_2 & \omega^2 y_3^u \tilde{\Phi}_3 \end{pmatrix}. \tag{2.59}$$

For the same VEV as before we have the matrix

$$M_u = \frac{1}{\sqrt{2}} \begin{pmatrix} y_1^u v & y_1^u v & y_1^u v \\ y_2^u v & \omega^2 y_2^u v & \omega y_2^u v \\ y_3^u v & \omega y_3^u v & \omega^2 y_3^u v \end{pmatrix}, \tag{2.60}$$

which gives

$$M_u M_u^\dagger = \frac{1}{2} \begin{pmatrix} |y_1^u v|^2 & 0 & 0 \\ 0 & |y_2^u v|^2 & 0 \\ 0 & 0 & |y_3^u v|^2 \end{pmatrix}. \tag{2.61}$$

For the charged leptons and this VEV choice we can not mimic a transposed down sector as we had done for $(v, 0, 0)$. We instead assign similarly the e^c as an anti-triplet and assign the $\bar{L}_1, \bar{L}_2, \bar{L}_3$, respectively, as $1_{00}, 1_{20}, 1_{10}$. With this choice we get

$$\begin{aligned} L_Y^e = y_1^d \bar{L}_1 & \left(\Phi_1 e_1^c + \Phi_2 e_2^c + \Phi_3 e_3^c \right)_{00} \\ & + y_2^d \bar{L}_2 \left(\Phi_1 e_1^c + \omega^2 \Phi_2 e_2^c + \omega \Phi_3 e_3^c \right)_{10} \\ & + y_3^d \bar{L}_3 \left(\Phi_1 e_1^c + \omega \Phi_2 e_2^c + \omega^2 \Phi_3 e_3^c \right)_{20} \end{aligned} \tag{2.62}$$

corresponding to Yukawa matrices of the form (LR convention)

$$Y_e = \begin{pmatrix} y_1^e\Phi_1 & y_1^e\Phi_2 & y_1^e\Phi_3 \\ y_2^e\Phi_1 & \omega^2 y_2^e\Phi_2 & \omega y_2^e\Phi_3 \\ y_3^e\Phi_1 & \omega y_3^e\Phi_2 & \omega^2 y_3^e\Phi_3 \end{pmatrix} \tag{2.63}$$

leading to a non-diagonal matrix M_e for the (v, v, v) VEV, but to a diagonal $M_e M_e^\dagger$ combination.

2.3.3 The $(\omega v, v, v)$ case

For $\bar{Q}_1, \bar{Q}_2, \bar{Q}_3$ chosen as, respectively, $1_{00}, 1_{02}, 1_{01}$ (i.e. the same choices as taken above for the $(v, 0, 0)$) the corresponding Yukawa matrices are the same in terms of Φ_i

$$Y_d = \begin{pmatrix} y_1^d\Phi_1 & y_1^d\Phi_2 & y_1^d\Phi_3 \\ y_2^d\Phi_3 & y_2^d\Phi_1 & y_2^d\Phi_2 \\ y_3^d\Phi_2 & y_3^d\Phi_3 & y_3^d\Phi_1 \end{pmatrix}, \tag{2.64}$$

$$Y_u = \begin{pmatrix} y_1^u\tilde{\Phi}_1 & y_1^u\tilde{\Phi}_2 & y_1^u\tilde{\Phi}_3 \\ y_2^u\tilde{\Phi}_2 & y_2^u\tilde{\Phi}_3 & y_2^u\tilde{\Phi}_1 \\ y_3^u\tilde{\Phi}_3 & y_3^u\tilde{\Phi}_1 & y_3^u\tilde{\Phi}_2 \end{pmatrix}. \tag{2.65}$$

Replacing field vector Φ_i with the VEV we get

$$M_d = \frac{1}{\sqrt{2}} \begin{pmatrix} y_1^d\omega v & y_1^d v & y_1^d v \\ y_2^d v & y_2^d\omega v & y_2^d v \\ y_3^d v & y_3^d v & y_3^d\omega v \end{pmatrix}, \tag{2.66}$$

$$M_u = \frac{1}{\sqrt{2}} \begin{pmatrix} y_1^u\omega^2 v & y_1^u v & y_1^u v \\ y_2^u v & y_2^u v & y_2^u\omega^2 v \\ y_3^u v & y_3^u\omega^2 v & y_3^u v \end{pmatrix} \tag{2.67}$$

giving (as in the previous cases) the diagonal products

$$M_d M_d^\dagger = \frac{1}{2} \begin{pmatrix} |y_1^d v|^2 & 0 & 0 \\ 0 & |y_2^d v|^2 & 0 \\ 0 & 0 & |y_3^d v|^2 \end{pmatrix}, \tag{2.68}$$

$$M_u M_u^\dagger = \frac{1}{2} \begin{pmatrix} |y_1^u v|^2 & 0 & 0 \\ 0 & |y_2^u v|^2 & 0 \\ 0 & 0 & |y_3^u v|^2 \end{pmatrix}. \tag{2.69}$$

For charged leptons and for this VEV we choose to assign the e^c as a $\bar{3}$, and the three $\bar{L}_1, \bar{L}_2, \bar{L}_3$, respectively, as singlets $1_{00}, 1_{02}, 1_{01}$. We therefore obtain exactly the same

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structures as for the down quarks

$$Y_e = \begin{pmatrix} y_1^e\Phi_1 & y_1^e\Phi_2 & y_1^e\Phi_3 \\ y_2^e\Phi_3 & y_2^e\Phi_1 & y_2^e\Phi_2 \\ y_3^e\Phi_2 & y_3^e\Phi_3 & y_3^e\Phi_1 \end{pmatrix}, \tag{2.70}$$

$$M_e = \frac{1}{\sqrt{2}} \begin{pmatrix} y_1^e\omega v & y_1^e v & y_1^e v \\ y_2^e v & y_2^e\omega v & y_2^e v \\ y_3^e v & y_3^e v & y_3^e\omega v \end{pmatrix}, \tag{2.71}$$

$$M_e M_e^\dagger = \frac{1}{2} \begin{pmatrix} |y_1^e v|^2 & 0 & 0 \\ 0 & |y_2^e v|^2 & 0 \\ 0 & 0 & |y_3^e v|^2 \end{pmatrix}. \tag{2.72}$$

2.4 Flavour Changing Neutral Currents

In order to see the structure of Flavour Changing Neutral Currents (FCNCs) in this model, let us make the following transformation among the Φ_j fields

$$\Phi' = O \cdot K\ \Phi, \tag{2.73}$$

with the matrices O and K given by

$$O = \begin{pmatrix} \frac{v_1}{v} & \frac{v_2}{v} & \frac{v_3}{v} \\ \frac{v_2}{v'} & -\frac{v_1}{v'} & 0 \\ \frac{v_1}{v''} & \frac{v_2}{v''} & \frac{-(v_1^2+v_2^2)/v_3}{v''} \end{pmatrix}, \qquad K = \begin{pmatrix} e^{-i\alpha_1} & 0 & 0 \\ 0 & e^{-i\alpha_2} & 0 \\ 0 & 0 & e^{-i\alpha_3} \end{pmatrix}, \tag{2.74}$$

where $v = \sqrt{v_1^2 + v_2^2 + v_3^2}$, $v' = \sqrt{v_1^2 + v_2^2}$ and $v'' = \sqrt{v_1^2 + v_2^2 + (v_1^2 + v_2^2)^2/v_3^2}$. The new components of the Φ' doublets are the primed scalar fields, together with G^0 and G^+

$$\begin{pmatrix} h' \\ R \\ R' \end{pmatrix} = O \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \end{pmatrix}, \qquad \begin{pmatrix} G^0 \\ I \\ I' \end{pmatrix} = O \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix}, \qquad \begin{pmatrix} G^+ \\ H_1'^+ \\ H_2'^+ \end{pmatrix} = O \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \\ \phi_3^+ \end{pmatrix}. \tag{2.75}$$

This transformation singles out h' as well as the neutral pseudo-Goldstone boson G^0 and the charged Goldstone boson G^+. The scalar field h' has couplings to the quarks which are proportional to the mass matrices and it is the only scalar field in this basis with triple couplings to a pair of gauge bosons. The other scalar fields only couple to a pair of gauge bosons through quartic couplings. As a result h' could be identified as the SM like Higgs boson if it were already a mass eigenstate. This fact results from the choice of the first row of the matrix O [41, 42], the choice of the other two rows is free provided that they respect the orthogonality relations. Therefore a transformation of this form may lead to many different scalar bases. It should be noticed that what characterises the rotation by the matrix $O \cdot K$ is the fact that in this new scalar basis only the first doublet acquires a VEV, v, different from zero. In these bases, the vacuum is of the form $(v, 0, 0)$, with v real.

In general three Higgs doublet models, h' obtained after this rotation, is not yet a mass eigenstate. In the CP-conserving case, in general, the physical neutral scalars are obtained, after further mixing among h', R and R', as well as mixing between I and I'. In the CP violating case all five neutral fields may mix among themselves. In this scalar basis, after the O rotation, FCNCs arise from the couplings to the remaining four neutral Higgs fields. The structure of the Higgs mediated FCNCs and the charged Higgs couplings to the quark mass eigenstates in models with three Higgs doublets are given by [43]

$$\begin{aligned} L_Y =& -\frac{\sqrt{2}H_1'^+}{v'}\bar{u}^p\left(V\mathcal{N}_d\gamma_R + \mathcal{N}_u^\dagger\, V\gamma_L\right)d^p - \frac{\sqrt{2}H_2'^+}{v''}\bar{u}^p\left(V\mathcal{N}_d'\gamma_R + \mathcal{N}_u'^\dagger\, V\gamma_L\right)d^p \\ &-\frac{h'}{v}\left(\bar{d_L}^p D_d\, d_R^p + \bar{u_L}^p D_u u_R^p\right) - \bar{d_L}^p D_d\, d_R^p - \bar{u_L}^p D_u u_R^p \\ &-\bar{d_L}^p\frac{1}{v'}\,\mathcal{N}_d(R+iI)d_R^p - \bar{u_L}^p\frac{1}{v'}\mathcal{N}_u(R-iI)u_R^p - \\ &-\bar{d_L}^p\frac{1}{v''}\,\mathcal{N}_d'(R'+iI')d_R^p - \bar{u_L}^p\frac{1}{v''}\,\mathcal{N}_u'(R'-iI')u_R^p + \text{h.c.}, \end{aligned} \tag{2.76}$$

where V represents the CKM matrix and we denote the physical mass eigenstates with a superscript p (e.g. d_R^p), and with

$$\mathcal{N}_d = \frac{1}{\sqrt{2}}U_{dL}^\dagger\,(v_2 e^{i\alpha_1}\Gamma_1 - v_1 e^{i\alpha_2}\Gamma_2)\,U_{dR}, \tag{2.77}$$

$$\mathcal{N}_u = \frac{1}{\sqrt{2}}U_{uL}^\dagger(v_2 e^{-i\alpha_1}\Delta_1 - v_1 e^{-i\alpha_2}\Delta_2)\,U_{uR}, \tag{2.78}$$

$$\mathcal{N}_d' = \frac{1}{\sqrt{2}}U_{dL}^\dagger\,(v_1 e^{i\alpha_1}\Gamma_1 + v_2 e^{i\alpha_2}\Gamma_2 + x e^{i\alpha_3}\Gamma_3)\,U_{dR}, \tag{2.79}$$

$$\mathcal{N}_u' = \frac{1}{\sqrt{2}}U_{uL}^\dagger\,(v_1 e^{-i\alpha_1}\Delta_1 + v_2 e^{-i\alpha_2}\Delta_2 + x e^{-i\alpha_3}\Delta_3)\,U_{uR}, \tag{2.80}$$

where $x \equiv -(v_1^2+v_2^2)/v_3$. Note these equations are only valid when $v_3 \neq 0$.

For completeness, let us consider $\mathcal{N}_d$ and $\mathcal{N}_d'$, which can be written as

$$\mathcal{N}_d = \frac{v_2}{v_1}D_d - \frac{v_2}{\sqrt{2}}\left(\frac{v_2}{v_1}+\frac{v_1}{v_2}\right)U_{dL}^\dagger e^{i\alpha_2}\Gamma_2\,U_{dR} - \frac{v_2\,v_3}{v_1\sqrt{2}}U_{dL}^\dagger e^{i\alpha_3}\Gamma_3 U_{dR} \tag{2.81}$$

$$\mathcal{N}_d' = D_d - \frac{v_3 - x}{\sqrt{2}}\,U_{dL}^\dagger e^{i\alpha_3}\Gamma_3\,U_{dR} \tag{2.82}$$

These expressions are general and can be particularised for our model.

In the SM at one loop level $b \to s\gamma$ is described by the Feynman diagrams of figure 1. In models with additional scalars one can in principle replace the W line by neutral scalars and the internal quark line by down type quarks. These diagrams can only exist if there are FCNC mediated by neutral scalars. Likewise, the W can also in principle, be replaced by a charged Higgs whenever the relevant couplings are present leaving the up-type quark in the internal fermion line.

2.4.1 The $(v, 0, 0)$ case

This particular vacuum configuration already corresponds to the Higgs basis, meaning that there is no need to apply the transformation given by eq. (2.73) and that one can identify

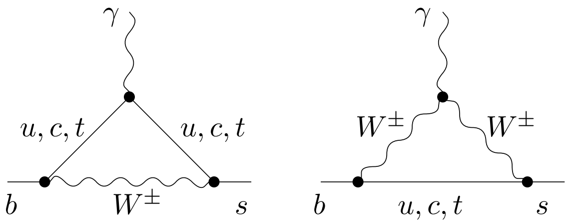


Figure 1. Leading-order Feynman diagrams for $b \to s\gamma$ in the SM.

Φ_1 with the doublet where the SM-like boson appears. In this case, from eq. (2.45), it can be seen that the matrices Γ_1, Γ_2 and Γ_3 are of the form:

$$\Gamma_1 = \begin{pmatrix} y_1^d & 0 & 0 \\ 0 & y_2^d & 0 \\ 0 & 0 & y_3^d \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & y_1^d & 0 \\ 0 & 0 & y_2^d \\ y_3^d & 0 & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} 0 & 0 & y_1^d \\ y_2^d & 0 & 0 \\ 0 & y_3^d & 0. \end{pmatrix}, \tag{2.83}$$

The mass matrix for the down quarks is given by eq. (2.46) and it is already diagonal. Therefore U_{dL} and U_{dR} in eq. (2.42), are the identity matrix and the physical quarks coincide with those in the flavour basis. Note that eqs. (2.79) and (2.80) are ill-defined in this case since $v_3 = 0$. However, both $\mathcal{N}_d$ and $\mathcal{N}_d'$ can be read directly from eq. (2.40) leading to the couplings $-\bar{d_L}^p\ \Gamma_2(R+iI)d_R^p$ and $-\bar{d_L}^p\ \Gamma_3(R'+iI')d_R^p$. Since the diagonal entries of Γ_2 and Γ_3 are zero only the d quark can be in the internal quark line of the diagrams of figure 1 with W replaced by the neutral scalars R, I, R' and I', therefore, leading to a strong suppression by its mass square divided by the square of the mass of the heavy neutral scalars.

For the up quark sector from eq. (2.48) we have:

$$\Delta_1 = \begin{pmatrix} y_1^u & 0 & 0 \\ 0 & 0 & y_2^u \\ 0 & y_3^u & 0 \end{pmatrix}, \quad \Delta_2 = \begin{pmatrix} 0 & y_1^u & 0 \\ y_2^u & 0 & 0 \\ 0 & 0 & y_3^u \end{pmatrix}, \quad \Delta_3 = \begin{pmatrix} 0 & 0 & y_1^u \\ 0 & y_2^u & 0 \\ y_3^u & 0 & 0 \end{pmatrix}, \tag{2.84}$$

The mass matrix for the up quarks is given by eq. (2.49). In this case both $\mathcal{M}_u\mathcal{M}_u^\dagger$ and $\mathcal{M}_u^\dagger\mathcal{M}_u$ are diagonal. In order for $\mathcal{M}_u$ to also become diagonal we need U_{uR} of the form:

$$U_{uR} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{2.85}$$

now we have from eq. (2.40), for the up sector:

$$L_Y^u = -\overline{Q_L^p}\ \Delta_1\ U_{uR}\ \tilde{\Phi}_1 u_R^p - \overline{Q_L^p}\ \Delta_2\ U_{uR}\ \tilde{\Phi}_2 u_R^p\ - \overline{Q_L^p}\ \Delta_3\ U_{uR}\ \tilde{\Phi}_3 u_R^p\ + \text{h.c.} \tag{2.86}$$

leading to the couplings $-\bar{u_L}^p\ \Delta_2 U_{uR}\ (R+iI)u_R^p$ and $-\bar{u_L}^p\ \Delta_3 U_{uR}\ (R'+iI')u_R^p$, with:

$$\Delta_2 U_{uR} = \begin{pmatrix} 0 & 0 & y_1^u \\ y_2^u & 0 & 0 \\ 0 & y_3^u & 0 \end{pmatrix}, \quad \Delta_3 U_{uR} = \begin{pmatrix} 0 & y_1^u & 0 \\ 0 & 0 & y_2^u \\ y_3^u & 0 & 0 \end{pmatrix}. \tag{2.87}$$

For the charged Higgs couplings we have:

$$L_Y^{\pm} = -\sqrt{2} H_1'^{+} \bar{u}^p \left(V \Gamma_2 \gamma_R + (\Delta_2 U_{uR})^\dagger \; V \gamma_L \right) d^p \\ -\sqrt{2} H_2'^{+} \bar{u}^p \left(V \Gamma_3 \gamma_R + (\Delta_3 U_{uR})^\dagger \; V \gamma_L \right) d^p + +\text{h.c.} \tag{2.88}$$

The fact that in this case all four matrices Γ_2, $(\Delta_2 U_{uR})^\dagger$, Γ_3 and $(\Delta_3 U_{uR})^\dagger$ have zero diagonal entries implies that in the approximation of V being the identity matrix the one loop diagrams of figure1 with W^+ replaced by a charged Higgs scalar would require a generation gap in each vertex. Therefore only the quark u could be in the internal quark line, leading to a very strong suppression due to its small mass when compared to the mass of the charged Higgs.

2.4.2 The (v, v, v) case

In this case the Yukawa matrices in the down sector have the structure given by eq. (2.55) with the Γ matrices of the form

$$\Gamma_1 = \begin{pmatrix} y_1^d & 0 & 0 \\ y_2^d & 0 & 0 \\ y_3^d & 0 & 0 \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & y_1^d & 0 \\ 0 & \omega^2 y_2^d & 0 \\ 0 & \omega y_3^d & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} 0 & 0 & y_1^d \\ 0 & 0 & \omega y_2^d \\ 0 & 0 & \omega^2 y_3 \end{pmatrix} \tag{2.89}$$

and similarly, from eq. (2.59), for the matrices Δ with the superscripts d replaced by u. In this case the diagonalisation equation is

$$U_{dL}^\dagger \; M_d \; U_{dR} = \mathcal{D}_d \tag{2.90}$$

with

$$U_{dL} = \mathbb{1}, \quad U_{dR} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}. \tag{2.91}$$

From eqs. (2.77) and (2.79) one obtains:

$$\mathcal{N}_d = \frac{1}{\sqrt{2}} v \left(\Gamma_1 - \Gamma_2 \right) U_{dR} = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{3}} \begin{pmatrix} 0 & y_1^d - y_1^d w & y_1^d - y_1^d w^2 \\ y_2^d - y_2^d w^2 & 0 & y_2^d - y_2^d w \\ y_3^d - y_3^d w & y_3^d - y_3^d w^2 & 0 \end{pmatrix} \tag{2.92}$$

$$\mathcal{N}_d' = \frac{1}{\sqrt{2}} v \left(\Gamma_1 + \Gamma_2 - 2\Gamma_3 \right) U_{dR} = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{3}} \begin{pmatrix} 0 & -3 y_1^d w^2 & -3 y_1^d w \\ -3 y_2^d w & 0 & -3 y_2^d w^2 \\ -3 y_3^d w^2 & -3 y_3^d w & 0 \end{pmatrix} \tag{2.93}$$

and similarly for the up sector with the index d replaced by u. It can be readily verified that in the up sector there is also cancellation of the diagonal terms whilst the non-diagonal terms are in general different from zero. This means that there are FCNCs mediated by R, I, R' and I' but there are no flavour diagonal couplings to these scalars. As a result $b \to s\gamma$ can only be mediated by neutral Higgs R, I, R' and I' (which we choose to be heavier than the SM-like one) via diagrams with two flavour changing neutral vertices. The quark inside the loop will have to be the down quark and a suppression factor of its mass square divided by the square of the mass of the heavy scalar will occur just as in the $(v,0,0)$ case. Notice that R, I, R' and I' may not yet be the physical fields but h' already is and therefore does not mix with them.

The fact that in this case the diagonal entries of $\mathcal{N}_d$, $\mathcal{N}_u$, $\mathcal{N}'_d$ and $\mathcal{N}'_u$ are zero also implies that in the approximation of V being the identity matrix the one loop diagram of figure 1 with W^+ replaced by a charged Higgs scalar would require a generation gap in each vertex. Therefore only the quark u could be in the internal quark line, leading to a very strong suppression due to its small mass when compared to the mass of the charged Higgs.

2.4.3 The $(\omega v, v, v)$ case

In this case the matrices Γ_1, Γ_2 and Γ_3 can be read from eq. (2.64) and are of the form

$$\Gamma_1 = \begin{pmatrix} y_1^d & 0 & 0 \\ 0 & y_2^d & 0 \\ 0 & 0 & y_3 \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & y_1^d & 0 \\ 0 & 0 & y_2^d \\ y_3^d & 0 & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} 0 & 0 & y_1^d \\ y_2^d & 0 & 0 \\ 0 & y_3^d & 0 \end{pmatrix}. \tag{2.94}$$

Now, the diagonalisation equation of the corresponding down quark mass matrix, written in eq. (2.66), requires:

$$U_{dL} = \mathbb{1}, \quad U_{dR} = \frac{1}{\sqrt{3}} \begin{pmatrix} \omega^2 & 1 & 1 \\ 1 & \omega^2 & 1 \\ 1 & 1 & \omega^2 \end{pmatrix}. \tag{2.95}$$

In this case we have for $\mathcal{N}_d$

$$\begin{aligned} \mathcal{N}_d &= \frac{1}{\sqrt{2}} v \left(\omega\Gamma_1 - \Gamma_2\right) U_{dR} \\ &= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{3}} v \begin{pmatrix} 0 & y_1^d\omega - y_1^d\omega^2 & y_1^d\omega - y_1^d \\ y_2^d\omega - y_2^d & 0 & y_2^d\omega - y_2^d\omega^2 \\ y_3^d\omega - y_3^d\omega^2 & y_3^d\omega - y_3^d & 0 \end{pmatrix}. \end{aligned} \tag{2.96}$$

In what concerns $\mathcal{N}'_d$ we now have

$$\begin{aligned} \mathcal{N}'_d &= \frac{1}{\sqrt{2}} v \left(\omega\Gamma_1 + \Gamma_2 - 2\Gamma_3\right) U_{dR} \\ &= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{3}} v \begin{pmatrix} 0 & -3y_1^d & -3\omega^2 y_1^d \\ -3\omega^2 y_2^d & 0 & -3y_2^d \\ -3y_3^d & -3\omega^2 y_3^d & 0 \end{pmatrix}. \end{aligned} \tag{2.97}$$

In these matrices there are no flavour diagonal couplings and therefore R, I, R' and I' can only mediate $b \to s\gamma$ transitions with two flavour changing neutral vertices as in the previous cases.

For the up quark sector, from eq. (2.65), we have:

$$\Delta_1 = \begin{pmatrix} y_1^u & 0 & 0 \\ 0 & 0 & y_2^u \\ 0 & y_3^u & 0 \end{pmatrix} \quad ; \quad \Delta_2 = \begin{pmatrix} 0 & y_1^u & 0 \\ y_2^u & 0 & 0 \\ 0 & 0 & y_3^u \end{pmatrix} \quad ; \quad \Delta_3 = \begin{pmatrix} 0 & 0 & y_1^u \\ 0 & y_2^u & 0 \\ y_3^u & 0 & 0 \end{pmatrix}. \tag{2.98}$$

Now, the diagonalisation equation of the corresponding up quark mass matrix, written in eq. (2.67), requires:

$$U_{uL} = \mathbb{1} \quad , \quad U_{uR} = \frac{1}{\sqrt{3}} \begin{pmatrix} \omega & 1 & 1 \\ 1 & 1 & \omega \\ 1 & \omega & 1 \end{pmatrix} \tag{2.99}$$

and $\mathcal{N}_u$ and $\mathcal{N}_u'$ are of the form

$$\begin{aligned} \mathcal{N}_u &= \frac{1}{\sqrt{2}} v \left(\omega^2 \Delta_1 - \Delta_2\right) U_{dR} \\ &= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{3}} v \begin{pmatrix} 0 & y_1^u \omega^2 - y_1^u & y_1^u \omega^2 - y_1^u \omega \\ y_2^u \omega^2 - y_2^u \omega & 0 & y_2^u \omega^2 - y_2^u \\ y_3^u \omega^2 - y_3^u & y_3^u \omega^2 - y_3^u \omega & 0 \end{pmatrix}, \end{aligned} \tag{2.100}$$

$$\begin{aligned} \mathcal{N}_u' &= \frac{1}{\sqrt{2}} v \left(\omega^2 \Delta_1 + \Delta_2 - 2\Delta_3\right) U_{dR} \\ &= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{3}} v \begin{pmatrix} 0 & -3y_1^u \omega & -3y_1^u \\ -3y_2^u & 0 & -3y_2^u \omega \\ -3y_3^u \omega & -3y_3^u & 0 \end{pmatrix}. \end{aligned} \tag{2.101}$$

There are several similarities in the structure of $\mathcal{N}_u$, $\mathcal{N}_u'$ and of $\mathcal{N}_d$ and $\mathcal{N}_d'$. In all cases the diagonal entries are zero and the implications for $b \to s\gamma$ are similar to those of the two vacua discussed previously.

3 Phenomenology of the model

With the minima and respective Yukawa couplings described in the previous section, we now proceed with a phenomenological analysis of each case. For each vacuum choice we scan over 3 independent masses while fixing the SM-like Higgs boson mass m_h to 125.25 GeV [44]. For numerical analysis we use a `SARAH` generated `SPheno` [45, 46] spectrum generator to compute STU parameters [47–49] and unitarity limit [50]. In the case of STU parameters we look for a region of masses within 3σ from best fit values of [44]

$$S = -0.01 \pm 0.10 \tag{3.1}$$
$$T = 0.03 \pm 0.12 \tag{3.2}$$
$$U = 0.02 \pm 0.11 \tag{3.3}$$

While checking if unitarity limit is fulfilled we do take into account also finite scattering energy contributions, i.e. also contributions to scattering amplitudes from trilinear couplings (see appendix B for details regarding `SPheno` setting). Finally, the Higgs sector is checked against experimental constraints using `HiggsBounds` *v5.10.2* [51], `HiggsSignals` *v2.6.2* [52] and also directly against experimental results if the above mentioned codes do not include a relevant analysis.

Because neither the module computing low energy observables nor the one computing unitarity constraints work with complex VEVs, in case of the $(\omega v, v, v)$ vacuum we use a rephasing freedom and analyse an equivalent case of a (v, v, v) vacuum with the replacement $\Phi_1 \to \omega\Phi_1$.[3]

Finally, we note that the point of view of figures shown below differs from case to case and was chosen to better show non-trivial features of allowed parameter space regions. In all cases charged Higgs bosons are mass degenerate (see section 2) and their common mass $m_{H_1^\pm} = m_{H_2^\pm}$ is given on the vertical axis. Horizontally, we show the 2 independent masses describing masses of the 4 remaining, pairwise degenerate, neutral states.

3.1 The $(v, 0, 0)$ case

In figure 2 we show masses allowed by the T-parameter and the unitarity constraint for the $(v, 0, 0)$ vacuum. In this case pseudoscalars A_1 and A_2 are, pairwise, mass degenerate with H_1 and H_2, i.e. $m_{H_1} = m_{A_1}$, $m_{H_2} = m_{A_2}$. Those masses are given in the horizontal planes of plots. S and U parameters are not constraining at all within the shown range of masses. This will also be the case for the remaining vacua.

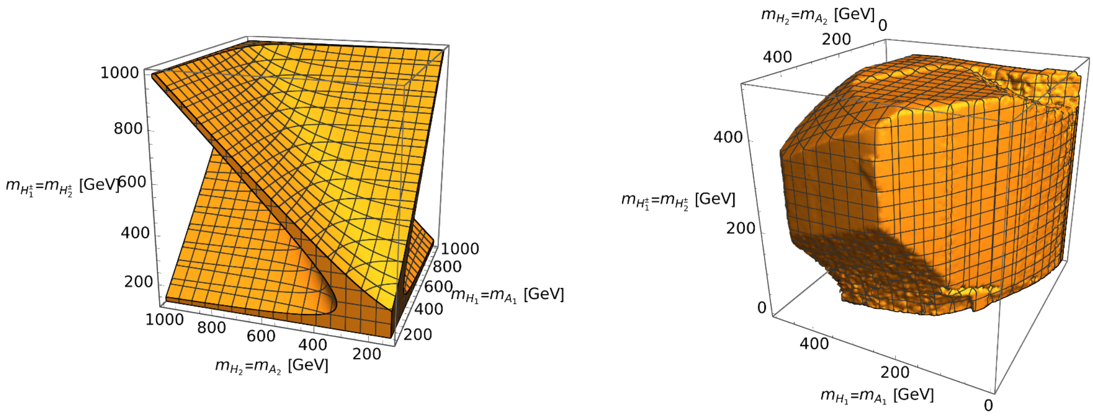
As seen in figure 2a the T-parameter forces either $m_{H_1} = m_{A_1} \approx m_{H^\pm}$ or $m_{H_2} = m_{A_2} \approx m_{H^\pm}$ or both. This is similar as in the case of the 2HDM as shown for example in [53]. Meanwhile, the unitarity constraints lead to the conclusion that the masses of $H^\pm$ and neutral Higgses must be limited to $\lesssim 500\,\text{GeV}$.

As far as direct experimental limits are concerned, non-SM Higgses decay to pairs consisting of an another Higgs and a gauge boson, or to fermions in the flavour-violating manner (see appendix C for an example decay pattern). Limits on neutral beyond SM Higgses are avoided because of both their non-standard decay patterns and because the leading production channels of H's and A's here are $\bar{b}q$ + h.c., where $q = d, s$ and not the gg fusion. Vector boson fusion (VBF) and Higgsstrahlung is also forbidden as beyond the SM Higgses do not have VEVs and therefore lack triple VVH couplings. Therefore typical limits, like the observed limit on the $\text{BR}(H \to e\mu) < 6.1 \cdot 10^{-5}$ from [54], do not apply as they assume SM-like Higgs boson with mass of 125 GeV and SM production channels. This allows them to evade experimental limits. No points are excluded by `HiggsBounds`, while `HiggsSignals` only requires that $\min(m_{H_1}, m_{H_2}, m_{H^\pm}) > m_h/2$ as otherwise 2 body decays of a SM-like Higgs to neutral or charged (or both) Higgses are open.[4] The example point fulfilling all constrains is $m_{H_1} = m_{A_1} = 134.2\,\text{GeV}$, $m_{H_2} = m_{A_2} = 139.3\,\text{GeV}$ and $m_{H^\pm} = 165.6\,\text{GeV}$, giving $T = 0.03$ and passing the unitarity check.

With the `SARAH` interface to `HiggsBounds` the LEP production cross sections of charged Higgses are not considered, only their partial widths (see comments in [55]). `HiggsSignals` puts a limit of $m_H^\pm \gtrsim 95\,\text{GeV}$. This broadly coincides with the LEP limit from direct H^+H^- production [56]. We emphasize that present limits on charged Higgs boson masses coming

[3] To the `arXiv` version of this work we attach the rephased (and not the original) model.

[4] We point out that due to a problem with tagging VEV-less fields as 'Higgses' in `SARAH`, in the case of $(v, 0, 0)$ vacuum fields H_i are not checked against experimental limits in `HiggsBounds` and `HiggsSignals`. There are checked in the remaining cases though. Since in those cases the only constraints on H_i come from them being a decay products of h (which is already taken into account in the $(v, 0, 0)$ case) and H_i decay patters do not differ between cases, we conclude that checking H_i against `HiggsSignals` and `HiggsBounds` in the $(v, 0, 0)$ case would not lead to any new constraints.



(a) T.

(b) Unitarity.

Figure 2. Regions allowed by the T-parameter at 99% level (a) and unitarity (b) for the $(v, 0, 0)$ vacuum in the $m_{H_1} = m_{A_1}$ *vs.* $m_{H_2} = m_{A_2}$ *vs.* $m_{H_1^\pm} = m_{H_2^\pm}$ space. The 4th free parameter, the mass of the SM-like Higgs m_h, is set to 125.25 GeV. On the left panel we additionally assume that all masses are larger than m_h while on the right one that they are larger than 1 GeV.

from the LHC analyses do not apply in our case. This is because the experimental searches were based on the assumption that the charged Higgs boson can couple to fermions of the same generation (like in supersymmetric or two-Higgs doublet models), see e.g. analyses for decay channels: $t \to bH^+ \to bc\bar{s}$ [57], $t \to bH^+ \to bW^+A$ with a subsequent flavour diagonal decay $A \to \mu^+\mu^-$ [58], $t \to H^+b \to b\bar{b}c$ [59] or $H^+ \to t\bar{b}$ [60]. In our case the charged Higgs boson couples to fermions from different generations giving rise to vastly different kinematic distributions even if the same final state could be produced. For example the final state in $t \to bc\bar{s}$ as in [57] can be produced in our model, however with the different decay chain $t \to sH^+ \to sc\bar{b}$, which means that the signal of H^+ should be looked at the invariant mass of $c\bar{b}$ jets instead of $c\bar{s}$. A proper reanalysis of the experimental data might provide limits applicable to our model.

3.2 The (v, v, v) case

In this case the scalar and pseudoscalar Higgses have, separately, common masses: $m_{H_1} = m_{H_2}$ and $m_{A_1} = m_{A_2}$. The T-parameter allowed region in figure 3a has the same features as the one in figure 2a, giving analogous relations between masses of $SU(2)_L$ doublet components. Unitarity also puts a similar limit on the masses requiring them to be smaller than about 500 GeV (figure 3b). No points are excluded by `HiggsBounds` and, similarly as before, `HiggsSignals` limits masses to $\min(m_{H_1}, m_{A_1}) > m_h/2$ and $m_{H^\pm} \gtrsim 95$ GeV. The same arguments hold as in the case of $(v, 0, 0)$, including lack of VBF and Higgsstrahlung processes. The lack of VBF and Higgsstrahlung processes for (v, v, v) case follows from the fact, as explained in section 2.4, that the SM-like Higgs does not change between the Higgs basis and mass eigenstates meaning that non-SM Higgses do not possess a VEV — the same as in the $(v, 0, 0)$ case.

The example point fulfilling unitarity and T-parameter constraint ($T = 0.03$) is $m_H = 198.6$ GeV, $m_A = 202.4$ GeV, $m_{H^\pm} = 172.3$ GeV.

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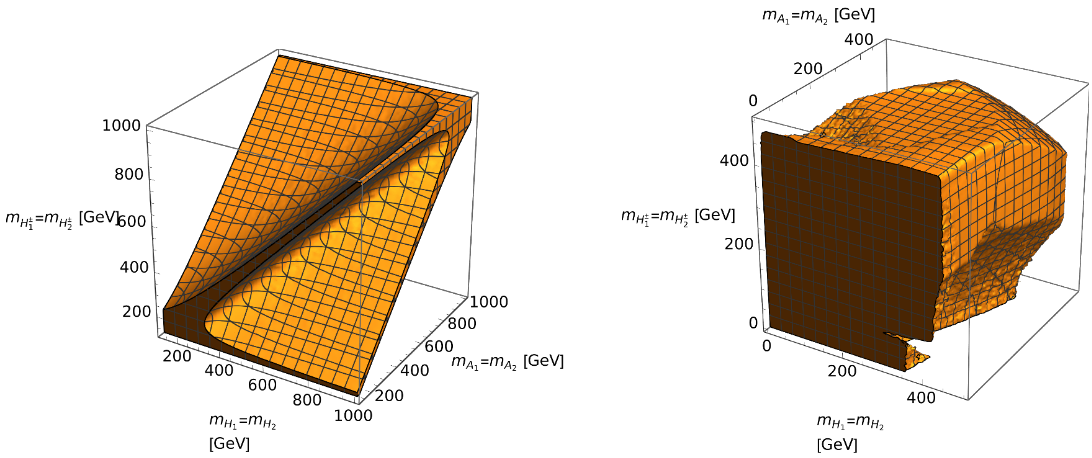
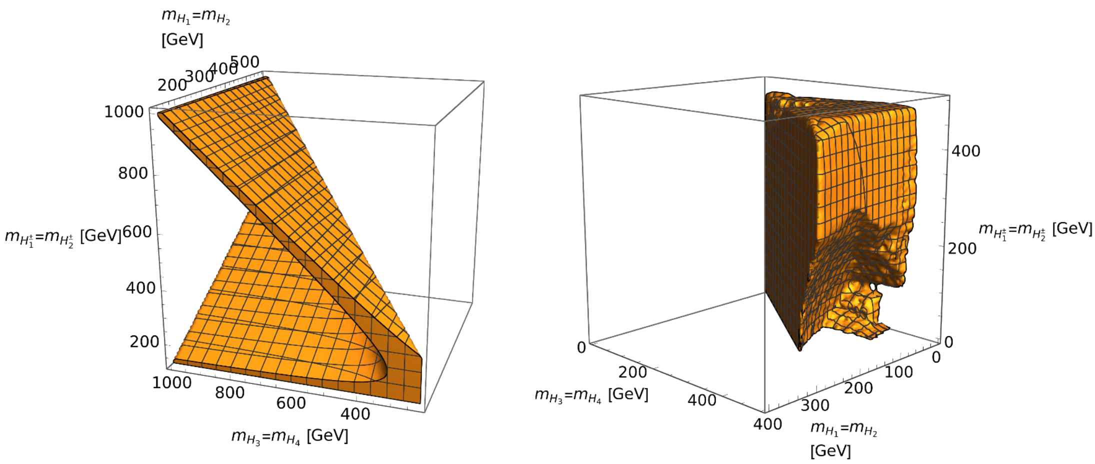


(a) T. (b) Unitarity.

Figure 3. Regions allowed by the T-parameter at 99% level (a) and unitarity (b) for the (v, v, v) vacuum in the $m_{H_1} = m_{H_2}$ *vs.* $m_{A_1} = m_{A_2}$ *vs.* $m_{H_1^\pm} = m_{H_2^\pm}$ space. The 4th free parameter, the mass of the SM-like Higgs m_h, is set to 125.25 GeV. On the left panel we additionally assume that all masses are larger than m_h while on the right one that they are larger than 1 GeV.



(a) T. (b) Unitarity.

Figure 4. Regions allowed by the T-parameter at 99% level (a) and unitarity (b) for the $(\omega v, v, v)$ vacuum in the $m_{H_1} = m_{H_2}$ *vs.* $m_{H_3} = m_{H_4}$ *vs.* $m_{H_1^\pm} = m_{H_2^\pm}$ space. The 4th free parameter, the mass of the SM-like Higgs m_h, is set to 125.25 GeV. On the left panel we additionally assume that all masses are larger than m_h while on the right one that they are larger than 1 GeV.

3.3 The $(\omega v, v, v)$ case

In this case CP is ill-defined so we label all neutral mass eigenstates beyond the SM-like state h as H_i $(i = 1 \ldots 4)$. We find pairwise mass degeneracies, with H_1, H_2 sharing one mass and H_3 and H_4 sharing another one. As explained in section 2.2.4 there is a double degeneracy in solution for potential parameters for a given set of masses. Qualitatively, the allowed region of masses is the same in both cases therefore in figure 4 we only plot one of the solutions. Most of the features of figure 4 where already present in figures 2 and 3 with the distinctive feature in the current case being mainly the condition $m_{H_3} > \sqrt{3}m_{H_1}$. Still, just as in previous cases (see again the discussion in section 2.4), there are no flavour-diagonal couplings of non-SM Higgses and hence the experimental limits on them are avoided.

An example point passing all phenomenological constraints is: $m_{H_1} = m_{H_2} = 158.6\,\mathrm{GeV}$, $m_{H_3} = m_{H_4} = 314.3\,\mathrm{GeV}$, $m_{H^\pm} = 149.7\,\mathrm{GeV}$ with $T = 0.03$, fulfilled unitarity bound and not excluded by `HiggsBounds` and `HiggsSignals`.

3.4 Comment on unitarity

As seen from the discussion in this section, the unitarity provides a very strong constraint, limiting all masses to $\lesssim 500\,\mathrm{GeV}$ irrespectively of the chosen vacuum. This is relatively easy to understand. As masses increase, so must the absolute values of couplings as the mass scale is fixed by the EWSB scale. Qualitatively, regions allowed by unitarity are roughly given by $max\{|s|, |r_1|, |r_2|, |d|\} \lesssim \pi$. We emphasise though that in our analysis we consider also finite energy contributions to the scattering matrix, making it sensitive also to 3-point interactions originating from eq. (2.2) after the electroweak symmetry breaking.

4 Conclusions

We have described an extension of the Standard Model with 3 Higgs doublets and the potential invariant under the $\Delta(27)$ symmetry. After covering the known tree-level minima of this potential, we gave formulas for the physical parameters (masses) in terms of the potential parameters and then proceed to constrain the parameter space of the model through STU, unitarity as well as certain flavour violation processes. We concluded that unitarity provides a rather strong constraint and keeps the scalar masses below around 500 GeV. We considered assignments of the fermions under the $\Delta(27)$ symmetry that give Yukawa structures leading to distinct masses for each generation and the unit CKM matrix (which we consider a reasonable leading order approximation). Based on that toy model of Yukawa couplings, we were able to test contributions of the additional scalars to flavour violating processes. Using $b \to s\gamma$ as an example, we concluded that flavour violating processes are tamed by the imposed discrete symmetry. The Higgs mediated neutral flavour changing contributions are suppressed by ratios of the mass of the down-type quark in the loop to the mass of the heavy Higgs. In our model, whenever such contributions are present, two flavour changing neutral vertices are needed implying that the quark in the loop is the down quark, which is extremely light compared to the mass of the heavy Higgs.

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A Group theory of $\boldsymbol{\Delta(27)}$

$\Delta(27)$ is a discrete subgroup of SU(3) with two complex 3-dimensional and 9 distinct 1-dimensional irreducible representations. Here we label one of the 3-dimensional ones as the anti-triplet $\bar{\mathbf{3}}$, the other as the triplet $\mathbf{3}$, and label the singlets as $\mathbf{1}_{k,l}$ $(k, l = 0, 1, 2)$ — where the labels k, l denote the transformation properties under the order 3 generators c, d of the group ($c^3 = 1$ and $d^3 = 1$). The generators for 1-dimensional representations are simply the phases (powers of $\omega \equiv e^{i\frac{2\pi}{3}}$). In a convenient basis, which we use throughout, for the triplet and anti-triplet representation we have

$$c_{3,\bar{3}} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \tag{A.1}$$

$$d_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \tag{A.2}$$

$$d_{\bar{3}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \tag{A.3}$$

With this notation, as the generators of the group are of order 3, the trivial singlet is obtained from products of singlets where the labels add up to zero modulo 3. The product of a triplet and anti-triplet results in all nine singlets whereas the product of two anti-triplets yields three triplets and vice-versa as follows

$$\begin{aligned}
\mathbf{3} \otimes \mathbf{3} &= \bar{\mathbf{3}}_{S_1} \oplus \bar{\mathbf{3}}_{S_2} \oplus \bar{\mathbf{3}}_A \\
\bar{\mathbf{3}} \otimes \bar{\mathbf{3}} &= \mathbf{3}_{S_1} \oplus \mathbf{3}_{S_2} \oplus \mathbf{3}_A \\
\mathbf{3} \otimes \bar{\mathbf{3}} &= \sum_{r=0}^{2} \mathbf{1}_{r,0} \oplus \sum_{r=0}^{2} \mathbf{1}_{r,1} \oplus \sum_{r=0}^{2} \mathbf{1}_{r,2} \\
\mathbf{1}_{k,\ell} \otimes \mathbf{1}_{k',\ell'} &= \mathbf{1}_{k+k' \ mod\ 3, \ell+\ell' \ mod\ 3}
\end{aligned} \tag{A.4}$$

The specific composition rules for the (anti-)triplet products are

$$
\begin{aligned}
(\mathbf{3}\otimes\mathbf{3})_{\bar{\mathbf{3}}_{S_1}} &= (x_1y_1, x_2y_2, x_3y_3)\,, \\
(\mathbf{3}\otimes\mathbf{3})_{\bar{\mathbf{3}}_{S_2}} &= \frac{1}{2}\,(x_2y_3 + x_3y_2, x_3y_1 + x_1y_3, x_1y_2 + x_2y_1)\,, \\
(\mathbf{3}\otimes\mathbf{3})_{\bar{\mathbf{3}}_{A}} &= \frac{1}{2}\,(x_2y_3 - x_3y_2, x_3y_1 - x_1y_3, x_1y_2 - x_2y_1)\,, \\
(\mathbf{3}\otimes\bar{\mathbf{3}})_{\mathbf{1}_{r,0}} &= x_1y_1 + \omega^{2r}x_2y_2 + \omega^{r}x_3y_3, \\
(\mathbf{3}\otimes\bar{\mathbf{3}})_{\mathbf{1}_{r,1}} &= x_1y_2 + \omega^{2r}x_2y_3 + \omega^{r}x_3y_1, \\
(\mathbf{3}\otimes\bar{\mathbf{3}})_{\mathbf{1}_{r,2}} &= x_1y_3 + \omega^{2r}x_2y_1 + \omega^{r}x_3y_2,
\end{aligned}
\tag{A.5}
$$

where $r = 0, 1, 2$, and (x_1, x_2, x_3) and (y_1, y_2, y_3) are the components of the (anti-)triplets in the product. Care should be taken in the product of a triplet and anti-triplet as the ordering is relevant. The subscript for the product of a triplet and an anti-triplet resulting in a singlet specifies which of the singlets it transforms as, whereas the product of two triplets (or two anti-triplets) results in distinct anti-triplets (or triplets), where the subscripts denote if it is a symmetric product (S_1, S_2) or an anti-symmetric one (A).

B `SPheno` setup

For numerical analysis we use `SPheno` in the `OnlyLowEnergySPheno` setup. Higgs boson masses are computed purely at the tree level and parameter points are specified in terms of physical Higgs masses, with scalar potential parameters extracted using eqs. (2.11)–(2.14), (2.21)–(2.24) or (2.36)–(2.39). We equate 3HDM VEVs (with obvious numerical prefactors) to the SM VEV fixed to $v_{\mathrm{SM}} \equiv \left(\sqrt{2}G_F\right)^{-1/2}$, with $G_F = 1.1663787{\cdot}10^{-5}\,\mathrm{GeV}^{-2}$. No RGE running is performed, so through calculation of all observables (unitarity, STU, etc.) the same values of s, r_1, r_2 and d are used. There is however a small difference in gauge and Yukawa couplings used in different parts of the code due to how `SPheno` extracts them from SM input parameters.

We use the following settings in `SPheno` to check the unitarity constraints

```
BLOCK SPhenoInput
  440 1                 # Tree-level unitarity constraints (limit s->infinity)
  441 1                 # Full tree-level unitarity constraints
  442 1.                # sqrt(s_min)
  443 5000.             # sqrt(s_max)
  444 -1000             # steps
  445 0                 # running
  445 2                 # Cut-Level for T/U poles
```

Technically, as for the (finite) maximal energy of 5 TeV, occasionally the maximal scattering eigenvalue $\lambda_{\max}^{\text{finite }S} < \lambda_{\max}^{S\to\inf}$ we plot the region where $max(\lambda_{\max}^{\text{finite }S}, \lambda_{\max}^{S\to\inf}) < \frac{1}{2}$. Here $\lambda_{\max}^{\text{finite }S}$ ($\lambda_{\max}^{S\to\inf}$) is the eigenvalue as given in block `TREELEVELUNITARITYwTRILINEARS` (`TREELEVELUNITARITY`) of `SPheno`'s `SLHA` [50, 61, 62] output.

C Example decay patterns of non-SM Higgs bosons

Example of decay patters of non-SM Higgses for the $(v,0,0)$ vacuum and $m_{H_1} = 200\,\mathrm{GeV}$, $m_{H_2} = 300\,\mathrm{GeV}$ and $m_{H_2^\pm} = 400\,\mathrm{GeV}$ as given by the created 3HDM `SPheno` spectrum generator. In the extract of the `SLHA` output below, `H0_2`, `A0_2`, `Hp_2` correspond to H_2, A_1 and $H_1^\pm$, respectively. For other particles standard `SPheno` symbols and indexing are used.

```
DECAY         45      7.97690384E+00     # H0_2
#     BR                NDA       ID1       ID2
      1.29114182E-02     2          36          23     # BR(H0_2 -> A0_2 VZ )
      3.23331057E-04     2          -1           5     # BR(H0_2 -> Fd_1^* Fd_3 )
      3.23497669E-04     2          -3           5     # BR(H0_2 -> Fd_2^* Fd_3 )
      3.23331057E-04     2          -5           1     # BR(H0_2 -> Fd_3^* Fd_1 )
      3.23497669E-04     2          -5           3     # BR(H0_2 -> Fd_3^* Fd_2 )
      2.46405648E-01     2          -2           6     # BR(H0_2 -> Fu_1^* Fu_3 )
      2.46424262E-01     2          -4           6     # BR(H0_2 -> Fu_2^* Fu_3 )
      2.46405648E-01     2          -6           2     # BR(H0_2 -> Fu_3^* Fu_1 )
      2.46424262E-01     2          -6           4     # BR(H0_2 -> Fu_3^* Fu_2 )
DECAY         36      7.60203352E-01     # A0_2
#     BR                NDA       ID1       ID2
      9.65338080E-04     2          -1           5     # BR(A0_2 -> Fd_1^* Fd_3 )
      9.65695420E-04     2          -3           5     # BR(A0_2 -> Fd_2^* Fd_3 )
      9.65338080E-04     2          -5           1     # BR(A0_2 -> Fd_3^* Fd_1 )
      9.65695420E-04     2          -5           3     # BR(A0_2 -> Fd_3^* Fd_2 )
      1.36165292E-04     2         -11          15     # BR(A0_2 -> Fe_1^* Fe_3 )
      1.36636215E-04     2         -13          15     # BR(A0_2 -> Fe_2^* Fe_3 )
      1.36165292E-04     2         -15          11     # BR(A0_2 -> Fe_3^* Fe_1 )
      1.36636215E-04     2         -15          13     # BR(A0_2 -> Fe_3^* Fe_2 )
      2.49066556E-01     2          -2           6     # BR(A0_2 -> Fu_1^* Fu_3 )
      2.48685848E-01     2          -4           6     # BR(A0_2 -> Fu_2^* Fu_3 )
      2.49066556E-01     2          -6           2     # BR(A0_2 -> Fu_3^* Fu_1 )
      2.48685848E-01     2          -6           4     # BR(A0_2 -> Fu_3^* Fu_2 )
DECAY         37      3.06760690E+01     # Hp_2
#     BR                NDA       ID1       ID2
      1.11313431E-01     2          36          24     # BR(Hp_2 -> A0_2 VWp )
      6.11469833E-03     2          46          24     # BR(Hp_2 -> A0_3 VWp )
      2.96841032E-04     2          -1           6     # BR(Hp_2 -> Fd_1^* Fu_3 )
      7.64780553E-01     2          -5           4     # BR(Hp_2 -> Fd_3^* Fu_2 )
      1.11313431E-01     2          35          24     # BR(Hp_2 -> H0_1 VWp )
      6.11469833E-03     2          45          24     # BR(Hp_2 -> H0_2 VWp )
```

The entirety of the above output is attached to the `arXiv` version of this work.

References

[1] J.F. Gunion, H.E. Haber, G.L. Kane and S. Dawson, *The Higgs hunter's guide*, *Front. Phys.* **80** (2000) 1 [INSPIRE].

[2] G.C. Branco, P.M. Ferreira, L. Lavoura, M.N. Rebelo, M. Sher and J.P. Silva, *Theory and phenomenology of two-Higgs-doublet models*, *Phys. Rept.* **516** (2012) 1 [arXiv:1106.0034] [INSPIRE].

- [3] I.P. Ivanov and E. Vdovin, *Classification of finite reparametrization symmetry groups in the three-Higgs-doublet model*, Eur. Phys. J. C **73** (2013) 2309 [arXiv:1210.6553] [INSPIRE].
- [4] N. Darvishi and A. Pilaftsis, *Classifying accidental symmetries in multi-Higgs doublet models*, Phys. Rev. D **101** (2020) 095008 [arXiv:1912.00887] [INSPIRE].
- [5] N. Darvishi, M.R. Masouminia and A. Pilaftsis, *Maximally symmetric three-Higgs-doublet model*, Phys. Rev. D **104** (2021) 115017 [arXiv:2106.03159] [INSPIRE].
- [6] R. de Adelhart Toorop, F. Bazzocchi, L. Merlo and A. Paris, *Constraining flavour symmetries at the EW scale. Part I. The A_4 Higgs potential*, JHEP **03** (2011) 035 [arXiv:1012.1791] [*Erratum ibid.* **01** (2013) 098] [INSPIRE].
- [7] R. de Adelhart Toorop, F. Bazzocchi, L. Merlo and A. Paris, *Constraining flavour symmetries at the EW scale. Part II. The fermion processes*, JHEP **03** (2011) 040 [arXiv:1012.2091] [INSPIRE].
- [8] G.C. Branco, J.M. Gerard and W. Grimus, *Geometrical T violation*, Phys. Lett. B **136** (1984) 383 [INSPIRE].
- [9] I. de Medeiros Varzielas, S.F. King and G.G. Ross, *Neutrino tri-bi-maximal mixing from a non-Abelian discrete family symmetry*, Phys. Lett. B **648** (2007) 201 [hep-ph/0607045] [INSPIRE].
- [10] E. Ma, *Neutrino mass matrix from $\Delta(27)$ symmetry*, Mod. Phys. Lett. A **21** (2006) 1917 [hep-ph/0607056] [INSPIRE].
- [11] I. de Medeiros Varzielas and D. Emmanuel-Costa, *Geometrical CP violation*, Phys. Rev. D **84** (2011) 117901 [arXiv:1106.5477] [INSPIRE].
- [12] I. de Medeiros Varzielas, D. Emmanuel-Costa and P. Leser, *Geometrical CP violation from non-renormalisable scalar potentials*, Phys. Lett. B **716** (2012) 193 [arXiv:1204.3633] [INSPIRE].
- [13] G. Bhattacharyya, I. de Medeiros Varzielas and P. Leser, *A common origin of fermion mixing and geometrical CP violation, and its test through Higgs physics at the LHC*, Phys. Rev. Lett. **109** (2012) 241603 [arXiv:1210.0545] [INSPIRE].
- [14] P.M. Ferreira, W. Grimus, L. Lavoura and P.O. Ludl, *Maximal CP violation in lepton mixing from a model with $\Delta(27)$ flavour symmetry*, JHEP **09** (2012) 128 [arXiv:1206.7072] [INSPIRE].
- [15] E. Ma, *Neutrino mixing and geometric CP violation with $\Delta(27)$ symmetry*, Phys. Lett. B **723** (2013) 161 [arXiv:1304.1603] [INSPIRE].
- [16] C.C. Nishi, *Generalized CP symmetries in $\Delta(27)$ flavor models*, Phys. Rev. D **88** (2013) 033010 [arXiv:1306.0877] [INSPIRE].
- [17] I. de Medeiros Varzielas and D. Pidt, *Towards realistic models of quark masses with geometrical CP violation*, J. Phys. G **41** (2014) 025004 [arXiv:1307.0711] [INSPIRE].
- [18] A. Aranda, C. Bonilla, S. Morisi, E. Peinado and J.W.F. Valle, *Dirac neutrinos from flavor symmetry*, Phys. Rev. D **89** (2014) 033001 [arXiv:1307.3553] [INSPIRE].
- [19] I. de Medeiros Varzielas and D. Pidt, *Geometrical CP violation with a complete fermion sector*, JHEP **11** (2013) 206 [arXiv:1307.6545] [INSPIRE].

[20] P.F. Harrison, R. Krishnan and W.G. Scott, *Deviations from tribimaximal neutrino mixing using a model with* $\Delta(27)$ *symmetry*, *Int. J. Mod. Phys. A* **29** (2014) 1450095 [arXiv:1406.2025] [INSPIRE].

[21] E. Ma and A. Natale, *Scotogenic* Z_2 *or* $\mathrm{U}(1)_D$ *model of neutrino mass with* $\Delta(27)$ *symmetry*, *Phys. Lett. B* **734** (2014) 403 [arXiv:1403.6772] [INSPIRE].

[22] M. Fallbacher and A. Trautner, *Symmetries of symmetries and geometrical CP violation*, *Nucl. Phys. B* **894** (2015) 136 [arXiv:1502.01829] [INSPIRE].

[23] M. Abbas, S. Khalil, A. Rashed and A. Sil, *Neutrino masses and deviation from tribimaximal mixing in* $\Delta(27)$ *model with inverse seesaw mechanism*, *Phys. Rev. D* **93** (2016) 013018 [arXiv:1508.03727] [INSPIRE].

[24] I. de Medeiros Varzielas, $\Delta(27)$ *family symmetry and neutrino mixing*, *JHEP* **08** (2015) 157 [arXiv:1507.00338] [INSPIRE].

[25] F. Björkeroth, F.J. de Anda, I. de Medeiros Varzielas and S.F. King, *Towards a complete* $\Delta(27) \times \mathrm{SO}(10)$ *SUSY GUT*, *Phys. Rev. D* **94** (2016) 016006 [arXiv:1512.00850] [INSPIRE].

[26] P. Chen, G.-J. Ding, A.D. Rojas, C.A. Vaquera-Araujo and J.W.F. Valle, *Warped flavor symmetry predictions for neutrino physics*, *JHEP* **01** (2016) 007 [arXiv:1509.06683] [INSPIRE].

[27] A.E. Cárcamo Hernández, H.N. Long and V.V. Vien, *A* 3-3-1 *model with right-handed neutrinos based on the* $\Delta(27)$ *family symmetry*, *Eur. Phys. J. C* **76** (2016) 242 [arXiv:1601.05062] [INSPIRE].

[28] A.E. Cárcamo Hernández, S. Kovalenko, J.W.F. Valle and C.A. Vaquera-Araujo, *Predictive Pati-Salam theory of fermion masses and mixing*, *JHEP* **07** (2017) 118 [arXiv:1705.06320] [INSPIRE].

[29] I. de Medeiros Varzielas, G.G. Ross and J. Talbert, *A unified model of quarks and leptons with a universal texture zero*, *JHEP* **03** (2018) 007 [arXiv:1710.01741] [INSPIRE].

[30] N. Bernal, A.E. Cárcamo Hernández, I. de Medeiros Varzielas and S. Kovalenko, *Fermion masses and mixings and dark matter constraints in a model with radiative seesaw mechanism*, *JHEP* **05** (2018) 053 [arXiv:1712.02792] [INSPIRE].

[31] I. De Medeiros Varzielas, M.L. López-Ibáñez, A. Melis and O. Vives, *Controlled flavor violation in the MSSM from a unified* $\Delta(27)$ *flavor symmetry*, *JHEP* **09** (2018) 047 [arXiv:1807.00860] [INSPIRE].

[32] A.E. Cárcamo Hernández, J.C. Gómez-Izquierdo, S. Kovalenko and M. Mondragón, $\Delta(27)$ *flavor singlet-triplet Higgs model for fermion masses and mixings*, *Nucl. Phys. B* **946** (2019) 114688 [arXiv:1810.01764] [INSPIRE].

[33] F. Björkeroth, I. de Medeiros Varzielas, M.L. López-Ibáñez, A. Melis and O. Vives, *Leptogenesis in* $\Delta(27)$ *with a universal texture zero*, *JHEP* **09** (2019) 050 [arXiv:1904.10545] [INSPIRE].

[34] F. Staub, *From superpotential to model files for FeynArts and CalcHep/CompHep*, *Comput. Phys. Commun.* **181** (2010) 1077 [arXiv:0909.2863] [INSPIRE].

[35] F. Staub, *Automatic calculation of supersymmetric renormalization group equations and self energies*, *Comput. Phys. Commun.* **182** (2011) 808 [arXiv:1002.0840] [INSPIRE].

JHEP02(2023)231

[36] F. Staub, *SARAH 3.2: Dirac gauginos, UFO output, and more*, Comput. Phys. Commun. **184** (2013) 1792 [arXiv:1207.0906] [INSPIRE].

[37] F. Staub, *SARAH 4: a tool for (not only SUSY) model builders*, Comput. Phys. Commun. **185** (2014) 1773 [arXiv:1309.7223] [INSPIRE].

[38] I. de Medeiros Varzielas, S.F. King, C. Luhn and T. Neder, *CP-odd invariants for multi-Higgs models: applications with discrete symmetry*, Phys. Rev. D **94** (2016) 056007 [arXiv:1603.06942] [INSPIRE].

[39] I. de Medeiros Varzielas, S.F. King, C. Luhn and T. Neder, *Minima of multi-Higgs potentials with triplets of* $\Delta(3n^2)$ *and* $\Delta(6n^2)$, Phys. Lett. B **775** (2017) 303 [arXiv:1704.06322] [INSPIRE].

[40] I.P. Ivanov and C.C. Nishi, *Symmetry breaking patterns in 3HDM*, JHEP **01** (2015) 021 [arXiv:1410.6139] [INSPIRE].

[41] J.F. Donoghue and L.F. Li, *Properties of charged Higgs bosons*, Phys. Rev. D **19** (1979) 945 [INSPIRE].

[42] H. Georgi and D.V. Nanopoulos, *Suppression of flavor changing effects from neutral spinless meson exchange in gauge theories*, Phys. Lett. B **82** (1979) 95 [INSPIRE].

[43] F.J. Botella, G.C. Branco and M.N. Rebelo, *Minimal flavour violation and multi-Higgs models*, Phys. Lett. B **687** (2010) 194 [arXiv:0911.1753] [INSPIRE].

[44] Particle Data Group collaboration, *Review of particle physics*, PTEP **2020** (2020) 083C01 [INSPIRE].

[45] W. Porod, *SPheno, a program for calculating supersymmetric spectra, SUSY particle decays and SUSY particle production at* e^+e^- *colliders*, Comput. Phys. Commun. **153** (2003) 275 [hep-ph/0301101] [INSPIRE].

[46] W. Porod and F. Staub, *SPheno 3.1: extensions including flavour, CP-phases and models beyond the MSSM*, Comput. Phys. Commun. **183** (2012) 2458 [arXiv:1104.1573] [INSPIRE].

[47] M.E. Peskin and T. Takeuchi, *A new constraint on a strongly interacting Higgs sector*, Phys. Rev. Lett. **65** (1990) 964 [INSPIRE].

[48] W.J. Marciano and J.L. Rosner, *Atomic parity violation as a probe of new physics*, Phys. Rev. Lett. **65** (1990) 2963 [*Erratum ibid.* **68** (1992) 898] [INSPIRE].

[49] D.C. Kennedy and P. Langacker, *Precision electroweak experiments and heavy physics: a global analysis*, Phys. Rev. Lett. **65** (1990) 2967 [*Erratum ibid.* **66** (1991) 395] [INSPIRE].

[50] M.D. Goodsell and F. Staub, *Unitarity constraints on general scalar couplings with SARAH*, Eur. Phys. J. C **78** (2018) 649 [arXiv:1805.07306] [INSPIRE].

[51] P. Bechtle, O. Brein, S. Heinemeyer, G. Weiglein and K.E. Williams, *HiggsBounds: confronting arbitrary Higgs sectors with exclusion bounds from LEP and the Tevatron*, Comput. Phys. Commun. **181** (2010) 138 [arXiv:0811.4169] [INSPIRE].

[52] P. Bechtle, S. Heinemeyer, O. Stål, T. Stefaniak and G. Weiglein, *HiggsSignals: confronting arbitrary Higgs sectors with measurements at the Tevatron and the LHC*, Eur. Phys. J. C **74** (2014) 2711 [arXiv:1305.1933] [INSPIRE].

[53] J. Haller, A. Hoecker, R. Kogler, K. Mönig, T. Peiffer and J. Stelzer, *Update of the global electroweak fit and constraints on two-Higgs-doublet models*, Eur. Phys. J. C **78** (2018) 675 [arXiv:1803.01853] [INSPIRE].

JHEP02(2023)231

[54] ATLAS collaboration, *Search for the Higgs boson decays $H \to ee$ and $H \to e\mu$ in pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector*, Phys. Lett. B **801** (2020) 135148 [arXiv:1909.10235] [INSPIRE].

[55] F. Staub, M. Goodsell and W. Porod, *SARAH wiki webpage*, https://gitlab.in2p3.fr/goodsell/sarah/-/wikis/HiggsBounds.

[56] ALEPH, DELPHI, L3, OPAL and LEP collaborations, *Search for charged Higgs bosons: combined results using LEP data*, Eur. Phys. J. C **73** (2013) 2463 [arXiv:1301.6065] [INSPIRE].

[57] CMS collaboration, *Search for a light charged Higgs boson in the $H^\pm \to cs$ channel in proton-proton collisions at $\sqrt{s} = 13$ TeV*, Phys. Rev. D **102** (2020) 072001 [arXiv:2005.08900] [INSPIRE].

[58] CMS collaboration, *Search for a light charged Higgs boson decaying to a W boson and a CP-odd Higgs boson in final states with $e\mu\mu$ or $\mu\mu\mu$ in proton-proton collisions at $\sqrt{s} = 13$ TeV*, Phys. Rev. Lett. **123** (2019) 131802 [arXiv:1905.07453] [INSPIRE].

[59] ATLAS collaboration, *Search for a light charged Higgs boson in $t \to H^+ b$ decays, with $H^+ \to cb$, in the lepton+jets final state in proton-proton collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector*, Tech. Rep. ATLAS-CONF-2021-037, CERN, Geneva, Switzerland (2021) [INSPIRE].

[60] ATLAS collaboration, *Search for charged Higgs bosons decaying into a top-quark and a bottom-quark at $\sqrt{s} = 13$ TeV with the ATLAS detector*, Tech. Rep. ATLAS-CONF-2020-039, CERN, Geneva, Switzerland (2020) [INSPIRE].

[61] P.Z. Skands et al., *SUSY Les Houches accord: interfacing SUSY spectrum calculators, decay packages, and event generators*, JHEP **07** (2004) 036 [hep-ph/0311123] [INSPIRE].

[62] B.C. Allanach et al., *SUSY Les Houches accord 2*, Comput. Phys. Commun. **180** (2009) 8 [arXiv:0801.0045] [INSPIRE].

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